Kinematic Calculation for Rehabilitation in Hemiplegic Lower Extremity

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ABSTRACT- An analytical method of inverse kinematics for human lower extremity is developed in this paper. With this method, the posture of lower extremity can be determined by using the position and orientation of foot. The kinematic calculations are performed on the model with six or seven degrees of freedom. Furthermore, example calculations of kinematics on 6DOF and 7DOF are given, respectively. The kinematics experiment of a mechanical model with 6DOF has been performed.

1 Introduction

It is well known that the exercise therapy is an effective method for rehabilitation in hemiplegic lower extremity of apoplexy patients. It is evident that increasing the times of repeated practice can approve the curative effect. However, since the human strength and time are restricted, it is hard for an apoplexy patient to do plenty of training under the assistance of a physical therapist. Therefore, the development of exercise assisting system has become of increasing importance.

For making progress in rehabilitation, the two respects are both of importance. The first is to ensure the repeated training number of times. The second is to let the patient take an active interest in training by biofeed back.

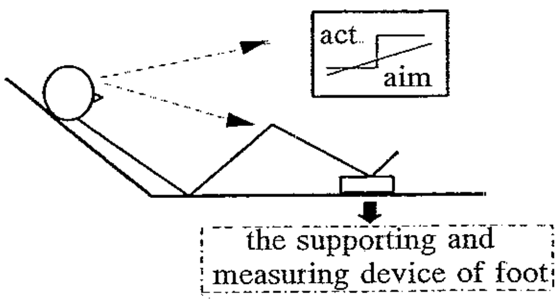

Figure 1: The sketch of the exercise assisting system

As represented in Fig.1, when a patient is undertaking the training, the lower extremity position and orientation of the patient are required to be shown on a computer screen. So that the patient can get his/her lower extremity information by his/her visual sense at any time. For this end, this paper trys to give a technic which determine the posture of lower extremity from the position and orientation of foot by using inverse kinematic calculation.

Recently, the human body kinematics has been studied extensively. The motion scopes of human joints and the motion principle of the human musculoskeletal system are studied and illustrated in detail [1]. About the kinematics of Human motion, joint geometry, joint kinematics, and biomechanics of human motion are analyzed [2], where, the inverse kinematics of human lower extremity is analyzed on 2DOF or 3DOF. In general, inverse kinematic problem on more than 3DOF is very difficult to solving by analytical method, but by numerical method. This paper proposes analytical methods of inverse kinematics for lower extremity with 6DOF or 7DOF.

2 Outline of the exercise assisting system

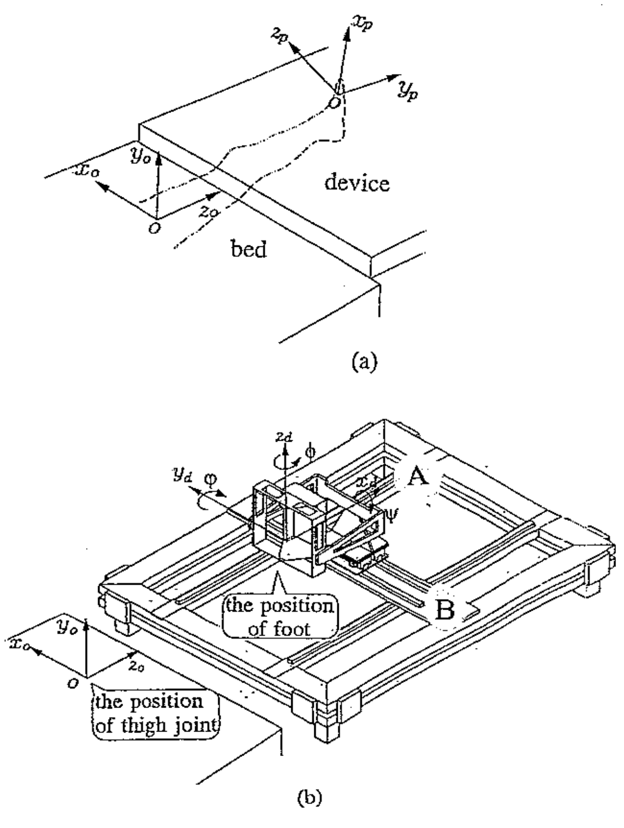

Figure 2: The machine frame of the exercise assisting system

We designed and made a device, shown as Fig.2, for obtaining the position and orientation of foot, This supporting and measuring device can be allowed to move in a horizontal plane. This device is composed of three parts, which can be allowed to rotate around the shafts x_d, y_d, z_d, respectively. We can measure the rotation angles ψ, φ, ϕ, by encoders installed on the rotation component parts, respectively. The position of the distal foot can be measured by another encoders installed on the places A and B of the device as shown in Fig.2. In this paper, the distal foot orientation will be described using Euler's angles α, β, γ. The Euler's angles α, β γ can be obtained from the angles ψ, φ, ϕ of the rotation component parts by conversion calculation.

3 Direct kinematic calculation

There are seven joints on human lower extremity, the knee motion involves two joints, extending joint and rotation joint. Because the displacement scope of the rotation joint is small, usually it can be neglected. In this case, the human lower extremity can be described with a 6DOF model, the inverse kinematics calculation can be uniquely solved from the position and orientation of the distal foot. For a precise results of kinematic calculation, we shall consider also setting up the model including the rotation joint of knee. Since there is one redundant DOF on human lower extremity with 7DOF, the method of inverse kinematics calculation for 6DOF can not be used to the inverse kinematics calculation on 7DOF. Therefore, in the ensuring subsection, we will perform the kinematic calculations on 6DOF or 7DOF, respectively.

3.1 The case of six degrees of freedom (6DOF)

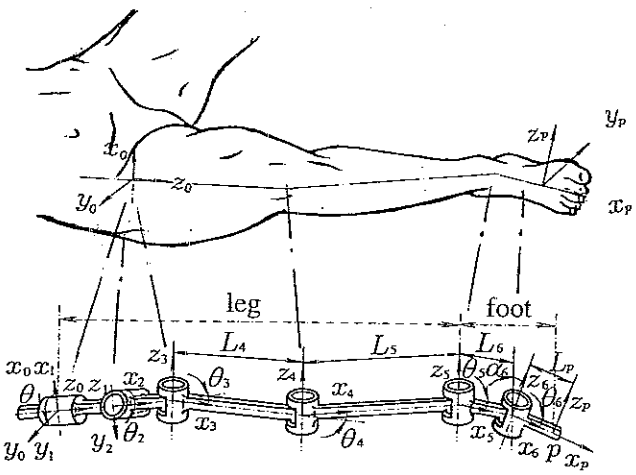

Figure. 3: Model of six degrees of freedom in space

Shown as Fig.3, an attitude of lower extremity with 6DOF is described by base coordinates $\Sigma_0(x_0, y_0, z_0)$. A proximal coordinates $\Sigma_1(x_1, y_1, z_1)$ originating at the thigh joint is fixed with the trunk, and coordinates $\Sigma_1(x_1, y_1, z_1)$ is set up to overlap coordinates $\Sigma_0(x_0, y_0, z_0)$. The terminal point of the distal lower extremity is designated by $\boldsymbol{p}$ with the coordinates $\Sigma\boldsymbol{p}(x, y, z)$. The links are numbered from zero to 6, where the lengths of links L_1, L_2 and L_2 equal zero, link 0 is an imaginary unmovable object connected with link 1. Except for the last link, each link of the chain has two joints connecting to the adjacent bodies. The joints are numbered from 1 between the links zero and 1, to 6, between links 5 and 6. Local coordinates $\Sigma_i(x_i, y_i, z_i)$ are attached to each link i. The position and orientation of each link can also be represented in the coordinates $\Sigma_0(x_0, y_0, z_0)$.

