

## Analysis of Finger Position Regions on Grasped Object with Multifingered Hand

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### Abstract

In this paper, an analytical approach for obtaining finger position regions of an object with multifingered hand is proposed. The successful grasp of the object can be achieved if the resultant force and resultant moment acting on the object are zero. At first, a method that can be used to determine which combination of the object edges touched by fingers is possible to be used for grasping, is given by using the force equilibrium condition. Then, Graspable Finger Position Region on an edge combination is defined where the object can be held successfully and that the region is bounded by plural boundary hyperplanes is shown. With the combining of these boundary hyperplanes, an algorithm for obtaining the Graspable Finger Position Region is proposed. Lastly, computing examples are performed to verify the effectiveness for the proposed approach.

### 1 Introduction

The grasp of an object by a robot hand is a primitive but very important subtask in automatic systems. Advanced applications sometimes require multiple robotic fingers to perform a task coordinately. If a multifingered robot hand is used to achieve greater generality, a procedure for computationally synthesizing successful grasps is required. Therefore, for a given object, the determining of the object grasp finger position regions is a fundamental and important question.

In literatures, finger position and finger force on an object were treated as variables simultaneously. The problem of solving the object grasp finger position regions was dealt with as nonlinear problem. The multifingered finger positions are computed generally by the method of scanning every sample points selected on the object, but this method is complicated and the computing spends too long time.

The studies on two, three, four finger's grasping have been researched. In [1] an algorithm is represented for directly constructing force-closure grasp based on the shape of the grasped object. The synthesis of force-closure grasps was used in to find independent regions of contact for the fingertips. In [2] an analytical method is represented to find an optimum force closure grasp of a planar polygon using a three-fingered robot hand. In [3] it has been shown that form-closure of a polygonal object with four fingers can be achieved. In [4] maximal object segments are obtained where fingers can be positioned independently while force-closure is found by linear optimization within the grasp finger position regions. In [5] Omata proposed an algorithm for approximately computing the positions of fingertips maintaining equilibrium when they grasp a polyhedral object.

About finger position regions for multifingered hand, many attempts have been made by analytical method

But most of these studies lay stress on the problem of grasp force analysis. The object grasp regions with the finger position has not been solved yet.

The purpose of this paper is to exactly determine the finger position regions of a given object. In this study, at first, we distinguish candidates from combinations of object edges. The forces and the moments exerted by the fingers are equilibrium if the grasp is successful. We use the force equilibrium condition to select candidates grasped successfully from combinations of object edges. This is the necessary condition for the successful grasp. Moreover, we use the analysis method to solve the problem of the object grasp finger position regions. For a candidate of the successful grasp edges, the regions of successful grasp finger position are analyzed by using the moment equilibrium condition. Therefore, the computational load can be reduced.

### 2 Force and Moment Equilibrium Conditions

The following discussion is based on two assumptions: (1) The object is a 2D polygon with definite geometric shape; (2) The fingertips of a robot hand will touch the object by point contact with Coulomb friction and the contact point number of fingertips on each edge is not more than one.

As shown in Fig 1, with respect to coordinates  $\Sigma_0$ ,  $f_i \in \mathbb{R}^2$  is the applied force of finger  $i$ , the direction of  $f_i$  is toward the object.  $e_{i1} \in \mathbb{R}^2$  and  $e_{i2} \in \mathbb{R}^2$  are the edge vectors (toward the object) of the friction cone.  $k_{i1}, k_{i2}$  are the components of the force  $f_i$  in  $e_{i1}, e_{i2}$  respectively. Therefore,  $f_i$  can be described by

$$f_i = k_{i1}e_{i1} + k_{i2}e_{i2}, \quad k_{i1}, k_{i2} \geq 0 \quad (1)$$

For a successful grasp with  $n$  fingers, the force equilibrium and moment equilibrium are required:

$$\sum_{i=1}^n f_i = \sum_{i=1}^n (k_{i1}e_{i1} + k_{i2}e_{i2}) = 0 \in \mathbb{R}^2, \quad k_{i1}, k_{i2} \geq 0, \quad (2)$$

