15-884/484 – Electric Power Systems 1: DC and AC Circuits

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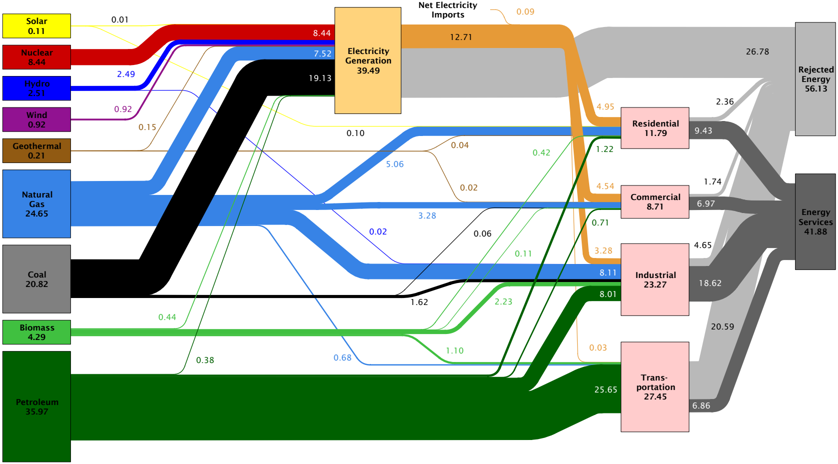
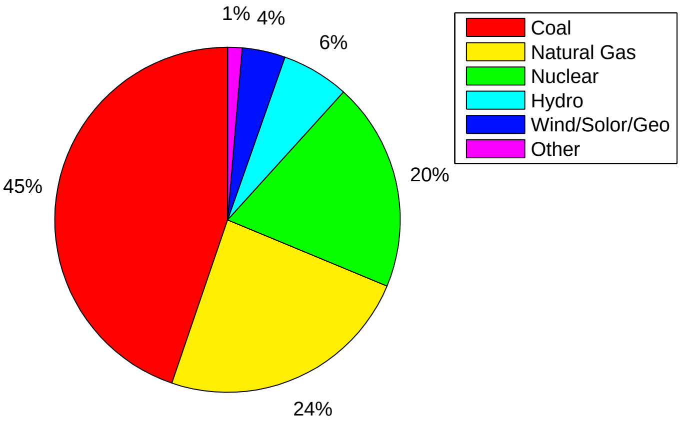
Estimated U.S. Energy Use in 2010: ~98.0 Quads



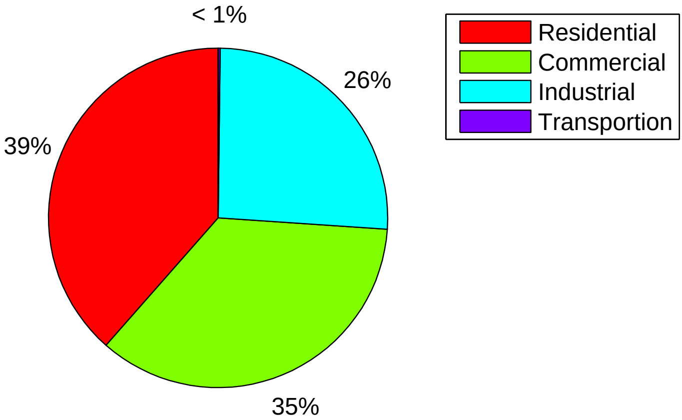
Source: LLNL 2011. Data is based on DOE/EIA-0384(2010), October 2011. If this information or a reproduction of it is used, credit must be given to the Lawrence Livermore National Laboratory and the Department of Energy, under whose auspices the work was performed. Distributed electricity represents only retail electricity sales and does not include self-generation. EIA reports flows for hydro, wind, solar and geothermal in BTU-equivalent values by assuming a typical fossil fuel plant "heat rate." (see EIA report for explanation of change to geothermal in 2010). The efficiency of electricity production is calculated as the total retail electricity delivered divided by the primary energy input into electricity generation. End use efficiency is estimated as 80% for the residential, commercial and industrial sectors, and as 25% for the transportation sector. Totals may not equal sum of components due to independent rounding. LLNL-MI-410527

