ECCV 2010 TUTORIAL

NONRIGID STRUCTURE FROM MOTION

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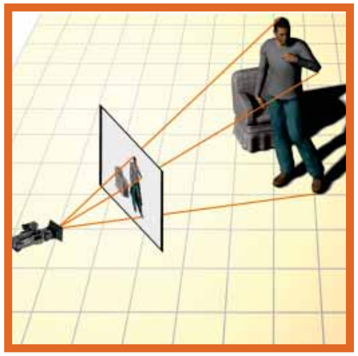
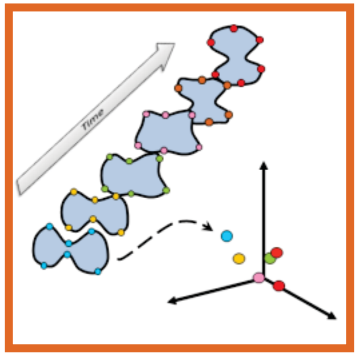
NONRIGID STRUCTURE FROM MOTION

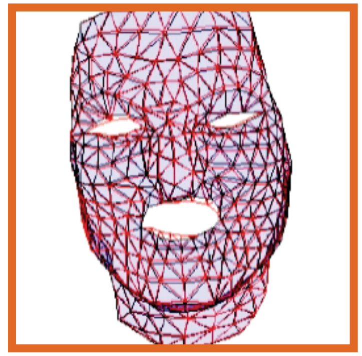
Tutorial Outline



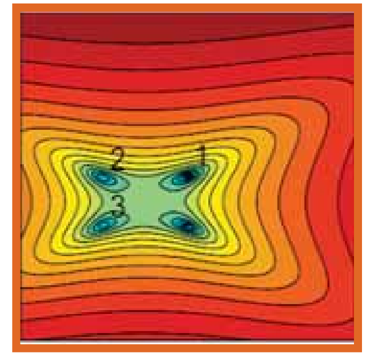
Introduction to Nonrigid SFM



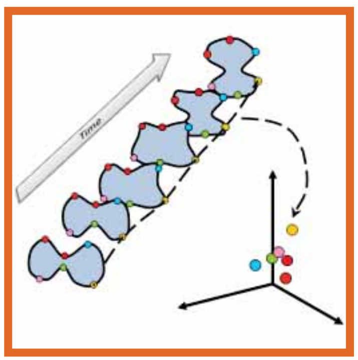
Shape Representation



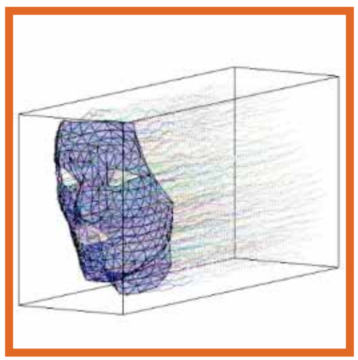
Shape Estimation



Ambiguity of Orthogonality Constraints

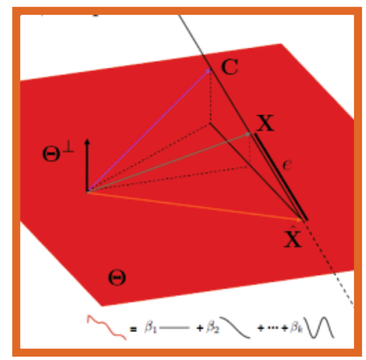


Trajectory Representation



Shape-Trajectory Duality

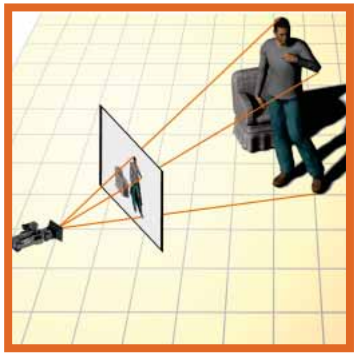
Trajectory Estimation



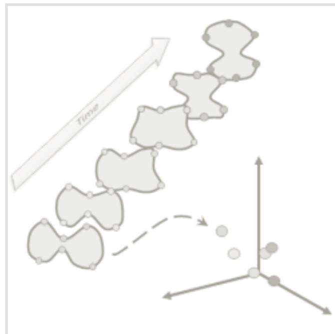
Reconstructibility and limitations

NONRIGID STRUCTURE FROM MOTION

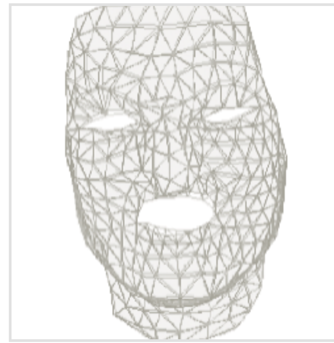
Tutorial Outline



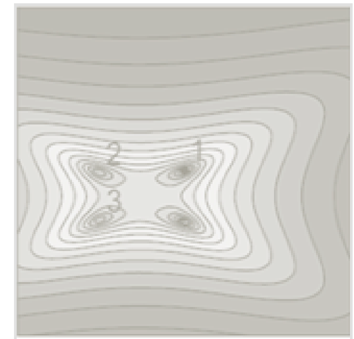
Introduction to Nonrigid SFM



Shape Representation

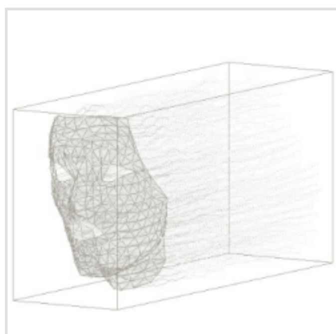


Shape Estimation



Ambiguity of Orthogonality Constraints

Trajectory Representation



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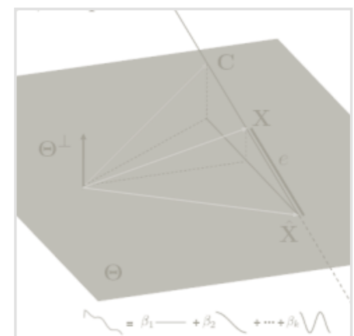
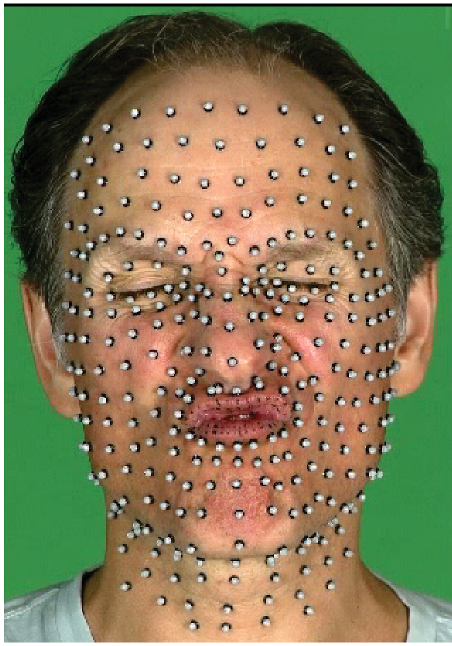
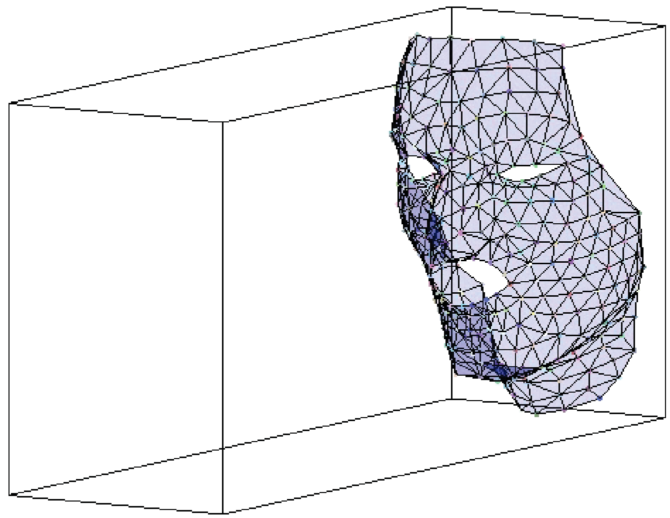


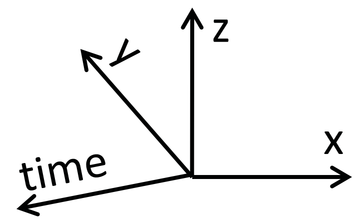
Reconstructibility and limitations

NONRIGID STRUCTURE

3D Structure That Deforms Over Time







4D DYNAMIC STRUCTURE

IMAGE MOTION

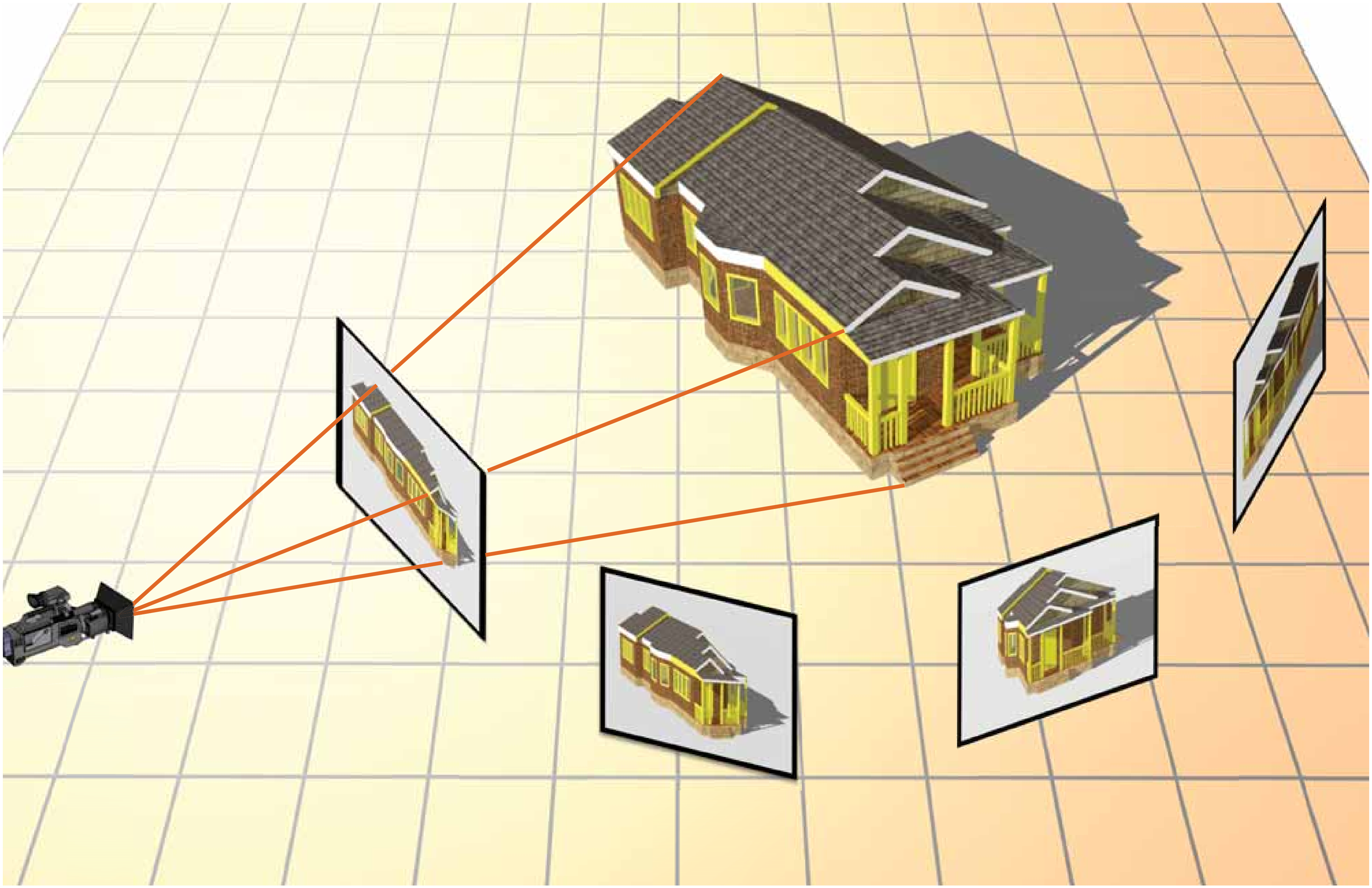
OBJECT MOTION

CAMERA MOTION

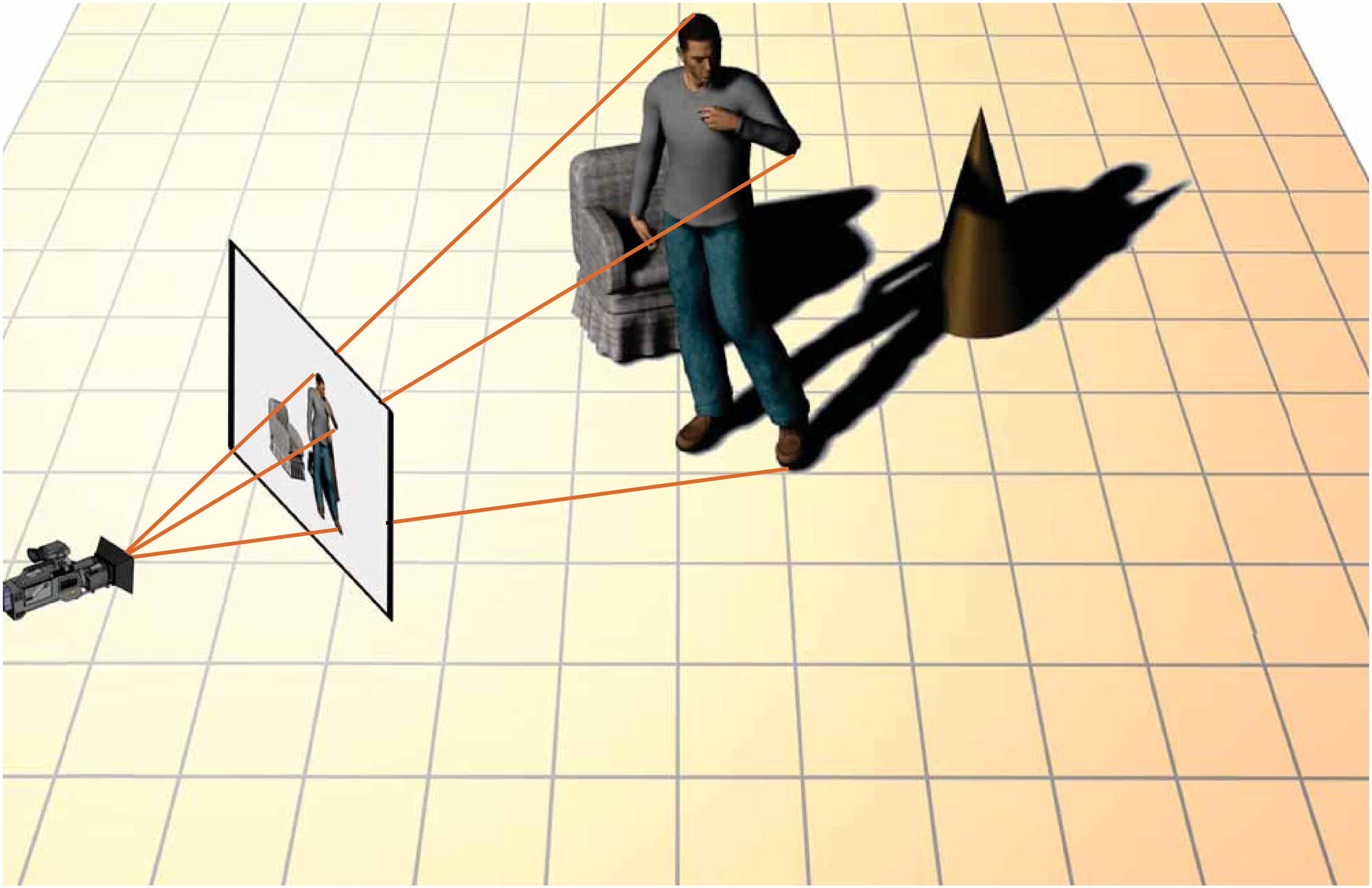
IMAGE MOTION

OBJECT MOTION AND **CAMERA** MOTION

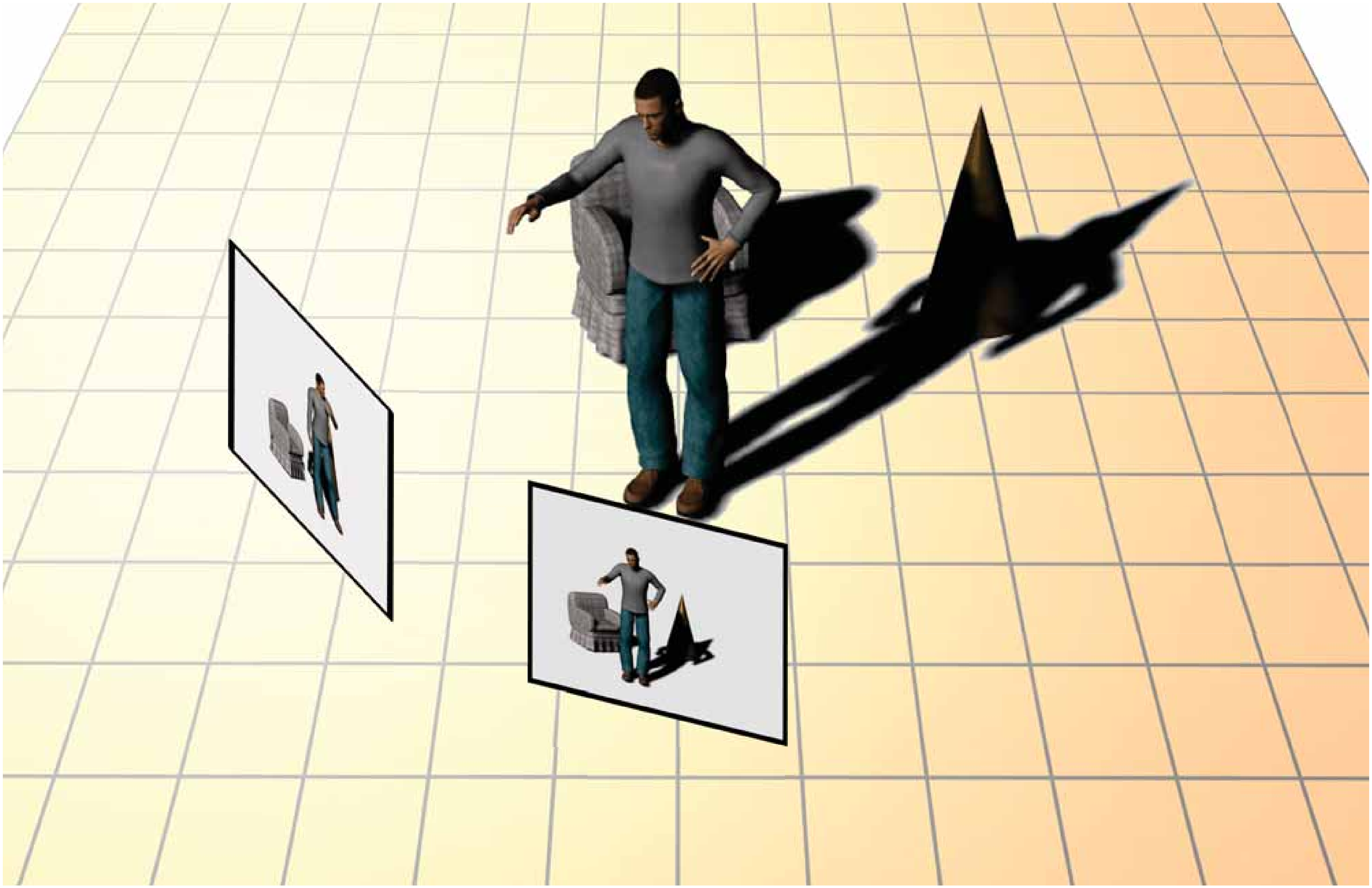
Rigid Structure from Motion



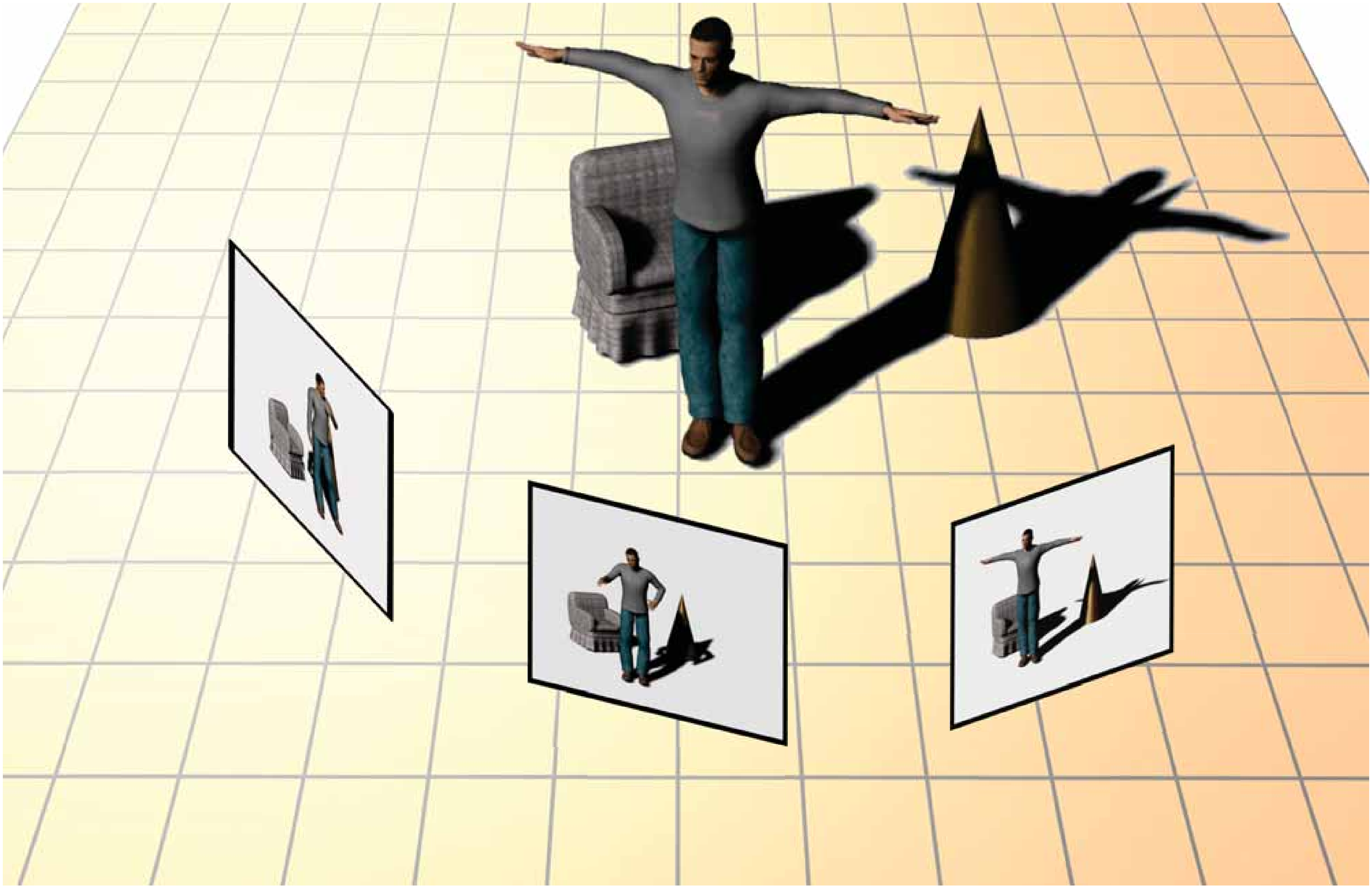
Nonrigid Structure from Motion



NONRIGID STRUCTURE FROM MOTION



NONRIGID STRUCTURE FROM MOTION



NONRIGID STRUCTURE FROM MOTION

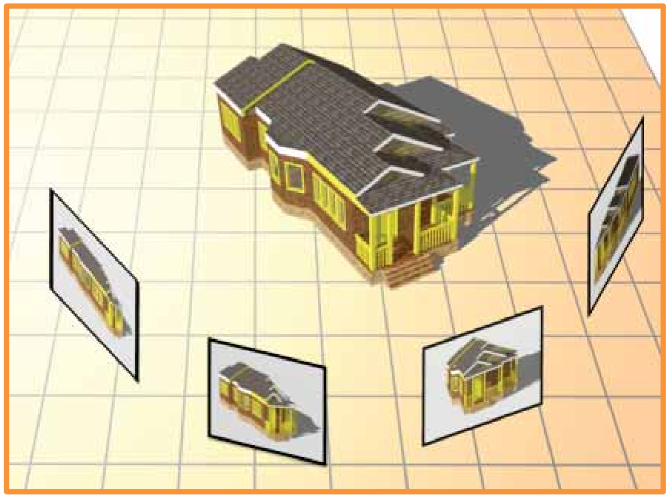
ONLY ONE VIEW PER 3D CONFIGURATON: ILL-POSED PROBLEM
EQUIVALENT TO FINDING 3D FROM SINGLE IMAGE

FACTORIZATION METHOD FOR RIGID SFM

Kontsevich *et al.* 1987, Tomasi and Kanade 1992

ASSUMPTIONS

- Orthographic Camera
- At least 3 images
- Rigid Scene
- Camera Motion
- Corresponding points available

