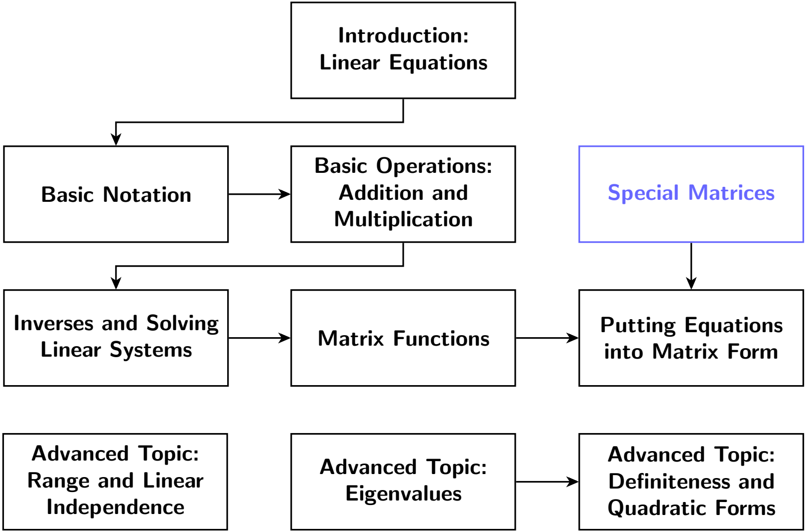
Linear Algebra Review



The Identity Matrix

$$I \in \mathbb{R}^{n \times n} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

- Has the property that for any $A \in \mathbb{R}^{m \times n}$

$$AI = A = IA$$

(note that the identity matrices on the left and right are *different sizes*, $n \times n$ versus $m \times m$)

The Zero Matrix

$$0 \in \mathbb{R}^{m \times n} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

- Useful in defining block forms for matrices; e.g. $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{p \times q}$

$$C \in \mathbb{R}^{(m+p) \times (n+q)} = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$$

The All-ones Vector

$$\mathbf{1} \in \mathbb{R}^n = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

- Useful, for example, in compactly representing sums

$$a \in \mathbb{R}^n, \quad \mathbf{1}^T a = \sum_{i=1}^{n} a_i$$

The Standard Basis Vector

$$e_i \in \mathbb{R}^n = \begin{bmatrix} \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \end{bmatrix} \leftarrow i\text{th row}$$

- Can be used to extract entries of a vector $x \in \mathbb{R}^n$

$$x^T e_i = x_i$$

Symmetric Matrices

- Symmetric matrix: $A \in \mathbb{R}^{n \times n}$ with $A = A^T$
- Arise naturally in many settings
  - For $A \in \mathbb{R}^{m \times n}$, $A^T A \in \mathbb{R}^{n \times n}$ is symmetric

Diagonal Matrices

- For $d \in \mathbb{R}^n$

$$\operatorname{diag}(d) \in \mathbb{R}^{n \times n} = \begin{bmatrix} d_1 & 0 & \cdots & 0 \\ 0 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d_n \end{bmatrix}$$

- For example, the identity is given by $I = \operatorname{diag}(1)$

- Multiplying $A \in \mathbb{R}^{m \times n}$ by a diagonal matrix $D \in \mathbb{R}^{n \times n}$ on the right scales the *columns* of A

$$AD = \begin{bmatrix} | & | & & | \\ d_1 a_1 & d_2 a_2 & \cdots & d_n a_n \\ | & | & & | \end{bmatrix}$$

- Multiplying by a diagonal matrix $D \in \mathbb{R}^{m \times m}$ on the left scales the *rows* of A

$$DA = \begin{bmatrix} — & d_1 a_1^T & — \\ — & d_2 a_2^T & — \\ & \vdots & \\ — & d_m a_m^T & — \end{bmatrix}$$