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An Analytical Grasp Planning on Given Object with Multifingered Hand

Ying Li, Yong Yu *and* Showzow Tsujio
Dept. of Mechanical Engineering, Kagoshima University, Japan

Abstract

In this paper, an analytical approach is proposed for planning finger positions of an object with multifingered hand. At first, a method is given to obtain which combination of the object edges is possible to be used for grasping. Then, Graspable Finger Position Region (GFPR) on a combination of edges is defined where the object can be held successfully. It is shown that the region is bounded by several boundary hyperplanes. With the combining these boundary hyperplanes, two propositions for analytically and exactly obtaining the GFPR are proposed. And, an algorithm is proposed to find the Stable GFPR that contains the biggest inscribed hypersphere of GFPR and has the larger volume. Lastly, numerical example is performed to show the effectiveness of the proposed grasp planning approach.

1 Introduction

The grasp of an object by a robot hand is a primitive but very important subtask in automatic systems. Advanced applications sometimes require multiple robotic fingers to perform a task coordinately. Therefore, determination of the stable grasp of an object is a fundamental and important issue.

In literatures, finger position and finger force on an object were treated as variables simultaneously. The problem of solving the graspable finger position regions of an object was dealt with as a nonlinear problem. The finger positions were computed by the method of scanning every sample points selected on the object, which is complicated and the computing load is large.

About the finger forces, the methods for constructing of the force-closure and the form-closure were proposed in [1]~[4]. About finger position regions for multifingered hand, many attempts have been made by analytical method. In [4], maximal object segments were obtained, where fingers were positioned independently while ensuring force-closure. Omata proposed an algorithm for approximately computing the positions of fingertips with maintaining equilibrium when a polyhedral object is grasped [5], but computation load dependents the sample points on each edge selected for scanning computation and is large [8]. The object grasp regions has not been exactly solved by analytical method yet.

Work on grasp stable analysis can be found in [3] and [6], they minimize the worst-case forces needed to balance any direction of forces acting on the center of gravity of the object, but these methods are only suitable for 3 or 4 fingers.

Thereby, the purpose of this paper is that: (1) we will exactly determine the graspable finger position regions of a given object; (2) we will find the stable finger position region using an analytical method. In this paper, we distinguish candidates from combinations of the object edges touched by fingertips. Then, we propose a novel method to solve the problem of the finger position regions for a grasp edge candidate. Lastly, we give an approach to find the stable graspable region from the obtained finger position region.

2 Graspable Edge Candidates

2.1 Force and Moment Equilibrium

In this paper, we address the problem of planning stable grasps of a flat polygonal object. We focus on force closure grasps, such that arbitrary planar forces and torques acting on the grasped object can be balanced by the contact forces exerted with the fingers.

The discussion is based on two assumptions: (1) the object is a 2D polygon with definite geometric shape; (2) at least 2 fingertips of a hand touch the object by point contact with Coulomb friction, and the contact point of fingertips on each edge is not more than one.

We assume that arbitrary fingertip contact forces satisfying the friction condition can be exerted at every contact point. The force closure can be achieved while the necessary and sufficient condition are satisfied as: (1) (General positioning condition of contact points) the set of contact points includes two different points in 2D case; (2) (Existence condition of non-zero internal force) there exists a set of contact forces such that the resultant force and moment are **0**, the friction condition is satisfied, and all of the finger forces are not **0** simultaneously. With respect to a frame Σ_o as shown in Fig.1, $\boldsymbol{f}_i \in \boldsymbol{R}^2$ is the applied force of ith finger, and its direction is inward the object. $e_{i1} \in \boldsymbol{R}^2$ and $e_{i2} \in \boldsymbol{R}^2$ are the edge vectors (inward the object) of the friction cone. k_{i1} and k_{i2} denote the magnitudes of the force $\boldsymbol{f}_i$ in e_{i1} and e_{i2} respectively. $\boldsymbol{f}_i$ can be described by the form

$$\boldsymbol{f}_i = k_{i1}e_{i1} + k_{i2}e_{i2}, \quad k_{i1}, k_{i2} \geq 0. \tag{1}$$

For a force closure grasp with $n(n \geq 2)$ fingers, the force equilibrium and moment equilibrium conditions

$$\sum_{i=1}^{n} \boldsymbol{f}_i = \sum_{i=1}^{n}(k_{i1}e_{i1}+k_{i2}e_{i2}) = 0 \in \boldsymbol{R}^2, \ k_{i1}, \ k_{i2} \geq 0, \tag{2}$$

$$\sum_{i=1}^{n} \boldsymbol{r}_i \otimes \boldsymbol{f}_i = \sum_{i=1}^{n} \boldsymbol{r}_i \otimes (k_{i1}e_{i1}+k_{i2}e_{i2}) = 0 \in \boldsymbol{R}^1, \ k_{i1}, \ k_{i2} \geq 0 \tag{3}$$

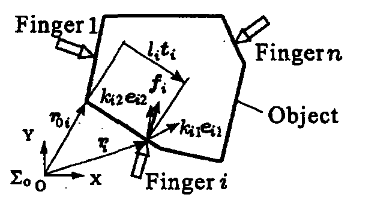

Figure 1: Fingertip forces and fingertip positions

must be satisfied, where $r_i \in R^2$ is the position vector of ith finger, all of k_{i1}, k_{i2} will not become 0 at the same time, and $\otimes$ denotes the cross product of vectors.