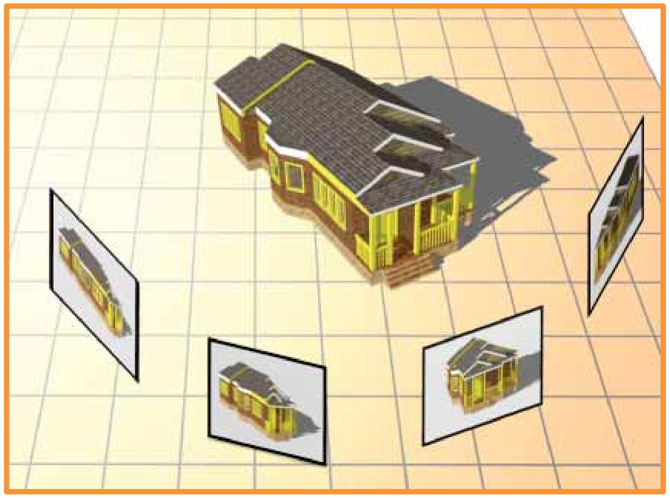


FACTORIZATION METHOD FOR RIGID SFM

Kontsevich *et al.* 1987, Tomasi and Kanade 1992

NOTATION

- P 3D points seen in F frames
- $\mathbf{X}_j = [X_j, Y_j, Z_j]$ is j^{th} 3D point
  $1 \leq j \leq P$
- $\mathbf{x}_{ij} = [x_{ij}, y_{ij}]$ is the projection of $\mathbf{X}_j$ in i^{th} frame $1 \leq i \leq F$
- $\mathbf{P}_i$ is the camera projection matrix if the i^{th} frame $1 \leq i \leq F$



FACTORIZATION METHOD FOR RIGID SFM

Kontsevich *et al.* 1987, Tomasi and Kanade 1992

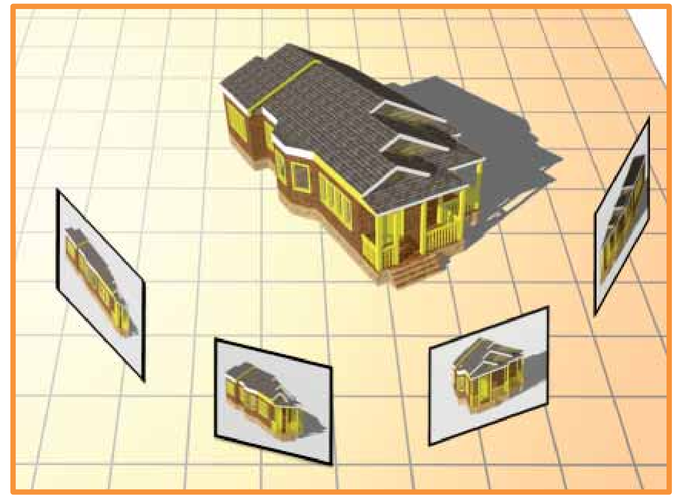
2D image point

orthographic projection matrix

3D scene point

$$\mathbf{x}_{ij} = \mathbf{P}_i \mathbf{X}_j$$

$2 \times 1 \qquad 2 \times 4 \quad 4 \times 1$



$$\mathbf{x}_{ij} = \mathbf{K}\left[\mathbf{R'}_i | \mathbf{T'}_i\right] \mathbf{X}_j$$

$$\begin{bmatrix} x_{ij} \\ y_{ij} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_i^1 & r_i^2 & r_i^3 & t_i^x \\ r_i^4 & r_i^5 & r_i^6 & t_i^y \\ r_i^7 & r_i^8 & r_i^9 & t_i^z \end{bmatrix} \begin{bmatrix} X_j \\ Y_j \\ Z_j \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_{ij} \\ y_{ij} \end{bmatrix} = \begin{bmatrix} r_i^1 & r_i^2 & r_i^3 \\ r_i^4 & r_i^5 & r_i^6 \end{bmatrix} \begin{bmatrix} X_j \\ Y_j \\ Z_j \end{bmatrix} + \begin{bmatrix} t_i^x \\ t_i^y \end{bmatrix}$$

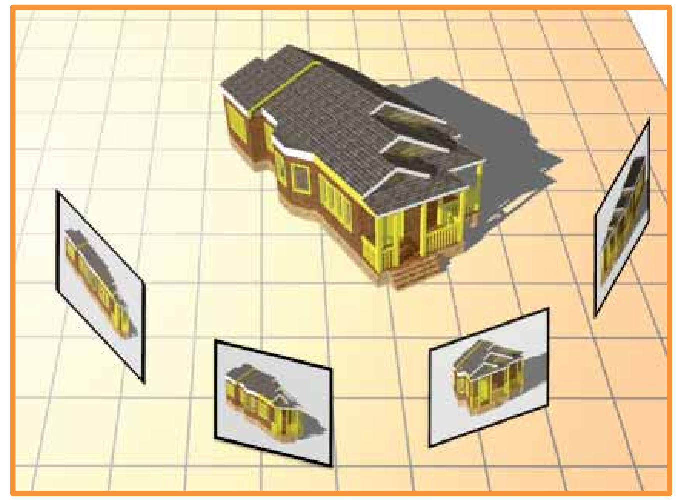
Factorization Method for Rigid SFM

Kontsevich *et al.* 1987, Tomasi and Kanade 1992

2 rows of a 3D rotation matrix

image offset

$$\underset{2\times 1}{\mathbf{x}_{ij}} = \underset{2\times 3}{\mathbf{R}_i}\,\underset{3\times 1}{\mathbf{X}_j} + \underset{2\times 1}{\mathbf{t}_i}$$



TRICK

- Choose scene origin to be center of 3D points
- Choose image origins to be center of 2D points
- Allows us to drop camera translation

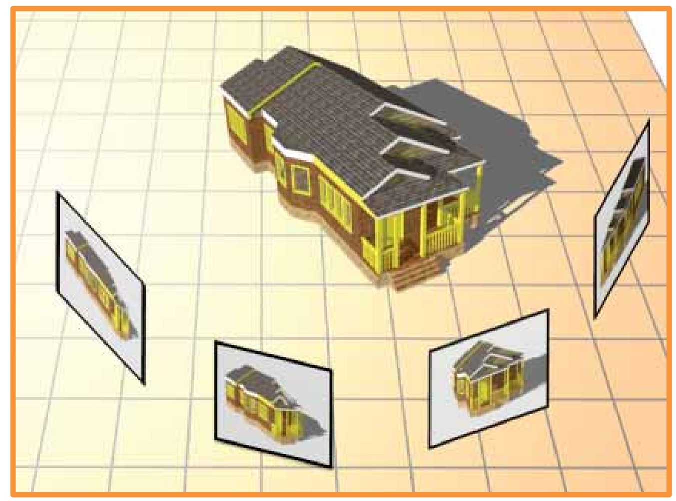
FACTORIZATION METHOD FOR RIGID SFM

Kontsevich *et al.* 1987, Tomasi and Kanade 1992

2 rows of a 3D rotation matrix

image offset

$$\underset{2\times 1}{\mathbf{x}_{ij}} = \underset{2\times 3}{\mathbf{R}_i}\underset{3\times 1}{\mathbf{X}_j} + \underset{2\times 1}{\cancel{\mathbf{t}_i}}$$



TRICK

- Choose scene origin to be center of 3D points
- Choose image origins to be center of 2D points
- Allows us to drop camera translation

FACTORIZATION METHOD FOR RIGID SFM

Kontsevich *et al.* 1987, Tomasi and Kanade 1992

PROJECTION OF P 3D POINTS IN i^{th} IMAGE

$$\underbrace{\begin{bmatrix} \mathbf{x}_{i1} & \mathbf{x}_{i2} & \dots & \mathbf{x}_{iP} \end{bmatrix}}_{2 \times P} = \underbrace{\mathbf{R}_i}_{2 \times 3} \underbrace{\begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \dots & \mathbf{X}_P \end{bmatrix}}_{3 \times P}$$

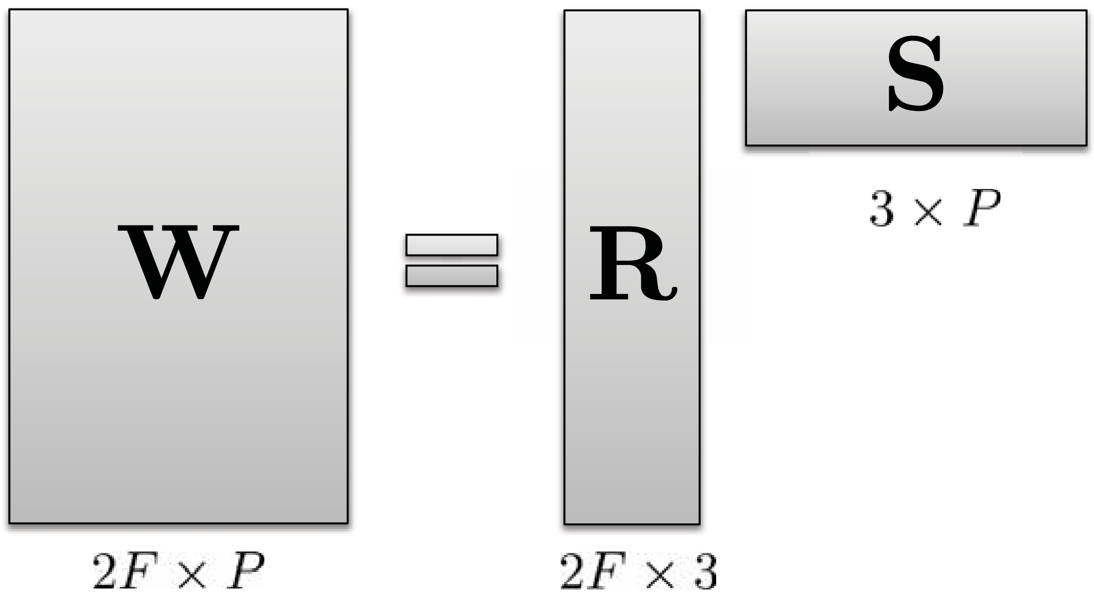
PROJECTION OF P 3D POINTS IN F IMAGES

$$\underbrace{\begin{bmatrix} \mathbf{x}_{11} & \mathbf{x}_{12} & \dots & \mathbf{x}_{1P} \\ \mathbf{x}_{21} & \mathbf{x}_{22} & \dots & \mathbf{x}_{2P} \\ \vdots & \vdots & & \vdots \\ \mathbf{x}_{F1} & \mathbf{x}_{F2} & \dots & \mathbf{x}_{FP} \end{bmatrix}}_{2F \times P} = \underbrace{\begin{bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \\ \vdots \\ \mathbf{R}_F \end{bmatrix}}_{2F \times 3} \underbrace{\begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \dots & \mathbf{X}_P \end{bmatrix}}_{3 \times P}$$

Factorization Method for Rigid SFM

Kontsevich *et al.* 1987, Tomasi and Kanade 1992

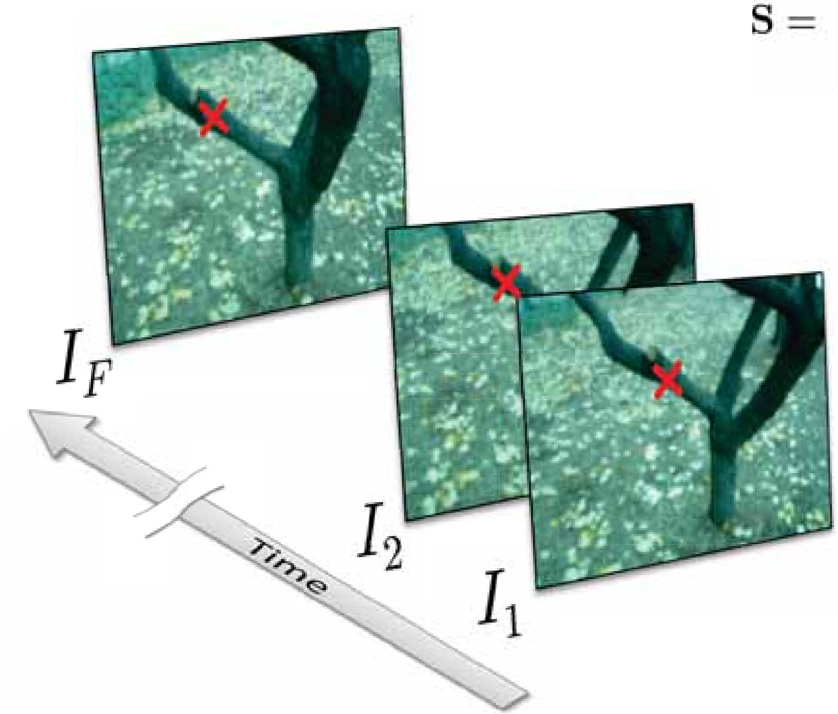
PROJECTION OF P 3D POINTS IN F IMAGES



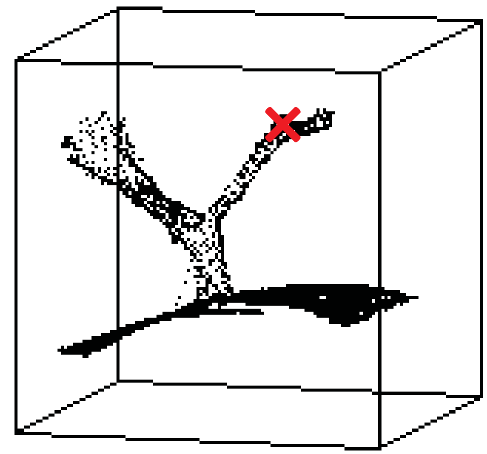
$$\mathbf{W}_{\text{measurement}} = \mathbf{R}_{\text{motion}} \times \mathbf{S}_{\text{shape}}$$

FACTORIZATION METHOD FOR RIGID SFM

Kontsevich *et al.* 1987, Tomasi and Kanade 1992



$$\mathbf{S} = \begin{bmatrix} X_1 & X_2 & \dots & X_P \\ Y_1 & Y_2 & \dots & Y_P \\ Z_1 & Z_2 & \dots & Z_P \end{bmatrix}$$



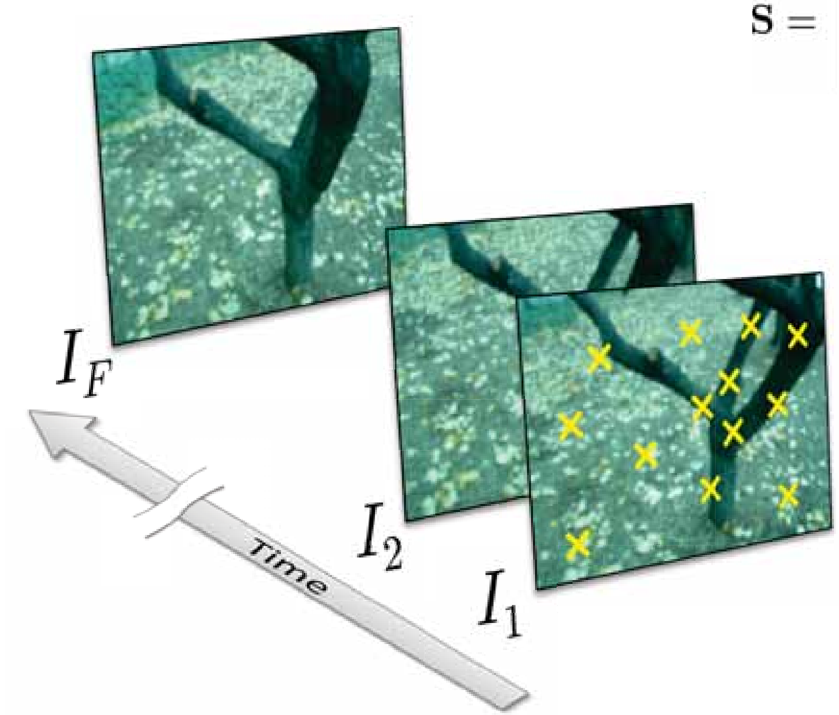
$$\begin{bmatrix} \mathbf{x}_{11} & \mathbf{x}_{12} & \dots & \mathbf{x}_{1P} \\ \mathbf{x}_{21} & \mathbf{x}_{22} & \dots & \mathbf{x}_{2P} \\ \vdots & \vdots & & \vdots \\ \mathbf{x}_{F1} & \mathbf{x}_{F2} & \dots & \mathbf{x}_{FP} \end{bmatrix}$$

Image Observations Matrix, $\mathbf{W}$

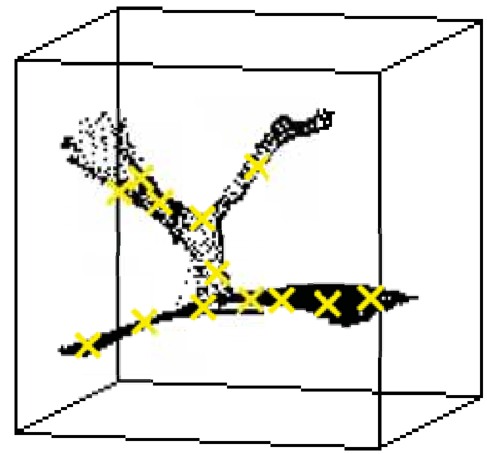
$2F \times P$

Factorization Method for Rigid SFM

Kontsevich *et al.* 1987, Tomasi and Kanade 1992



$$\mathbf{S} = \begin{bmatrix} X_1 & X_2 & \dots & X_P \\ Y_1 & Y_2 & \dots & Y_P \\ Z_1 & Z_2 & \dots & Z_P \end{bmatrix}$$



$$\begin{bmatrix} \mathbf{x}_{11} & \mathbf{x}_{12} & \dots & \mathbf{x}_{1P} \\ \mathbf{x}_{21} & \mathbf{x}_{22} & \dots & \mathbf{x}_{2P} \\ \vdots & \vdots & & \vdots \\ \mathbf{x}_{F1} & \mathbf{x}_{F2} & \dots & \mathbf{x}_{FP} \end{bmatrix}$$

Image Observations Matrix, $\mathbf{W}$

$2F \times P$

FACTORIZATION METHOD FOR RIGID SFM

Kontsevich *et al.* 1987, Tomasi and Kanade 1992

HOW TO SOLVE FOR $\mathbf{Q}$

- Observation: The correct $\mathbf{Q}$ will result in an $\mathbf{R}$ whose rows are pair-wise orthonormal

$$\mathbf{R} = \hat{\mathbf{R}}\mathbf{Q}$$

- The i^{th} image results in the following 3 constraints on $\mathbf{Q}$

$$\mathbf{R}_{2i-1:2i}\mathbf{R}_{2i-1:2i}^T = \mathbf{I}_{2\times 2} = \left(\hat{\mathbf{R}}_{2i-1:2i}\mathbf{Q}\right)\left(\hat{\mathbf{R}}_{2i-1:2i}\mathbf{Q}\right)^T$$

ORTHONORMALITY CONSTRAINTS ⇨

$$\hat{\mathbf{R}}_{2i-1:2i}\mathbf{Q}\mathbf{Q}^T\hat{\mathbf{R}}_{2i-1:2i} = \mathbf{I}_{2\times 2}$$

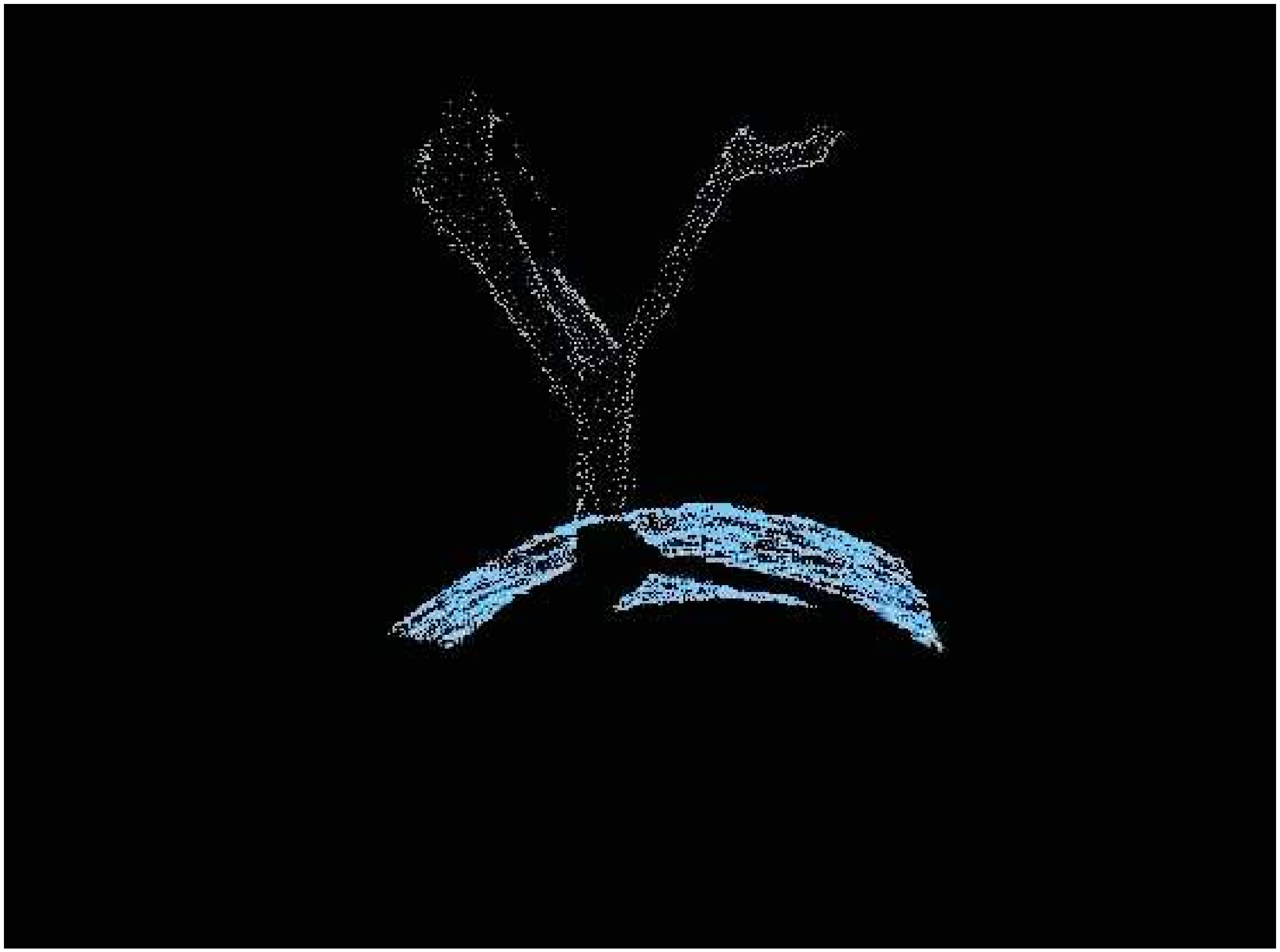
- Total $3F$ constraints on 6 terms of $\mathbf{Q}\mathbf{Q}^T$
- Can be solved linearly for $\mathbf{G} = \mathbf{Q}\mathbf{Q}^T$ for $F \geq 3$

FACTORIZATION METHOD FOR RIGID SFM

Kontsevich *et al.* 1987, Tomasi and Kanade 1992

FACTORIZATION METHOD FOR RIGID SFM

Kontsevich *et al.* 1987, Tomasi and Kanade 1992

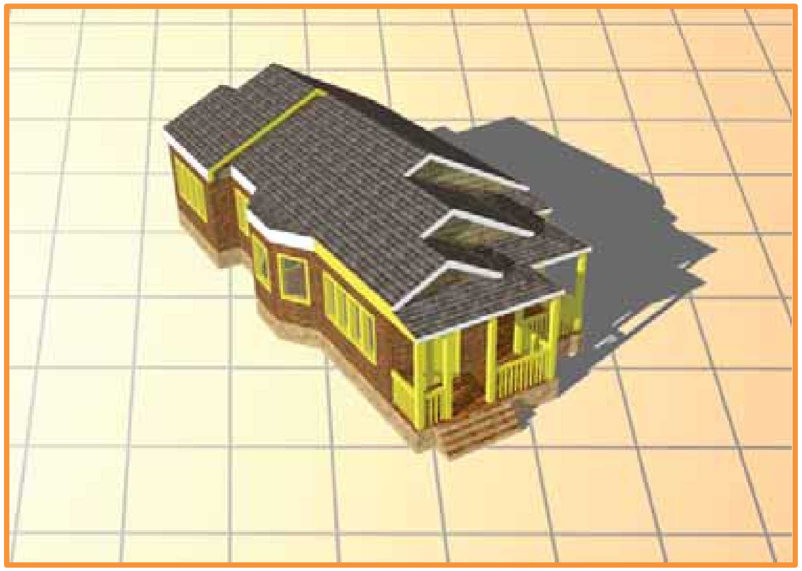


NonRigid Structure

3D Structure That Deforms Over Time

RIGID STRUCTURE

$$\mathbf{S}_{3\times P} = \begin{bmatrix} X_1 & X_2 & \ldots & X_P \\ Y_1 & Y_2 & \ldots & Y_P \\ Z_1 & Z_2 & \ldots & Z_P \end{bmatrix}$$