The link lengths are L_4, L_5, L_6, L_p, and joints 1, 2, 3 are connected at one point. α_6 is a twisted angle of link 6. The external (or anatomic) angles are $\theta_1, \theta_2, \cdots, \theta_6$. Show as Fig 3, joints 1, 2, 3 are in "hip", joint 4 is in "knee", joints 5, 6 are in "foot", and point $\boldsymbol{p}$ is the distal foot.

When angles $\theta_1, \theta_2, \cdots, \theta_6$, are known, the position and orientation of point $\boldsymbol{p}$ on the tip of the chain can be solved easily [3] [4].

The position and orientation of coordinates Σ_i relative to the previous coordinates Σ_{i-1} can be described by a 4×4 transformation matrix ${}^{i-1}\boldsymbol{T}_i$. Then, the position and orientation of point $\boldsymbol{p}$ are defined as the composition of sequential coordinate transformations ${}^{i-1}\boldsymbol{T}_i$, where i runs from 1 to 6. Because the composition of several displacements is given by matrix multiplication of the corresponding transformation matrices, the homogeneous transformation $\boldsymbol{T}(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$ of Σ_p with respect to Σ_0 can be expressed by

$$\boldsymbol{T}(\theta_1, \cdots, \theta_6) = {}^0\boldsymbol{T}_1\ {}^1\boldsymbol{T}_2\ {}^2\boldsymbol{T}_3\ {}^3\boldsymbol{T}_4\ {}^4\boldsymbol{T}_5\ {}^5\boldsymbol{T}_6\ {}^6\boldsymbol{T}_p. \quad (1)$$

Eq.(1) is termed by the structure equation of the model. Calculating the right-hand of eq (1), can be described by the form

$$\boldsymbol{T}(\theta_1, \cdots, \theta_6) = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

Since $\theta_1, \cdots, \theta_6$ are known, elements of matrix $\boldsymbol{T}$ can be obtained. By describing the distal foot orientation using Euler's angles, the orientation can be expressed as

$$\alpha = \tan^{-1} 2\ (\pm\, r_{23},\ \pm\, r_{13}), \quad (3)$$

$$\beta = \tan^{-1} 2\ (\pm \sqrt{r_{13}^2 + r_{23}^2},\ r_{33}), \quad (4)$$

$$\gamma = \tan^{-1} 2\ (\pm\, r_{32},\ \mp\, r_{31}). \quad (5)$$

The distal foot position is expressed by the form

$$\boldsymbol{p}_{\Sigma 0} = [\ x \quad y \quad z\ ]^T. \quad (6)$$

3.2 The case of even degrees of freedom (7DOF)

Really, human knee motion involves two joints θ_4, $\theta_{4'}$. For a precise result of kinematic calculation, we shall consider to set up the model including $\theta_{4'}$.

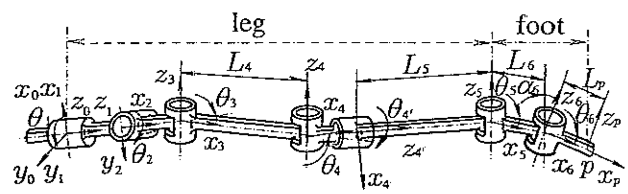

Figure. 4: Model of seven degrees of freedom in space

As shown in Fig 4, we insert $\Sigma_{4'}(x_{4'}, y_{4'}, z_{4'})$ between local coordinate systems $\Sigma_4(x_4, y_4, z_4)$ and $\Sigma_5(x_5, y_5, z_5)$ on the model of 6DOF. Similar to **3.1**, joints 1, 2, 3 is also in "hip", joint 4, 4′ are in "knee", joints 5, 6 are in "foot", and point $\boldsymbol{p}$ is the distal foot(Fig.4).

When angles $\theta_1, \theta_2, \theta_3, \theta_4, \theta_{4'}, \theta_5, \theta_6$are known, the position and orientation of point $\boldsymbol{p}$ the tip of the model, can be solved easily [3] [4].

The homogeneous transformation, $T(\theta_1, \theta_2, \theta_3, \theta_4, \theta_{4'}, \theta_5, \theta_6)$, of Σ_p with respect to Σ_0 can be expressed by the form

$$T(\theta_1, \cdots, \theta_4', \theta_5, \theta_6) = {}^0T_1 \cdots {}^4T_{4'}\ {}^{4'}T_5\ {}^5T_6\ {}^6T_p \quad (7)$$

By the same method of **3.1**, the position and orientation of distal foot can be obtained.

4 Inverse kinematic calculation (analytic method)

When position and orientation of the point p is known, the process of solving angles $\theta_1, \theta_2, \cdots, \theta_6$ is defined as inverse kinematics. Because of the complexity of the inverse calculation, in general, the inverse kinematic problem on more than 3DOF is solved by numerical methods So far, there is no analytic inverse kinematic calculation method for human lower extremity with more than 3DOF by analytic methods. The purpose of this paper is to explore an analytic method of the inverse kinematic calculation for human lower extremity with 6DOF or 7DOF

4.1 The case of six degrees of freedom (6DOF)

Relative to coordinates Σ_0, the distal foot position $p_{\Sigma 0}$ and orientation (α, β, γ) are known The joint angles, $(\theta_1, \cdots, \theta_6)$, of human lower extremity well be solved in this subsection.

4.1.1 solving process of angles θ_6, θ_2, θ_1

Form the position (x, y, z) and orientation (α, β, γ), we can obtained the homogeneous transformation, $T(\alpha, \beta, \gamma, x, y, z)$, and it can be expressed as follows [1] [2],

$$T(\alpha,\ \beta,\ \gamma,\ x,\ y,\ z)$$

$$= \begin{bmatrix} C_\alpha C_\beta C_\gamma - S_\alpha S_\gamma & -C_\alpha C_\beta S_\gamma - S_\alpha C_\gamma & C_\alpha S_\beta & x \\ S_\alpha C_\beta C_\gamma + C_\alpha S_\gamma & -S_\alpha C_\beta S_\gamma + C_\alpha C_\gamma & S_\alpha S_\beta & y \\ -S_\beta C_\gamma & S_\beta S_\gamma & C_\beta & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & x \\ a_{21} & a_{22} & a_{23} & y \\ a_{31} & a_{32} & a_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (8)$$

where, C_* and S_* denote $\cos\theta_*$ and $\sin\theta_*$, repectively, and a_{**} denotes the elements of matrix $T(\alpha,\ \beta,\ \gamma, x, y, z)$ Taking into account eq.(1), we will substitute $T(\alpha,\ \beta,\ \gamma,\ x,\ y,\ z)$ for $T(\theta_1, \cdots, \theta_6)$.