2.2 Selecting Edge Candidates

We select candidates for the successful grasp from all combinations of the object edges by using the force equilibrium condition eq.(2), that can be rewritten as

$$E_1 k = 0 \in R^2, \quad k \geq 0, \tag{4}$$

$$E_1 \triangleq [e_{11} \;\; e_{12} \;\; e_{21} \;\; e_{22} \; \cdots \; e_{n1} \;\; e_{n2}] \in R^{2\times 2n}, \tag{5}$$

$$k \triangleq [k_{11} \;\; k_{12} \;\; k_{21} \;\; k_{22} \; \cdots \; k_{n1} \;\; k_{n2}]^T \in R^{2n}. \tag{6}$$

The solution set of k in eq.(4) can be represented by a convex polyhedral cone [7], that is

$$k = H_1\alpha = [h_{11}, \; h_{12}, \; \cdots, \; h_{1m}]\alpha, \quad \alpha \geq 0, \tag{7}$$

$$\alpha \triangleq \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_m \end{bmatrix}^T \in R^m, \tag{8}$$

where $h_{1j}, j = 1, 2, \cdots, m$ are span vectors of the polyhedral cone. α is a coefficient vector representing the component of $h_{1j}, j = 1, 2, \cdots, m$. If k does not exist, the combination of edges can be excluded. If k exists, the combination of edges will be one of graspable candidates.

3 Graspable Finger Position Regions

3.1 Boundary Hyperplanes of Finger Position Region

The fingertip position vector on ith edge can be described as

$$r_i = r_{0i} + l_i t_i, \quad i = 1, 2, \cdots, n, \tag{9}$$

where $r_{0i} \in R^2$ is a vertex position vector of ith edge, $t_i \in R^2$ the direction vector of the edge, and l_i the position variable (see Fig.1). When the length of ith edge is L_i, the bound of l_i is $0 \leq l_i \leq L_i$.

For n edges touched by n fingers, let

$$l \triangleq [l_1 \;\; l_2 \;\; \cdots \;\; l_n]^T \in R^n \tag{10}$$

refer to a *Finger Position Vector*, whose bounds are

$$0 \leq l \leq L, \tag{11}$$

$$L \triangleq [L_1 \;\; L_2 \;\; \cdots \;\; L_n]^T \in R^n. \tag{12}$$

In this paper, the permissible region of l is called as *Graspable Finger Position Region* (GFPR hereafter) that meets the force closure grasp, the force and the moment equilibrium, and the edge length bounds for the stable grasp of a object.

Substituting eqs.(7) and (9) into eq.(3), the equation of variables l and α can be obtained as

$$(l^T A + b)k = (l^T A + b)H_1\alpha = 0, \quad \alpha \geq 0, \tag{13}$$

where A and b are denoted as

$$A \triangleq \begin{bmatrix} [t_1\otimes] & 0_{1\times 2} & \cdots & 0_{1\times 2} \\ 0_{1\times 2} & [t_2\otimes] & \cdots & 0_{1\times 2} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{1\times 2} & 0_{1\times 2} & \cdots & [t_n\otimes] \end{bmatrix} E_2 \in R^{n\times 2n}, \tag{14}$$

$$E_2 \triangleq \begin{bmatrix} e_{11} & e_{12} & 0_{2\times 1} & 0_{2\times 1} & \cdots & 0_{2\times 1} & 0_{2\times 1} \\ 0_{2\times 1} & 0_{2\times 1} & e_{21} & e_{22} & \cdots & 0_{2\times 1} & 0_{2\times 1} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{2\times 1} & 0_{2\times 1} & 0_{2\times 1} & 0_{2\times 1} & \cdots & e_{n1} & e_{n2} \end{bmatrix} \in R^{2n\times 2n}, \tag{15}$$

$$b \triangleq \begin{bmatrix} [r_{01}\otimes] & [r_{02}\otimes] & \cdots & [r_{0n}\otimes] \end{bmatrix} E_2 \in R^{1\times 2n}, \tag{16}$$

$$[t_i \otimes] = [-t_{iy} \;\; t_{ix}] \in R^{1\times 2}, \tag{17}$$

$$[r_{0i} \otimes] = [-r_{0iy} \;\; r_{0ix}] \in R^{1\times 2}. \tag{18}$$

Eq.(13) shows a nonlinear problem with respect to the variables l and α. To solve the problem linearly, we introduce boundary hyperplanes of l corresponding to the span vectors of polyhedral cone of eq.(7). Then, we derive an algorithm to determine the GFPR using the boundary hyperplanes.