NONRIGID STRUCTURE

3D Structure That Deforms Over Time

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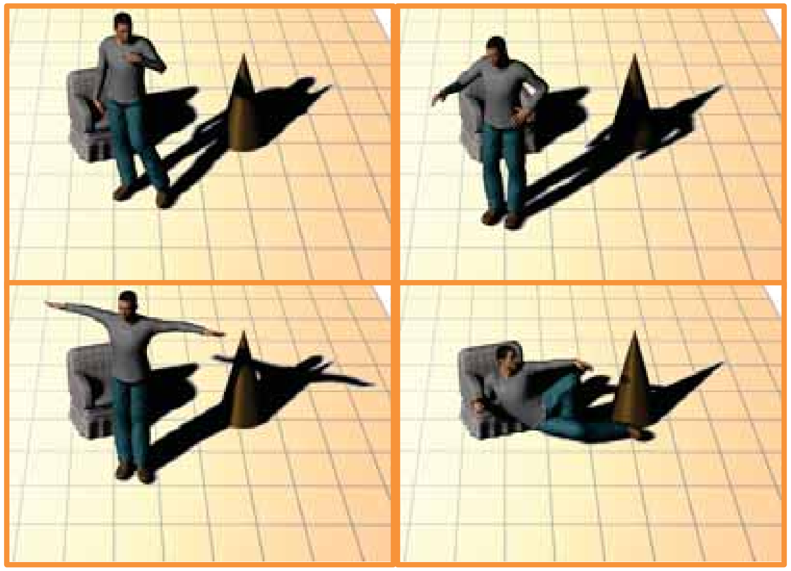
$$\mathbf{S}_{3\times P} = \begin{bmatrix} X_1 & X_2 & \dots & X_P \\ Y_1 & Y_2 & \dots & Y_P \\ Z_1 & Z_2 & \dots & Z_P \end{bmatrix}$$



NONRIGID STRUCTURE

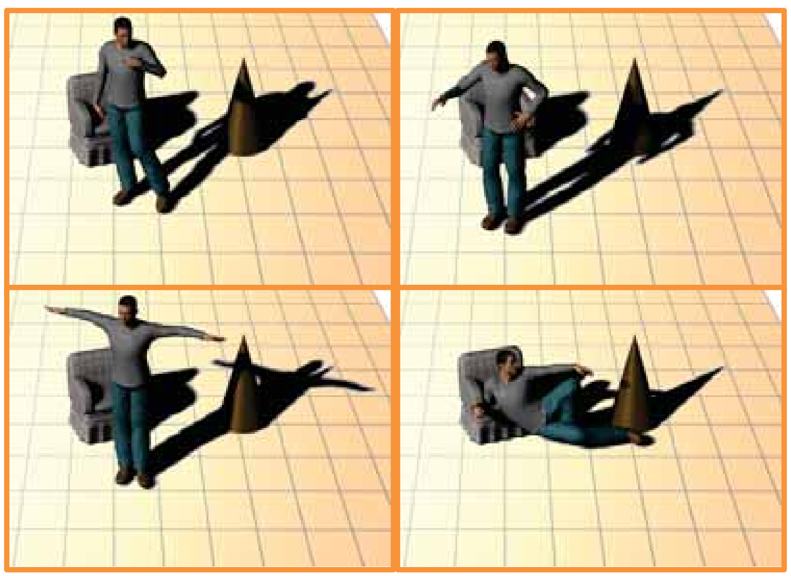
$$\mathbf{S}_{3F\times P} = \begin{bmatrix} \begin{bmatrix} X_{11} & X_{12} & \dots & X_{1P} \\ Y_{11} & Y_{12} & \dots & Y_{1P} \\ Z_{11} & Z_{12} & \dots & Z_{1P} \end{bmatrix}_{3\times P} \\ \begin{bmatrix} X_{21} & X_{22} & \dots & X_{2P} \\ Y_{21} & Y_{22} & \dots & Y_{2P} \\ Z_{21} & Z_{22} & \dots & Z_{2P} \end{bmatrix}_{3\times P} \\ \vdots \\ \begin{bmatrix} X_{F1} & X_{F2} & \dots & X_{FP} \\ Y_{F1} & Y_{F2} & \dots & Y_{FP} \\ Z_{F1} & Z_{F2} & \dots & Z_{FP} \end{bmatrix}_{3\times P} \end{bmatrix}$$

NonRigid Structure

3D Structure That Deforms Over Time

RIGID STRUCTURE

$$\mathbf{S}_{3\times P} = \begin{bmatrix} X_1 & X_2 & \ldots & X_P \\ Y_1 & Y_2 & \ldots & Y_P \\ Z_1 & Z_2 & \ldots & Z_P \end{bmatrix}$$



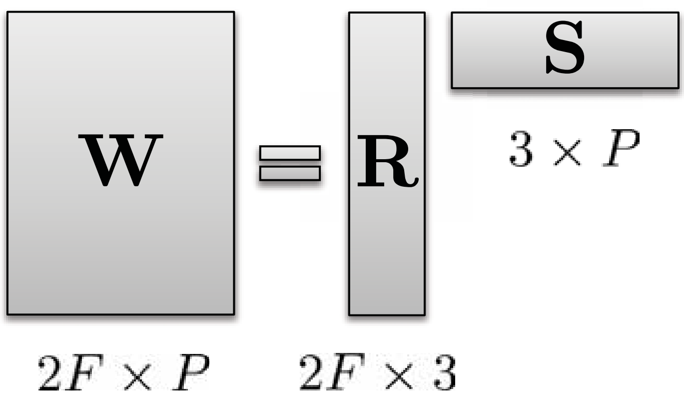
NONRIGID STRUCTURE

$$\mathbf{S}_{3F\times P} = \begin{bmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} & \cdots & \mathbf{X}_{1P} \\ \mathbf{X}_{21} & \mathbf{X}_{22} & \cdots & \mathbf{X}_{2P} \\ \vdots & \vdots & & \vdots \\ \mathbf{X}_{F1} & \mathbf{X}_{F2} & \cdots & \mathbf{X}_{FP} \end{bmatrix}$$

NonRigid Structure From Motion

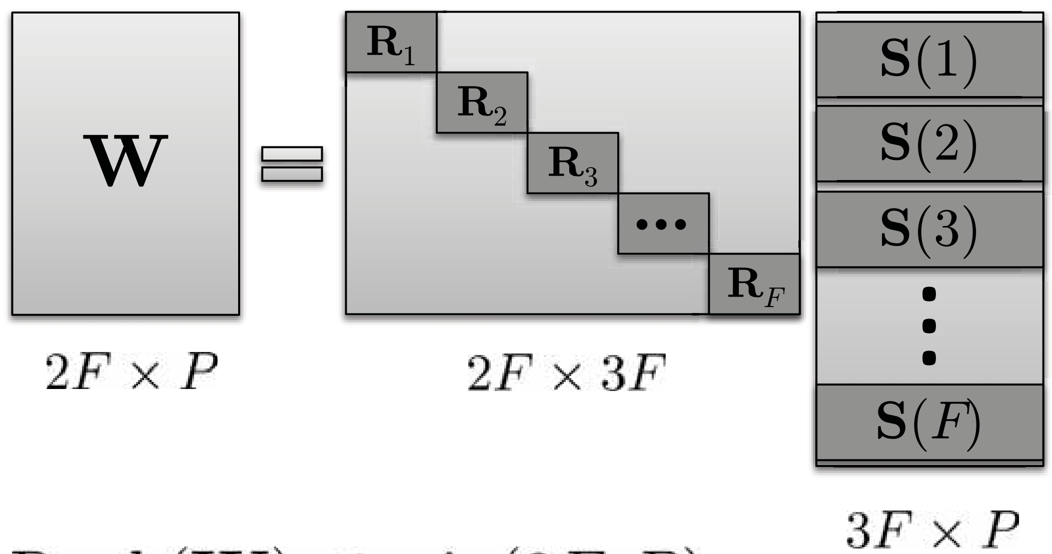
Comparison with Rigid Structure from Motion

RIGID SFM



$$\text{Rank}(\mathbf{W}) \leq 3$$

NONRIGID SFM



$$\text{Rank}(\mathbf{W}) \leq \min(2F, P)$$

NONRIGID STRUCTURE FROM MOTION

Explosion of Unknowns

Example: Given a 40 second video with 100 tracked points

RIGID SFM

- Inputs:

  100 pts x 40 sec x 30 fps x 2 (x, y)

  = 240,000 observations

- Unknowns:

  100 points x 3 (X,Y,Z)

  = **300** unknowns

NONRIGID SFM

- Inputs:

  100 pts x 40 sec x 30 fps x 2

  = 240,000 observations

- Unknowns:

  100 points x 40 sec x 30 fps x 3

  = **360,000** unknowns

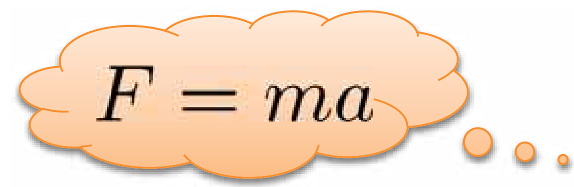
Nonrigid Structure From Motion

Explosion of Unknowns

IN GENERAL, NRSFM HAS MORE UNKNOWNS THAN CONSTRAINTS

ILL-POSED PROBLEM: Additional assumptions are necessary to constrain the solution.

HOWEVER...



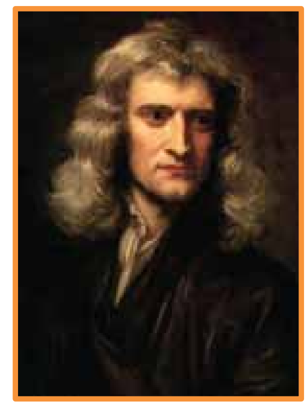
Motion is not random:

3D points are often highly correlated in space and time

Points move because an actuator exerts force on them

Hence their acceleration is limited by the actuating force

Therefore, shape does not deform arbitrarily over time



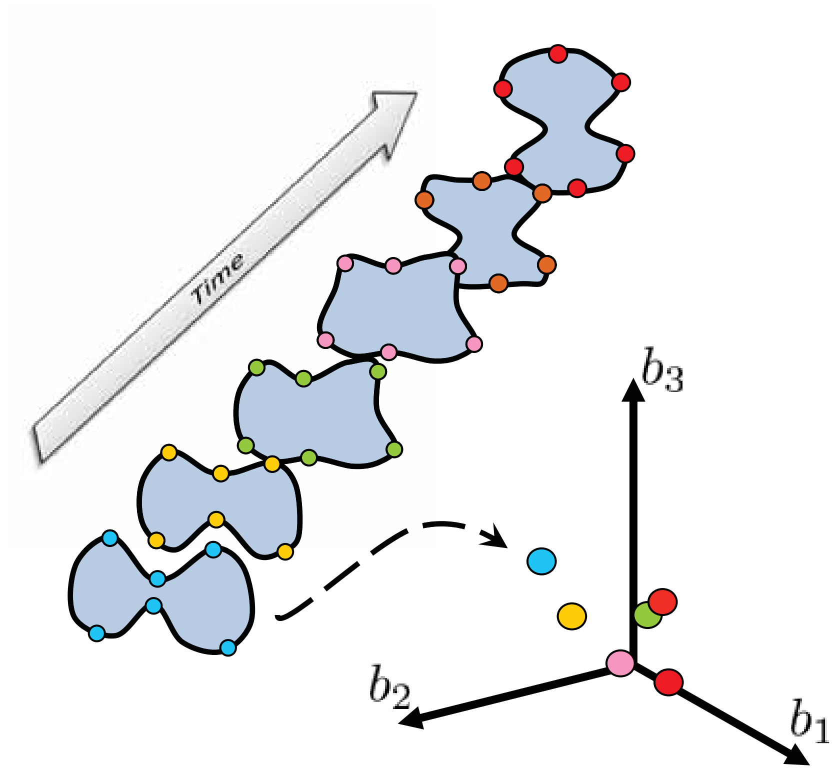
4D STRUCTURE OFTEN LIES IN A LOW DIMENSIONAL SUBSPACE