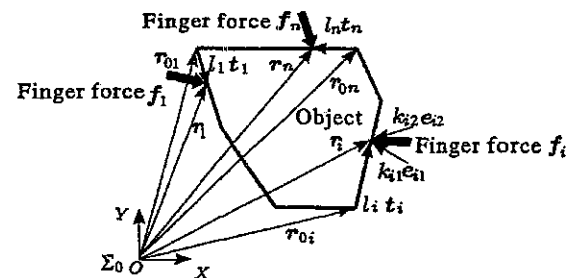


Figure 1: Fingertip forces and fingertip positions

$$\sum_{i=1}^n \mathbf{r}_i \times \mathbf{f}_i = \sum_{i=1}^n \mathbf{r}_i \times (k_{i1} \mathbf{e}_{i1} + k_{i2} \mathbf{e}_{i2}) = \mathbf{0} \in \mathbf{R}^1, \quad k_{i1}, k_{i2} \geq 0. \quad (3)$$

where  $\mathbf{r}_i \in \mathbf{R}^2$  is the position vector of finger  $i$ .

Next, to determine the grasp finger position regions for a given object, the force equilibrium condition is used firstly to select which combination of the object edges is suitable for grasping.

### 3 Selecting the Edge Candidates

When the grasp finger position regions are computed by the scanning method, it is necessary to compute all the sample points selected on every edge. In this section, we consider to select candidates of edges for the successful grasp from combinations of the object edges using the force equilibrium condition. Consequently, the grasp finger position regions can be computed only for the selected edge candidates by using the moment equilibrium condition.

The force equilibrium condition eq.(2) of  $n$  fingers can be expressed as

$$\mathbf{E}_1 \mathbf{k} = \mathbf{0} \in \mathbf{R}^2, \quad \mathbf{k} \geq \mathbf{0}, \quad (4)$$

$$\mathbf{E}_1 \triangleq [\mathbf{e}_{11} \ \mathbf{e}_{12} \ \mathbf{e}_{21} \ \mathbf{e}_{22} \ \cdots \ \mathbf{e}_{n1} \ \mathbf{e}_{n2}] \in \mathbf{R}^{2 \times 2n}, \quad (5)$$

$$\mathbf{k} \triangleq [k_{11} \ k_{12} \ k_{21} \ k_{22} \ \cdots \ k_{n1} \ k_{n2}]^T \in \mathbf{R}^{2n}. \quad (6)$$

In order to select candidates of graspable edges, we solve the  $\mathbf{k}$  in eq.(4). The set of  $\mathbf{k}$  is a convex polyhedral cone [10] and can be given by

$$\mathbf{k} = \mathbf{H} \boldsymbol{\alpha} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_J] \boldsymbol{\alpha}, \quad \boldsymbol{\alpha} \geq \mathbf{0}, \quad (7)$$

$$\boldsymbol{\alpha} \triangleq [\alpha_1 \ \alpha_2 \ \cdots \ \alpha_J]^T \in \mathbf{R}^J, \quad (8)$$

where,  $\mathbf{h}_j$ ,  $j = 1, 2, \cdots, J$  are span vectors of the polyhedral cone.  $\boldsymbol{\alpha}$  is a coefficient vector for representing the magnitude in  $\mathbf{h}_j$ . Note that, the obtained  $\mathbf{k}$  represents the possible solution set where some  $\mathbf{k}$  is not permissible since the moment equilibrium condition is not considered.

If a robot hand with  $n$  fingers is used to grasp an object that has  $d$  edges, the number of the combinations of the edges is  ${}_n C_d$ . For each combination of edges,  $\mathbf{k}$  is computed by eq.(4). If the  $\mathbf{k}$  does not exist, the combination of edges can be excluded. If the  $\mathbf{k}$  exists, the combination of edges will be graspable candidate. In the next section, The grasp finger position regions will be determined based on the edge candidates by using moment equilibrium condition.

### 4 Determining the Finger Position Regions

When  $n$  fingers are used to grasp an object, the fingertips are located on  $n$  edges of the object respectively. As it is shown by Fig.1, the fingertip position vector on  $i$ th edges with respect to  $\Sigma_o$  can be described as

$$\mathbf{r}_i = \mathbf{r}_{0i} + l_i \mathbf{t}_i, \quad i = 1, 2, \cdots, n, \quad (9)$$

where,  $\mathbf{r}_{0i} \in \mathbf{R}^2$  is a vertex position vector of the edge where  $i$ th finger acts on,  $\mathbf{t}_i \in \mathbf{R}^2$  is  $i$ th edge vector from the vertex,  $l_i$  is a position variable of  $i$ th finger contact

point. When the length of  $i$ th edge is  $L_i$ , the limit of  $l_i$  is  $0 < l_i < L_i$ .