Eq.(13) represents a hyperplane of l for a given α. From eq.(7), the solution set of k is a convex polyhedral expressed by m span vectors h_{1j}, $j = 1, 2, \cdots, m$. For one span vector h_{1j} where $\alpha_j = 1$, $\alpha_s = 0$, $s = 1, 2, \cdots, m$, $s \neq j$, we can obtain one hyperplane P_j. For m span vectors of k, we can obtain m hyperplanes

$$P_j = \{l | (l^T A + b)h_{1j} = 0\}, \quad j = 1, 2, \cdots, m. \tag{19}$$

Each P_j for span vector h_{1j} is a boundary hyperplane of the GFPR and divides the finger position configuration space R^n into two hemi-spaces

$$P_j^+ = \{l | (l^T A + b)h_{1j} \geq 0\}, \quad j = 1, 2, \cdots, m, \tag{20}$$

$$P_j^- = \{l | (l^T A + b)h_{1j} \leq 0\}, \quad j = 1, 2, \cdots, m. \tag{21}$$

3.2 GFPR Formed by Two Hyperplanes

According to eq.(13), the graspable finger position vector l corresponding two span vectors h_{1q} and h_{1r} of k exists in the following set (see Fig.2)

$$W_{qr} = \{l | (l^T A + b)h_{1q}\alpha_q + (l^T A + b)h_{1r}\alpha_r = 0, \; \alpha_q, \alpha_r \geq 0\}. \tag{22}$$

Eq.(22) shows that W_{qr} is the linear combination of $(l^T A + b)h_{1q}$ and $(l^T A + b)h_{1r}$ depending on α_q and α_r. Because of α_q, $\alpha_r \geq 0$, the set of l satisfying eq.(22) can be obtained by the following proposition.

Proposition 1: Corresponding to the region between the two span vectors h_q and h_r of k, the set W_{qr} can be represented using the set of U_{qr} between the boundary hyperplanes P_q and P_r, where U_{qr} is expressed as follows (which has been proved in [8])

$$U_{qr} = U_{qr}^1 \cup U_{qr}^2, \tag{23}$$

$$U_{qr}^1 = P_q^+ \cap P_r^-, \quad U_{qr}^2 = P_q^- \cap P_r^+. \tag{24}$$

From the length bounds of object edge, $0 \leq l \leq L$,

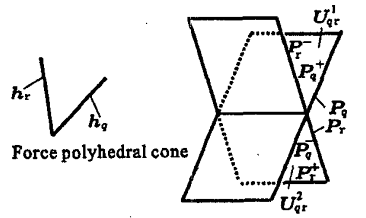

Figure 2: Force polyhedral cone and boundary hyperplanes

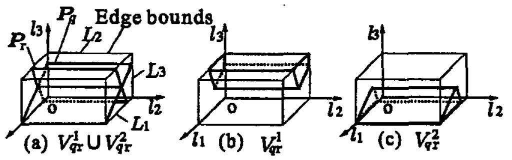

Figure 3: Two convex polyhedrons

U_{qr} will be divided into 2 convex polyhedrons(shown in Fig.3) as the following:

$$V^1_{qr}=\{l|(l^TA+b)h_{1q}\geq 0,\ (l^TA+b)h_{1r}\leq 0,\ 0\leq l\leq L\}, \tag{25}$$

$$V^2_{qr}=\{l|(l^TA+b)h_{1q}\leq 0,\ (l^TA+b)h_{1r}\geq 0,\ 0\leq l\leq L\}. \tag{26}$$

Thus, the GFPR can be represented by

$$V_{qr} = V^1_{qr} \cup V^2_{qr}, \tag{27}$$

and V^1_{qr} or V^2_{qr} can be represented such as

$$l = \sum_{d=1}^{D} l_{vd}\beta_d,\ \ \beta_d \geq 0,\ \ \sum_{d=1}^{D}\beta_d = 1, \tag{28}$$

where, l_{vd}, $d = 1, 2, \cdots, D$ are the vertexes, β_d is the barycentric coordinates. The GFPRs are located in the union set of the all of the V^1_{qr} and V^2_{qr}. Generally the union set is a polyhedron but not a convex.

3.3 GFPR Formed by m Hyperplanes

Now, we give a method for obtaining GFPR formed by m hyperplanes. According to eq.(13), for the set of k formed by m span vectors, the set of l can be represented by

$$W_l=\left\{ l| \sum_{j=1}^{m}(l^TA+b)h_{1j}\alpha_j=0,\ \alpha_j\geq 0,\ j=1,2,\cdots,m\right\}. \tag{29}$$

From m span vectors of k, m hyperplanes of $P_j, j = 1, 2, \cdots, m$ are obtained.