There are two characters for the human lower extremity. First, the rotation shafts of angles, $\theta_1, \theta_2, \theta_3$, intersect at one point Secondly, the rotation shafts of angles $\theta_3, \theta_4, \theta_5$, are in the same direction. In consideration of two characters, eq.(1) can be rewritten with the following form,

$$[\ {}^0T_1\ {}^1T_2\ {}^2T_3\ ]^{-1}\ T(\alpha,\ \beta,\ \gamma,\ x,\ y,\ z) = {}^3T_4\ {}^4T_5\ {}^5T_6\ {}^6T_p. \quad (9)$$

Terms on the left-hand of eq (9) only consist of functions of angles $\theta_1, \theta_2, \theta_3$. Matrix elements on two sides of eq.(9) are equal, respectively. Moreover, the third row elements of the two matrix only consist of functions of angles $\theta_1, \theta_2, \theta_6$ Accordingly,

$$a_{11}\ C1\ C2 + a_{21}\ C2\ S1 + a_{31}\ S2 = S\alpha 6\ S6, \quad (10)$$

$$a_{12}\ C1\ C2 + a_{22}\ C2\ S1 + a_{32}\ S2 = S\alpha 6\ C6, \quad (11)$$

$$a_{13}\ C1\ C2 + a_{23}\ C2\ S1 + a_{33}\ S2 = C\alpha 6, \quad (12)$$

$$x\ C1\ C2 + y\ C2\ S1 + z\ S2 = L_p\ S\alpha 6\ S6, \quad (13)$$

can be obtained, where $C\alpha 6$ and $S\alpha 6$ denote $\cos\alpha_6$ and $\sin\alpha_6$. Eliminating terms $C1C2$, $C2S1$, $S2$ from eq.(10) $\sim$ (13), the equation about θ_6 can be obtained as

$$A_6\ S6 + B_6\ C6 = D_6, \quad (14)$$

where, A_6, B_6, D_6 are expressed in detail as follows:

$$\begin{aligned} A_6 &= B\ S\alpha 6\ a_{13}(a_{22}a_{13} - a_{12}a_{23}) \\ &\quad - A\ S\alpha 6\ a_{13}(a_{13}y - a_{23}x - L_p a_{21}a_{13} + L_p a_{11}a_{23}), \\ B_6 &= -B\ S\alpha 6\ a_{13}(a_{21}a_{13} - a_{11}a_{23}), \\ D_6 &= B\ C\alpha 6\ a_{11}(a_{22}a_{13} - a_{12}a_{23}) \\ &\quad - B\ C\alpha 6\ a_{12}(a_{21}a_{13} - a_{11}a_{23}) \\ &\quad - A\ C\alpha 6\ a_{11}(a_{13}y - a_{23}x) + A\ C\alpha 6\ x(a_{21}a_{13} - a_{11}a_{23}), \\ A &= (a_{31}a_{13} - a_{11}a_{33})(a_{22}a_{13} - a_{12}a_{23}) \\ &\quad - (a_{21}a_{13} - a_{11}a_{23})(a_{32}a_{13} - a_{12}a_{33}), \\ B &= (a_{31}a_{13} - a_{11}a_{33})(a_{13}y - a_{23}x) \\ &\quad - (a_{21}a_{13} - a_{11}a_{23})(a_{13}z - a_{33}x) \end{aligned}$$

From eq (14), θ_6 is formulated in the form

$$\theta_6 = \tan^{-1}2(A_6, B_6) \pm \tan^{-1}2(\sqrt{A_6^2 + B_6^2 - D_6^2}, D_6) \quad (15)$$

Substituting θ_6 into eqs (10) $\sim$ (13), $S2$ is formulated in the form

$$S2 = (B_2\ S6 + D_2\ C6 + E_2)/A_2, \quad (16)$$

where, B_2, D_2, E_2, A_2 are expressed in detail as follows:

$$\begin{aligned} B_2 &= (a_{22}\ a_{13} - a_{12}\ a_{23})a_{13}\ S\alpha 6, \\ D_2 &= (a_{21}\ a_{13} - a_{11}\ a_{23})a_{13}\ S\alpha 6, \\ E_2 &= (a_{21}\ a_{13} - a_{11}\ a_{23})a_{12}\ C\alpha 6 \\ &\quad - (a_{22}\ a_{13} - a_{12}\ a_{23})a_{11}\ C\alpha 6, \\ A_2 &= (a_{22}\ a_{13} - a_{12}\ a_{23})(a_{31}\ a_{13} - a_{11}\ a_{33}) \\ &\quad - (a_{21}\ a_{13} - a_{11}\ a_{23})(a_{32}\ a_{13} - a_{12}\ a_{33}). \end{aligned}$$

Using the obtained the $\theta_6, S2$, $C2S1$ is then formulated in the form:

$$C2S1 = (S\alpha 6\ a_{13}\ S6 - C\alpha 6\ a_{11} - B_1\ S2)/A_1, \quad (17)$$

where, B_1, A_1 are expressed in detail as follows:

$$\begin{aligned} B_1 &= (a_{31}\ a_{13} - a_{11}\ a_{33}), \\ A_1 &= (a_{21}\ a_{13} - a_{11}\ a_{23}). \end{aligned}$$

From the obtained $\theta_6, S2, C2S1$, finally, $C1C2$ is formulated in the form

$$C1C2 = (S\alpha 6\ S6 - a_{21}\ C2S1 - a_{31}\ S2)/a11 \quad (18)$$

If $\cos\theta_2 \neq 0$, θ_1 can be formulated in the form

$$\theta_1 = \tan^{-1}2\ (\pm C2S1,\ \pm C1S2) \quad (19)$$

When $\theta_2 = \frac{\pi}{2}$ or $-\frac{\pi}{2}$, θ_1 can not be determined uniquely.