Nonrigid Structure From Motion

Two Major Approaches

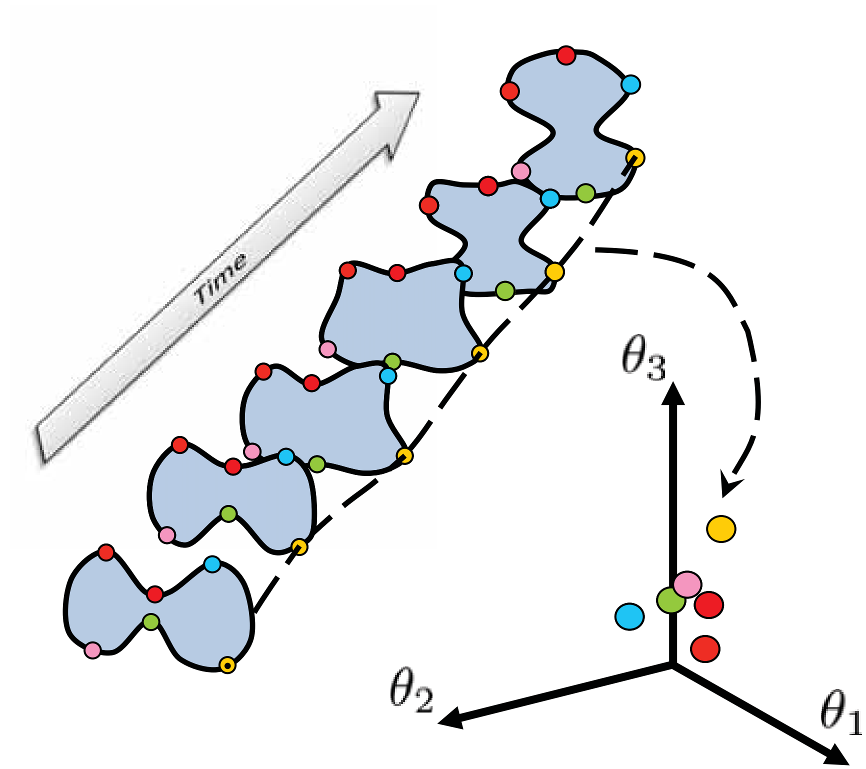
Shape Basis

3D points at each time instant lie in a low dimensional subspace



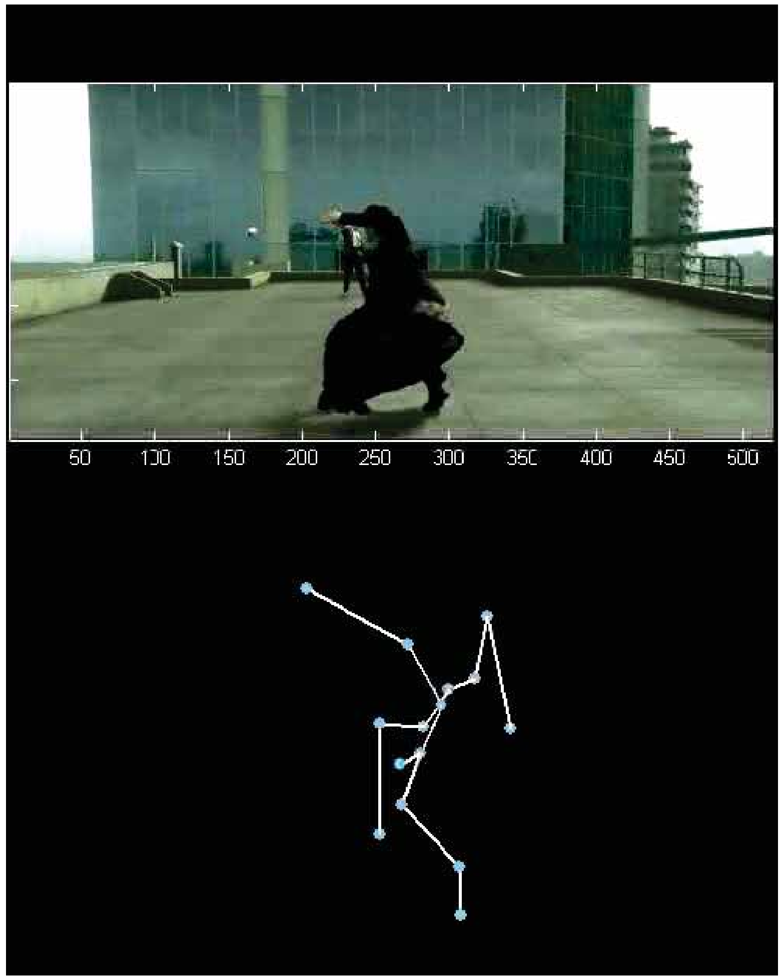
Trajectory Basis

Trajectory of each point over time lies in a low dimensional subspace



EXAMPLES OF APPLICATIONS

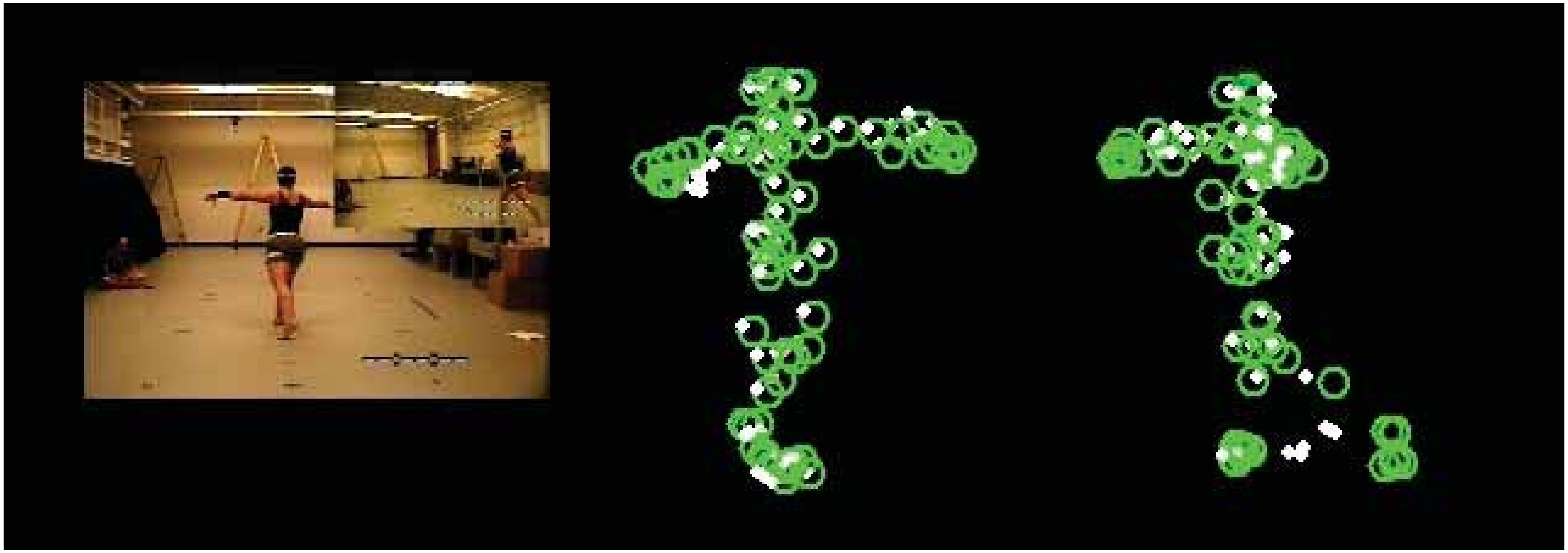
Match Moving in Movies



Akhter *et al.* NIPS 2008

EXAMPLES OF APPLICATIONS

Motion-Capture

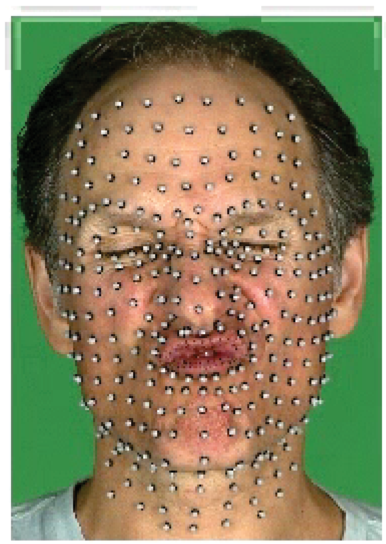


Input Video — Two views of the reconstruction

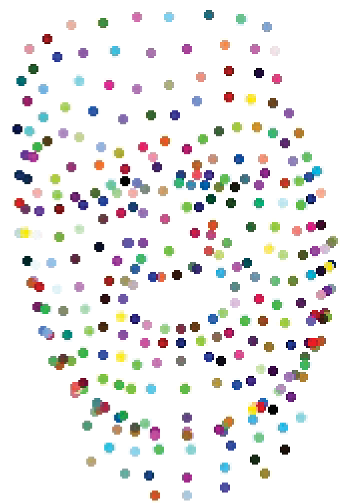
Akhter *et al.* NIPS 2008

EXAMPLES OF APPLICATIONS

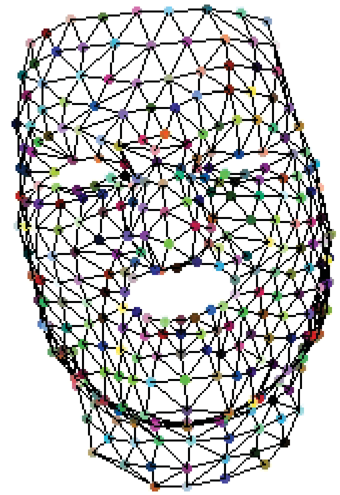
Motion-Capture Cleanup



Video



Unlabeled Data
Input

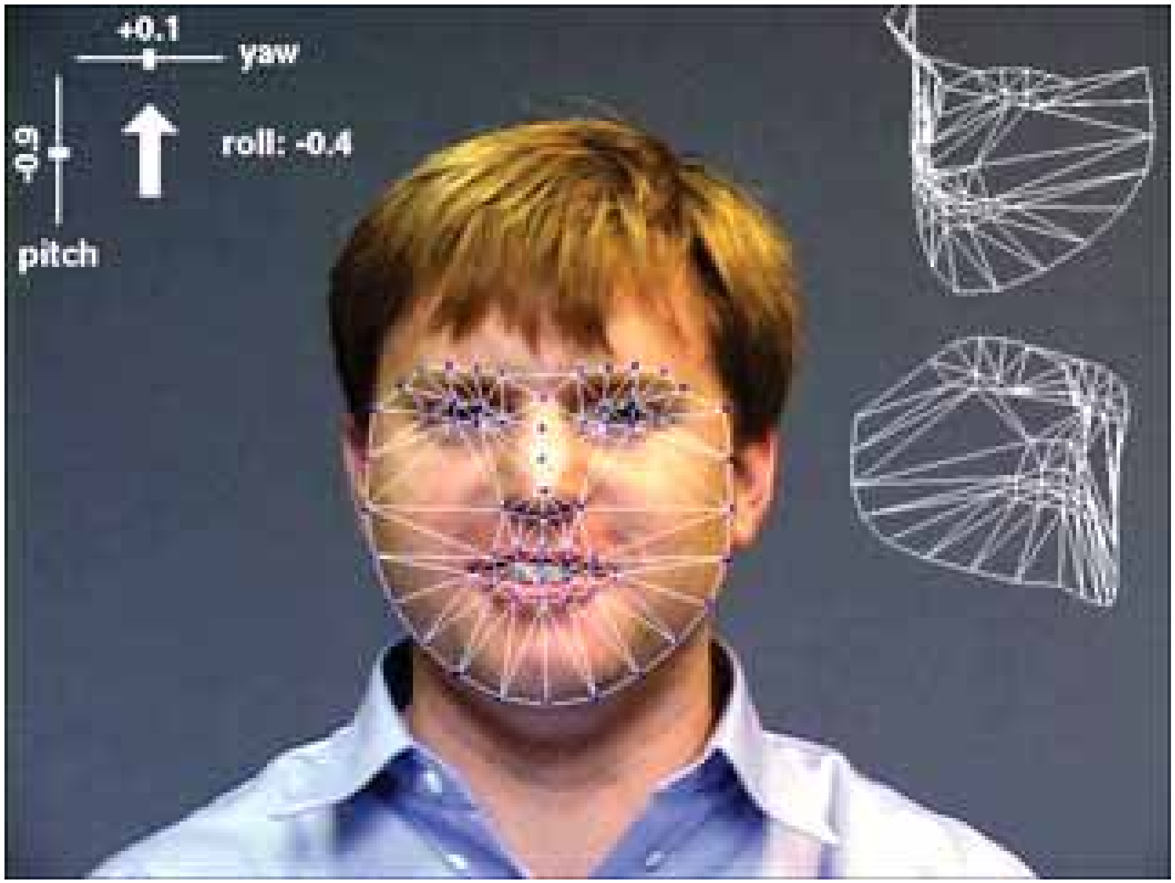


Reconstruction
Output

Disney Research, Pittsburgh

EXAMPLES OF APPLICATIONS

Tracking in 2D and 3D



Credit: Iain Matthews

Examples of Applications

Animation

Jain *et al.* SCA 2010

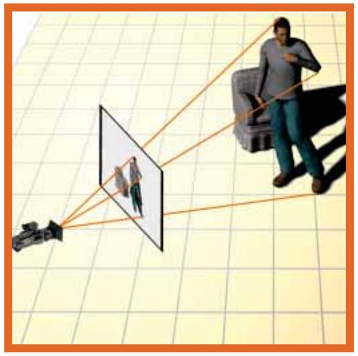
Examples of Applications

Browsing Image Collections

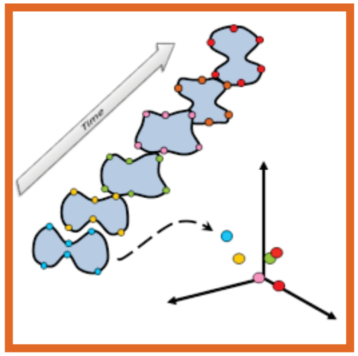
Credit: Hyun Soo Park

NONRIGID STRUCTURE FROM MOTION

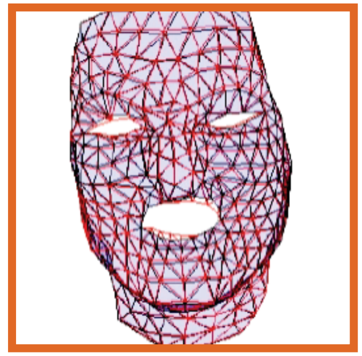
Tutorial Outline



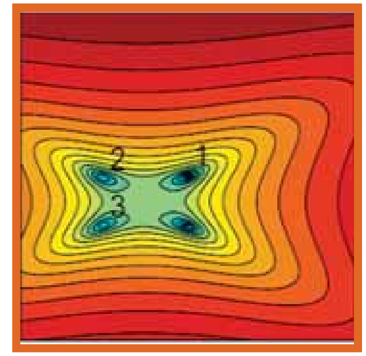
Introduction to Nonrigid SFM



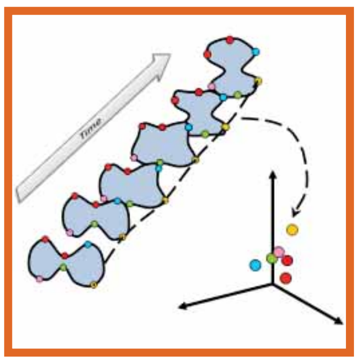
Shape Representation



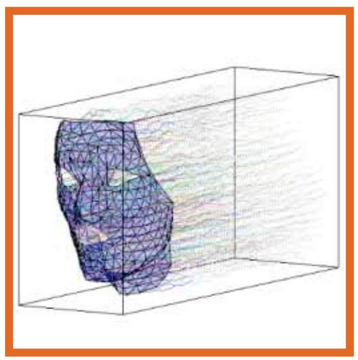
Shape Estimation



Ambiguity of Orthogonality Constraints

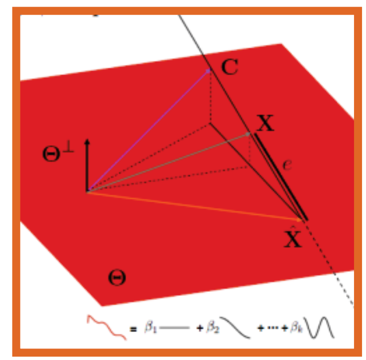


Trajectory Representation



Shape-Trajectory Duality

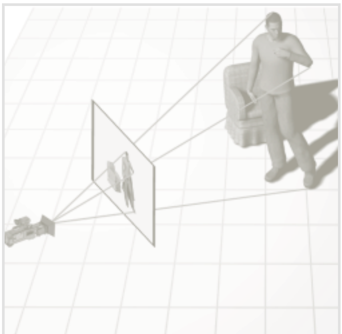
Trajectory Estimation



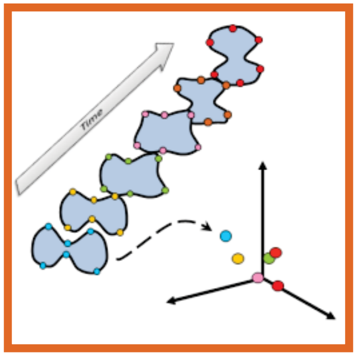
Reconstructibility and limitations

NONRIGID STRUCTURE FROM MOTION

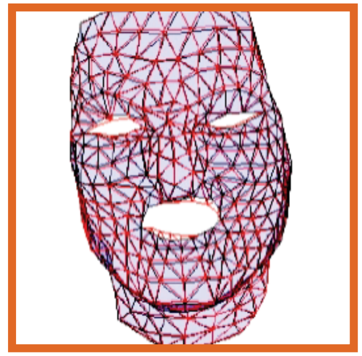
Tutorial Outline



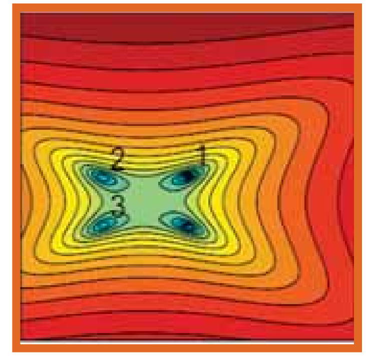
Introduction to Nonrigid SFM



Shape Representation

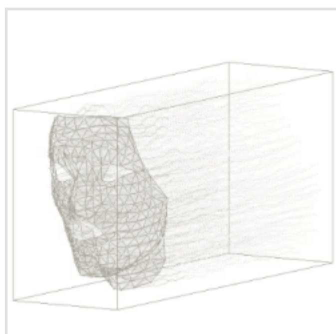


Shape Estimation



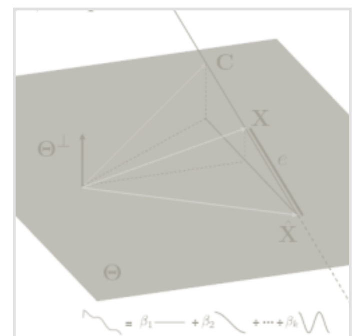
Ambiguity of Orthogonality Constraints

Trajectory Representation



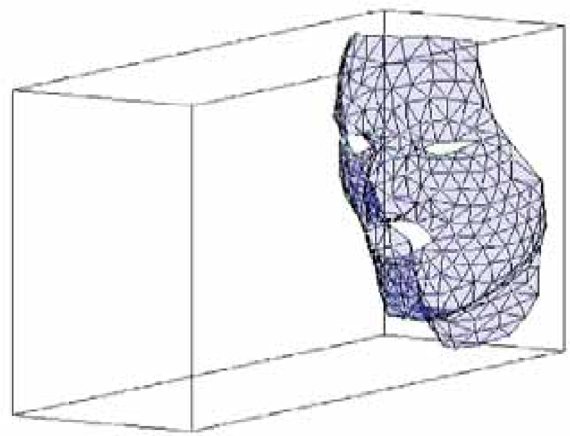
Shape-Trajectory Duality

Trajectory Estimation



Reconstructibility and limitations

DYNAMIC STRUCTURE



$$\mathbf{S}_{3F\times P} = \begin{bmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} & \cdots & \mathbf{X}_{1P} \\ \mathbf{X}_{21} & \mathbf{X}_{22} & \cdots & \mathbf{X}_{2P} \\ \vdots & \vdots & & \vdots \\ \mathbf{X}_{F1} & \mathbf{X}_{F2} & \cdots & \mathbf{X}_{FP} \end{bmatrix}$$

space

time

DYNAMIC STRUCTURE

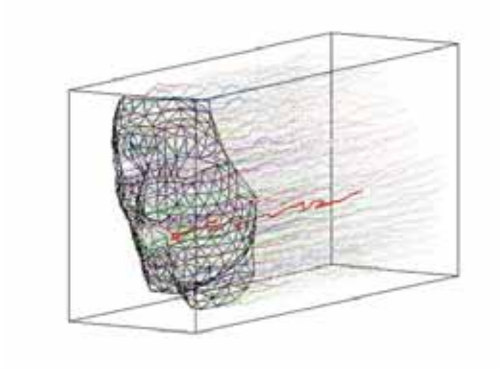
UNDER ORTHOGRAPHIC PROJECTION

$$\begin{bmatrix} \mathbf{x}_{11} & \cdots & \mathbf{x}_{1P} \\ \mathbf{x}_{21} & & \mathbf{x}_{2P} \\ \vdots & & \vdots \\ \mathbf{x}_{F1} & \cdots & \mathbf{x}_{FP} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_1 & & & \\ & \mathbf{R}_2 & & \\ & & \ddots & \\ & & & \mathbf{R}_F \end{bmatrix} \begin{bmatrix} \mathbf{X}_{11} & \cdots & \mathbf{X}_{1P} \\ \mathbf{X}_{21} & & \mathbf{X}_{2P} \\ \vdots & & \vdots \\ \mathbf{X}_{F1} & \cdots & \mathbf{X}_{FP} \end{bmatrix}$$

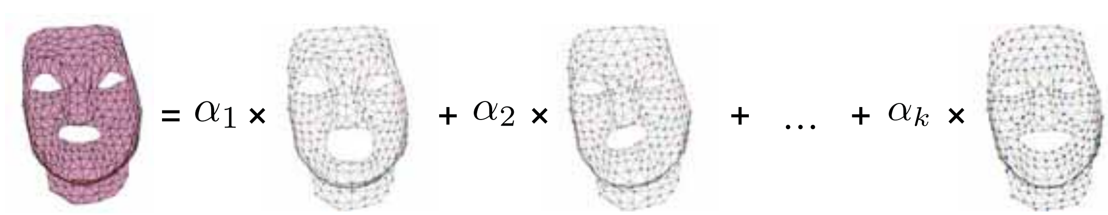
$$\mathbf{W} = \mathbf{R}\mathbf{X}$$

LINEAR SHAPE MODEL

[T. Cootes et al. 91, Bregler et al. 97]



$$\begin{bmatrix} \mathbf{X}_{11} & \cdots & \mathbf{X}_{1P} \\ \mathbf{X}_{21} & & \mathbf{X}_{2P} \\ \vdots & & \vdots \\ \mathbf{X}_{F1} & \cdots & \mathbf{X}_{FP} \end{bmatrix}$$



LINEAR SHAPE MODEL

$$\begin{bmatrix} \mathbf{X}_{11} & \cdots & \mathbf{X}_{1P} \\ \mathbf{X}_{21} & & \mathbf{X}_{2P} \\ \vdots & & \vdots \\ \mathbf{X}_{F1} & \cdots & \mathbf{X}_{FP} \end{bmatrix} = \begin{bmatrix} \omega_{11} & \cdots & \omega_{1k} \\ \omega_{21} & & \omega_{2k} \\ \vdots & & \vdots \\ \omega_{F1} & \cdots & \omega_{Fk} \end{bmatrix} \begin{bmatrix} -\mathbf{b_1}- \\ -\mathbf{b_2}- \\ \vdots \\ -\mathbf{b_k}- \end{bmatrix}$$

LINEAR SHAPE MODEL

RECONSTRUCTION

5 Basis 15 Basis 25 Basis

LINEAR SHAPE MODEL

UNDER ORTHOGRAPHIC PROJECTION

$$\underbrace{\begin{bmatrix} \mathbf{x}_{11} & \cdots & \mathbf{x}_{1P} \\ \mathbf{x}_{21} & & \mathbf{x}_{2P} \\ \vdots & & \vdots \\ \mathbf{x}_{F1} & \cdots & \mathbf{x}_{FP} \end{bmatrix}}_{2F \times P} = \underbrace{\begin{bmatrix} \mathbf{R}_1 & & & \\ & \mathbf{R}_2 & & \\ & & \ddots & \\ & & & \mathbf{R}_F \end{bmatrix}}_{2F \times 3F\ (6F)} \underbrace{\begin{bmatrix} \mathbf{X}_{11} & \cdots & \mathbf{X}_{1P} \\ \mathbf{X}_{21} & & \mathbf{X}_{2P} \\ \vdots & & \vdots \\ \mathbf{X}_{F1} & \cdots & \mathbf{X}_{FP} \end{bmatrix}}_{3F \times P}$$

$$= \underbrace{\begin{bmatrix} \mathbf{R}_1 & & & \\ & \mathbf{R}_2 & & \\ & & \ddots & \\ & & & \mathbf{R}_F \end{bmatrix}}_{2F \times 3F\ (6F)} \underbrace{\begin{bmatrix} \omega_{11} & \cdots & \omega_{1k} \\ \omega_{21} & & \omega_{2k} \\ \vdots & & \vdots \\ \omega_{F1} & \cdots & \omega_{Fk} \end{bmatrix}}_{3F \times 3k} \underbrace{\begin{bmatrix} -\mathbf{b_1}- \\ -\mathbf{b_2}- \\ \vdots \\ -\mathbf{b_k}- \end{bmatrix}}_{3k \times P}$$

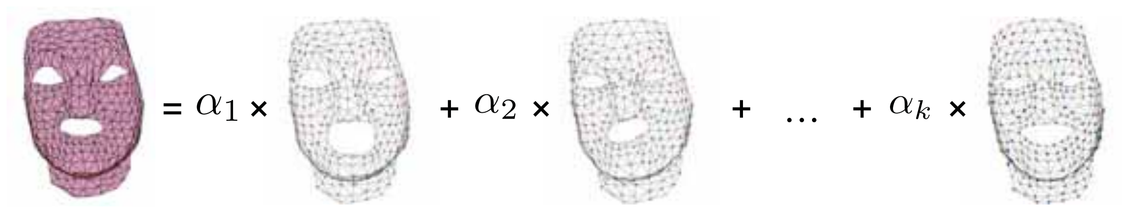
KNOWNS VS UNKNOWNS

KNOWNS: $2F \times P$

UNKNOWNS: $6F + (3F \times k) + (k \times P)$

$$2F \times P \geq 6F + (3F \times k) + (k \times P)$$

LINEAR SHAPE MODEL



RIGID COMPONENT

$$\mathbf{Y} = \mathbf{R}\left(\sum_{i=1}^{K} \omega_i \mathbf{b}_i\right) + \mathbf{T}$$

NONRIGID COMPONENT

IDEA: RIGID COMPONENT GETS FOLDED INTO PROJECTION

CHALLENGE

TRILINEAR ESTIMATION

$$\begin{bmatrix} \mathbf{R}_1 & & & \\ & \mathbf{R}_2 & & \\ & & \ddots & \\ & & & \mathbf{R}_F \end{bmatrix} \begin{bmatrix} \omega_{11} & \cdots & \omega_{1k} \\ \omega_{21} & & \omega_{2k} \\ \vdots & & \vdots \\ \omega_{F1} & \cdots & \omega_{Fk} \end{bmatrix} \begin{bmatrix} -\mathbf{b_1}- \\ -\mathbf{b_2}- \\ \vdots \\ -\mathbf{b_k}- \end{bmatrix}$$

$$\mathbf{W} = \mathbf{R \Omega B}$$

BREGLER *et al.* 2000

Nested SVD

$$\begin{bmatrix} \mathbf{x}_{11} & \cdots & \mathbf{x}_{1P} \\ \mathbf{x}_{21} & & \mathbf{x}_{2P} \\ \vdots & & \vdots \\ \mathbf{x}_{F1} & \cdots & \mathbf{x}_{FP} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_1 & & & \\ & \mathbf{R}_2 & & \\ & & \ddots & \\ & & & \mathbf{R}_F \end{bmatrix} \begin{bmatrix} \omega_{11} & \cdots & \omega_{1k} \\ \omega_{21} & & \omega_{2k} \\ \vdots & & \vdots \\ \omega_{F1} & \cdots & \omega_{Fk} \end{bmatrix} \begin{bmatrix} -\mathbf{b_1}- \\ -\mathbf{b_2}- \\ \vdots \\ -\mathbf{b_k}- \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \omega_{11}\mathbf{R}_1 & \cdots & \omega_{1k}\mathbf{R}_1 \\ \omega_{21}\mathbf{R}_2 & & \omega_{2k}\mathbf{R}_2 \\ \vdots & & \vdots \\ \omega_{F1}\mathbf{R}_F & \cdots & \omega_{Fk}\mathbf{R}_F \end{bmatrix}}_{2F \times 3k} \underbrace{\begin{bmatrix} -\mathbf{b}_1- \\ -\mathbf{b}_2- \\ \vdots \\ -\mathbf{b}_k- \end{bmatrix}}_{3k \times P}$$

BREGLER *et al.* 2000

Outer SVD

$$\underset{\textstyle\left[\begin{array}{ccc}\mathbf{x}_{11} & \cdots & \mathbf{x}_{1P}\\ \mathbf{x}_{21} & & \mathbf{x}_{2P}\\ \vdots & & \vdots\\ \mathbf{x}_{F1} & \cdots & \mathbf{x}_{FP}\end{array}\right]}{\mathbf{W}} = \underset{2F \times 3k}{\underbrace{\overset{\mathbf{H}}{\left[\begin{array}{ccc}\omega_{11}\mathbf{R}_1 & \cdots & \omega_{1k}\mathbf{R}_1\\ \omega_{21}\mathbf{R}_2 & & \omega_{2k}\mathbf{R}_2\\ \vdots & & \vdots\\ \omega_{F1}\mathbf{R}_F & \cdots & \omega_{Fk}\mathbf{R}_F\end{array}\right]}}} \underset{3k \times P}{\underbrace{\overset{\mathbf{B}}{\left[\begin{array}{c}-\mathbf{b}_1-\\ -\mathbf{b}_2-\\ \vdots\\ -\mathbf{b}_k-\end{array}\right]}}}$$

SVD

$$\mathbf{W} = \mathbf{U}\mathbf{D}\mathbf{V}^T$$

$$\mathbf{W} = (\mathbf{U}\mathbf{D}^{\frac{1}{2}})(\mathbf{D}^{\frac{1}{2}}\mathbf{V}^T)$$

$$\mathbf{W} = \hat{\mathbf{H}}\hat{\mathbf{B}}$$

BREGLER *et al.* 2000

Inner SVD

$$\mathbf{W} = \hat{\mathbf{H}}\hat{\mathbf{B}}$$

$$\mathbf{H} = \begin{bmatrix} \omega_{11}\mathbf{R}_1 & \cdots & \omega_{1k}\mathbf{R}_1 \\ \omega_{21}\mathbf{R}_2 & & \omega_{2k}\mathbf{R}_2 \\ \vdots & & \vdots \\ \omega_{F1}\mathbf{R}_F & \cdots & \omega_{Fk}\mathbf{R}_1 \end{bmatrix}$$

$$\mathbf{h}_1 = \begin{bmatrix} \omega_{11}r_1^1 & \omega_{11}r_1^2 & \omega_{11}r_1^3 & \cdots & \omega_{1k}r_1^1 & \omega_{1k}r_1^2 & \omega_{1k}r_1^3 \\ \omega_{11}r_1^4 & \omega_{11}r_1^5 & \omega_{11}r_1^6 & \cdots & \omega_{1k}r_1^4 & \omega_{1k}r_1^5 & \omega_{1k}r_1^6 \end{bmatrix}$$

$$\mathbf{h}'_1 = \underbrace{\begin{bmatrix} \omega_{11}r_1^1 & \omega_{11}r_1^2 & \omega_{11}r_1^3 & \omega_{11}r_1^4 & \omega_{11}r_1^5 & \omega_{11}r_1^6 \\ \omega_{12}r_1^1 & \omega_{12}r_1^2 & \omega_{12}r_1^3 & \omega_{12}r_1^4 & \omega_{12}r_1^5 & \omega_{12}r_1^6 \\ \vdots & & & & & \vdots \\ \omega_{1k}r_1^1 & \omega_{1k}r_1^2 & \omega_{1k}r_1^3 & \omega_{1k}r_1^4 & \omega_{1k}r_1^5 & \omega_{1k}r_1^6 \end{bmatrix}}_{\text{rank 1}} = \begin{bmatrix} \omega_{11} \\ \omega_{12} \\ \vdots \\ \omega_{1k} \end{bmatrix} \begin{bmatrix} r_1^1 & r_1^2 & r_1^3 & r_1^4 & r_1^5 & r_1^6 \end{bmatrix}$$

SVD $\mathbf{h}'_1 = \mathbf{u}\mathbf{d}\mathbf{v}^T = \hat{\omega}\hat{\mathbf{r}}$

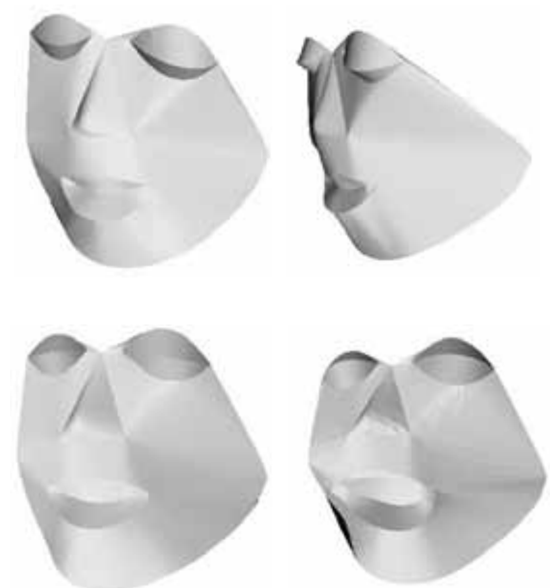
METRIC RECTIFICATION USING ORTHONORMALITY CONSTRAINTS

BREGLER *et al.* 2000

OVERVIEW

- OUTER SVD: PERFORM SVD ON **W** TO GET ESTIMATES OF:
  - **H**: CAMERA PROJECTIONS AND COEFFICIENTS
    - INNER SVD: PERFORM SVD ON **H** TO GET ESTIMATES OF:
      - OMEGA: COEFFICIENTS
      - **R**: CAMERA PROJECTIONS
    - METRIC RECTIFY USING ORTHONORMALITY CONSTRAINTS
  - **B**: THE SHAPE BASIS

RESULTS



BREGLER *et al.* 2000
IN PERSPECTIVE

- **SEMINAL WORK**: SHOWED THAT FACTORIZATION METHODS CAN BE APPLIED TO NONRIGID OBJECTS
- **CASCADING ERROR**: ANY OUTER SVD ESTIMATION ERROR CASCADES INTO INNER SVD ESTIMATION
- **AMBIGUITY ERROR**: ESTIMATION OF METRIC RECTIFICATION
- **NUMBER OF BASIS**: LARGE NUMBER OF BASIS REQUIRED
- **MISSING DATA**: NEEDS COMPLETE **W** MATRIX

METRIC RECTIFICATION

AMBIGUITY

$$\mathbf{W} = \hat{\mathbf{H}}\hat{\mathbf{B}}$$

$$\mathbf{W} = \underline{\hat{\mathbf{H}}\mathbf{G}}\;\underline{\mathbf{G}^{-1}\hat{\mathbf{B}}}$$

$$\mathbf{H} = \hat{\mathbf{H}}\mathbf{G} \qquad \mathbf{B} = \mathbf{G}^{-1}\hat{\mathbf{B}}$$

$$\mathbf{H} = \begin{bmatrix} & \hat{\mathbf{H}} & \end{bmatrix} \underbrace{\begin{bmatrix} | & | & & | \\ \mathbf{g}_1 & \mathbf{g}_2 & \cdots & \mathbf{g}_k \\ | & | & & | \end{bmatrix}}_{\mathbf{G}_{3k \times 3k}} = \begin{bmatrix} \omega_{11}\mathbf{R}_1 & \cdots & \omega_{1k}\mathbf{R}_1 \\ \omega_{21}\mathbf{R}_2 & & \omega_{2k}\mathbf{R}_2 \\ \vdots & & \vdots \\ \omega_{F1}\mathbf{R}_F & \cdots & \omega_{Fk}\mathbf{R}_1 \end{bmatrix}$$

METRIC RECTIFICATION

ORTHONORMALITY CONSTRAINT

$$\mathbf{H} = \begin{bmatrix} & \hat{\mathbf{H}} & \end{bmatrix} \begin{bmatrix} | & | & & | \\ \mathbf{g}_1 & \mathbf{g}_2 & \cdots & \mathbf{g}_k \\ | & | & & | \end{bmatrix} = \begin{bmatrix} \omega_{11}\mathbf{R}_1 & \cdots & \omega_{1k}\mathbf{R}_1 \\ \omega_{21}\mathbf{R}_2 & & \omega_{2k}\mathbf{R}_2 \\ \vdots & & \vdots \\ \omega_{F1}\mathbf{R}_F & \cdots & \omega_{Fk}\mathbf{R}_1 \end{bmatrix}$$

$$\mathbf{R}_i\mathbf{R}_i^T = \mathbf{I}$$

ORTHONORMALITY CONSTRAINT

$$\begin{bmatrix} & \hat{\mathbf{H}} & \end{bmatrix} \begin{bmatrix} | \\ \mathbf{g}_k \\ | \end{bmatrix} = \begin{bmatrix} \omega_{1k}\mathbf{R}_1 \\ \omega_{2k}\mathbf{R}_2 \\ \vdots \\ \omega_{Fk}\mathbf{R}_F \end{bmatrix}$$

$$\omega_{ik}\mathbf{R}_i = \hat{\mathbf{H}}_{2i-1:2i}\mathbf{g}_k$$

$$\mathbf{H}_{2i-1:2i}\mathbf{g}_k\mathbf{g}_k^T\hat{\mathbf{H}}_{2i-1:2i} = \omega_{ik}^2\mathbf{I}$$

METRIC RECTIFICATION

ORTHONORMALITY CONSTRAINT

$$\mathbf{R}_i \mathbf{R}_i^T = \mathbf{I}$$

ORTHONORMALITY CONSTRAINT

$$\mathbf{H}_{2i-1:2i} \mathbf{g}_k \mathbf{g}_k^T \hat{\mathbf{H}}_{2i-1:2i} = \omega_{ik}^2 \mathbf{I} = \begin{bmatrix} \omega_{ik}^2 & 0 \\ 0 & \omega_{ik}^2 \end{bmatrix}$$

$$\mathbf{H}_{2i} \mathbf{g}_k \mathbf{g}_k^T \hat{\mathbf{H}}_{2i-1} = \mathbf{0}$$