U.S. Electricity Generation



Data: EIA Electric Power Annual 2010

U.S. Electricity Consumption



Data: EIA Electric Power Annual 2010

Basics of Electrical Power

- **Charge:** property of matter that causes it to experience force when near other charge
  - Measured in *coulombs* (C), charge equal to that of 6.25×10^{18} protons

- **Voltage:** electric potential energy, measured in *volts* (V), and denoted with symbol v or V

$$1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

  - Voltage really a measure of *difference* in electric potential, we talk of "voltage drop" between two points in a circuit

- **Current:** Flow of charge through a material, measured in *amperes* (A), and denoted with symbol i or I

$$1 \text{ ampere} = \frac{1 \text{ coulomb}}{1 \text{ second}}$$

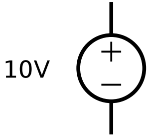
  - Unlike voltage, current measured at a single point in a circuit

- Electrical power, still measured in *watts* (W), denoted p or P

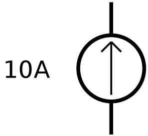
$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ second}} = 1 \text{ volt} \cdot 1 \text{ ampere} \Longleftrightarrow P = IV$$

Direct Current (DC) Circuits

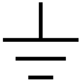
- **Voltage Source:** Maintains fixed voltage drop across two ends



- **Current Source:** Maintains fixed current through this point in the circuit



- **Ground:** Specifies reference voltage ($= 0$) at this point



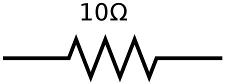
- **Resistor:** "Resists" flow of electricity
  - Resistance measured in *ohms* (Ω), denoted with symbol R

$$1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}}$$

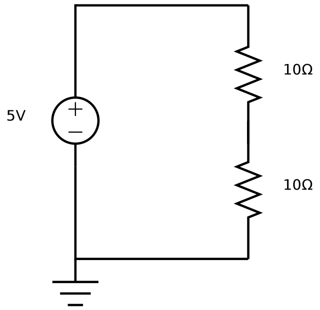
  - Relates current and voltage via *Ohm's law*

$$V = IR$$

  - Symbol in circuit diagrams



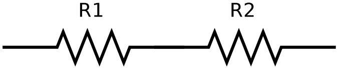
- A simple DC circuit



- Goal of *linear circuit analysis*: given knowledge of voltages (currents) in circuit, compute currents (voltages) in circuit
  - Called *linear* circuit analysis because solution is given by a set of linear equations

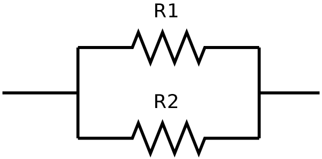
$$V = ZI, \;\; V, I \in \mathbb{R}^n, Z \in \mathbb{R}^{n \times n} \text{ (impedance matrix)}$$