From eqs. (17), (18), $C2$, is obtained in the from

$$C2 = \pm\sqrt{(C2S1)^2 + (C1S2)^2} \quad (20)$$

From eqs (16), (20), θ_2 can be formulated in the form

$$\theta_2 = \tan^{-1}2\ (S2,\ C2) \quad (21)$$

4.1.2 solving process of angles θ_3, θ_4, θ_5

Now, by using the known joint angles θ_1, θ_2, θ_6, let us begin to solve angles θ_3, θ_4, θ_5. Because joint angles θ_1, θ_2 are known, similar to subsection 4.1.1, eq. (1) can be rewritten in the form

$$[\ {}^0\boldsymbol{T}_1\ {}^1\boldsymbol{T}_2\ ]^{-1}\ \boldsymbol{T}(\alpha,\ \beta,\ \gamma,\ x,\ y,\ z) = {}^2\boldsymbol{T}_3\ {}^3\boldsymbol{T}_4\ {}^4\boldsymbol{T}_5\ {}^5\boldsymbol{T}_6\ {}^6\boldsymbol{T}_p \quad (22)$$

Terms on the left-hand of eq.(22) only consist of functions of angles θ_1, θ_2, and terms on the right-hand of equation consist of functions of unknown $\theta_3, \theta_4, \theta_5$. Matrix elements on two sides of equation are equal. According to elements of third and forth columns on the first row, third and forth columns elements on the third row,

$$S\alpha6\ S(3{+}4{+}5) = a_{33}C2 - a_{13}C1S2 - a_{23}S1S2, \quad (23)$$

$$L_4C3 + L_5C(3{+}4) + (L_6 + L_pC6)\ C(3{+}4{+}5) - L_pC\alpha6\ S6\ S(3{+}4{+}5) = -C1S2x - S1S2y + C2z, \quad (24)$$

$$-S\alpha6\ C(3{+}4{+}5) = -a_{23}C1 + a_{13}S1, \quad (25)$$

$$L_4S3 + L_5S(3{+}4) + (L_6 + L_pC6)\ S(3{+}4{+}5) + L_pC\alpha6\ S6\ C(3{+}4{+}5) = S1x - C1y, \quad (26)$$

are obtained, where $S3, C3, S(3{+}4), C(3{+}4), S(3{+}4{+}5), C(3{+}4{+}5)$ are be treated as new variables. Eliminating $S(3{+}4{+}5), C(3{+}4{+}5)$ from eq.(23) $\sim$ (26),

$$L_4\ C3 + L_5\ C(3+4) = E_3, \quad (27)$$

$$L_4\ S3 + L_5\ S(3+4) = F_3, \quad (28)$$

are obtained, where, E_3, F_3 are

$$E_3 = -C1S2x - S1S2y + C2z + (L_6 + L_pC6)(-a_{23}C1 + a_{13}S1)/S\alpha6 + L_pC\alpha6S6(a_{33}C2 - a_{13}C1S2 - a_{23}S1S2)/S\alpha6,$$

$$F_3 = S1x - C1y - (L_6 + L_pC6)(a_{33}C2 - a_{13}C1S2 - a_{23}S1S2)/S\alpha6 + L_pC\alpha6S6(-a_{23}C1 + a_{13}S1)/S\alpha6$$

From eqs (27) and (28), the equation about θ_3 is obtained. θ_3 is formulated in the form

$$A_3\ S3 + B_3\ C3 = D_3, \quad (29)$$

where, A_3, B_3, and D_3 are

$$A_3 = 2\ F_3\ L_4,$$
$$B_3 = 2\ E_3\ L_4,$$
$$D_3 = E_3^2 + F_3^2 + L_4^2 - L_5^2$$

Then θ_3 is formulated in the form

$$\theta_3 = \tan^{-1}2(A_3, B_3) \pm \tan^{-1}2(\sqrt{A_3^2 + B_3^2 - D_3^2}, D_3) \quad (30)$$

Also, from equation (27),(28), the equation about $(\theta_3 + \theta_4)$ is obtained as

$$A_4\ S(3+4) + B_4\ C(3+4) = D_4, \quad (31)$$

where, A_4, B_4, and D_4 are

$$A_4 = 2\ F_3\ L_5,$$
$$B_4 = 2\ E_3\ L_5,$$
$$D_4 = E_3^2 + F_3^2 + L_5^2 - L_4^2$$

Therefore, $(\theta_3 + \theta_4)$ is formulated in the form

$$(\theta_3 + \theta_4) = \tan^{-1}2(A_4, B_4) \pm \tan^{-1}2(\sqrt{A_4^2 + B_4^2 - D_4^2}, D_4) \quad (32)$$

From equations (30) and (32), θ_4 can be obtained by

$$\theta_4 = \tan^{-1}2(A_4, B_4) \pm \tan^{-1}2(\sqrt{A_4^2 + B_4^2 - D_4^2}, D_4) - \tan^{-1}2(A_3, B_3) \mp \tan^{-1}2(\sqrt{A_3^2 + B_3^2 - D_3^2}, D_3) \quad (33)$$

From eq. (23) and (25), the equation about $(\theta_3 + \theta_4 + \theta_5)$ can be formulated by

$$(\theta_3 + \theta_4 + \theta_5) = \tan^{-1}2\ (A_5, B_5).$$

Thereby, we have

$$\theta_5 = \tan^{-1}2(A_5, B_5) - \theta_5 - \theta_5, \quad (34)$$

where A_5, B_5 are expressed as follows:

$$A_5 = (a_{33}C2 - a_{13}C1S2 - a_{23}S1S2)/S\alpha6,$$
$$B_5 = (a_{23}C1 - a_{13}S1)/S\alpha6.$$

Till now, for human lower extremity with 6DOF, the method for obtaining the angles $\theta_1, \cdots, \theta_6$ from the position and orientation of the distal foot is derived.

4.1.3 Example calculation

Let us consider an example, where the structural parameters of a lower extremity are

L_4	L_5	L_6	L_p	α_6
37cm	38cm	4cm	12cm	15 °

,

the position and orientation of the distal foot are

x	y	z	α	β	γ
−19cm	−5cm	86cm	27 °	78 °	165 °

,

By the proposed analytic method of inverse kinematic calculation an unique solution of joint angles is obtained as follows:

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
11.66 °	12.44 °	−12.22 °	29.44 °	−17.27 °	−11.69 °

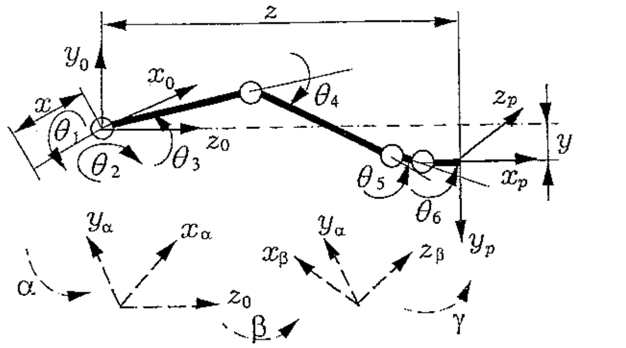

Figure 5: Schematic illustration of the example calculation on 6DOF

where, the displacement scopes of joint angles are given respectively by

$$0 \le \theta_1 \le 2\pi, \quad -\tfrac{\pi}{2} < \theta_2 < \tfrac{\pi}{2}, \quad -\tfrac{\pi}{2} \le \theta_3 \le \tfrac{\pi}{2},$$
$$0 \le \theta_4 \le \pi, \quad -\tfrac{\pi}{2} \le \theta_5 \le \tfrac{\pi}{2}, \quad -\tfrac{\pi}{2} \le \theta_6 \le \tfrac{\pi}{2}$$

Structural parameters, position and orientation of the distal foot and the joint angles are shown in Fig 5.