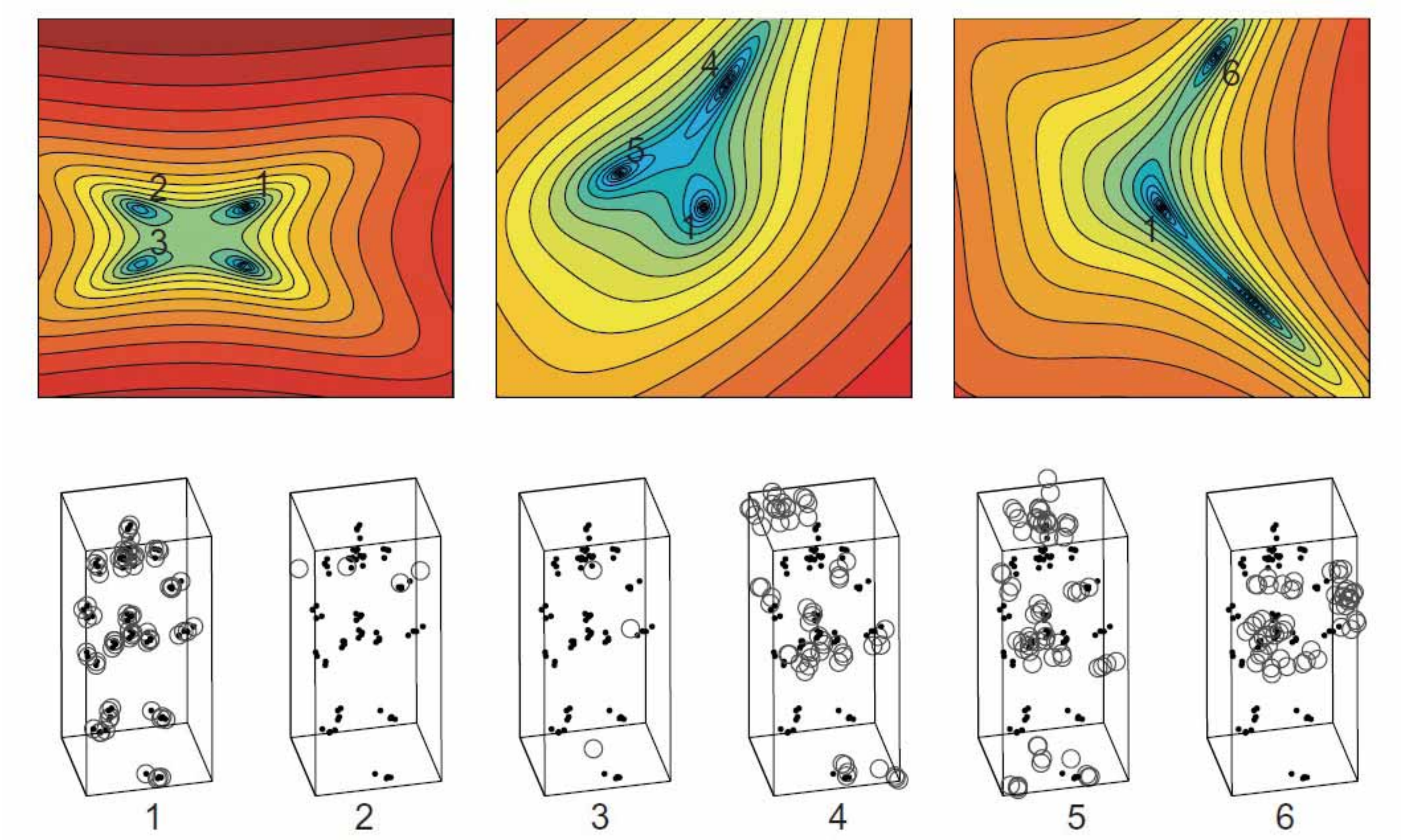
$$\mathbf{H}_{2i-1} \mathbf{g}_k \mathbf{g}_k^T \hat{\mathbf{H}}_{2i-1} = \omega_{ik}^2 \qquad \mathbf{H}_{2i} \mathbf{g}_k \mathbf{g}_k^T \hat{\mathbf{H}}_{2i} = \omega_{ik}^2$$

$$\mathbf{H}_{2i-1} \mathbf{g}_k \mathbf{g}_k^T \hat{\mathbf{H}}_{2i-1} = \mathbf{H}_{2i} \mathbf{g}_k \mathbf{g}_k^T \hat{\mathbf{H}}_{2i}$$

CHALLENGE?

AMBIGUITY

OPTIMIZATION



CHALLENGE

MISSING DATA

- A.M. Buchanan and A.W. Fitzgibbon, "Damped Newton Algorithms for Matrix Factorization with Missing Data," IEEE International Conference on Computer Vision and Pattern Recognition, 2005.
- L. Torresani, A. Hertzmann, and Christoph Bregler, "Nonrigid Structure-from-Motion: Estimating Shape and Motion with Hierarchical Priors," Transactions on Pattern Analysis and Machine Intelligence, 2008.
- SPANISH FOLKS
- CVPR 2010 BEST PAPER
- BRANCH AND BOUND

CHALLENGES

OVERVIEW

- MISSING DATA
- BEST *K*
- TRILINEAR OPTIMIZATION

LINEAR SHAPE MODEL

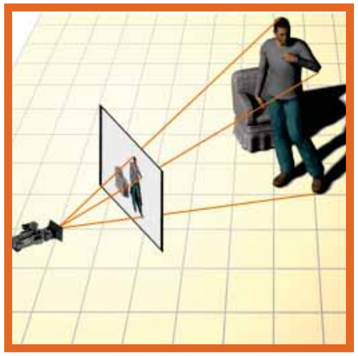
PERSPECTIVE PROJECTION

LINEAR SHAPE MODEL

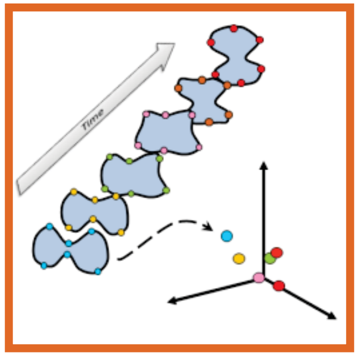
MAXIMUM LIKELIHOOD SOLUTION

Nonrigid Structure From Motion

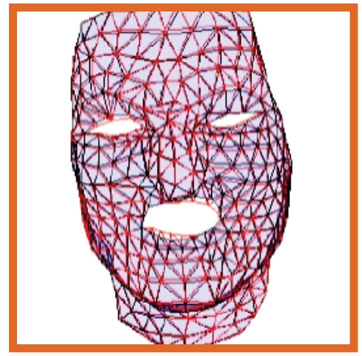
Tutorial Outline



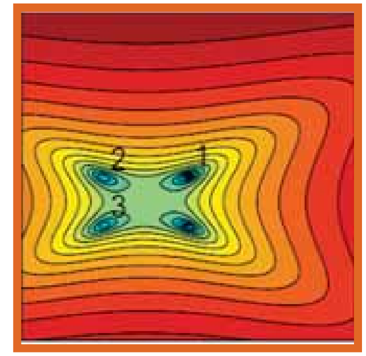
Introduction to Nonrigid SFM



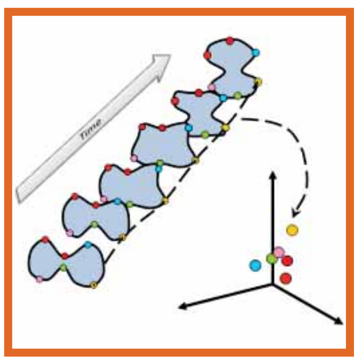
Shape Representation



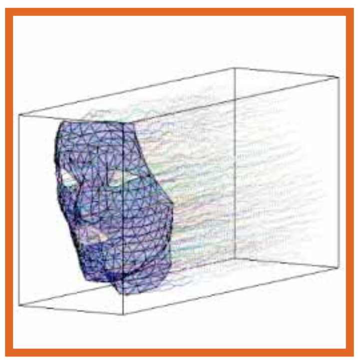
Shape Estimation



Ambiguity of Orthogonality Constraints

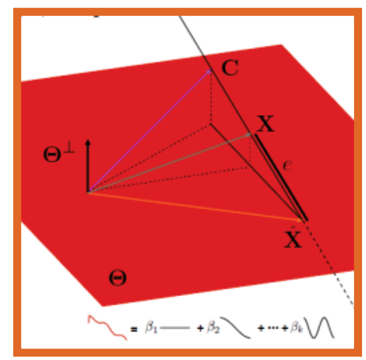


Trajectory Representation



Shape-Trajectory Duality

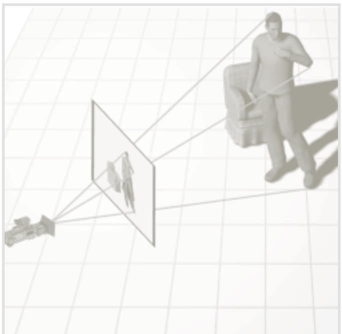
Trajectory Estimation



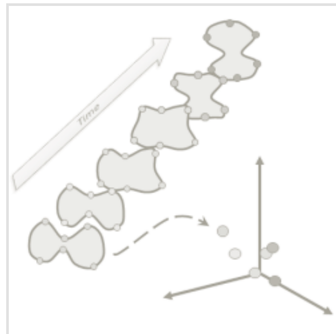
Reconstructibility and limitations

NONRIGID STRUCTURE FROM MOTION

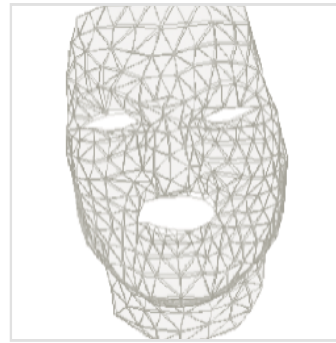
Tutorial Outline



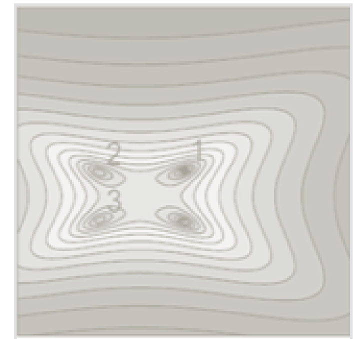
Introduction to Nonrigid SFM



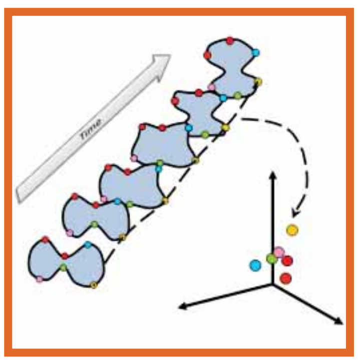
Shape Representation



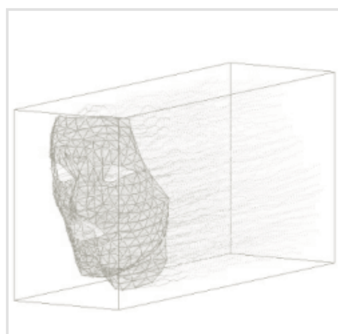
Shape Estimation



Ambiguity of Orthogonality Constraints

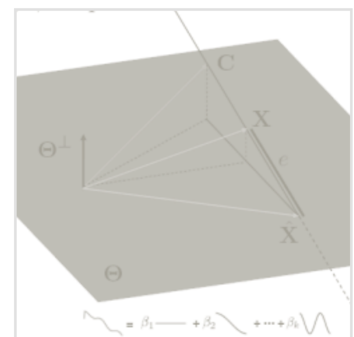


Trajectory Representation



Shape-Trajectory Duality

Trajectory Estimation



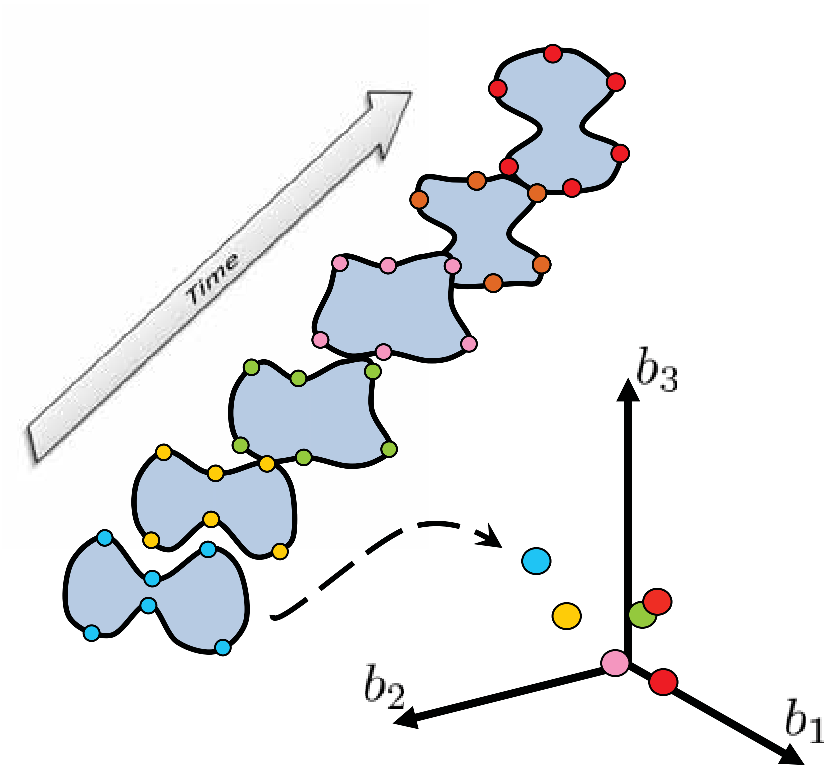
Reconstructibility and limitations

Nonrigid Structure From Motion

Two Major Approaches

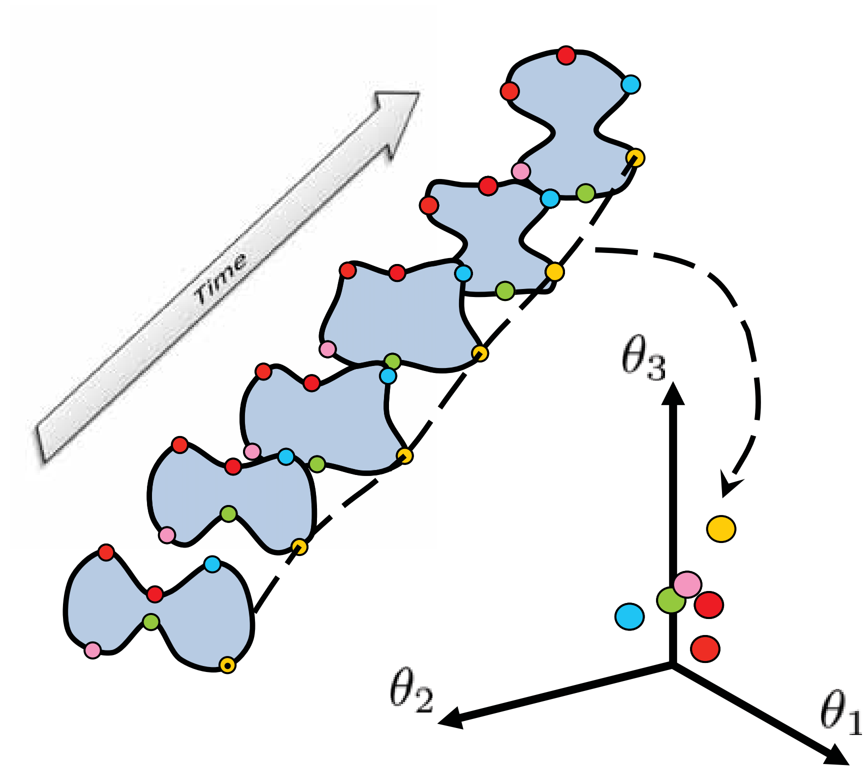
Shape Basis

3D points at each time instant lie in a low dimensional subspace

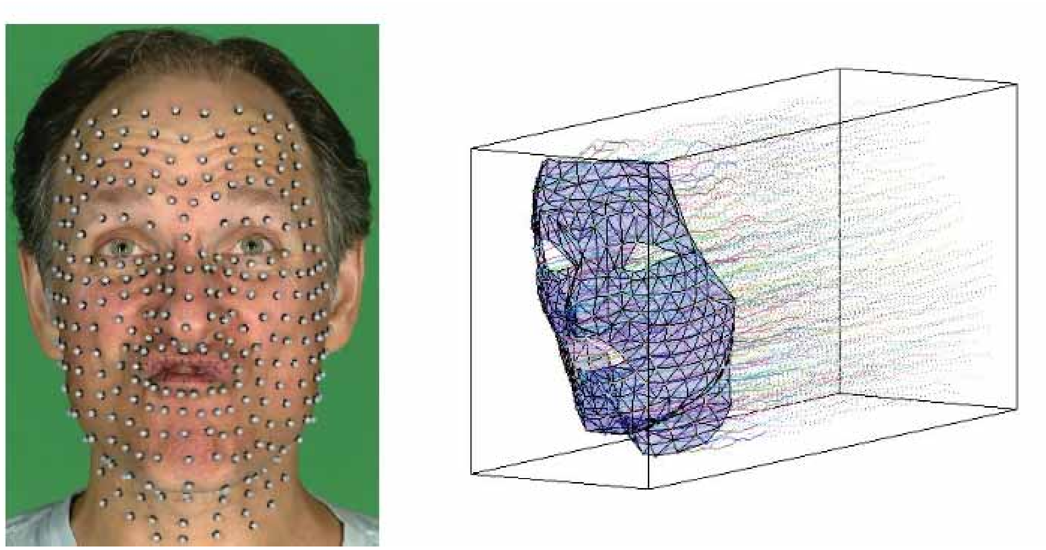


Trajectory Basis

Trajectory of each point over time lies in a low dimensional subspace



DYNAMIC STRUCTURE



$$\mathbf{S}_{3F\times P} = \begin{bmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} & \cdots & \mathbf{X}_{1P} \\ \mathbf{X}_{21} & \mathbf{X}_{22} & \cdots & \mathbf{X}_{2P} \\ \vdots & \vdots & & \vdots \\ \mathbf{X}_{F1} & \mathbf{X}_{F2} & \cdots & \mathbf{X}_{FP} \end{bmatrix}$$

Trajectory

Shape

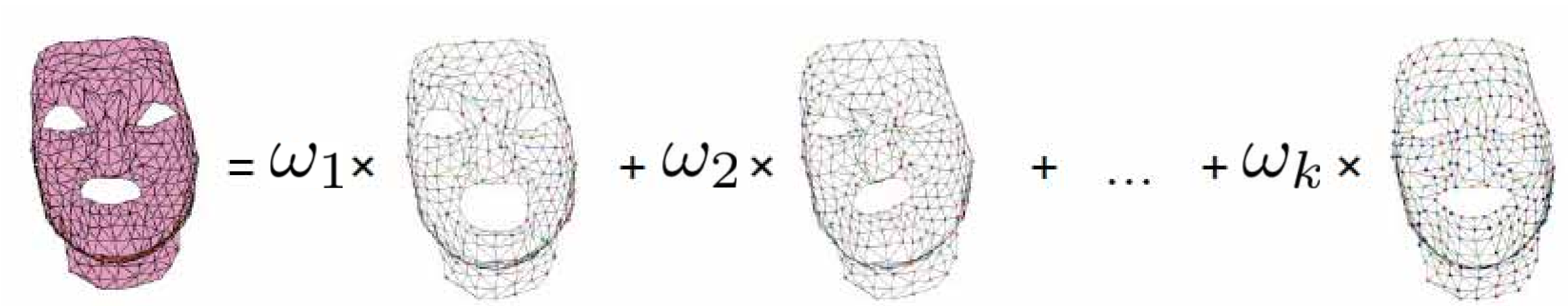
DYNAMIC STRUCTURE

Shape Representation

$$\mathbf{S}_{3F\times P} = \begin{bmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} & \cdots & \mathbf{X}_{1P} \\ \mathbf{X}_{21} & \mathbf{X}_{22} & \cdots & \mathbf{X}_{2P} \\ \vdots & \vdots & & \vdots \\ \mathbf{X}_{F1} & \mathbf{X}_{F2} & \cdots & \mathbf{X}_{FP} \end{bmatrix}$$

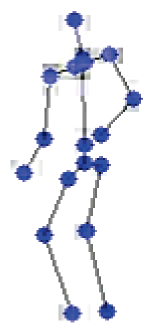
Shape

LINEAR SHAPE MODEL



DYNAMIC STRUCTURE

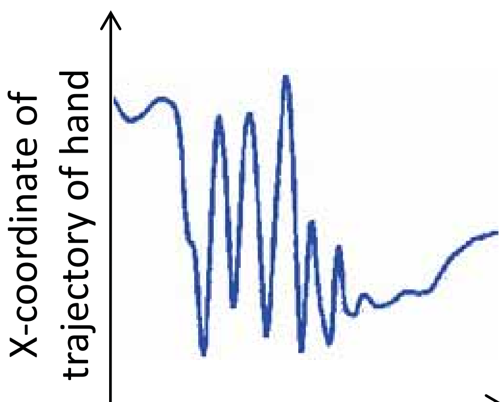
Trajectory Representation

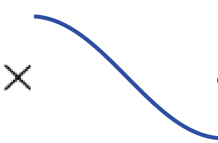
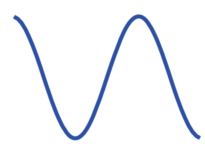


$$\mathbf{S}_{3F\times P} = \begin{bmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} & \cdots & \mathbf{X}_{1P} \\ \mathbf{X}_{21} & \mathbf{X}_{22} & \cdots & \mathbf{X}_{2P} \\ \vdots & \vdots & & \vdots \\ \mathbf{X}_{F1} & \mathbf{X}_{F2} & \cdots & \mathbf{X}_{FP} \end{bmatrix}$$

Trajectory

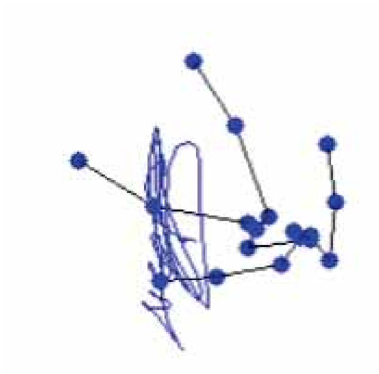
LINEAR TRAJECTORY MODEL



$= a_1 \times$ $+ a_2 \times$  $+ \ldots + a_k \times$ 

Dynamic Structure

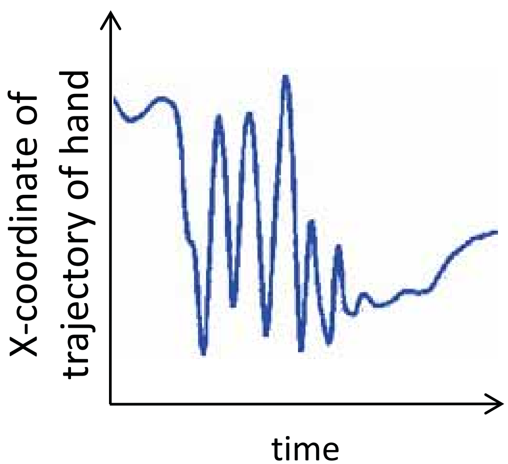
Trajectory Representation



$$\mathbf{S}_{3F\times P} = \begin{bmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} & \cdots & \mathbf{X}_{1P} \\ \mathbf{X}_{21} & \mathbf{X}_{22} & \cdots & \mathbf{X}_{2P} \\ \vdots & \vdots & & \vdots \\ \mathbf{X}_{F1} & \mathbf{X}_{F2} & \cdots & \mathbf{X}_{FP} \end{bmatrix}$$

Trajectory

LINEAR TRAJECTORY MODEL



$$T_j^X = \sum_{k=1}^{K} a_{jk}^X \theta^k$$

Trajectory Coefficient
Contribution of k^{th} basis in the trajectory of j^{th} point

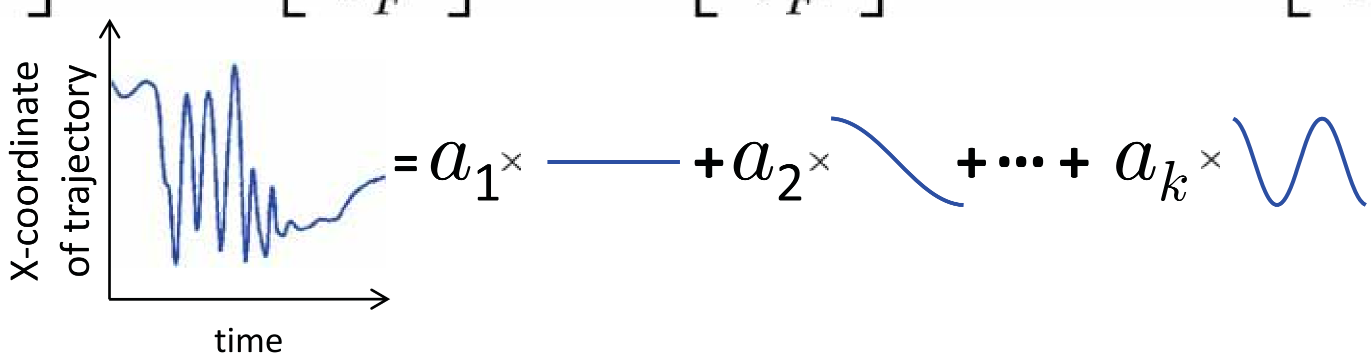
k^{th} trajectory basis vector

Trajectory of j^{th} point (X-component only)