For  $n$  edges touched by  $n$  fingers, let

$$\mathbf{l} \triangleq [l_1 \ l_2 \ \cdots \ l_n]^T \in \mathbf{R}^n \quad (10)$$

refer to a *Finger Position Vector*, its limit is

$$\mathbf{0} \leq \mathbf{l} \leq \mathbf{L}, \quad (11)$$

$$\mathbf{L} \triangleq [L_1 \ L_2 \ \cdots \ L_n]^T \in \mathbf{R}^n. \quad (12)$$

In this paper, the permissible regions of  $\mathbf{l}$  is defined as *Graspable Finger Position Regions* that meet the force equilibrium, the moment equilibrium and the edge length limits, for grasping an object stably.

Substituting eq.(7) and (9) into eq.(3), the equation on variables  $\mathbf{l}$  and  $\boldsymbol{\alpha}$  can be obtained as

$$\mathbf{l}^T \mathbf{A} + \mathbf{b} \boldsymbol{\alpha} = \mathbf{0}, \quad \boldsymbol{\alpha} \geq \mathbf{0}, \quad (13)$$

where,  $\mathbf{A}$  and  $\mathbf{b}$  are denoted as

$$\mathbf{A} \triangleq \begin{bmatrix} [\mathbf{t}_1 \otimes] & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & [\mathbf{t}_2 \otimes] & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & [\mathbf{t}_n \otimes] \end{bmatrix} \mathbf{E}_2 \mathbf{H} \in \mathbf{R}^{n \times J}, \quad (14)$$

$$\mathbf{E}_2 \triangleq \begin{bmatrix} \mathbf{e}_{11} & \mathbf{e}_{12} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{e}_{21} & \mathbf{e}_{22} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{e}_{n1} & \mathbf{e}_{n2} \end{bmatrix} \in \mathbf{R}^{2n \times 2n}, \quad (15)$$

$$\mathbf{b} \triangleq [b_1 \ b_2 \ \cdots \ b_J] \\ = [[\mathbf{r}_{01} \otimes] \ [\mathbf{r}_{02} \otimes] \ \cdots \ [\mathbf{r}_{0n} \otimes]] \mathbf{E}_2 \mathbf{H} \in \mathbf{R}^{1 \times J}, \quad (16)$$

$$[\mathbf{t}_i \otimes] = [-t_{iy} \ t_{ix}] \in \mathbf{R}^{1 \times 2}, \quad (17)$$

$$[\mathbf{r}_{0i} \otimes] = [-r_{0iy} \ r_{0ix}] \in \mathbf{R}^{1 \times 2}. \quad (18)$$

From eq.(13), we know that determining  $\mathbf{l}$  and  $\boldsymbol{\alpha}$  simultaneously is a nonlinear problem. Corresponding to the span vectors of polyhedral cone  $\mathbf{k}$ , we introduce boundary hyperplanes of  $\mathbf{l}$ . Then, we derive an algorithm to determine grasp finger position regions using the boundary hyperplanes.

Analyzing the eq.(13), one can be find that eq.(13) represents a hyperplane of  $\mathbf{l}$  for a given  $\boldsymbol{\alpha}$ . For one span vector  $\mathbf{h}_j$  by setting  $\alpha_j = 1$ ,  $\alpha_i = 0$ , ( $i \neq j$ ), we can obtain one hyperplane  $p_j$ . For  $J$  span vectors of  $\mathbf{k}$ , we can obtain  $J$  hyperplanes of  $\mathbf{l}$  correspondingly,

$$\mathbf{l}^T \mathbf{A}_j + b_j = 0, \quad j = 1, 2, \cdots, J. \quad (19)$$

These hyperplanes are boundary hyperplanes of the grasp finger position regions. As shown in Fig.2, for the two span vectors of  $\mathbf{k}$ , for example  $\mathbf{h}_q$  and  $\mathbf{h}_r$ , the two boundary hyperplanes  $p_q$  and  $p_r$  can be obtained. When a  $\mathbf{k}$  exists between  $\mathbf{h}_q$  and  $\mathbf{h}_r$ , the corresponding  $\mathbf{l}$  exists between  $p_q$  and  $p_r$ . Therefore, corresponding to the region between the two span vectors  $\mathbf{h}_q$  and  $\mathbf{h}_r$  of the polyhedral cone, the finger positions of successful grasps are located in the region between  $p_q$  and  $p_r$ . The hyperplanes  $p_q$  and  $p_r$  can be represented as follows:

$$\mathbf{l}^T \mathbf{A}_q + b_q = 0, \quad (20)$$

$$\mathbf{l}^T \mathbf{A}_r + b_r = 0, \quad (21)$$

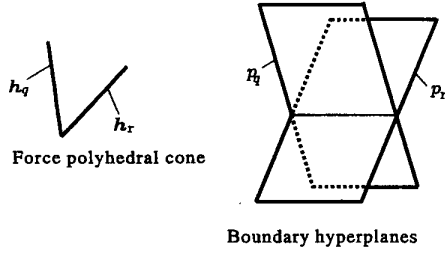


Figure 2: Force polyhedral cone and boundary hyperplanes

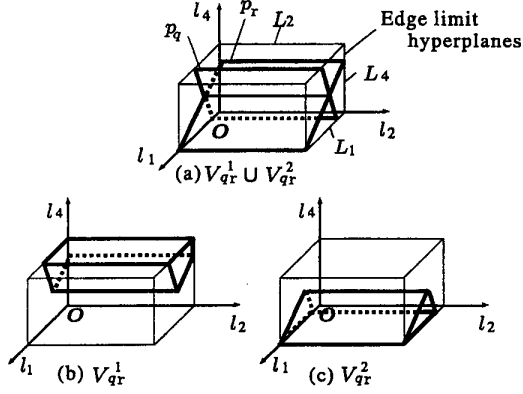


Figure 3: Boundary hyperplanes and edge length limit hyperplanes

As shown in Fig.3, the length limit  $0 \leq l \leq L$  of edges are presented as the hyperplanes forming the rectangular parallelepiped in the case of three fingers. When the length limit condition is considered, the graspable finger position region between the two boundary hyperplanes will be cut by the hyperplanes of the edge limit  $l$ . Namely, The graspable finger position region will be determined by the boundary hyperplanes and the length limit hyperplanes.

Generally, the region bounded by two boundary hyperplanes is divided into two convex polyhedrons by an intersection line of the hyperplanes  $p_q$  and  $p_r$ . The two polyhedrons  $V_{qr}^1$  and  $V_{qr}^2$  can be represented by

$$V_{qr}^1 = \{l | l^T A_q + b_q \geq 0, l^T A_r + b_r \leq 0, 0 \leq l \leq L\} \quad (22)$$

$$q, r = 1, 2, \dots, J, \quad q \neq r,$$

$$V_{qr}^2 = \{l | l^T A_q + b_q \leq 0, l^T A_r + b_r \geq 0, 0 \leq l \leq L\} \quad (23)$$

$$q, r = 1, 2, \dots, J, \quad q \neq r.$$

The finger positions of a stable grasp are located elementarily in one region enclosed by two hyperplanes. For  $J$  boundary hyperplanes, the grasp finger position regions are located in the union set of the all of the both hyperplane's combinations. Therefore, the region of  $l$  is given as the following from

$$V_l = \{l | \bigcup_{q,r=1,q \neq r}^J (V_{qr}^1 \cup V_{qr}^2)\}. \quad (24)$$

$V_l$  is the graspable finger position region on a given object. An arbitrary element of  $V_l$ , is one combination of successful grasp finger positions.

## 5 Determining Grasp Finger Forces

According to eq.(1) and eq.(7), the finger forces of  $n$  fingers can be represented by the equations

$$F = E_2 k = E_2 H \alpha \in \mathbb{R}^{2n}, \quad \alpha \geq 0, \quad (25)$$

$$F \triangleq [f_1 \ f_2 \ \dots \ f_n]^T, \quad (26)$$

where,  $H$  had been obtained by eq.(7). Note that, the obtained  $F$  represents the possible region where some  $F$  is not permissible since the moment equilibrium condition is not considered.

Substituting an  $l$  of  $V_l$  into eq.(13), the  $\alpha$  can be obtained that meet the moment equilibrium. Then substituting the obtained  $\alpha$  into eq.(25), corresponding possible finger forces can be obtained.