To examine the solving process, we calculated position and orientation of the distal foot using direct kinematic calculation from obtained joint angles $(\theta_1, \theta_2, \cdots, \theta_6)$. The obtained results $(x,\ y,\ z,\ \alpha,\ \beta,\ \gamma)$ are equal to given $(x,\ y,\ z,\ \alpha,\ \beta,\ \gamma)$. This indicates that the solving process proposed here is correct according to the Fig 3

4.2 The case of even of degrees of freedom (7DOF)

As mentioned, the inverse kinematics calculation on 6DOF can be uniquely solved from the position and orientation of the distal foot. This method, however, can not be used for the model of human lower extremity with 7DOF, since there is one redundant DOF. In this subsection, the position of knee will be added in the ensuing calculation on 7DOF. The knee position will be measured by 3D digitizer and dual receiver motion tracker installed on the knee

4.2.1 solving process of angle θ_6

Represented in Fig 4, relative to coordinates Σ_0, the origin position of coordinate $\Sigma_4(x_4, y_4, z_4)$, the distal foot position $\boldsymbol{p}_{\Sigma 0}$ and orientation (α, β, γ) are known, and joint angles, $(\theta_1, \cdots, \theta_4', \theta_5, \theta_6)$, of human lower extremity are solved as the following.

By the same method of **4.1**, we will substitute $\boldsymbol{T}(\alpha, \beta, \gamma, x, y, z)$ for $\boldsymbol{T}(\theta_1, \cdots, \theta_{4'}, \theta_5, \theta_6)$ in eq (7).

Since the origin position of coordinates $\Sigma_4(x_4, y_4, z_4)$ was given, the homogeneous transformation , ${}^0\boldsymbol{T}_4(\theta_1, \theta_2, \theta_3, \theta_4)$, of Σ_4 with respect to Σ_0 can be described by the form

$$
{}^0\boldsymbol{T}_4(\theta_1, \theta_2, \theta_3, \theta_4) = {}^0\boldsymbol{T}_1\ {}^1\boldsymbol{T}_2\ {}^2\boldsymbol{T}_3\ {}^3\boldsymbol{T}_4
$$
$$
= \begin{bmatrix} b_{11} & b_{12} & b_{13} & x_4 \\ b_{21} & b_{22} & b_{23} & y_4 \\ b_{31} & b_{32} & b_{33} & z_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{35}
$$

where, b_{**} denote the elements of matrix ${}^0\boldsymbol{T}_4(\theta_1, \theta_2, \theta_3, \theta_4)$, $x_4,\ y_4,\ z_4$ are expressed in detail by

$$x_4 = L_4(-C1S2C3 + S1S3) \tag{36}$$
$$y_4 = L_4(-S1S2C3 + C1S3) \tag{37}$$
$$z_4 = L_4C2C3 \tag{38}$$

If the structural parameters of a lower extremity were given, the origin position $(x_4,\ y_4,\ z_4)$ of coordinate Σ_4 is determined by $\theta_1, \theta_2, \theta_3$, but the angles $\theta_1, \theta_2, \theta_3$, can not be determined by $(x_4,\ y_4,\ z_4)$ uniquely.

There by, let us notice that the rotation shafts of angles, $\theta_1, \theta_2, \theta_3$, also intersect at one point. In consideration of the character of human lower extremity and the origin position of coordinate Σ_4 was known, eq (7) can be rewritten with the following form,

$$
[\ {}^0\boldsymbol{T}_1\ {}^1\boldsymbol{T}_2\ {}^2\boldsymbol{T}_3\ ]^{-1}\boldsymbol{T}(\alpha,\ \beta,\ \gamma,\ x,\ y,\ z) = {}^3\boldsymbol{T}_4\ {}^4\boldsymbol{T}_{4'}\ {}^{4'}\boldsymbol{T}_5\ {}^5\boldsymbol{T}_6\ {}^6\boldsymbol{T}_p \tag{39}
$$

The terms on the right-hand of eq (39) only consist of functions of angles $\theta_1, \theta_2, \theta_3$, and the matrix elements of the first row on the right-hand of eq (39) can be obtained by eqs (36),(37) and (38). According to the two matrix elements of the first, the second, and the third columns on the first row:

$$
\begin{aligned}
&C6(C4C5 - S4C4'S5) + S\alpha6S6(S4S4') \\
&+S\alpha6S6(-S4C4'C5 - C4S5) \\
&\qquad = a_{31}C2C3 + a_{21}(-S1S2C3 - C1S3) \\
&\qquad\ +a_{11}(-C1S2C3 + S1S3),
\end{aligned} \tag{40}
$$
$$
\begin{aligned}
&-S6(C4C5 - S4C4'S5) + S\alpha6C6(S4S4') \\
&+C\alpha6S6(-S4C4'C5 - C4S5) \\
&\qquad = a_{32}C2C3 + a_{22}(-S1S2C3 - C1S3) \\
&\qquad\ +a_{12}(-C1S2C3 + S1S3),
\end{aligned} \tag{41}
$$
$$
\begin{aligned}
&C\alpha6(S4S4') + S\alpha6(-S4C4'C5 - C4S5) \\
&\qquad = a_{32}C2C3 + a_{22}(-S1S2C3 - C1S3) \\
&\qquad\ +a_{12}(-C1S2C3 + S1S3).
\end{aligned} \tag{42}
$$

Substituting eqs (36), (37) and (38) into the terms on the right-hand of eqs (40), (41) and (42), and defining $M_1,\ N_1,\ P_1$ as follows:

$$
\begin{aligned}
M_1 &= (a_{31}z_4 + a_{21}y_4 + a_{11}x_4)/L_4, \\
N_1 &= (a_{32}z_4 + a_{22}y_4 + a_{12}x_4)/L_4, \\
P_1 &= (a_{33}z_4 + a_{23}y_4 + a_{13}x_4)/L_4,
\end{aligned}
$$

then, eqs (40), (41) and (42) can be rewritten as

$$
\begin{aligned}
&C6(C4C5 - S4C4'S5) + S\alpha6S6(S4S4') \\
&\qquad +S\alpha6S6(-S4C4'C5 - C4S5) = M_1,
\end{aligned} \tag{43}
$$
$$
\begin{aligned}
&-S6(C4C5 - S4C4'S5) + S\alpha6C6(S4S4') \\
&\qquad +C\alpha6S6(-S4C4'C5 - C4S5) = N_1,
\end{aligned} \tag{44}
$$
$$
C\alpha6(S4S4') + S\alpha6(-S4C4'C5 - C4S5) = P_1, \tag{45}
$$

where, $(C4C5 - S4C4'S5), (S4S4')$ and $(-S4C4'C5 - C4S5)$ are be treated as new variables. Eliminating $(C4C5 - S4C4'S5)$ and $(-S4C4'C5 - C4S5)$ from eqs (43), (44) and (45), the equation about $S4S4'$, θ_6 can be obtained as

$$S4S4' = S\alpha6M1S6 + S\alpha6N1C6 + C\alpha6P1 \tag{46}$$

In the follows, we will derive another the equation about $S4S4'$, θ_6 Take account into that the rotation shafts of angles, $\theta_1, \theta_2, \theta_3$, intersect at one point, and the rotation shafts of angles $\theta_4, \theta_{4'}$, intersect at another point eq (7) can be rewritten with the following form

$$
{}^{4'}\boldsymbol{T}_5\ {}^5\boldsymbol{T}_6\ {}^6\boldsymbol{T}_p = [\ {}^0\boldsymbol{T}_1\ {}^1\boldsymbol{T}_2\ {}^2\boldsymbol{T}_3\ {}^3\boldsymbol{T}_4\ {}^4\boldsymbol{T}_{4'}\ ]^{-1} \\ \boldsymbol{T}(\alpha,\ \beta,\ \gamma,\ x,\ y,\ z) \quad (47)
$$