Trajectory Representation of Dynamic Structure

$$T_j^X = \sum_{k=1}^{K} a_{jk}^X \theta^k \qquad T_j^Y = \sum_{k=1}^{K} a_{jk}^Y \theta^k \qquad T_j^Z = \sum_{k=1}^{K} a_{jk}^Z \theta^k$$

$$\begin{bmatrix} X_{1j} \\ X_{2j} \\ \vdots \\ X_{Fj} \end{bmatrix} = a_{j1}^X \begin{bmatrix} \theta_1^1 \\ \theta_2^1 \\ \vdots \\ \theta_F^1 \end{bmatrix} + a_{j2}^X \begin{bmatrix} \theta_1^2 \\ \theta_2^2 \\ \vdots \\ \theta_F^2 \end{bmatrix} + \ldots + a_{jK}^X \begin{bmatrix} \theta_1^K \\ \theta_2^K \\ \vdots \\ \theta_F^K \end{bmatrix}$$



Trajectory Representation of Dynamic Structure

$$\begin{bmatrix} X_{1j} \\ X_{2j} \\ \vdots \\ X_{Fj} \end{bmatrix} = a_{j1}^X \begin{bmatrix} \theta_1^1 \\ \theta_2^1 \\ \vdots \\ \theta_F^1 \end{bmatrix} + a_{j2}^X \begin{bmatrix} \theta_1^2 \\ \theta_2^2 \\ \vdots \\ \theta_F^2 \end{bmatrix} + \ldots + a_{jK}^X \begin{bmatrix} \theta_1^K \\ \theta_2^K \\ \vdots \\ \theta_F^K \end{bmatrix}$$

X-component of trajectory of jth point as linear combination of K basis trajectories

X-component of trajectory of **all** point as linear combination of K basis trajectories

$$\begin{bmatrix} X_{11} & X_{12} & \ldots & X_{1P} \\ X_{21} & X_{22} & \ldots & X_{2P} \\ \vdots & \vdots & \vdots & \vdots \\ X_{F1} & X_{F2} & \ldots & X_{FP} \end{bmatrix} = \begin{bmatrix} \theta_1^1 & \theta_1^2 & \ldots & \theta_1^K \\ \theta_2^1 & \theta_2^2 & \ldots & \theta_2^K \\ \vdots & \vdots & \vdots & \vdots \\ \theta_F^1 & \theta_F^2 & \ldots & \theta_F^K \end{bmatrix} \begin{bmatrix} a_{11}^X & a_{21}^X & \ldots & a_{P1}^X \\ a_{12}^X & a_{22}^X & \ldots & a_{P2}^X \\ \vdots & \vdots & \vdots & \vdots \\ a_{1K}^X & a_{2K}^X & \ldots & a_{PK}^X \end{bmatrix}$$

$$\underset{F \times P}{\mathbf{S}^X} = \underset{F \times K}{\mathbf{\Theta}^X} \times \underset{K \times P}{\mathbf{A}^X}$$

X-component of trajectory of all points

$$
\begin{bmatrix}
X_{11} & X_{12} & \dots & X_{1P} \\
X_{21} & X_{22} & \dots & X_{2P} \\
\vdots & \vdots & \vdots & \vdots \\
X_{F1} & X_{F2} & \dots & X_{FP}
\end{bmatrix}
=
\begin{bmatrix}
\theta_1^1 & \theta_1^2 & \dots & \theta_1^K \\
\theta_2^1 & \theta_2^2 & \dots & \theta_2^K \\
\vdots & \vdots & \vdots & \vdots \\
\theta_F^1 & \theta_F^2 & \dots & \theta_F^K
\end{bmatrix}
\begin{bmatrix}
a_{11}^X & a_{21}^X & \dots & a_{P1}^X \\
a_{12}^X & a_{22}^X & \dots & a_{P2}^X \\
\vdots & \vdots & \vdots & \vdots \\
a_{1K}^X & a_{2K}^X & \dots & a_{PK}^X
\end{bmatrix}
$$

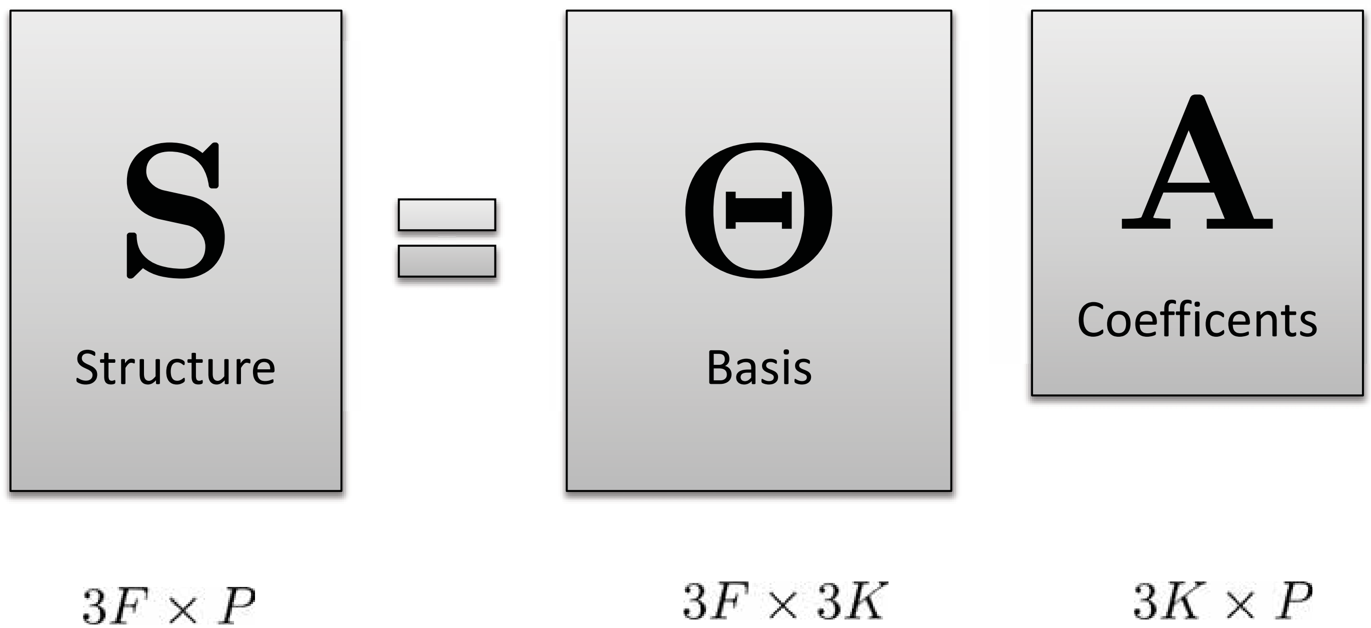
X, Y and Z-components of trajectory of all points

$$
\begin{bmatrix}
X_{11} & \dots & X_{1P} \\
Y_{11} & \dots & Y_{1P} \\
Z_{11} & \dots & Z_{1P} \\
X_{21} & \dots & X_{2P} \\
Y_{21} & \dots & Y_{2P} \\
Z_{21} & \dots & Z_{2P} \\
\vdots & & \vdots \\
X_{F1} & \dots & X_{FP} \\
Y_{F1} & \dots & Y_{FP} \\
Z_{F1} & \dots & Z_{FP}
\end{bmatrix}
=
\begin{bmatrix}
\theta_1^1 \dots \theta_1^K & & \\
& \theta_1^1 \dots \theta_1^K & \\
& & \theta_1^1 \dots \theta_1^K \\
\theta_2^1 \dots \theta_2^K & & \\
& \theta_2^1 \dots \theta_2^K & \\
& & \theta_2^1 \dots \theta_2^K \\
\vdots & \vdots & \vdots \\
\theta_F^1 \dots \theta_F^K & & \\
& \theta_F^1 \dots \theta_F^K & \\
& & \theta_F^1 \dots \theta_F^K
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
a_{11}^X & a_{21}^X & \dots & a_{P1}^X \\
\vdots & \vdots & \vdots & \vdots \\
a_{1K}^X & a_{2K}^X & \dots & a_{PK}^X
\end{bmatrix}_{A^X} \\
\begin{bmatrix}
a_{11}^Y & a_{21}^Y & \dots & a_{P1}^Y \\
\vdots & \vdots & \vdots & \vdots \\
a_{1K}^Y & a_{2K}^Y & \dots & a_{PK}^Y
\end{bmatrix}_{A^Y} \\
\begin{bmatrix}
a_{11}^Z & a_{21}^Z & \dots & a_{P1}^Z \\
\vdots & \vdots & \vdots & \vdots \\
a_{1K}^Z & a_{2K}^Z & \dots & a_{PK}^Z
\end{bmatrix}_{A^Z}
\end{bmatrix}
$$

$$
\mathbf{S}_{3F\times P} = \Theta_{3F\times 3K} \qquad \mathbf{A}_{3K\times P}
$$

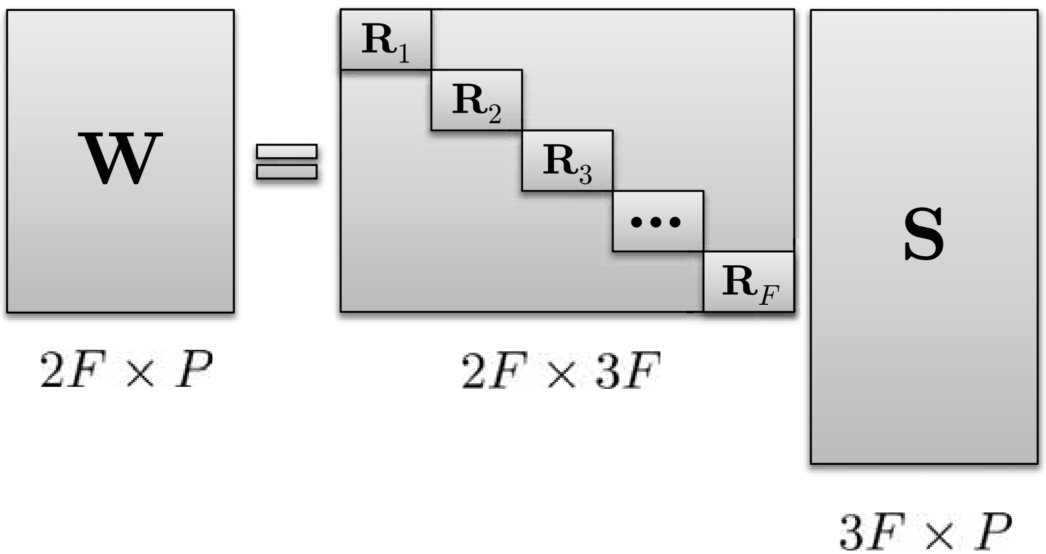
TRAJECTORY REPRESENTATION

of Dynamic Structure



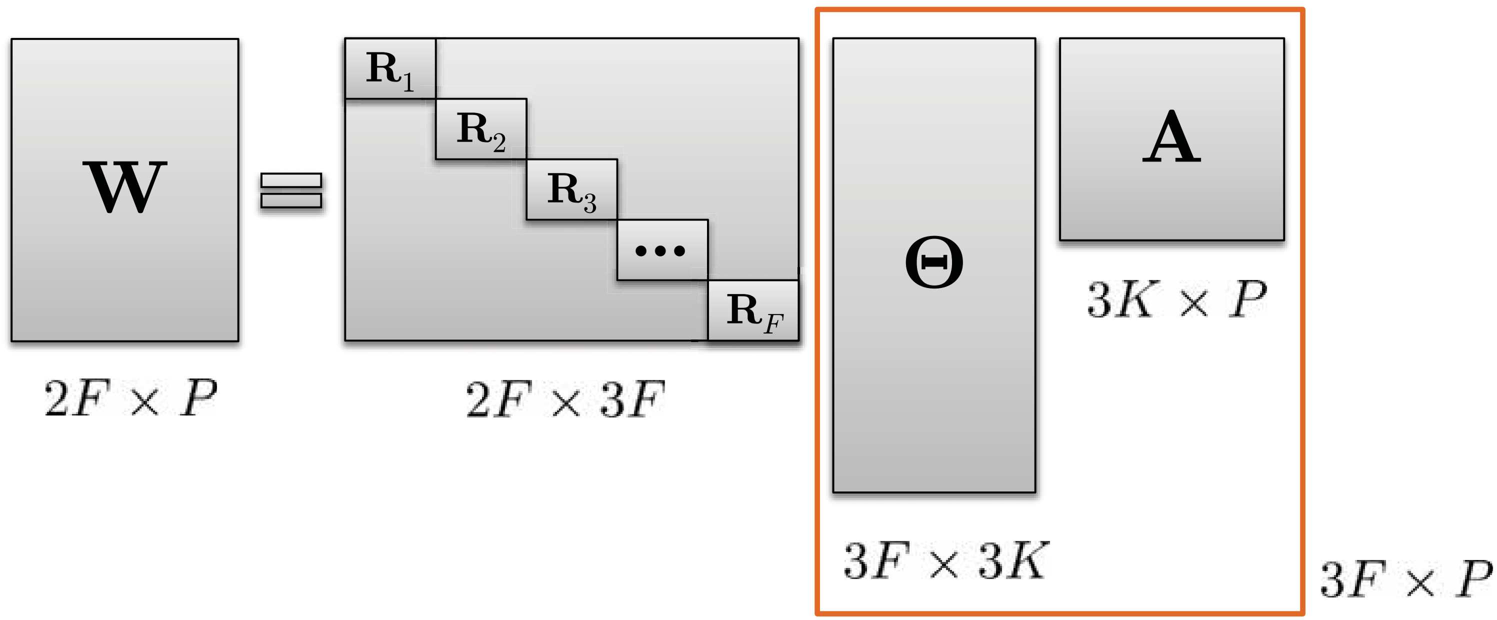
TRAJECTORY REPRESENTATION

of Dynamic Structure *Under Orthographic Projection*



Trajectory Representation

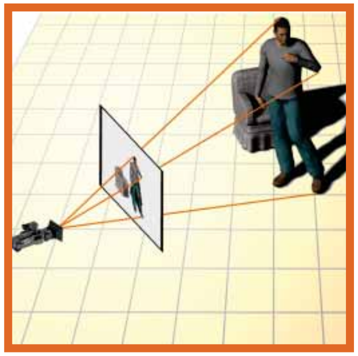
of Dynamic Structure *Under Orthographic Projection*



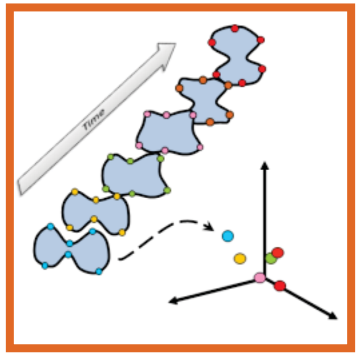
Structure $\mathbf{S}$, in trajectory subspace represented by K trajectory basis

NONRIGID STRUCTURE FROM MOTION

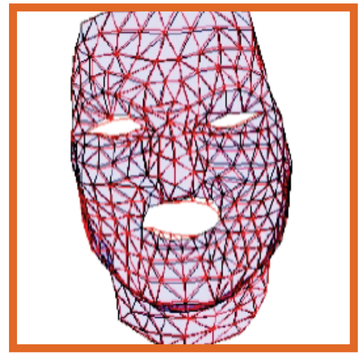
Tutorial Outline



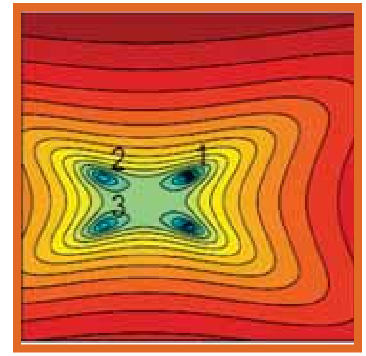
Introduction to Nonrigid SFM



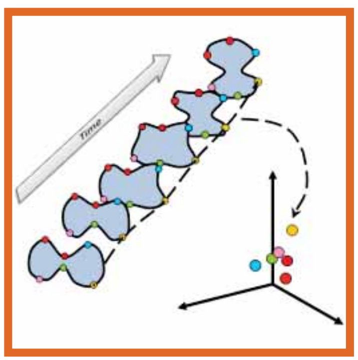
Shape Representation



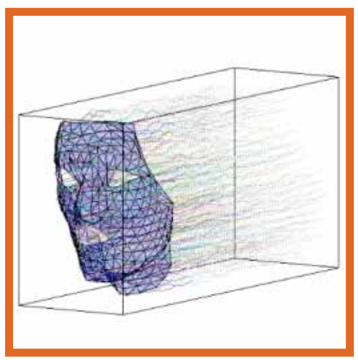
Shape Estimation



Ambiguity of Orthogonality Constraints

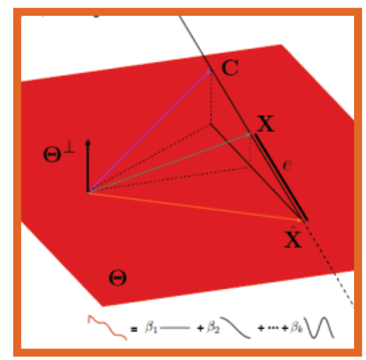


Trajectory Representation



Shape-Trajectory Duality

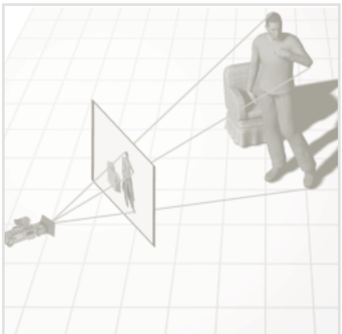
Trajectory Estimation



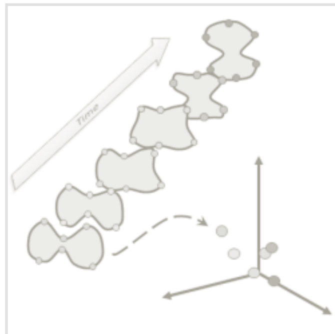
Reconstructibility and limitations

Nonrigid Structure From Motion

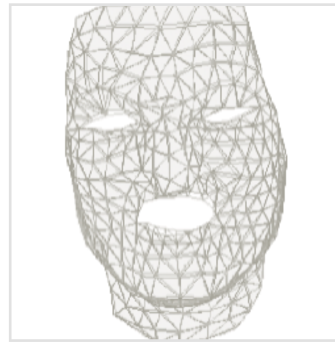
Tutorial Outline



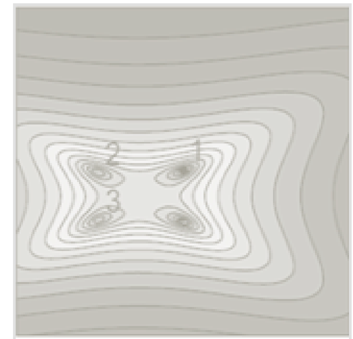
Introduction to Nonrigid SFM



Shape Representation

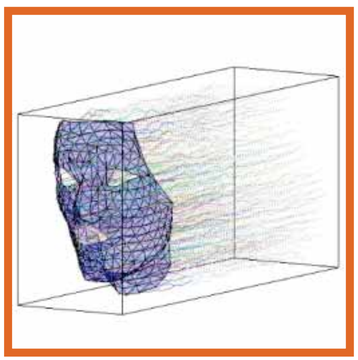


Shape Estimation



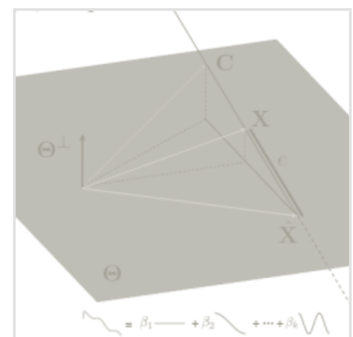
Ambiguity of Orthogonality Constraints

Trajectory Representation



Shape-Trajectory Duality

Trajectory Estimation

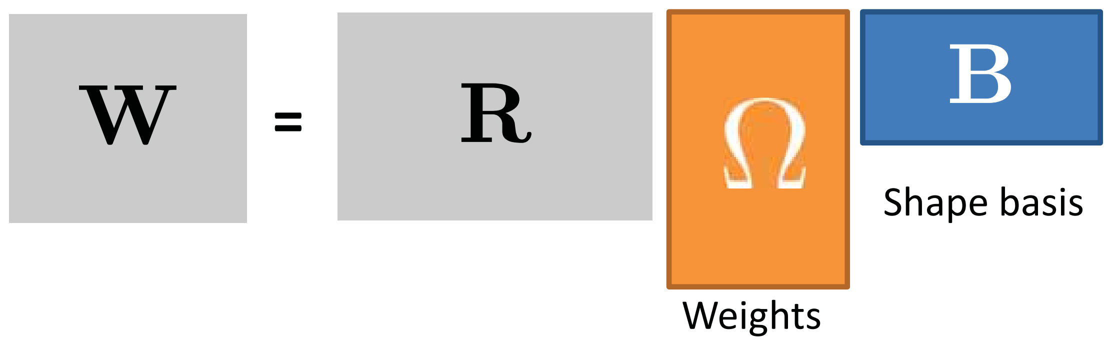


Reconstructibility and limitations

Duality

Weights and Bases

SHAPE FACTORIZATION

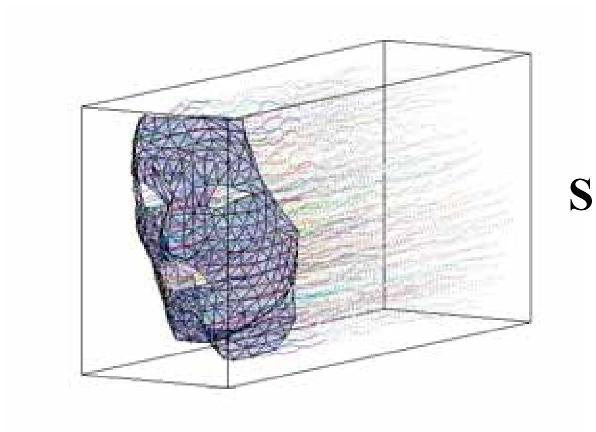


TRAJECTORY FACTORIZATION

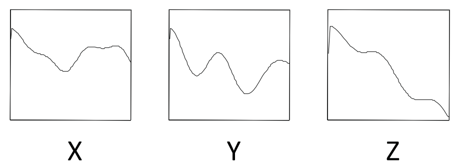
Shape weights are trajectory basis and trajectory weights are shape basis

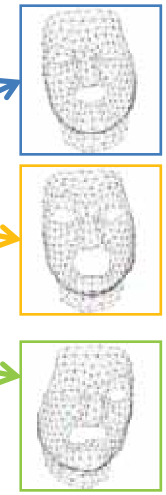
DUALITY

Weights and Bases



$$\mathbf{S}^{*}_{F\times 3P} = \begin{bmatrix} X_{11} & Y_{11} & Z_{11} & \cdots & X_{1P} & Y_{1P} & Z_{1P} \\ X_{21} & Y_{21} & Z_{21} & \cdots & X_{2P} & Y_{2P} & Z_{2P} \\ & & & \vdots & & & \\ X_{F1} & Y_{F1} & Z_{F1} & \cdots & X_{FP} & Y_{FP} & Z_{FP} \end{bmatrix}$$





- rank of columns = rank of rows
- Shape model and trajectory model has equal compaction power

Proof of Duality

Weights and Bases

Consider rearranged structure matrix $\mathbf{S}^*$

$$\mathbf{S}^*_{F\times 3P} = \begin{bmatrix} X_{11} & Y_{11} & Z_{11} & \cdots & X_{1P} & Y_{1P} & Z_{1P} \\ X_{21} & Y_{21} & Z_{21} & \cdots & X_{2P} & Y_{2P} & Z_{2P} \\ & & & \vdots & & & \\ X_{F1} & Y_{F1} & Z_{F1} & \cdots & X_{FP} & Y_{FP} & Z_{FP} \end{bmatrix}$$

$$\mathbf{S}^*_{F\times 3P} = \Omega^* \times \mathbf{B}^*$$

Proof of Duality

Weights and Bases

Consider rearranged structure matrix S*

$$\mathbf{S}^*_{F\times 3P} = \Omega^* \times \mathbf{B}^*$$

where

$$\Omega^* = \begin{bmatrix} \omega_{11} & \cdots & \omega_{1K} \\ \vdots & & \vdots \\ \omega_{F1} & \cdots & \omega_{FK} \end{bmatrix}$$

1st shape basis

$$\mathbf{B}^* = \begin{bmatrix} b^X_{11} & b^Y_{11} & b^Z_{11} & \cdots & b^X_{1P} & b^Y_{1P} & b^Z_{1P} \\ & \vdots & & \cdots & & \vdots & \\ b^X_{K1} & b^Y_{K1} & b^Z_{K1} & \cdots & b^X_{KP} & b^Y_{KP} & b^Z_{KP} \end{bmatrix}$$

Proof of Duality

Weights and Bases

$$\mathbf{S}^* = \Omega^* \times \mathbf{B}^* = \begin{bmatrix} \omega_{11} & \dots & \omega_{1K} \\ \vdots & & \vdots \\ \omega_{F1} & \dots & \omega_{FK} \end{bmatrix} \begin{bmatrix} b^X_{11} & b^Y_{11} & b^Z_{11} & \dots & b^X_{1P} & b^Y_{1P} & b^Z_{1P} \\ & \vdots & & \dots & & \vdots & \\ b^X_{K1} & b^Y_{K1} & b^Z_{K1} & \dots & b^X_{KP} & b^Y_{KP} & b^Z_{KP} \end{bmatrix}$$

To link shape to j^{th} trajectory, we select the coefficients related to j^{th} point

$$\begin{bmatrix} T^X_j & T^Y_j & T^Z_j \end{bmatrix} = \begin{bmatrix} \omega_{11} & \dots & \omega_{1K} \\ \vdots & & \vdots \\ \omega_{F1} & \dots & \omega_{FK} \end{bmatrix} \begin{bmatrix} b^X_{1j} & b^Y_{1j} & b^Z_{1j} \\ \vdots & \vdots & \vdots \\ b^X_{Kj} & b^Y_{Kj} & b^Z_{Kj} \end{bmatrix}$$