In the following section, we will give numerical examples using this proposed approach to determine the grasp finger position regions

## 6 Implementation

For a polygon object shown as Fig.4, we plan to determine its grasp finger position regions with 2 or 3 fingers respectively.

With respect to  $\Sigma_o$ , the vertex positions of the object are:

$$r_{o1} = \begin{bmatrix} 2.000 \\ 7.000 \end{bmatrix}, \quad r_{o2} = \begin{bmatrix} 2.000 \\ 2.000 \end{bmatrix}, \quad r_{o3} = \begin{bmatrix} 13.000 \\ 2.000 \end{bmatrix}, \quad r_{o4} = \begin{bmatrix} 8.000 \\ 7.000 \end{bmatrix}.$$

The direction vectors of the edges are:

$$t_1 = \begin{bmatrix} 0.000 \\ -1.000 \end{bmatrix}, \quad t_2 = \begin{bmatrix} 1.000 \\ 0.000 \end{bmatrix}, \quad t_3 = \begin{bmatrix} -0.707 \\ 0.707 \end{bmatrix}, \quad t_4 = \begin{bmatrix} -1.000 \\ 0.000 \end{bmatrix}.$$

The coefficient of friction between the object and fingers is set as 0.5 so that we have

$$e_{11} = \begin{bmatrix} 0.894 \\ -0.447 \end{bmatrix}, \quad e_{12} = \begin{bmatrix} 0.894 \\ 0.447 \end{bmatrix}, \quad e_{21} = \begin{bmatrix} 0.447 \\ 0.894 \end{bmatrix},$$

$$e_{22} = \begin{bmatrix} -0.447 \\ 0.894 \end{bmatrix}, \quad e_{31} = \begin{bmatrix} -0.949 \\ -0.316 \end{bmatrix}, \quad e_{32} = \begin{bmatrix} -0.316 \\ -0.949 \end{bmatrix},$$

$$e_{41} = \begin{bmatrix} -0.447 \\ -0.894 \end{bmatrix}, \quad e_{42} = \begin{bmatrix} 0.447 \\ -0.894 \end{bmatrix}.$$

on the 4 edges and the lengths of the edges are:

$$L_1 = 5.000, \quad L_2 = 11.000, \quad L_3 = 7.071, \quad L_4 = 6.000.$$

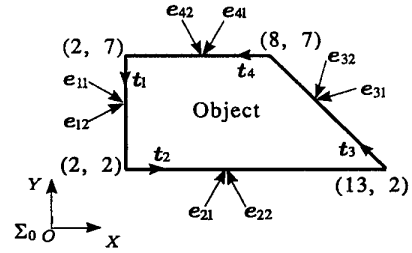


Figure 4: The Polygonal Object of 4 edges

## 6.1 Selecting Edge Candidates

### 6.1.1 Case for two fingers

The combinations of edges are  ${}_4C_2 = 6$  for the given object when the object is grasped by two fingers. By using eq.(4), The edge candidates can be obtained if the solution set of  $k$  exists. As a result, the combinations of edges 1-3 and 2-4 are the candidates. For example, for the combination of the edges 2-4, the grasp force component  $k$  is

$$k = \begin{bmatrix} k_{21} \\ k_{22} \\ k_{41} \\ k_{42} \end{bmatrix} = \begin{bmatrix} 0.000 & 1.000 \\ 1.000 & 0.000 \\ 0.000 & 1.000 \\ 1.000 & 0.000 \end{bmatrix} \alpha, \quad \alpha \geq 0, \quad (27)$$

$$\alpha = [\alpha_1 \quad \alpha_2]^T. \quad (28)$$

### 6.1.2 Case for three fingers

The combinations of edges are  ${}_4C_3 = 4$  when the object is grasped by a robot hand with three fingers. According to eq.(4), all combinations of edges are the candidates. For example, for the combination of the edges 1-2-4, its  $k$  is obtained as

$$k = \begin{bmatrix} k_{11} \\ k_{12} \\ k_{21} \\ k_{22} \\ k_{41} \\ k_{42} \end{bmatrix} = \begin{bmatrix} 0.000 & 0.000 & 0.000 & 1.333 \\ 0.000 & 0.000 & 0.800 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 \\ 1.000 & 0.000 & 0.600 & 1.667 \\ 0.000 & 1.000 & 1.000 & 1.000 \\ 1.000 & 0.000 & 0.000 & 0.000 \end{bmatrix} \alpha, \quad (29)$$

$$\alpha = [\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4]^T, \quad \alpha \geq 0. \quad (30)$$

The obtained edge candidates will be used to determine the graspable finger position regions in the next section.