Matrix elements on two sides of eq (47) are equal, respectively. According to the two matrix elements on the first row:

$$
\begin{aligned}
a_{31}V_1 + a_{11}V_2 + a_{21}V_3 &= -S\alpha 6S6, & (48)\\
a_{32}V_1 + a_{12}V_2 + a_{22}V_3 &= -S\alpha 6C6, & (49)\\
a_{33}V_1 + a_{13}V_2 + a_{23}V_3 &= -C\alpha 6, & (50)\\
zV_1 + xV_2 + yV_3 &= -L_6S\alpha 6S6 - L_4S4S4', & (51)
\end{aligned}
$$

where, V_1, V_2, V_3 are be treated as new variables representing

$$
\begin{aligned}
V_1 &= -(C4'S2) - C2(C4S3S4' + C3S4S4'),\\
V_2 &= S1(C3C4S4' - S3S4S4') - C1(C2C4'\\
&\quad -S2(S3C4S4' + C3S4S4')),\\
V_3 &= C1(C3C4S4' - S3S4S4') - S1(C2C4'\\
&\quad -S2(S3C4S4' + C3S4S4')).
\end{aligned}
$$

Eliminating the new variables from eqs (48) $\sim$ (51), the equation about $S4S4', \theta_6$ can be obtained as

$$
Q_2S4S4' = M_2S6 + N_2C6 + P_2, \quad (52)
$$

where, Q_2, M_2, N_2, P_2 are expressed in detail as follows:

$$
\begin{aligned}
Q_2 &= L_4a_{21}(a_{11}a_{22} - a_{12}a_{21})((a_{22}a_{31} - a_{21}a_{32})(a_{11}a_{23}\\
&\quad -a_{21}a_{13}) - (a_{23}a_{31} - a_{21}a_{33})(a_{11}a_{23} - a_{12}a_{21})),\\
M_2 &= E_2'((a_{22}a_{31} - a_{21}a_{32})(a_{11}a_{23} - a_{21}a_{13})\\
&\quad -(a_{23}a_{31} - a_{21}a_{33})(a_{11}a_{23} - a_{12}a_{21}))\\
&\quad -E_1'((a_{22}a_{31} - a_{21}a_{32})(a_{11}y - a_{21}x)\\
&\quad -(a_{31}y - a_{21}z)(a_{11}a_{23} - a_{12}a_{21})),\\
N_2 &= (a_{11}y - a_{21}x)((a_{22}a_{31} - a_{21}a_{32})(a_{11}a_{23} - a_{21}a_{13})\\
&\quad -(a_{23}a_{31} - a_{21}a_{33})(a_{11}a_{23} - a_{12}a_{21})) - (a_{11}a_{23}\\
&\quad -a_{21}a_{13})((a_{22}a_{31} - a_{21}a_{32})(a_{11}y - a_{21}x)\\
&\quad -(a_{31}y - a_{21}z)(a_{11}a_{23} - a_{12}a_{21}))S\alpha 6a_{21},\\
P_2 &= C\alpha 6a_{21}(a_{11}a_{22} - a_{12}a_{21})((a_{22}a_{31} - a_{21}a_{32})\\
&\quad (a_{11}y - a_{21}x) - (a_{31}y - a_{21}z)(a_{11}a_{23} - a_{12}a_{21})),\\
E_1' &= -S\alpha 6a_{22}(a_{11}a_{23} - a_{21}a_{13}) + S\alpha 6a_{23}(a_{11}a_{22} - a_{12}a_{21}),\\
E_2' &= -S\alpha 6a_{22}(a_{11}y - a_{21}x) + S\alpha 6y(a_{11}a_{22} - a_{12}a_{21})\\
&\quad -L_6S\alpha 6a_{21}(a_{11}a_{22} - a_{12}a_{21})
\end{aligned}
$$

From eqn (46), (52), the equation only about θ_6 can be obtained as:

$$
A_6'\ S6 + B_6'\ C6 = D_6' \quad (53)
$$

where, A_6', B_6', D_6' are expressed in detail as follows,

$$
\begin{aligned}
A_6' &= S\alpha 6M_1Q_2 - M_2,\\
B_6' &= S\alpha 6N_1Q_2 - N_2,\\
D_6' &= P_2 - C\alpha 6P_1Q_2.
\end{aligned}
$$

From eqn(53), θ_6 is formulated in the form,

$$
\theta_6 = \tan^{-1}2(A_6', B_6') \pm \tan^{-1}2(\sqrt{A_6'^2 + B_6'^2 - D_6'^2}, D_6') \quad (54)
$$

4.2.2 solving process of angle θ_5

Now, by using the known angle θ_6, let us solve angle θ_5 The origin position of coordinate Σ_4 was given, taking this into consideration again, eq (9) can be rewritten with the following form,

$$
\boldsymbol{T}(\alpha,\ \beta,\ \gamma,\ x,\ y,\ z)[\ {}^4\boldsymbol{T}_{4'}\ {}^{4'}\boldsymbol{T}_5\ {}^5\boldsymbol{T}_6\ {}^6\boldsymbol{T}_p\ ]^{-1} \\ = {}^0\boldsymbol{T}_1\ {}^1\boldsymbol{T}_2\ {}^2\boldsymbol{T}_3\ {}^3\boldsymbol{T}_4. \quad (55)
$$

Matrix elements on two-hand of eq (55) are equal, respectively In consideration of eqs (35), (39), according to the elements of the second and the third rows on the fourth column, the equations only about $S5$, $C5$ can be obtained as

$$
\begin{aligned}
M_3S5 + N_3C5 &= P_3, & (56)\\
M_4S5 + N_4C5 &= P_4, & (57)
\end{aligned}
$$

where, $M_3, N_3, P_3, M_4, N_4, P_4$ are be expressed in detail as follows:

$$
\begin{aligned}
M_3 &= L_5(a_{22}C\alpha 6C6 - a_{23}S\alpha 6 + a_{21}C\alpha 6S6),\\
M_4 &= L_5(a_{32}C\alpha 6C6 - a_{33}S\alpha 6 + a_{31}C\alpha 6S6),\\
N_3 &= -L_5(a_{21}C6 - a_{22}S6),\\
N_4 &= -L_5(a_{31}C6 - a_{32}S6),\\
P_3 &= y_4 - y + L_6a_{21}C6 + L_pa_{21} - L_6a_{22}S6,\\
P_4 &= z_4 - z + L_6a_{31}C6 + L_pa_{31} - L_6a_{32}S6.
\end{aligned}
$$