PROOF OF DUALITY

Weights and Bases

$$\begin{bmatrix} T_j^X & T_j^Y & T_j^Z \end{bmatrix} = \begin{bmatrix} \omega_{11} & \dots & \omega_{1K} \\ \vdots & & \vdots \\ \omega_{F1} & \dots & \omega_{FK} \end{bmatrix} \begin{bmatrix} b_{1j}^X & b_{1j}^Y & b_{1j}^Z \\ \vdots & \vdots & \vdots \\ b_{Kj}^X & b_{Kj}^Y & b_{Kj}^Z \end{bmatrix}$$

Can be rewritten as

$$T_j^X = \sum_{k=1}^{K} b_{kj}^X \omega^k$$

$$T_j^Y = \sum_{k=1}^{K} b_{kj}^Y \omega^k$$

$$T_j^Z = \sum_{k=1}^{K} b_{kj}^Z \omega^k$$

Compare to Trajectory Representation

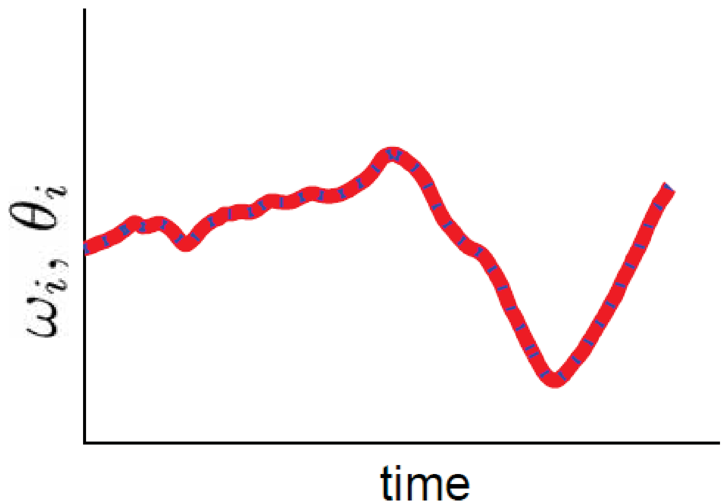
$$T_j^X = \sum_{k=1}^{K} a_{jk}^X \theta^k$$

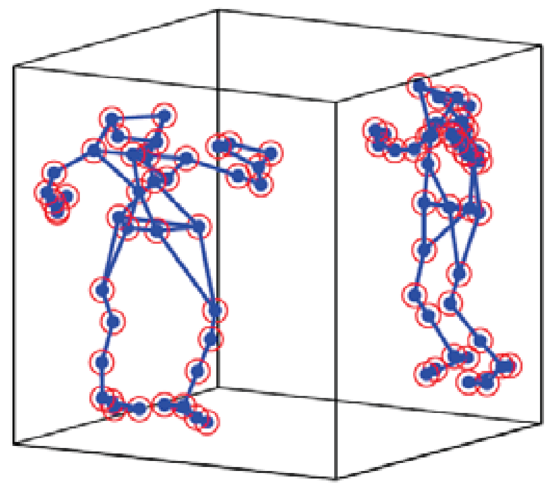
$$T_j^Y = \sum_{k=1}^{K} a_{jk}^Y \theta^k$$

$$T_j^Z = \sum_{k=1}^{K} a_{jk}^Z \theta^k$$

Illustration of Duality

SVD Shape and Trajectory Basis for Mocap Structure



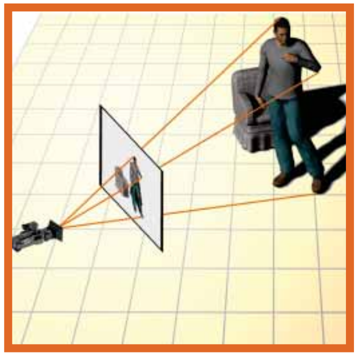


Shape Coefficients ≡ Trajectory Basis

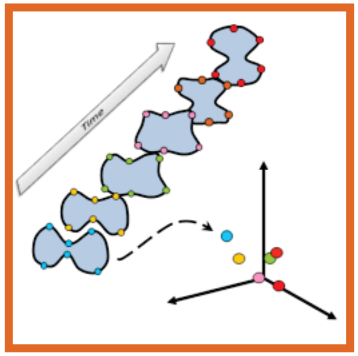
Trajectory Coefficients ≡ Shape Basis

Nonrigid Structure From Motion

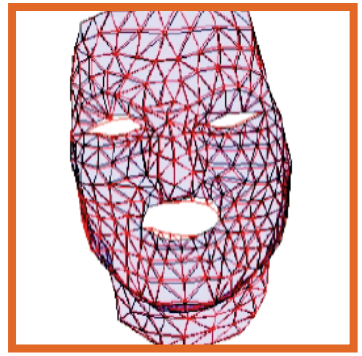
Tutorial Outline



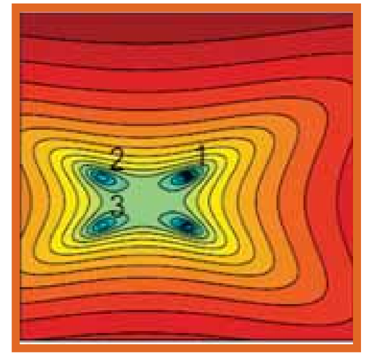
Introduction to Nonrigid SFM



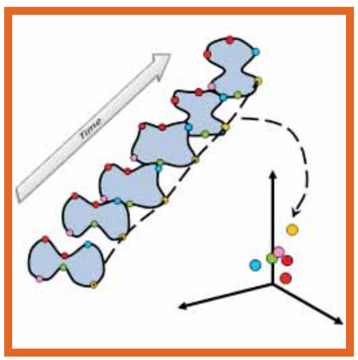
Shape Representation



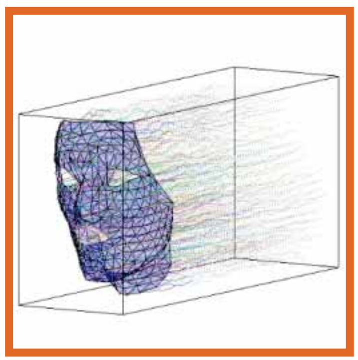
Shape Estimation



Ambiguity of Orthogonality Constraints

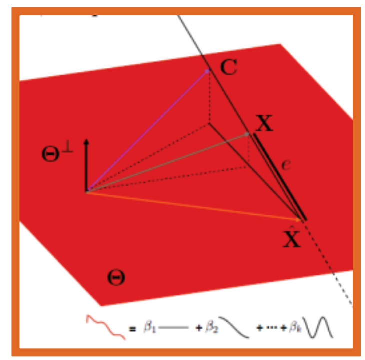


Trajectory Representation



Shape-Trajectory Duality

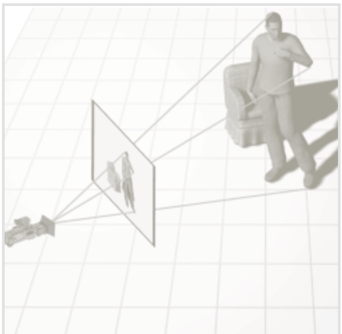
Trajectory Estimation



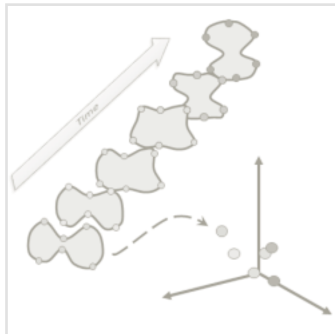
Reconstructibility and limitations

Nonrigid Structure From Motion

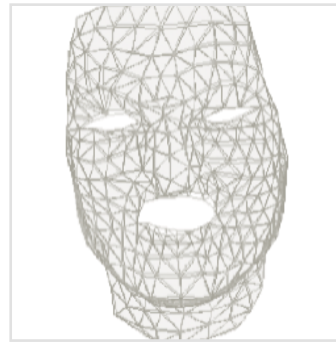
Tutorial Outline



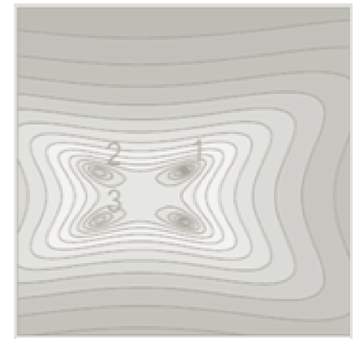
Introduction to Nonrigid SFM



Shape Representation

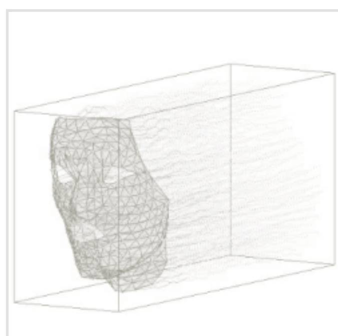


Shape Estimation



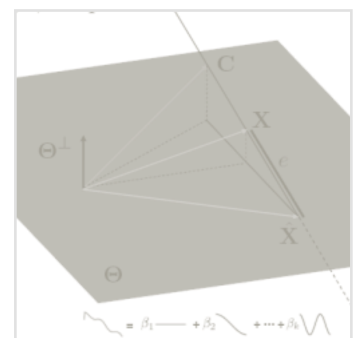
Ambiguity of Orthogonality Constraints

Trajectory Representation



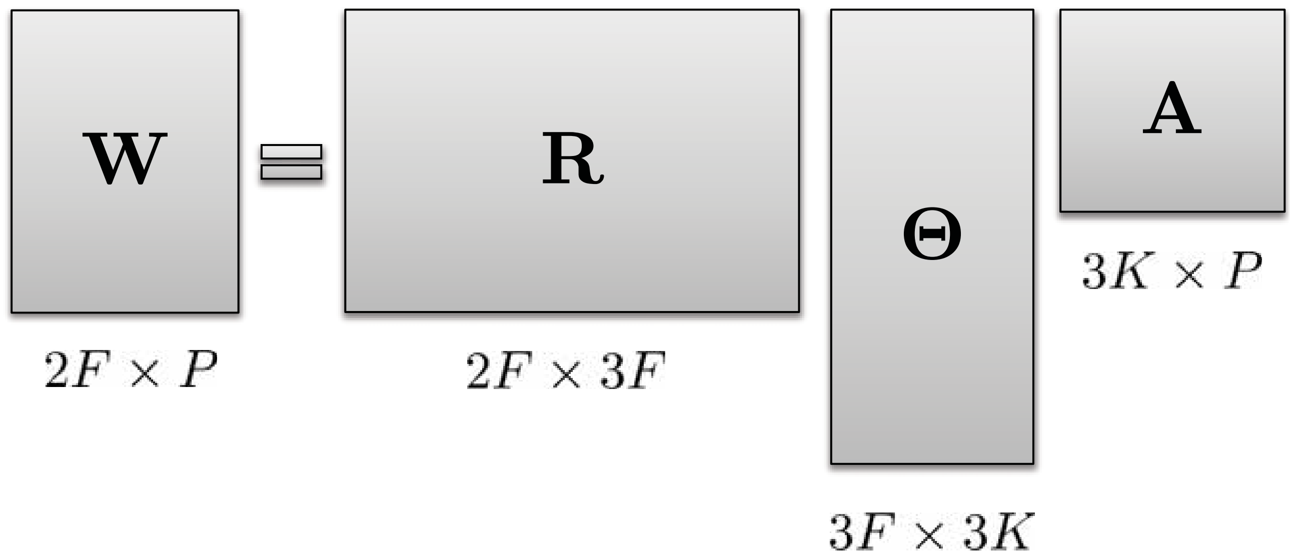
Shape-Trajectory Duality

Trajectory Estimation

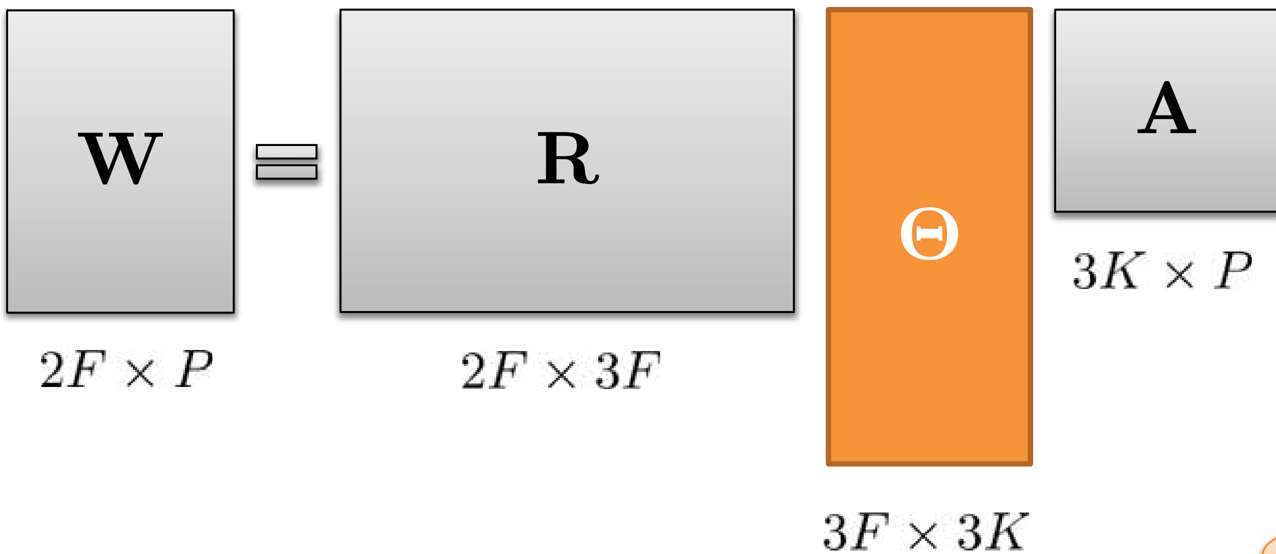


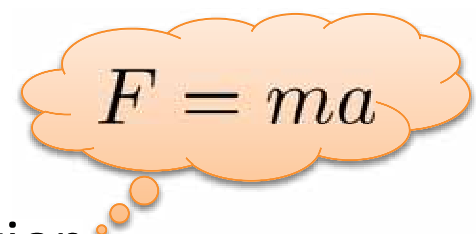
Reconstructibility and limitations

Estimating Structure via Trajectory Model



ESTIMATING STRUCTURE VIA TRAJECTORY MODEL

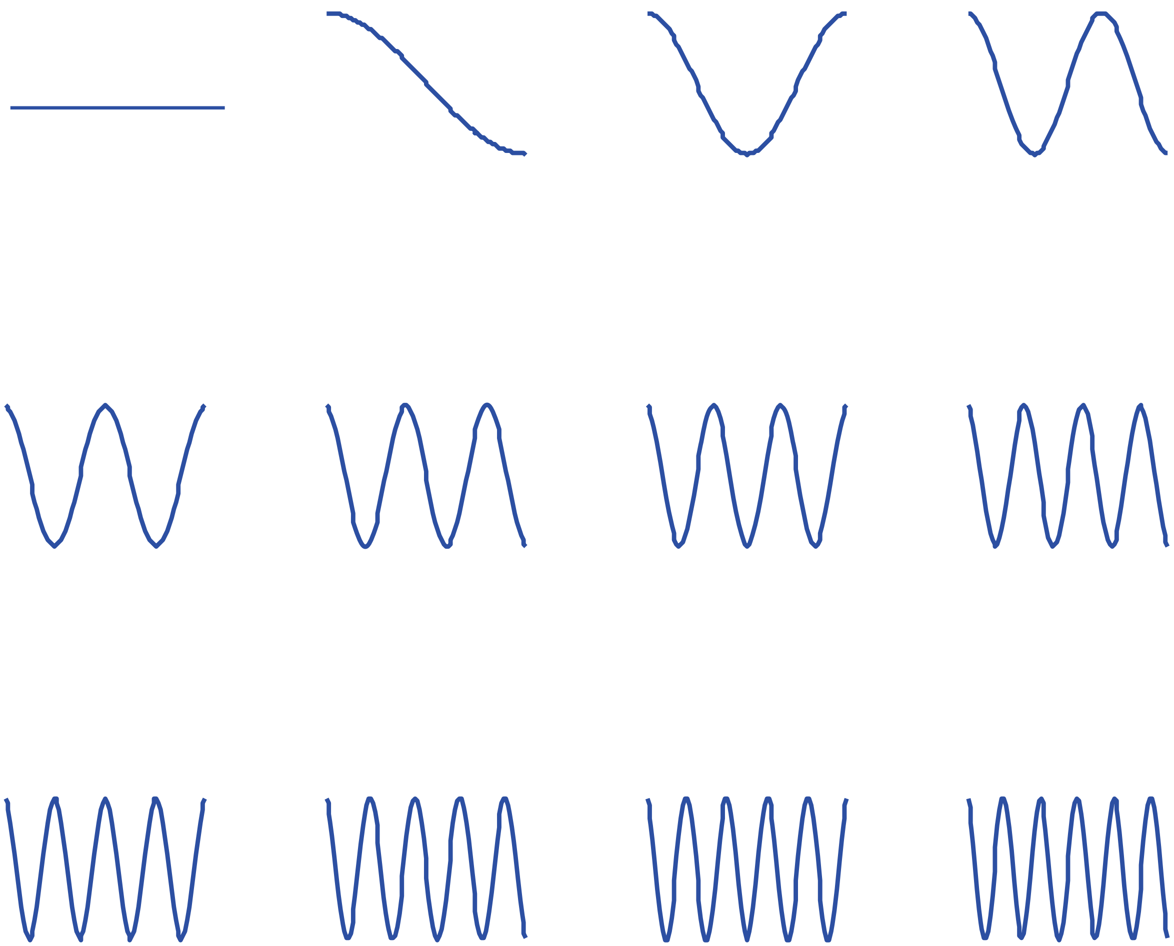




Object Independent Basis

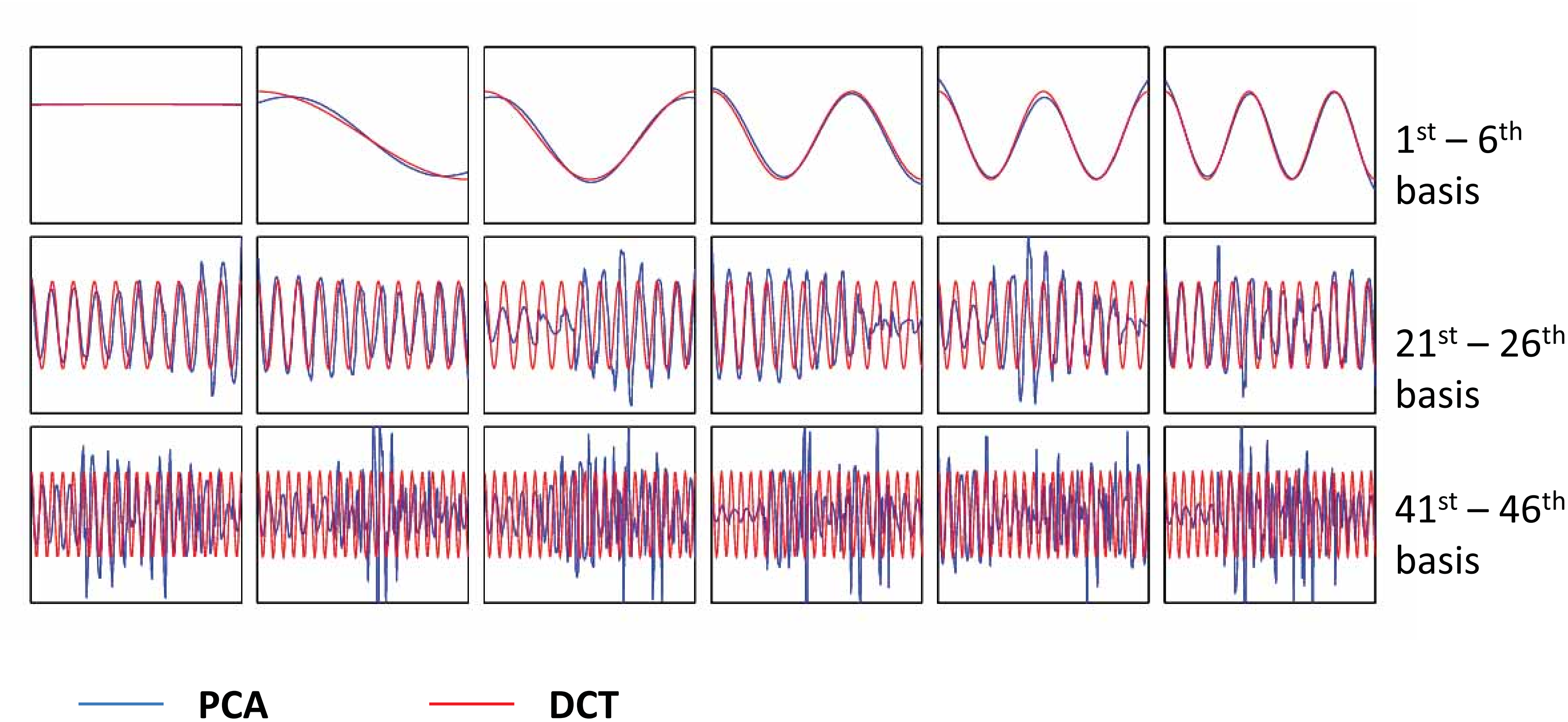
1. Deformation constrained by physical actuation
2. Trajectories vary smoothly and not randomly
3. Can be compactly represented by predefined basis
   e.g. Discrete Cosine Transform

DCT Basis

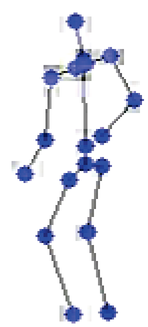


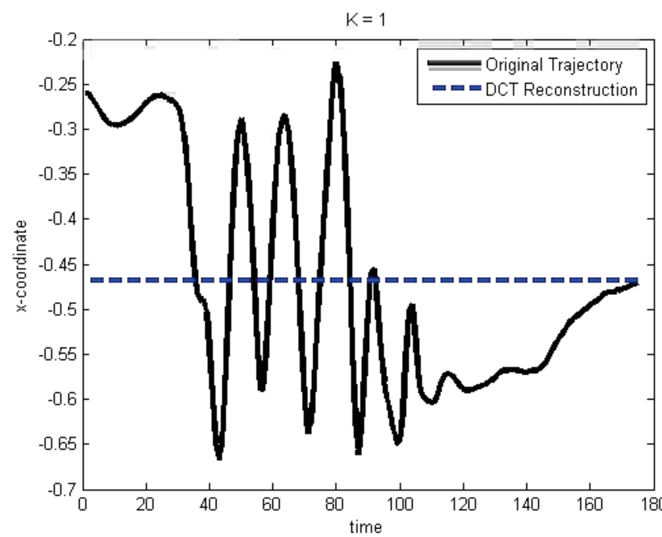
PREDEFINING TRAJECTORY BASIS

- We showed that PCA approaches DCT (Discrete Cosine Transform) on CMU's body MOCAP database.