Proposition 2: For the set of k formed by m span vectors, the set W_l can be expressed by the union set U_l of all of the set U_{qr}

$$U_l = \bigcup_{q,r=1,q\neq r}^{m} U_{qr} = \bigcup_{q,r=1,q\neq r}^{m} (U^1_{qr} \cup U^2_{qr}), \tag{30}$$

where U_{qr} is formed by 2 arbitrary hyperplanes P_q and P_r, $q, r = 1, 2, \cdots, m,\ q \neq r$ (which has been proved in the literature [8]).

Moreover, by considering the length bounds of object edges, from proposition 2 and eq.(27), the GFPR can be expressed as

$$V_l = \bigcup_{q,r=1,q\neq r}^{m} (V^1_{qr} \cup V^2_{qr}). \tag{31}$$

4 Determining Stable GFPR

In the previous section, the GFPR has been obtained as an union set of convex polyhedrons, but is not a convex polyhedron. In general, the stabler grasp can be achieved at these finger configuration points where the distances between the points and each hyperplane of GFPR polyhedron are farther. If we can find the biggest inscribed hypersphere of the GFPR polyhedron, the grasp near to the center of the hypersphere will be stabler, when a few errors of the contact points on the actual object exist. In addition, if the volume of a convex polyhedron of GFPR containing the hypersphere is larger, the stable finger position region is larger.

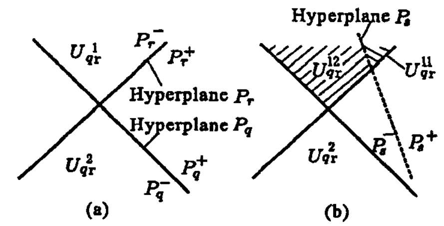

Figure 4: Convex set formed by 2 and 3 boundary hyperplanes

From what has been said above, for the obtained GFPR polyhedron, the *Stable GFPR* is defined as the convex polyhedron that contains the biggest inscribed hypersphere of the GFPR, and has larger volume.

4.1 Convex Polyhedron of Containing Biggest Hypersphere of GFPR

In fact, for a GFPR formed by m boundary hyperplanes, the convex polyhedrons in the GFPR are encircled by the plural boundary hyperplanes from 2 to m and object edge length bounds. In this paper, we give a proposition for considering what convex polyhedron the biggest inscribed hypersphere may lie in.

proposition 3: For the finger position set formed by m boundary hyperplanes, the convex set U^1_{qr} is enclosed by the 2 arbitrary boundary hyperplanes P^+_q and P^-_r, and the U^2_{qr} is enclosed by P^-_q and P^+_r. The convex sets, that are enclosed by plural boundary hyperplanes including P^+_q and P^-_r, belong to U^1_{qr}. The convex sets, that are enclosed by plural boundary hyperplanes including P^-_q and P^+_r, belong to U^2_{qr}.

proof At first, we consider the convex sets formed by 3 boundary hyperplanes. A hyperplane P_s is chosen from m hyperplanes arbitrarily. Because P_s does not parallel to P_q or P_r, the 2 convex sets that are enclosed by P^+_q, P^-_r and $P_s (q \neq r \neq s)$ are represented as

$$U^{11}_{qr} = (P^+_q \cap P^-_r \cap P^+_s) \subset (P^+_q \cap P^-_r) = U^1_{qr}, \tag{32}$$

$$U^{12}_{qr} = (P^+_q \cap P^-_r \cap P^-_s) \subset (P^+_q \cap P^-_r) = U^1_{qr}. \tag{33}$$

Similarly, the 2 convex sets that are enclosed by P^-_q, P^+_r and $P_s (q \neq r \neq s)$ are represented as

$$U^{21}_{qr} = (P^-_q \cap P^+_r \cap P^+_s) \subset (P^-_q \cap P^+_r) = U^2_{qr}, \tag{34}$$

$$U^{22}_{qr} = (P^-_q \cap P^+_r \cap P^-_s) \subset (P^-_q \cap P^+_r) = U^2_{qr}. \tag{35}$$

From eqs.(32)~(35), we have

$$U^{11}_{qr} \subset U^1_{qr},\ \ U^{12}_{qr} \subset U^1_{qr},\ \ U^{21}_{qr} \subset U^2_{qr},\ \ U^{22}_{qr} \subset U^2_{qr}. \tag{36}$$

Thus, the convex sets enclosed by P^+_q, P^-_r and P_s $(q \neq r \neq s)$ belong to the convex set U^1_{qr} enclosed by P^+_q, P^-_r. The convex sets enclosed by P^-_q, P^+_r and P_s $(q \neq r \neq s)$ belong to the convex set U^2_{qr} enclosed by P^-_q, P^+_r.