- Some simple rules for combining circuit elements
  - Resitors in series

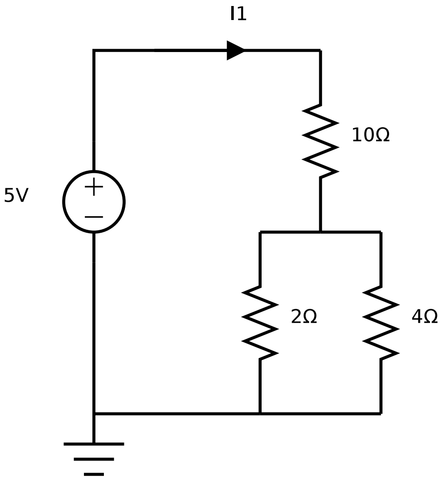


$$R = R_1 + R_2$$

  - Resistors in parallel

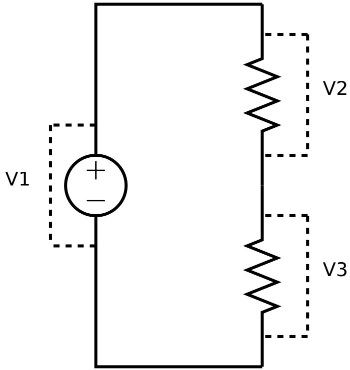


$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$



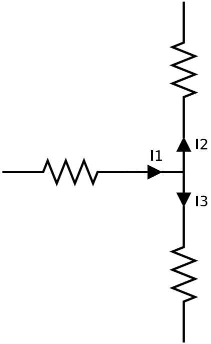
$$I_1 = ?$$

- **Kirchhoff's voltage law (KVL):** voltage around any closed loop sums to zero



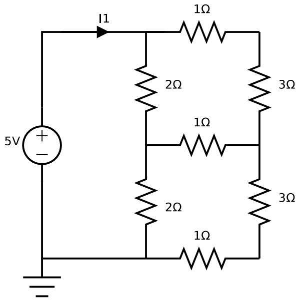
$$V_1 + V_2 + V_3 = 0$$

- **Kirchhoff's current law (KCL):** current entering and exiting any node sums to zero



$$I_1 - I_2 - I_3 = 0$$

- Kirchhoff's and Ohm's laws let us solve any linear circuit, but quickly becomes tedious

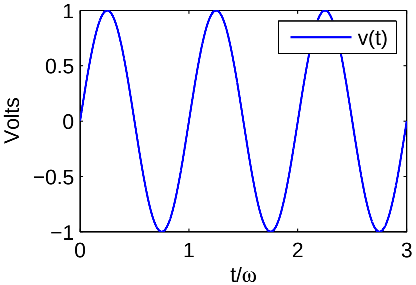


$$I_1 = ?$$

- Many circuit simulation programs can easily convert problems to linear system of equations and solve

Alternating Current (AC) Circuits

- Voltage/current varies sinusoidally with time

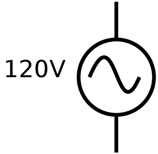


$$v(t) = V_{\max} \sin(\omega t + \phi)$$

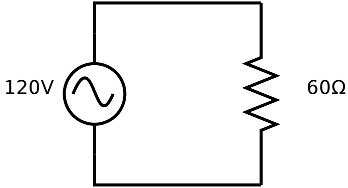
$V_{\max}$: peak voltage, ω: frequency (e.g., $60 \cdot 2\pi$), ϕ: phase angle

- Two conventions for reporting magnitude, peak $V_{\max}$ and root mean squared $V_{\text{rms}} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V_{\max}^2 \sin^2 t dt} = \frac{1}{\sqrt{2}} V_{\max}$

- **AC voltage source** - maintains sinusoidally alternating voltage



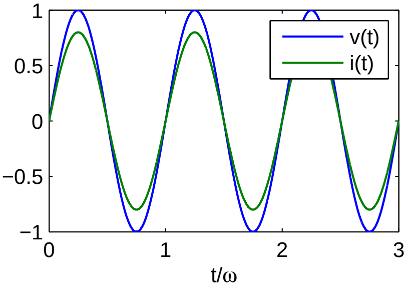
- Example AC circuit



- Resistive AC circuits: instantaneous current/voltage follow Ohm's law

$$v(t) = i(t)R$$

$$v(t) = V_{\max}\sin(\omega t + \phi) \Longrightarrow i(t) = \frac{V_{\max}}{R}\sin(\omega t + \phi)$$



- Voltage and current are *in phase*

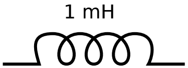
- **Inductors:** resists change in current
  - Simplest inductor is a coil of wire, resitance to current change due to magenetic field created by current
  - Inductance measured in *henries* (H), denoted with symbol L

$$1 \text{ henry} = 1 \text{ second} \cdot 1 \text{ ohm}$$

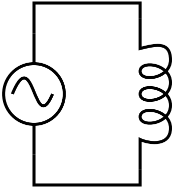
  - Relates current and voltage via the relationship

