Linear Algebra Review

Introduction:
Linear Equations

Basic Notation

Basic Operations:
Addition and Multiplication

Inverses and Solving
Linear Systems

Matrix Functions

Special Matrices

Putting Equations into Matrix Form

Advanced Topic:
Range and Linear Independence

Advanced Topic:
Eigenvalues

Advanced Topic:
Definiteness and Quadratic Forms
Quadratic Forms

• A quadratic form is a function \( f : \mathbb{R}^n \rightarrow \mathbb{R} \)

\[
f(x) = x^T A x
\]

for some \( A \in \mathbb{R}^{n \times n} \)

• Can take \( A \) to be symmetric, since

\[
x^T A x = (x^T A x)^T = x^T A^T x = x^T \frac{1}{2} (A + A^T) x
\]
• $A \in \mathbb{R}^{n \times n}$ is positive definite (positive semidefinite) if $x^T Ax > 0$ ($x^T Ax \geq 0$) for all $x \in \mathbb{R}^n \neq 0$

• $A \in \mathbb{R}^{n \times n}$ is negative definite (negative semidefinite) if $x^T Ax < 0$ ($x^T Ax \leq 0$) for all $x \in \mathbb{R}^n \neq 0$

• $A$ is indefinite if neither positive nor negative semidefinite
• Definiteness is characterized by eigenvalues of $A$

  - $A$ positive definite $\iff \lambda_i > 0$, $\forall i$
  
  - $A$ positive semidefinite $\iff \lambda_i \geq 0$, $\forall i$
  
  - $A$ negative definite $\iff \lambda_i < 0$, $\forall i$
  
  - $A$ negative semidefinite $\iff \lambda_i \leq 0$, $\forall i$