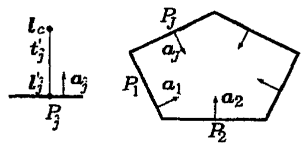

Figure 5: Distance from point to hyperplane

By the same way, we can also draw the conclusion that the convex sets enclosed by plural hyperplanes including P_q^+, P_r^- (or P_q^-, P_r^+) belong to the convex sets U_{qr}^1 (or U_{qr}^2). □

From proposition 3, taking the object edge length limits $0 \le l \le L$ into account, for the GFPR formed by m hyperplanes, the convex sets that are encircled by more than 2 hyperplanes including P_q, P_r, and object edge length bounds, belong to the convex polyhedrons V_{qr}^1 or V_{qr}^2 ($q,\ r = 1,\ 2, \cdots,\ m,\ q \ne r$).

Therefore, the biggest inscribed hypersphere of the convex polyhedron, that is enclosed by more than 2 hyperplanes including P_q, P_r and edge length bounds, belongs to the convex polyhedron V_{qr}^1 or V_{qr}^2. It can be seen that the biggest inscribed hypersphere of the GFPR lies in the convex polyhedrons V_{qr}^1 or V_{qr}^2 ($q,\ r = 1,\ 2, \cdots, m,\ q \ne r$).

4.2 Distances from One Point to Hyperplanes of Convex Polyhedron

The radius of an inscribed hypersphere is the distance from the hypersphere center to the tangent hyperplane of the convex polyhedron. To find the biggest inscribed hypersphere of a convex polyhedron, we consider the distances from one point to the hyperplanes.

As shown in Fig.5, each convex polyhedron V_{qr}^1 or V_{qr}^2 of GFPR is formed by the boundary hyperplanes

$$(l^TA + b)h_{1j} = 0,\quad j = q, r,\quad q \ne r,\quad q, r = 1, 2, \cdots, m, \tag{37}$$

and the edge bound hyperplanes

$$l_i = 0,\quad l_i = L_i,\quad i = 1, 2, \cdots, n. \tag{38}$$

We can rewrite these hyperplanes as

$$(a_{\hat{j}},\ l) = \hat{b}_{\hat{j}},\quad \hat{j} = 1, 2, \cdot, J, \tag{39}$$

where, J is the number of the hyperplanes forming the convex polyhedron. $a_{\hat{j}}$ is the normal vector (inward the convex polyhedron shown in Fig.5) of the hyperplane $P_{\hat{j}}$. $a_{\hat{j}}$ and $\hat{b}_{\hat{j}}$ for eq.(37) can be obtained as

$$a_{\hat{j}} = Ah_{1j},\quad \hat{b}_{\hat{j}} = -bh_{1j},\quad \text{if}\quad (l^TA + b)h_{1j} \ge 0, \tag{40}$$

$$a_{\hat{j}} = -Ah_{1j},\quad \hat{b}_{\hat{j}} = bh_{1j},\quad \text{if}\quad (l^TA + b)h_{1j} \le 0, \tag{41}$$

for eq.(38), $a_{\hat{j}}$ and $\hat{b}_{\hat{j}}$ can be obtained as

$$a_{\hat{j}} = [\underbrace{0\ \cdots\ 0\ 1}_{\hat{j}}\ 0 \cdots 0]^T \in R^{n\times 1};\quad \hat{b}_{\hat{j}} = 0,\ \text{if}\ l_i = 0, \tag{42}$$

$$a_{\hat{j}} = [\underbrace{0\ \cdots\ 0\ -1}_{\hat{j}}\ 0 \cdots 0]^T \in R^{n\times 1};\quad \hat{b}_{\hat{j}} = -L_{\hat{j}},\ \text{if}\ l_i = L_i \tag{43}$$

As shown in Fig.5, $l'_{\hat{j}}$ is the intersection of the hyperplane $P_{\hat{j}}$ and the normal line $l_c l'_{\hat{j}}$ of $P_{\hat{j}}$. $t'_{\hat{j}}$ is the distance from the point l_c to $P_{\hat{j}}$. We have

$$l_c - l'_{\hat{j}} = t'_{\hat{j}} a_{\hat{j}}. \tag{44}$$

Since the point $l'_{\hat{j}}$ is on $P_{\hat{j}}$, from eq.(39) we have

$$(a_{\hat{j}},\ l'_{\hat{j}}) = \hat{b}_{\hat{j}}. \tag{45}$$

Eqs.(44) and (45) give the distance $t'_{\hat{j}}$ as

$$t'_{\hat{j}} = (a_{\hat{j}},\ l_c) - \hat{b}_{\hat{j}},\quad \hat{j} = 1, 2, \cdots, J. \tag{46}$$

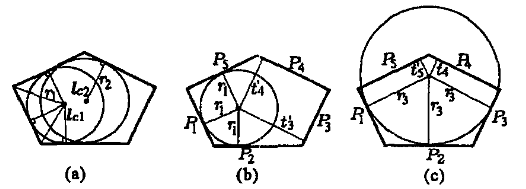

Figure 6: Inscribed hyperspheres of 2D polyhedron

4.3 Biggest Inscribed Hypersphere of Convex Polyhedron

Let us consider the biggest inscribed hypersphere for each convex polyhedron V_{qr}^1 or V_{qr}^2 ($q,\ r = 1, 2, \cdots, m,\ q \ne r$). Then, we will determine the biggest inscribed hypersphere of GFPR.