$$v = L\frac{di}{dt}$$

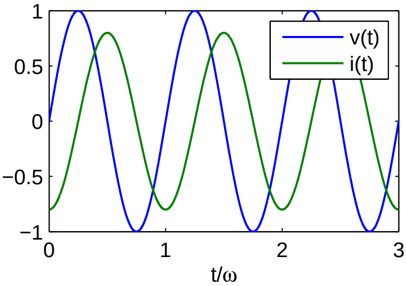
  - Symbol in circuits



- Inductor causes AC current to *lag* 90 degrees behind voltage



$$\frac{di}{dt}L = V_{\max}\sin(\omega t + \phi)$$
$$i(t) = \frac{V_{\max}}{L}\int \sin(\omega t + \phi)dt$$
$$= -\frac{V_{\max}}{L\omega}\cos(\omega t + \phi)$$
$$= \frac{V_{\max}}{L\omega}\sin(\omega t + \phi - \frac{\pi}{2})$$



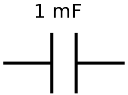
- **Capacitors:** store electric charge
  - Simple capacitor is two plates made of conducting material placed close together, but not touching
  - Capacitance measured in *farads* (F), denoted with symbol C

$$1 \text{ farad} = \frac{1 \text{ second}}{1 \text{ ohm}}$$

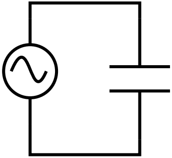
  - Relates current and voltage via the relationship

$$i = C\frac{dv}{dt}$$

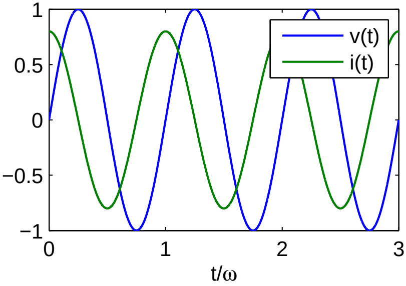
  - Symbol in circuits



- Capacitor causes AC current to *lead* voltage by 90 degrees



$$
\begin{aligned}
i(t) &= C V_{\max} \frac{d}{dt} \sin(\omega t + \phi) \\
&= C \omega V_{\max} \cos(\omega t + \phi) \\
&= C \omega V_{\max} \sin(\omega t + \phi + \frac{\pi}{2})
\end{aligned}
$$



- Working with sinusoidal equations gets tedious quickly

- Sinusoids are expressed entirely by their *magnitude* A and *phase angle* ϕ (assuming the same frequency over sinusoids)

$$f(t) = A \sin(\omega t + \phi)$$

- It is helpful to express these quantities in terms of *complex numbers*

- We can express voltage/current in terms of complex exponential

$$v(t) = \mathrm{Re}\{V_{\max} e^{j(\omega t+\phi)}\}, \quad \text{where } j = \sqrt{-1}$$

  using Euler's equation $e^{j\phi} = \cos\phi + j\sin\phi$

- For convenience, we'll use V and I to refer to the entire *complex* quantity, i.e.

$$V = V_{\max} e^{j(\omega t+\phi)}$$

  - When computing steady state characteristics, we can effectively ignore time, and represent voltage/curent with complex numbers

- This representation gives simple expressions for inductance and capacitance

$$V = j\omega L I, \quad V = -j\frac{1}{\omega C} I$$

- Some rules regarding complex numbers $x = a + jb$, $y = c + jd$

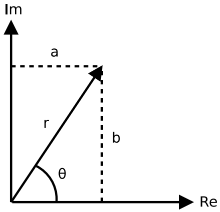
$$\bar{x} = a - jb \text{ (complex conjugate)}$$

$$x + y = (a + b) + j(c + d)$$

$$x \cdot y = (a + jb)(c + jd) = ab - bd + j(bc + ad)$$

$$\frac{1}{x} = \frac{a}{a^2 + b^2} + j\frac{-b}{a^2 + b^2} \quad \left(\frac{x}{y} = x \cdot \frac{1}{y}\right)$$