From eqs (56) and (57), S5 and C5 can be expressed by:

$$
\begin{aligned}
S5 &= (N_4P_3 - N_3P_4)/(M_3N_4 - M_4N_3), & (58)\\
C5 &= (M_4P_3 - M_3P_4)/(M_4N_3 - M_3N_4). & (59)
\end{aligned}
$$

From eqs (58) and (59), θ_5 is formulated in the form,

$$
\theta_5 = \tan^{-1}2(S5,\ C5). \quad (60)
$$

4.2.3 solving process of angles θ_4, and θ_4'

Now, by using the known angles θ_6 and θ_5, let us solve angles θ_4 and $\theta_{4'}$. the eqs (43), (44) and (45) can be rewritten in the forms:

$$
\begin{aligned}
&(C5C6 - C\alpha 6S5S6)C4 - (S5C6\\
&\quad +C\alpha 6C5S6)S4C4' + (S\alpha 6S7)S4S4' = M_1, \quad (61)\\
&-(C5S6 + C\alpha 6S5C6)C4 + (S5S6\\
&\quad -C\alpha 6C5C6)S4C4' + (C\alpha 6S7)S4S4' = N_1, \quad (62)\\
&(S\alpha 6S5)C4 + (S\alpha 6C5)S4C4' + C\alpha 6S4S4' = P_1. \quad (63)
\end{aligned}
$$

Substituting angles θ_6 and θ_5, into eqs (61), (62) and(63), moreover treating $C4$, $S4C4'$ and $S4S4'$ as new variables $C4$, $S4C4'$ and $S4S4'$ can be obtained and expressed in the from:

$$
\begin{aligned}
S4S4' &= K_1 = (H_2J_2 - H_1J_1)/(H_2I_1 - H_1I_2), & (64)\\
S4C4' &= K_2 = (J_2 - I_2K_1)/H_2, & (65)\\
C4 &= (P_1 - C\alpha 6K_1 - S\alpha 6C5K_2)/S\alpha 6S5, & (66)
\end{aligned}
$$

where, H_1, I_1, J_1, H_2, I_2, J_2 are expressed as follows:

$$H_1 = (C5S6 + C\alpha6S5C6)(S5C6 + C\alpha6C5S6) \\ -(C5C6 - C\alpha6S5S6)(S5S6 - C\alpha6C5C6),$$

$$I_1 = -(C5S6 + C\alpha6S5C6)S\alpha6S6 \\ -(C5C6 - C\alpha6S5S6)C\alpha6,$$

$$J_1 = -(C5S6 + C\alpha6S5C6)M_1 - (C5C6 - C\alpha6S5S6)N_1,$$

$$H_2 = -S\alpha6S5(S5C6 + C\alpha6C5S6) \\ -(C5C6 - C\alpha6S5S6)C\alpha6,$$

$$I_2 = S\alpha6^2S5S6 - (C5C6 - C\alpha6S5S6)C\alpha6,$$

$$J_2 = S\alpha6M_1 - (C5C6 - C\alpha6S5S6)P_1$$

From eqs (64), (65), $S4$ can be expressed by

$$S4 = \pm\sqrt{K_1^2 + K_2^2} \quad (67)$$

From eqs (66), (67), θ_4 can be formulated in the form:

$$\theta_4 = \tan^{-1}2(S4,\ C4) \quad (68)$$

If $\theta_4 \neq 0$ or π, from eqs (64) and (65), $\theta_{4'}$ can be formulated in the form:

$$\theta_{4'} = \tan^{-1}2(\pm K_1,\ \pm K_2) \quad (69)$$

When $\theta_4 = 0$ or π, $\theta_{4'}$ can not be determined uniquely In the extending state of human lower extremity, joint angle $\theta_{4'}$ becames zero and is unable to displace, this is ingenious character of human lower extremity structure

4.2.4 solving process of angles θ_1, θ_2, θ_3

Now, by using the known angles θ_4, $\theta_{4'}$, θ_5, θ_6, we will solve the angles θ_1, θ_2, θ_3. Accordingly, eq (7) can be rewritten with the following form:

$$^0\boldsymbol{T}_1\ ^1\boldsymbol{T}_2\ ^2\boldsymbol{T}_3 \\ = \boldsymbol{T}(\alpha,\ \beta,\ \gamma,\ x,\ y,\ z)[\ ^3\boldsymbol{T}_4\ ^4\boldsymbol{T}_{4'}\ ^{4'}\boldsymbol{T}_5\ ^5\boldsymbol{T}_6\ ^6\boldsymbol{T}_p\ ]^{-1} \quad (70)$$

Terms on the left-hand side of equation (70) only consist of functions of angles $\theta_1, \theta_2, \theta_3$ All of the terms on the left-hand side of equation (70) was known Eq.(70) can be described in the following form:

$$\begin{bmatrix} -C1S2C3+S1S3 & S1C3+C1S2S3 & C1C2 & 0 \\ -S1S2C3-C1S3 & -C1C3+S1S2S3 & S1C2 & 0 \\ C2C3 & -C2S3 & S2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (71)$$

Matrix elements on two-hand sides of equation (71) are equal, respectively Accordingly, the equations:

$$C1C2 = c_{13}, \quad (72)$$

$$S1C2 = c_{23}, \quad (73)$$

$$S2 = c_{33}, \quad (74)$$

$$C2C3 = c_{31}, \quad (75)$$

$$-C2S3 = c_{32}, \quad (76)$$

are obtained, where,

$$c_{13} = C4'(a_{13}C\alpha6+a_{12}S\alpha6C6+a_{11}S\alpha6S6)+S4'(S5(a_{11}C6-a_{12}S6)+C5(a_{12}C\alpha6C6-a_{13}S\alpha6+a_{11}C\alpha6S6)),$$

$$c_{23} = C4'(a_{23}C\alpha6+a_{22}S\alpha6C6+a_{21}S\alpha6S6)+S4'(S5(a_{21}C6-a_{22}S6)+C5(a_{22}C\alpha6C6-a_{23}S\alpha6+a_{21}C\alpha6S6)),$$

$$c_{33} = C4'(a_{33}C\alpha6+a_{32}S\alpha6C6+a_{31}S\alpha6S6)+S4'(S5(a_{31}C6-a_{32}S6)+C5(a_{32}C\alpha6C6-a_{33}S\alpha6+a_{31}C\alpha6S6)),$$

$$c_{31} = C4(C5(a_{31}C6-a_{32}S6)-S5(a_{32}C\alpha6C6-a_{33}S\alpha6+a_{31}C\alpha6S6))+S4(S4'(a_{33}C\alpha6+a_{32}C6S\alpha6+a_{31}S\alpha6S6)-C4'(S5(a_{31}C6-a_{32}S6)+C5(a_{32}C\alpha6C6-a_{33}S\alpha6+a_{31}C\alpha6S6)),$$

$$c_{32} = S4(C5(a_{31}C6-a_{32}S6)-S5(a_{32}C\alpha6C6-a_{33}S\alpha6+a_{31}C\alpha6S6))+C4(S4'(a_{33}C\alpha6+a_{32}C6S\alpha6+a_{31}S\alpha6S6)-C4'(S5(a_{31}C6-a_{32}S6)+C5(a_{32}C\alpha6C6-a_{33}S\alpha6+a_{31}C\alpha6S6))$$