As shown in Fig.6(a), for a 2D polyhedron, the center of an inscribed hypersphere is determined by 2 bisectors of 2 interior angles formed by 3 hyperplanes. In the same way, for an nD polyhedron, the center of an inscribed hypersphere is determined by n bisectors of n interior angles formed by $(n+1)$ hyperplanes.

Now, we consider the method for determining the inscribed hyperspheres of an nD convex polyhedron formed by $J(J \ge (n+1))$ hyperplanes. At first, $(n+1)$ hyperplanes are chosen arbitrarily from J hyperplanes. The inscribed hypersphere for these $(n+1)$ hyperplanes can be solved. Since the distances from the center of the inscribed hypersphere to the tangent hyperplanes are radiuses and meet

$$t'_1 = t'_2 = \cdots\cdots = t'_{(n+1)} = r_c. \tag{47}$$

From eq.(46), we rewrite the above equation as

$$(a_1,\ l_c) - \hat{b}_1 = (a_2,\ l_c) - \hat{b}_2, \tag{48}$$

$$(a_1,\ l_c) - \hat{b}_1 = (a_3,\ l_c) - \hat{b}_3, \tag{49}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots,$$

$$(a_1,\ l_c) - \hat{b}_1 = (a_{(n+1)},\ l_c) - \hat{b}_{(n+1)}. \tag{50}$$

The inscribed hypersphere satisfying eqs.(48) $\sim$ (50) is one hypersphere inscribed the $(n+1)$ hyperplanes chosen from J hyperplanes of the nD convex polyhedron. But it may not be an inscribed hypersphere of the convex polyhedron (reference Fig.6(c)).

Then, according to eq.(46), the coefficient of the distance $t'_{\hat{j}}$ from l_c to hyperplanes forming convex polyhedron and radius r_c can be obtained. If

$$r_c \le t'_{\hat{j}},\quad \hat{j} = 1, 2, \cdots, J \tag{51}$$

is met, the hypersphere is one of inscribed hyperspheres of the convex polyhedron (see Fig.6(b)).

For a convex polyhedron V_{qr}^1 or V_{qr}^2 formed by J hyperplanes, the number of inscribed hyperspheres for hyperplane combinations is ${}_JC_{(n+1)}$. If the number of the hyperspheres satisfying eqs.(48)$\sim$(51) is N, the radius of the biggest inscribed hypersphere is given as

$$r_{c\max} = \max\{r_1,\ r_2,\ \cdots, r_N\}, \tag{52}$$

where, $r_1,\ r_2,\ \cdots, r_N$ represent the radiuses of the inscribed hyperspheres for hyperplane combinations.

Furthermore, for the GFPR polyhedron, the biggest inscribed hyperspheres for V_{qr}^1 and $V_{qr}^2 (q,\ r = 1,\ 2,\ \cdots, m,\ q \neq r)$ can be obtained. If the radiuses of these hyperspheres are given as $r_{c\max 1},\ r_{c\max 2},\ \cdots, r_{c\max \bar{N}}$, the radius of the biggest inscribed hypersphere for GFPR is

$$r_{c\max} = \max\{r_{c\max 1},\ r_{c\max 2},\ \cdots, r_{c\max \bar{N}}\}. \tag{53}$$

The convex polyhedron containing the above hypersphere will be selected. If plural convex polyhedrons containing the biggest hypersphere with the same radius exist, it is need to compare their volumes further.

4.4 Volume of Convex Polyhedron

The volumes of convex polyhedrons can be obtained by the method expressed in [9].

To solve the volume for an nD convex polyhedron, at first, we must subdivide the nD convex polyhedron into simplexes, each of which can be given by its vertexes. For example, an nD simplex is given by the its vertexes as $l_{v1},\ l_{v2},\ \cdots, l_{v(n+1)}$. Then, its volume $\overline{V}_s$ is

$$\overline{V}_s = \frac{1}{n!}\,\mathrm{abs}\left(\begin{vmatrix} 1 & 1 & \cdots & 1 \\ l_{v1} & l_{v2} & \cdots & l_{v(n+1)} \end{vmatrix}\right). \tag{54}$$

If a nD convex polyhedron is subdivided into S simplexes, the volume of the polyhedron can be given as

$$\overline{V} = \sum_{s=1}^{S} \overline{V}_s. \tag{55}$$

5 Numerical Example

We give a numerical example using the proposed approach to determine the Stable GFPR with 3 fingers.