## 6.2 Determining Graspable Finger Position Regions

### 6.2.1 Case for 2 fingers

From eq.(27), for edges 2-4, for let  $\alpha_1 = [1 \ 0]^T$  and  $\alpha_2 = [0 \ 1]^T$ , we have

$$k_1 = h_1 = [0.000 \ 1.000 \ 0.000 \ 1.000]^T, \quad (31)$$

$$k_2 = h_2 = [1.000 \ 0.000 \ 1.000 \ 0.000]^T. \quad (32)$$

Correspondingly, two boundary hyperplanes are obtained by eq.(19):

$$0.894 l_2 + 0.894 l_4 = 7.602, \quad (33)$$

$$0.894 l_2 + 0.894 l_4 = 3.130, \quad (34)$$

which are two parallel boundary lines shown in Fig.5. Taking into account the length limits of the edges  $0 \leq l_2 \leq 11.000, 0 \leq l_4 \leq 6.000$ , for edges 2-4, the object grasp finger position region will be determined by both 2 boundary lines and 4 length limit lines. From eq.(22)

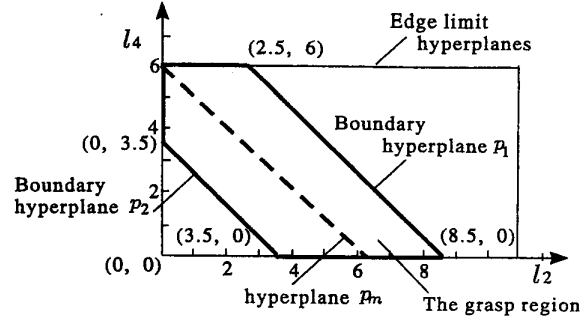


Figure 5: Graspable finger position regions of 2 fingers

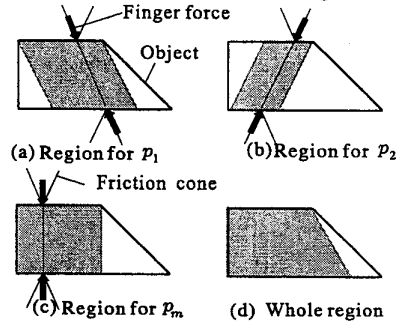


Figure 6: Graspable finger positions on actual object

(23) and (24), the graspable finger position region  $l$  is a convex polyhedron as the following form

$$V_l = [l_2 \ l_4]^T = \begin{bmatrix} 8.500 & 2.500 & 0.000 & 0.000 & 3.500 \\ 0.000 & 6.000 & 6.000 & 3.500 & 0.000 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_5 \end{bmatrix}, \quad (35)$$

$$\beta_1, \dots, \beta_5 \geq 0, \quad \beta_1 + \dots + \beta_5 = 1.$$

As shown in Fig.5 and 6, corresponding to the boundary hyperplane  $p_1$  in finger position space, the object graspable finger positions are in the region of Fig.6(a) on the actual object. Similarly, for the  $p_2$ , we have Fig.6(b), for a combination of  $p_1$  and  $p_2$ , we have Fig.6(c). Therefore, the whole graspable finger positions are in the region of Fig.6(d).

### 6.2.2 Case for three fingers

Similarly, for edges 1-2-4, corresponding to the four span vectors of  $k$  in eq.(29), four boundary hyperplanes are obtained as

$$0.894 l_2 + 0.894 l_4 = 7.602, \quad (36)$$

$$0.894 l_2 + 0.894 l_4 = 3.130, \quad (37)$$

$$0.716 l_1 + 0.537 l_2 + 0.894 l_4 = 6.708, \quad (38)$$

$$1.192 l_1 + 1.491 l_2 + 0.894 l_4 = 9.091. \quad (39)$$

The length limits of the edges are:  $0 \leq l_1 \leq 5.000, 0 \leq l_2 \leq 11.000, 0 \leq l_4 \leq 6.000$ . According to eq.(22) and (23), there are  $J = 4, q, r = 1, 2, 3, 4, q \neq r$ . For