- Often useful to express complex numbers in *polar* form



$$a + jb = re^{j\theta} \equiv r\angle\theta$$

$$\text{where } r = \sqrt{a^2 + b^2}, \theta = \tan^{-1} b/a$$

$$r_1\angle\theta_1 \cdot r_1\angle\theta_1 = r_1 \cdot r_2\angle(\theta_1 + \theta_2)$$

$$\frac{r_1\angle\theta_1}{r_1\angle\theta_1} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2)$$

- Generalization of Ohm's law for AC circuits, covers combination of resistance, inductance, capacitance

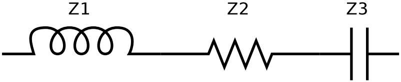
$$V = ZI$$

where Z is known as the *impedance*

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

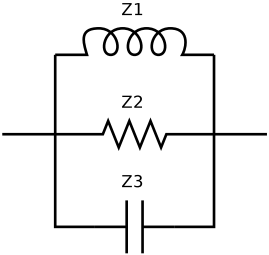
- Lets us find steady-state solutions for AC circuits using just linear (complex) equations

- Like resistance, impedance in series sum to total impedance

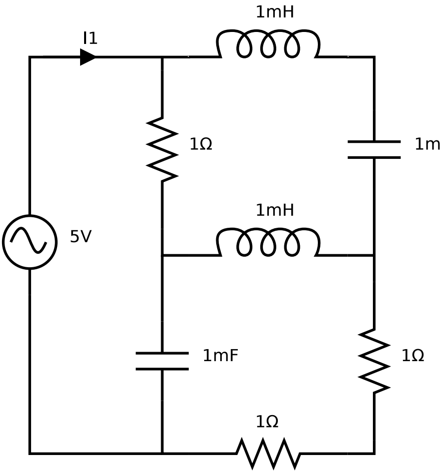


$$Z = Z_1 + Z_2 + Z_3$$

- Impedance in parallel sum inverses



$$Z = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$



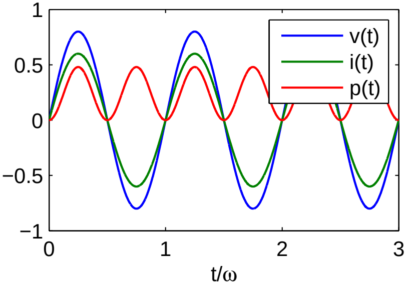
$$I_1 = ?$$

AC Power

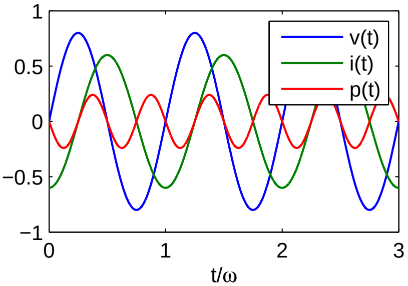
- Instantaneous power still given by equation

$$p(t) = v(t)i(t)$$

- When current/voltage are in phase, power is always positive



- When current current/voltage are out of phase, power can be negative



- *Real power* is RMS value of the positive, “consumed” portion of power

- *Reactive power* is RMS value of power that is regenerated every cycle

- Using complex voltage/current, we get an expression for *complex power*

$$S = \frac{1}{2}\bar{I}V = P + jQ = |S|\angle\theta$$

($\frac{1}{2}$ term comes from representing current/voltage with peak values, using RMS values removes this term)

- In equation above, θ is known as *power angle*

- *Apparent power* is absolute magnitude of power

$$|S| = \sqrt{P^2 + Q^2}$$

- Real power $= P = |S|\cos\theta$, reactive power $= Q = |S|\sin\theta$

- *Power factor* is ratio of real to apparent power

$$\text{p.f.} = \frac{P}{|S|} = \cos\theta$$

- Real, reactive, and apparent power all have the same units (volts $\cdot$ amperes $=$ watts).

- However, to differentiate, we use different names
  - Real power is measured in *watts* (W)
  - Apparent power is measured in *volt amperes* (VA)
  - Reactive power is measured in *volt amperes reactive* (VAR)