For the object shown in Fig.7, the vertex positions of the object with respect to Σ_o are:

$$r_{01}=\begin{bmatrix}2.000\\7.000\end{bmatrix},\ r_{02}=\begin{bmatrix}2.000\\2.000\end{bmatrix},\ r_{03}=\begin{bmatrix}13.000\\2.000\end{bmatrix},\ r_{04}=\begin{bmatrix}8.000\\7.000\end{bmatrix}.$$

The direction vectors of the edges are:

$$t_1=\begin{bmatrix}0.000\\-1.000\end{bmatrix},\ t_2=\begin{bmatrix}1.000\\0.000\end{bmatrix},\ t_3=\begin{bmatrix}-0.707\\0.707\end{bmatrix},\ t_4=\begin{bmatrix}-1.000\\0.000\end{bmatrix}.$$

The lengths of the edges are:

$$L_1 = 5.000,\quad L_2 = 11.000,\quad L_3 = 7.071,\quad L_4 = 6.000.$$

The coefficient of friction between the object and fingers is set as 0.5, so that we have

$$e_{11}=\begin{bmatrix}0.894\\0.447\end{bmatrix},\ e_{12}=\begin{bmatrix}0.894\\-0.447\end{bmatrix},\ e_{21}=\begin{bmatrix}-0.447\\0.894\end{bmatrix},\ e_{22}=\begin{bmatrix}0.447\\0.894\end{bmatrix},$$

$$e_{31}=\begin{bmatrix}-0.316\\-0.949\end{bmatrix},\ e_{32}=\begin{bmatrix}-0.949\\-0.316\end{bmatrix},\ e_{41}=\begin{bmatrix}0.447\\-0.894\end{bmatrix},\ e_{42}=\begin{bmatrix}-0.447\\-0.894\end{bmatrix}.$$

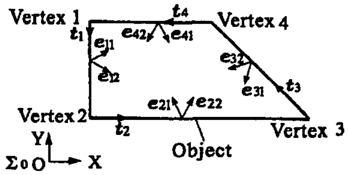

Figure 7: Polygonal object of example

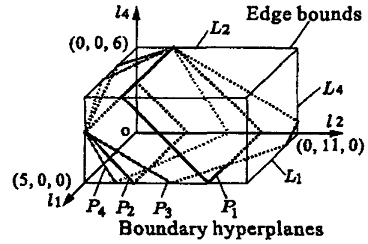

Figure 8: Boundary hyperplanes and edge bounds of 3 finger grasp

5.1 Determining the GFPR

The number of the edge combinations are ${}_4C_3 = 4$ when the object is grasped by a robot hand with 3 fingers. According to eq.(7), all the edge combinations are the candidates. For example, for the edge combination 1-2-4, its $\boldsymbol{k}$ is given as

$$\boldsymbol{k} = \begin{bmatrix}k_{11}\\k_{12}\\k_{21}\\k_{22}\\k_{41}\\k_{42}\end{bmatrix} = \begin{bmatrix}0.000 & 0.000 & 0.566 & 0.000\\0.000 & 0.000 & 0.000 & 0.566\\0.707 & 0.000 & 0.424 & 0.707\\0.000 & 0.707 & 0.000 & 0.000\\0.707 & 0.000 & 0.000 & 0.000\\0.000 & 0.707 & 0.707 & 0.424\end{bmatrix}\alpha,\ \alpha \geq 0, \tag{56}$$

$$\alpha = \begin{bmatrix}\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4\end{bmatrix}^T. \tag{57}$$

Corresponding to the 4 span vectors of $\boldsymbol{k}$, 4 boundary hyperplanes are obtained from eq.(19) as

$$P_1 = \{\, l \mid [0.000\ 0.707\ 0.707]l - 6.010 = 0 \},\tag{58}$$

$$P_2 = \{\, l \mid [0.000\ 0.707\ 0.707]l - 2.475 = 0 \},\tag{59}$$

$$P_3 = \{\, l \mid [0.566\ 0.424\ 0.707]l - 5.303 = 0 \},\tag{60}$$

$$P_4 = \{\, l \mid [0.506\ 0.707\ 0.424]l - 4.313 = 0 \},\tag{61}$$

where $l = [l_1\ \ l_2\ \ l_4]^T$ and shown in Fig.8. Taking into account $0 \leq l_i \leq L_i,\ i = 1, 2, 4$, and according to proposition 2 as well as eqs.(27) and (31), we can obtain convex polyhedrons V_{qr}^1 and V_{qr}^2, $(q, r = 1, 2, 3, 4,\ q \neq r)$, from each of both hyperplane combinations, which are

$$V_{12}^1 = \phi, \tag{62}$$

$$V_{12}^2 = \{\, l \mid l = \begin{bmatrix}0.000 & 0.000 & 5.000 & 0.000 & 5.000\\3.500 & 8.500 & 8.500 & 2.500 & 2.500\\0.000 & 0.000 & 0.000 & 6.000 & 6.000\end{bmatrix}$$

$$\begin{bmatrix}5.000 & 0.000 & 5.000 & 0.000 & 5.000\\3.500 & 0.000 & 0.000 & 0.000 & 0.000\\0.000 & 3.500 & 3.500 & 6.000 & 6.000\end{bmatrix}\begin{bmatrix}\beta_1\\ \vdots \\ \beta_{10}\end{bmatrix},$$

$$\beta_1, \cdots, \beta_{10} \geq 0,\quad \beta_1 + \cdots + \beta_{10} = 1\}, \tag{63}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots,$$