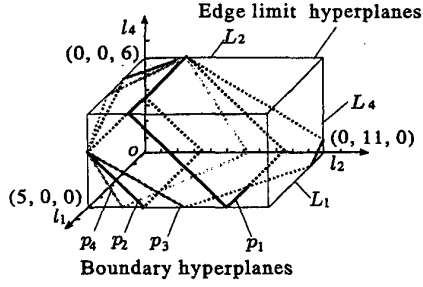


Figure 7: Boundary hyperplanes and edge limit hyperplanes of three finger grasp

each of both hyperplane combinations, we can obtain convex polyhedrons  $V_{qr}^1$  and  $V_{qr}^2$  separately, which are

$$V_{12}^1 = \phi, \quad (40)$$

$$V_{12}^2 = \{ l \mid l = \begin{bmatrix} 0.000 & 5.000 & 0.000 & 5.000 & 0.000 \\ 3.500 & 3.500 & 0.000 & 0.000 & 8.500 \\ 0.000 & 0.000 & 3.500 & 3.500 & 0.000 \\ 5.000 & 0.000 & 5.000 & 0.000 & 5.000 \\ 8.500 & 2.500 & 2.500 & 0.000 & 0.000 \\ 0.000 & 6.000 & 6.000 & 6.000 & 6.000 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_8 \end{bmatrix}, \quad (41)$$

$$\beta_1, \dots, \beta_8 \geq 0, \quad \beta_1 + \dots + \beta_8 = 1\},$$

$$V_{34}^1 = \{ l \mid l = \begin{bmatrix} 0.000 & 5.000 & 3.125 & 1.875 \\ 2.500 & 0.000 & 0.000 & 0.000 \\ 6.000 & 3.500 & 6.000 & 6.000 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_4 \end{bmatrix}, \quad (42)$$

$$\beta_1, \dots, \beta_4 \geq 0, \quad \beta_1 + \dots + \beta_4 = 1\},$$

$$V_{34}^2 = \{ l \mid l = \begin{bmatrix} 0.000 & 5.000 & 1.125 & 0.000 \\ 2.500 & 0.000 & 11.000 & 6.098 \\ 6.000 & 3.500 & 0.000 & 0.000 \\ 5.000 & 5.000 & 0.000 & 0.000 \\ 2.099 & 5.833 & 11.000 & 11.000 \\ 0.000 & 0.000 & 0.900 & 0.000 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_8 \end{bmatrix}, \quad (43)$$

$$\beta_1, \dots, \beta_8 \geq 0, \quad \beta_1 + \dots + \beta_8 = 1\}.$$

Therefore, the region  $V_l$  has the following form

$$V_l = \bigcup_{q,r=1,q \neq r}^4 (V_{qr}^1 \cup V_{qr}^2). \quad (44)$$

The regions of  $V_{qr}^1$ ,  $V_{qr}^2$  and  $V_l$  are illustrated in Fig.8, where  $V_{qr}^1$  and  $V_{qr}^2$  are convex polyhedrons respectively, while the union set  $V_l$  is a polyhedron but not a convex one (see Fig.8).

The  $V_l$  is the whole graspable finger position regions in finger position space. For an arbitrary element in the  $V_l$ , grasp finger force can be obtained correspondingly. For example, in  $V_{34}^2$  of eq.(53), when