From eqs (72), (73), $C2$ can be expressed by:

$$C2 = \pm\sqrt{c_{13}^2 + c_{12}^2} \quad (77)$$

From eqs (74), (77), θ_2 is formulated by:

$$\theta_2 = \tan^{-1}2(S2,\ C2) \quad (78)$$

If $\theta_2 \neq 0$ or π, from eqs (72), (73), (75) and (76) θ_1, θ_3 can be formulated in the forms:

$$\theta_1 = \tan^{-1}2(\pm c_{23},\ \pm c_{13}), \quad (79)$$

$$\theta_3 = \tan^{-1}2(\mp c_{32},\ \pm c_{31}), \quad (80)$$

When $\theta_2 = \frac{\pi}{2}$ or $-\frac{\pi}{2}$, θ_1 and θ_3 can not be determined uniquely

Till now, we had derived the method for obtaining the angles $\theta_1, \ldots, \theta_4', \theta_5, \theta_6$ from the position and orientation of the distal foot and the position of knee, for human lower extremity with 7DOF.

4.2.5 Example Calculation

Where, we give the same structural parameters of a lower extremity as 3.1 3, the position of knee joint is given by

x_4	y_4	z_4
−8.8cm	6.6cm	35.4cm

,

the position and orientation of the distal foot are given by

x	y	z	α	β	γ
−18.8cm	−5.7cm	85.9cm	31.3 °	77.0 °	164.4 °

By proposed analytic method of inverse kinematic calculation, an unique solution of joint angles is obtained as follows:

θ_1	θ_2	θ_3	θ_4	θ_4'	θ_5	θ_6
10.7 °	12.0 °	−11.9 °	30.0 °	5.3 °	−17.3 °	−11.6 °

where, the displacement scopes of joint angles are given respectively by

$$0 \le \theta_1 \le 2\pi \quad -\frac{\pi}{2} < \theta_2 < \frac{\pi}{2} \quad -\frac{\pi}{2} \le \theta_3 \le \frac{\pi}{2}$$
$$0 < \theta_4 < \pi \quad -\frac{\pi}{4} \le \theta_4' \le \frac{\pi}{4} \quad -\frac{\pi}{2} \le \theta_5 \le \frac{\pi}{2}$$
$$-\frac{\pi}{2} \le \theta_6 \le \frac{\pi}{2}$$

Structural parameters, the position of knee, position and orientation of the distal foot, joint angles are shown in Fig.6.

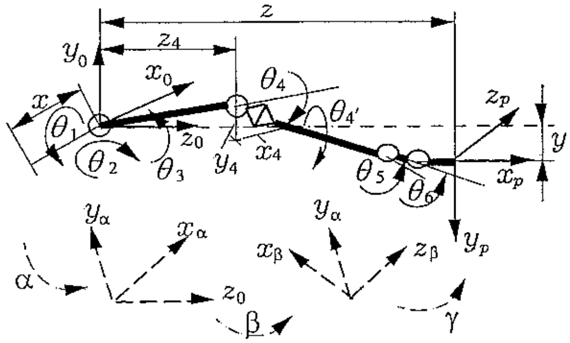

Figure 6: Schematic illustration of the example calculation on 7DOF

To examine the solving, similar to 4.1.3 we also calculated the position and orientation of the distal foot and the position of knee using direct kinematic calculation from obtained joint angles. The obtained results indicate that the solving process proposed here is correct according to the Fig.4.

5 Experiment

In order to examine the proposed analytic method and calculation program, as shown in Fig.7, we made a simple model of human lower extremity with 6DOF, where, the joint parts are designed for protractors easily to be clung to them. These angles can be precisely measured by the protractors.

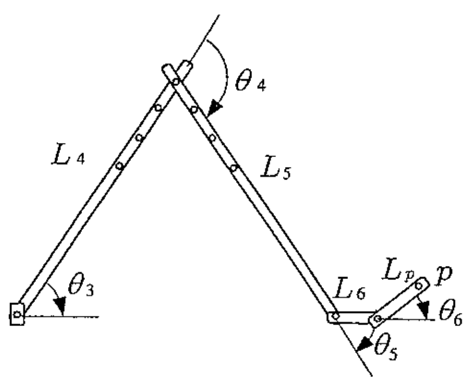

Figure 7: The mechanical model of human lower extremity with 6DOF

We have performed a lot of experiments on the model by changing link lengths and joint angles, let us give an example of the experiments, where, we set up: L_4=35cm, L_5=45cm, L_6=6cm, L_p=7cm, $\theta_1 = \theta_2 = 0\ ^\circ$, $\theta_6 = -50\ ^\circ$.

The results of calculation and measurement are shown in Tab.1. From the Tab.1, we can know that all of the data errors are within $\pm 2^\circ$. Therefore, it is indicated that the proposed analytic method is effective for lower extremity kinematic calculation.

Table. 1: The results of calculation and measurement

Data Number	θ 3[deg]		
	Calculation	Measurement	Error[%]
1	−82.285	−82	−0.348
2	−74.374	74	−0.505
3	−70.653	−71.5	1.184
4	−55.824	−56	0.314
5	−30.783	−31	0.7
6	−13.51	−14	3.5

Data Number	θ 4[deg]		
	Calculation	Measurement	Error[%]
1	132.294	132	0.222
2	122.489	122.5	−0.009
3	118.561	118.5	0.051
4	95.551	95.5	0.053
5	53.956	54	−0.081
6	23.714	25	−5.144

Data Number	θ 5[deg]		
	Calculation	Measurement	Error[%]
1	−51.644	−51.5	−0.279
2	−49.746	−50	0.508
3	−46.995	−47	0.01
4	−41.341	−41	−0.831
5	−24.804	−25	0.784
6	−11.835	−11	−7.59

The Experiment of the mechanical model on 7DOF just is under process.

6 Conclusion

We proposed an analytical method of inverse kinematics for human lower extremity with 6DOF or 7DOF. By using the proposed analytical method, inverse kinematic calculation is performed. For examining the proposed method, direct kinematic calculation is also performed. The direct kinematic calculation results are correspond with the inverse kinematic calculation results. This indicates that the proposed analytical methods are correct according to the mathematical model with 6DOF or 7DOF. Furthermore, the kinematics experiment of a simple model with 6DOF is performed, we confirmed the proposed analytical method for human lower extremity with 6DOF is correct.

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