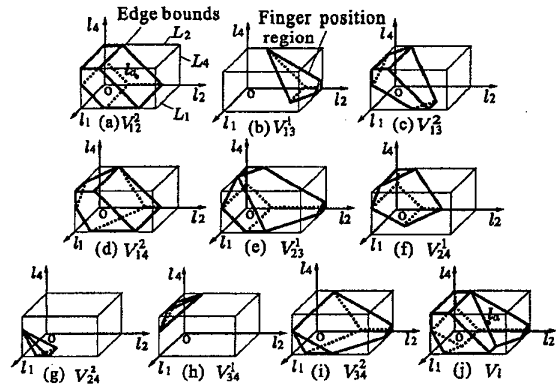

Figure 9: Graspable finger position regions for 3 fingers

$$V_{34}^2 = \{\, l \mid l = \begin{bmatrix} 0.000 & 5.000 & 1.125 & 0.000 \\ 2.500 & 0.000 & 11.000 & 6.098 \\ 6.000 & 3.500 & 0.000 & 0.000 \end{bmatrix}$$

$$\begin{bmatrix} 5.000 & 5.000 & 0.000 & 0.000 \\ 2.099 & 5.833 & 11.000 & 11.000 \\ 0.000 & 0.000 & 0.900 & 0.000 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_8 \end{bmatrix},$$

$$\beta_1, \cdots, \beta_8 \geq 0, \quad \beta_1 + \cdots + \beta_8 = 1\}. \tag{64}$$

The GFPR V_l therefore has the following form

$$V_l = \bigcup_{q,r=1,q\neq r}^{4} (V_{qr}^1 \cup V_{qr}^2). \tag{65}$$

The regions of V_{qr}^1 and V_{qr}^2 $(q, r = 1, 2, 3, 4,\ q \neq r)$ and V_l are illustrated in Fig.9, where V_{qr}^1 and V_{qr}^2, $(q, r = 1, 2, 3, 4,\ q \neq r)$ are convex polyhedrons respectively, while the union set V_l is a polyhedron but not a convex polyhedron.

5.2 Determining the Stable GFPR

For the convex polyhedrons V_{qr}^1 and V_{qr}^2 $(q, r = 1, 2, 3, 4,\ q \neq r)$, the biggest inscribed hypersphere and volume of each polyhedron can be obtained based on the computing result. Let us see 3 bigger convex polyhedrons having bigger hypersphere shown in Fig.10.

For the convex polyhedron V_{12}^2, the radius of the biggest inscribed hypersphere and volume $\overline{V}_{12}^2$ are

$$r_{c1} = 1.768, \quad \overline{V}_{12}^2 = 134.375. \tag{66}$$

For the convex polyhedron V_{14}^2, the radius r_{c2} of the biggest inscribed hypersphere and volume $\overline{V}_{14}^2$ are

$$r_{c2} = 1.589, \quad \overline{V}_{14}^2 = 94.833. \tag{67}$$

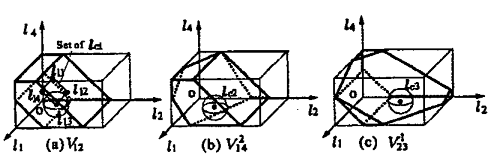

Figure 10: Inscribed hyperspheres of convex polyhedrons for edges 1-2-4

For V_{23}^1, the radius r_{c3} of the biggest inscribed hypersphere and volume $\overline{V}_{23}^1$ are

$$r_{c3} = 1.559, \quad \overline{V}_{23}^1 = 99.532. \tag{68}$$

It can be seen that the radius value r_{c1} is the biggest. Therefore, the convex polyhedron V_{12}^2 is the Stable GFPR for the edge candidate 1-2-4. If the radiuses for plural convex polyhedrons are the same, furthermore, the volumes of them are compared.

6 Conclusion

We presented an analytical approach to plan the stable Graspable Finger Position Region (GFPR). At first, we selected graspable candidates from all of the combinations of the object edges using the force equilibrium condition. Then, for a selected candidate, the regions of graspable finger position was analyzed by using the moment equilibrium condition. It was shown that the region is bounded by several boundary hyperplanes. With the combining these boundary hyperplanes, two propositions were proposed in order to obtain the GFPR exactly. The region was a polyhedron but not a convex polyhedron. Furthermore, the Stable GFPR was given by comparing the biggest inscribed hyperspheres and the volumes of the convex polyhedrons contained in the GFPR. The center part of the the biggest inscribed hypersphere is the stable finger position region. Lastly, numerical example were performed to show the effectiveness of the proposed approach.

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