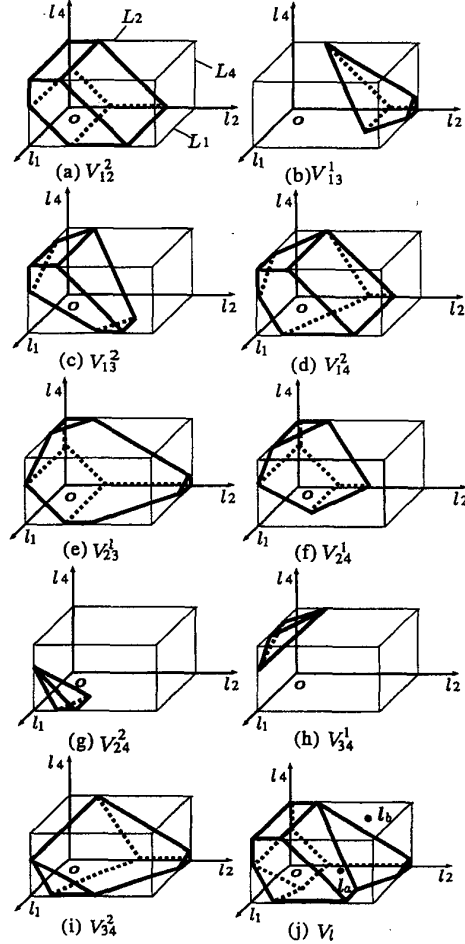


Figure 8: Graspable finger position regions of 3 finger grasp

$\beta_1 = \beta_2 = \dots = \beta_8 = 0.125$ , the correspondingly graspable finger positions  $l_a$  (see Fig.8(j) and 9)

$$l_a = \begin{bmatrix} l_1 \\ l_2 \\ l_4 \end{bmatrix} = \begin{bmatrix} 2.016 \\ 6.191 \\ 1.300 \end{bmatrix}. \quad (45)$$

According to eq.(13) and (25), the grasp finger force  $f_a$  is a polyhedral cone and given by

$$F_a = \begin{bmatrix} f_1 \\ f_2 \\ f_4 \end{bmatrix} = \begin{bmatrix} 0.967 & 0.290 & 3.272 & 0.000 \\ 0.232 & -0.145 & 1.636 & 0.000 \\ -0.425 & -0.629 & -0.780 & -0.334 \\ 0.851 & 1.257 & 3.349 & 1.120 \\ -0.541 & 0.338 & -2.492 & 0.334 \\ -1.083 & -1.112 & -4.985 & -1.120 \end{bmatrix} \alpha_a, \quad (46)$$

$$\alpha_a = [\alpha_{a1} \quad \alpha_{a2} \quad \alpha_{a3} \quad \alpha_{a4}]^T, \quad \alpha \geq 0.$$

As shown in Fig.10, the columns of  $F_a$  represent respectively the force and moment equilibriums in the cases

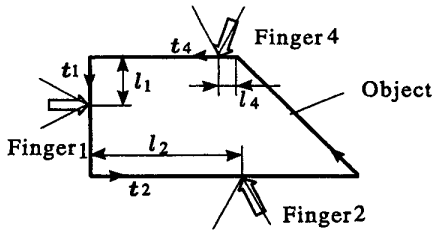


Figure 9: Finger positions of  $l_a$

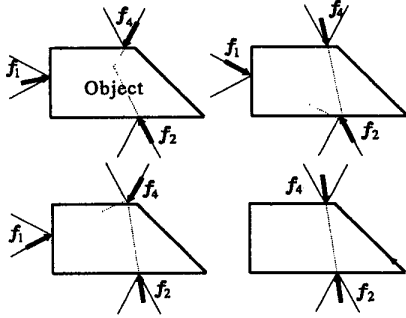


Figure 10: Finger forces from position  $l_a$

that the finger forces are located in the edge directions of the friction cone.

As shown in Fig.8 (j), if we select one point  $l_b$  outside of the  $V_l$ , for example,

$$l_b = \begin{bmatrix} l_1 \\ l_2 \\ l_4 \end{bmatrix} = \begin{bmatrix} 1.000 \\ 8.000 \\ 5.000 \end{bmatrix}. \quad (47)$$

substituting the element  $l_b$  into eq.(13), the  $\alpha$  that meet the moment equilibrium does not exist, as shown in Fig.11. Therefore,  $l_b$  is not successful grasp finger positions.

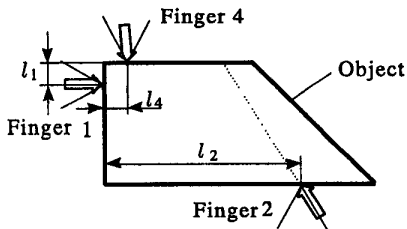


Figure 11: Finger positions of  $l_b$

## 7 Conclusion

We have presented a novel analytical method to determine candidates of graspable edges and graspable finger position regions for a given object.

Some features of the analytical method presented in this paper are summarized as follows:

- The permissible regions of the position variables of contact points were defined as graspable finger position regions.

- The object graspable finger position regions are determined exactly by using linear programming method. The union set of the object grasp regions is a geometrical polyhedron but not a convex one.
- The grasp edge candidates can be selected before the problem of the object grasp regions are determined. The computational load of the finger position regions will be reduced considerably.

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