COMPACTNESS OF DCT BASIS

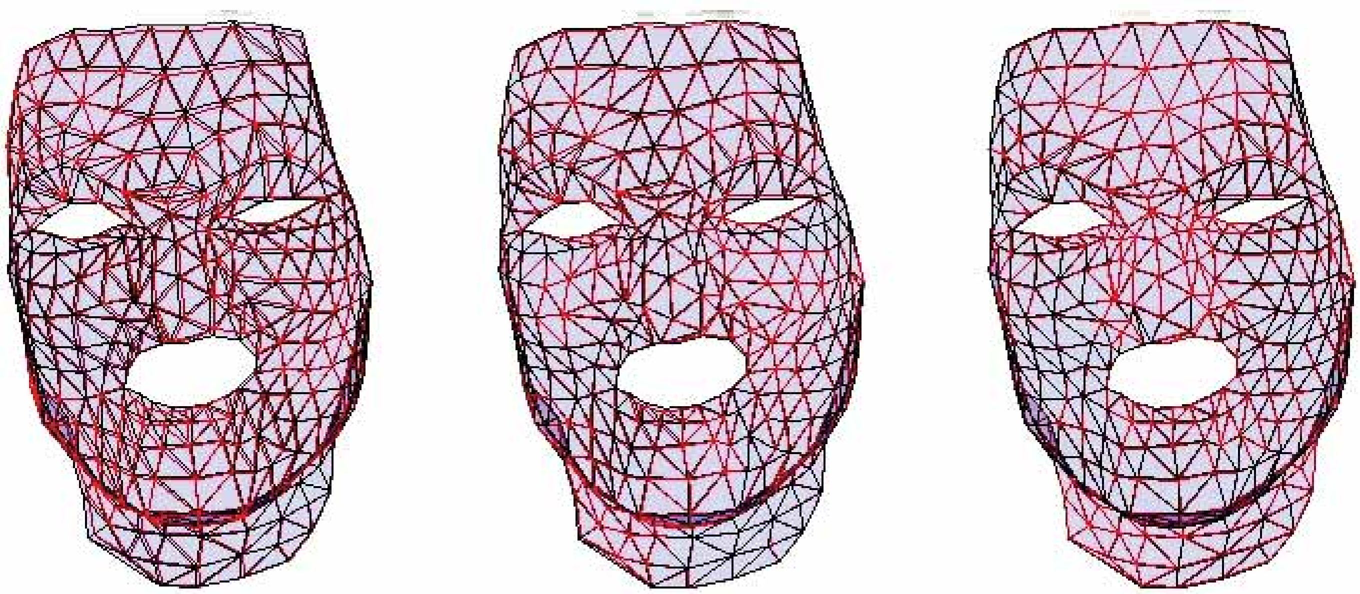




DCT RECONSTRUCTION

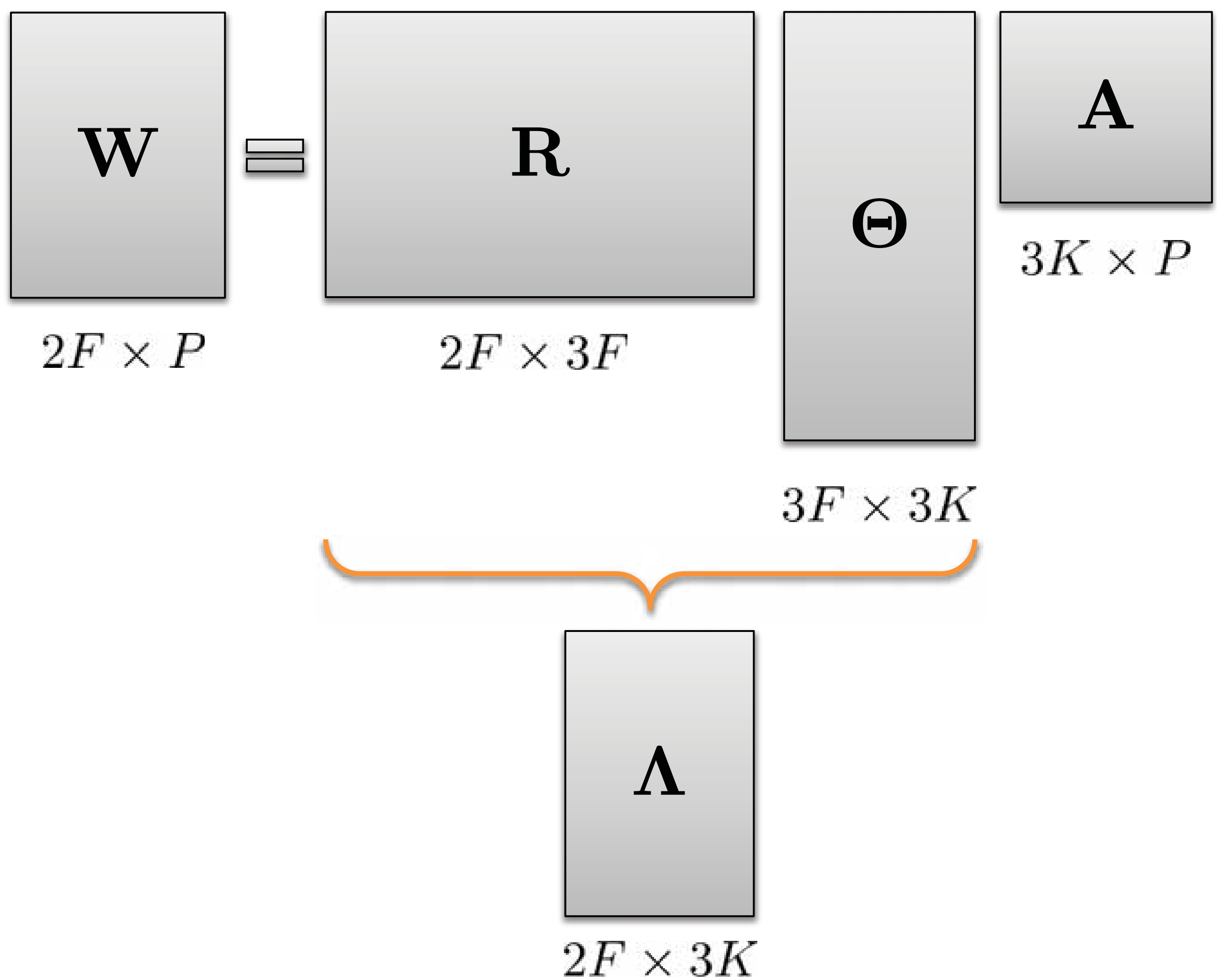
$$A = \Theta \setminus S$$

$$\hat{S} = \Theta A$$

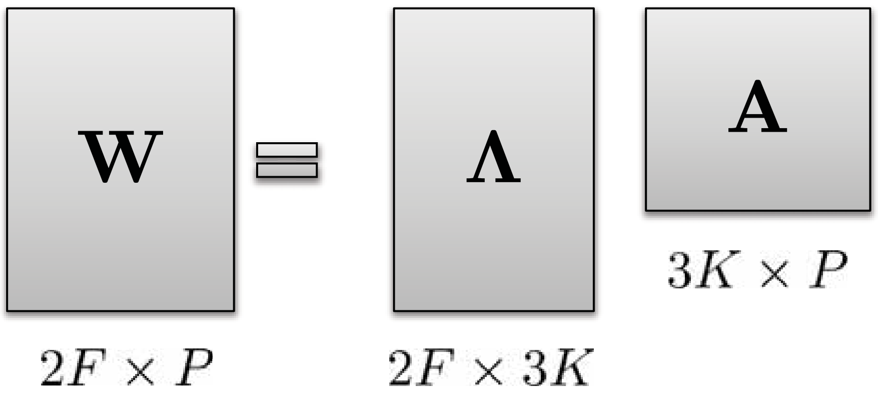


35 Basis | 50 Basis | 65 Basis

Estimating Structure via Trajectory Model



Estimating Structure via Trajectory Model



$$\mathrm{Rank}(\mathbf{W}) \leq 3K$$

$$3K < \min(2F, P)$$

Solution

1. By SVD, compute $\hat{\Lambda}, \hat{\mathbf{A}}$ $\quad\quad \mathbf{W} = \hat{\Lambda}\hat{\mathbf{A}}$
2. Correct solution differs by a linear transform

$$\Lambda = \hat{\Lambda}\mathbf{Q} \qquad \mathbf{A} = \mathbf{Q}^{-1}\hat{\mathbf{A}}$$

$3K \times 3K$

3. Solving for $\mathbf{Q}$?

FINDING $\mathbf{Q}$

The correct Q will yield the correct form of Λ

$$\Lambda = \begin{bmatrix} \theta_{11}R_1 & \dots & \theta_{1K}R_1 \\ & \vdots & \\ \theta_{F1}R_F & \dots & \theta_{FK}R_F \end{bmatrix}$$

We can just estimate first 3 columns of Q instead of estimating full Q

$$\hat{\Lambda}\mathbf{Q}_{|||} = \begin{bmatrix} \theta_{11}R_1 \\ \vdots \\ \theta_{F1}R_F \end{bmatrix}$$

If $\mathbf{Q}_{|||}$ is known:

- Compute $\mathbf{R}$
- Compute $\mathbf{\Lambda}$ $\quad \mathbf{\Lambda}_{2F\times 3K} = \mathcal{R}_{2F\times 3F}\mathbf{\Theta}_{3F\times 3K}$
- Compute $\mathbf{A}$

$$\Lambda_{2F\times 3K}\mathbf{A}_{3K\times P} = \mathbf{W}_{2F\times P}$$

FINDING $\mathbf{Q}_{|||}$

The correct Q will yield the correct form of Λ

$$\mathbf{\Lambda} = \begin{bmatrix} \theta_{11}R_1 & \dots & \theta_{1K}R_1 \\ & \vdots & \\ \theta_{F1}R_F & \dots & \theta_{FK}R_F \end{bmatrix}$$

Orthonormality Constraints

$$\hat{\mathbf{\Lambda}}_{2i-1:2i}\mathbf{Q}_{|||}\mathbf{Q}_{|||}^T\hat{\mathbf{\Lambda}}_{2i-1:2i}^T = \theta_{i1}^2 I_{2\times 2}$$

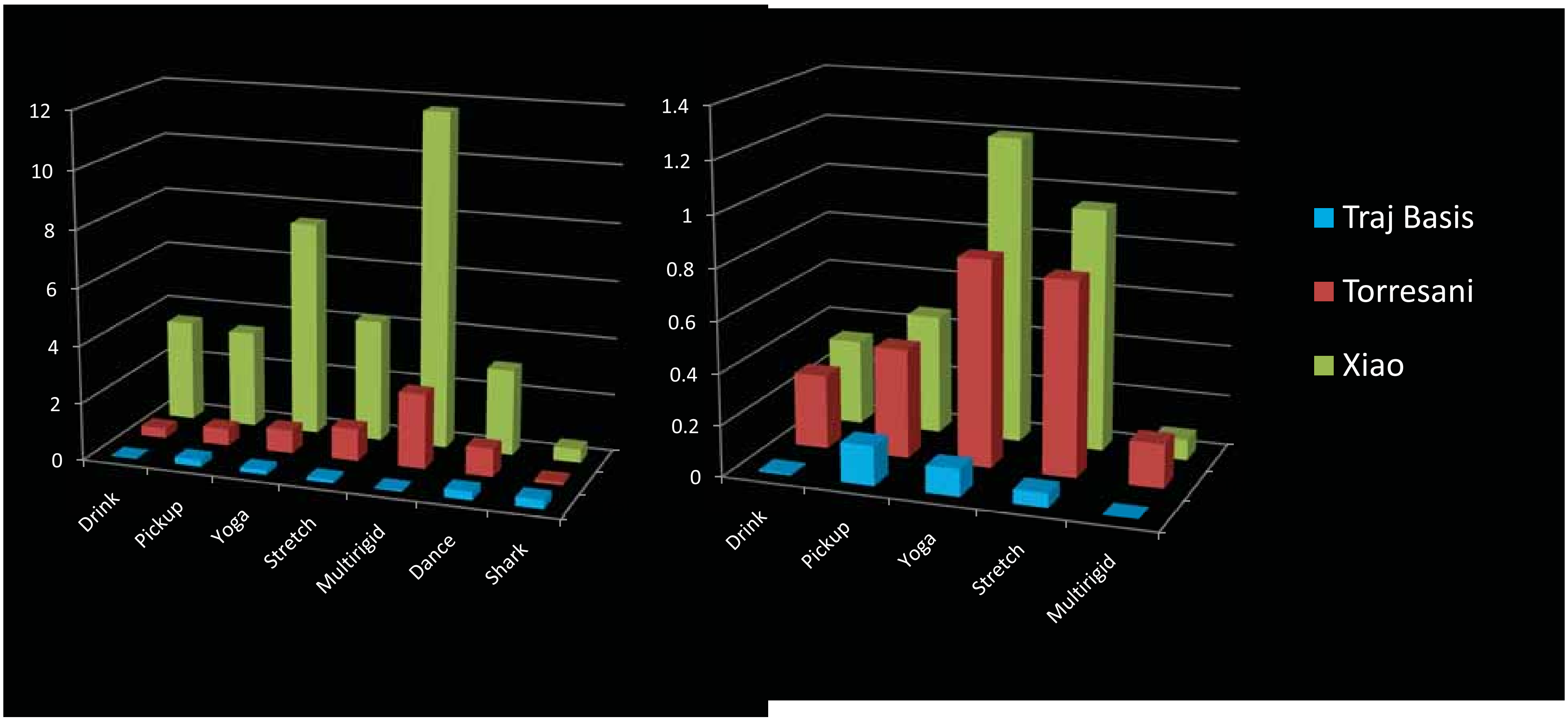
Each image yields 3 constraints because θ is known

F images yield 3F constraints

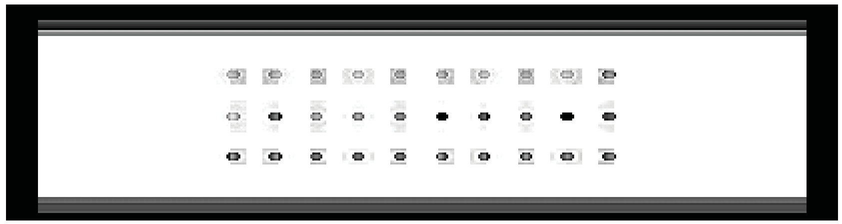
At least 3K images needed to constrain the solution

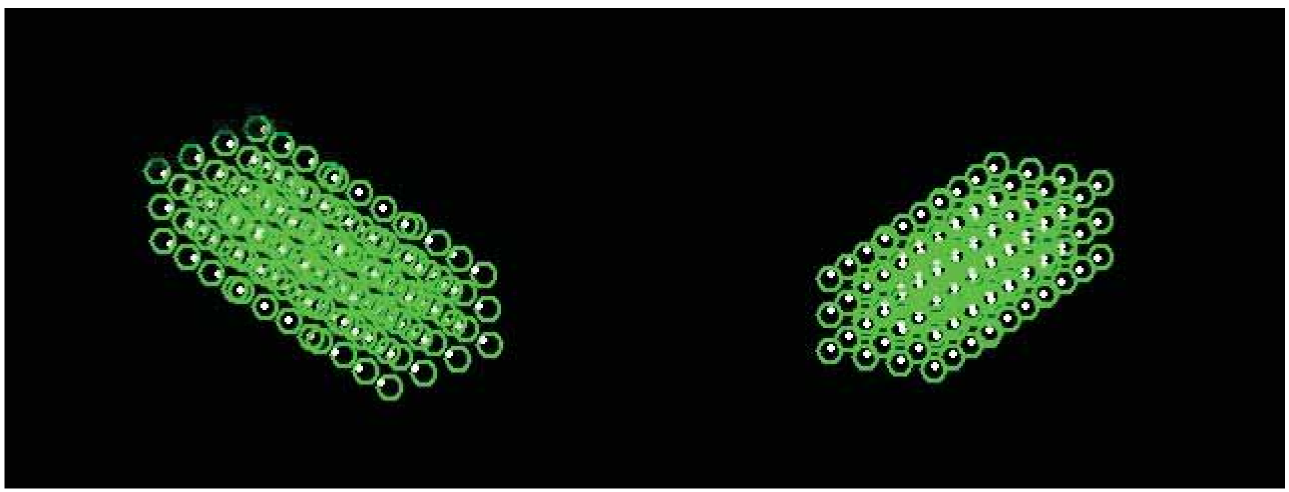
RESULTS

QUANTITATIVE RESULTS



We use synthetic and Motion captured data for quantitative experiments

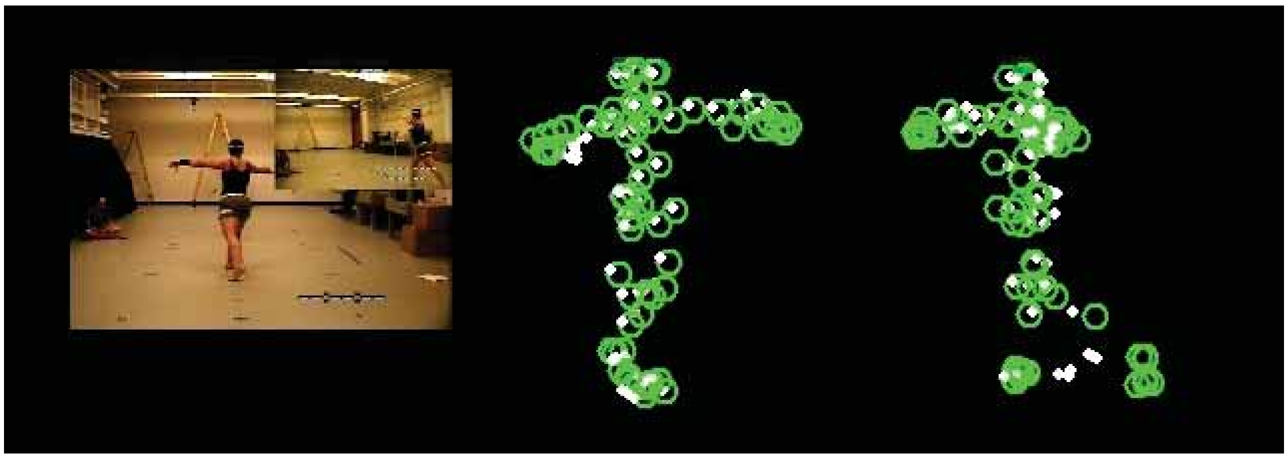




MOTION CAPTURE DATASETS

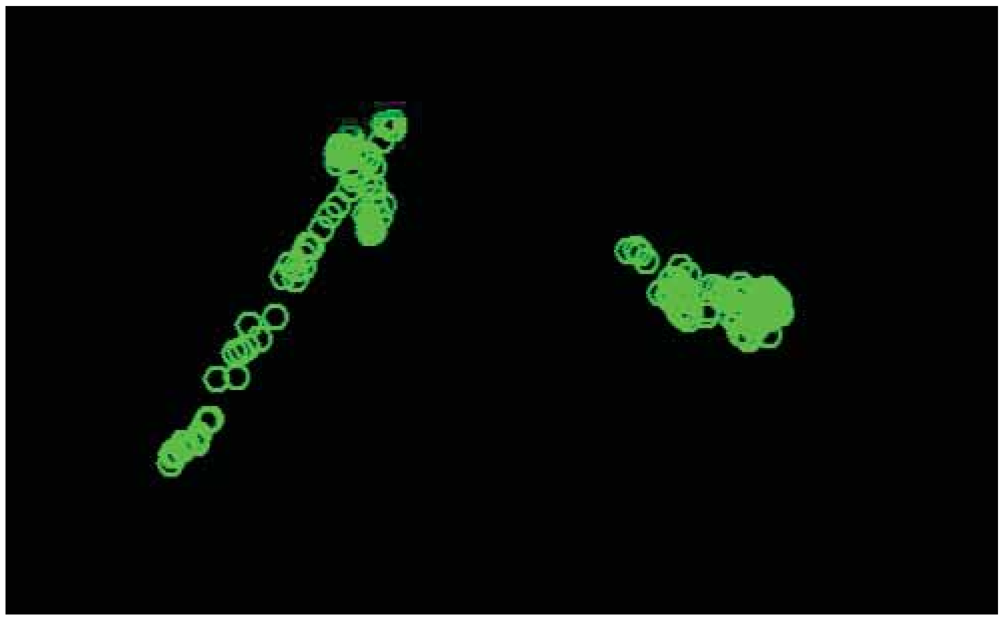
DANCE DATASET

75 points, 264 frames, K=5



Input Video Two views of the reconstruction

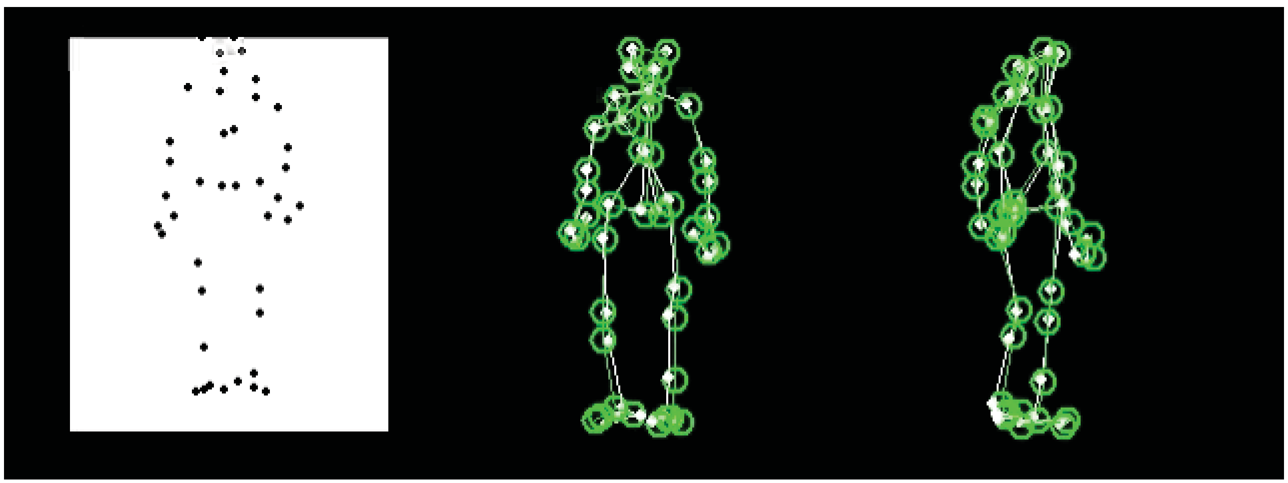
Torresani *et al.* 2005 Xiao *et al.* 2004



MOTION CAPTURE DATASETS

STRETCH DATASET

41 points, 370 frames, K=12



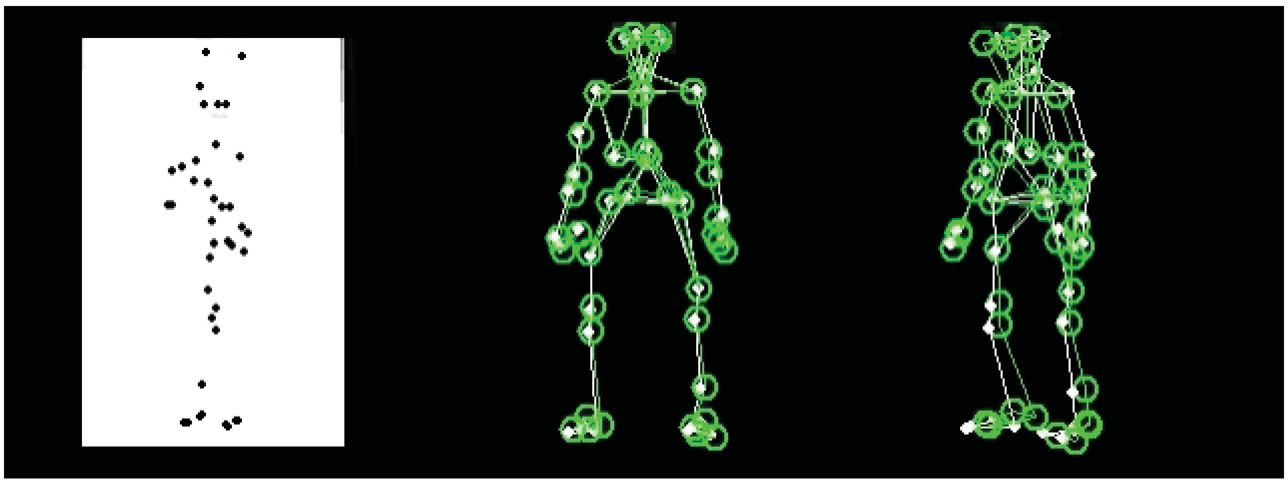
Input Data

Two views of the reconstruction

MOTION CAPTURE DATASETS

PICKUP DATASET

41 points, 357 frames, K=12



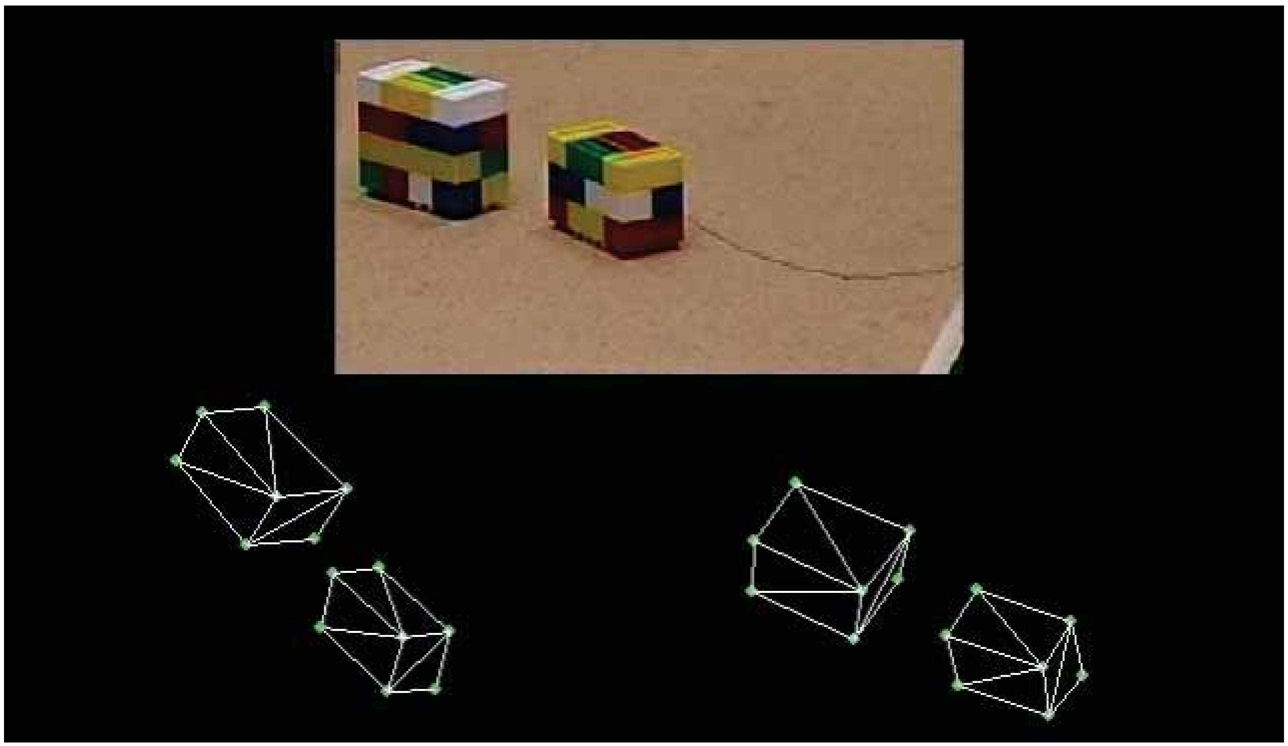
Input Data

Two views of the reconstruction

RESULTS ON REAL VIDEOS

CUBES SEQUENCES

14 points, 200 frames, K=2

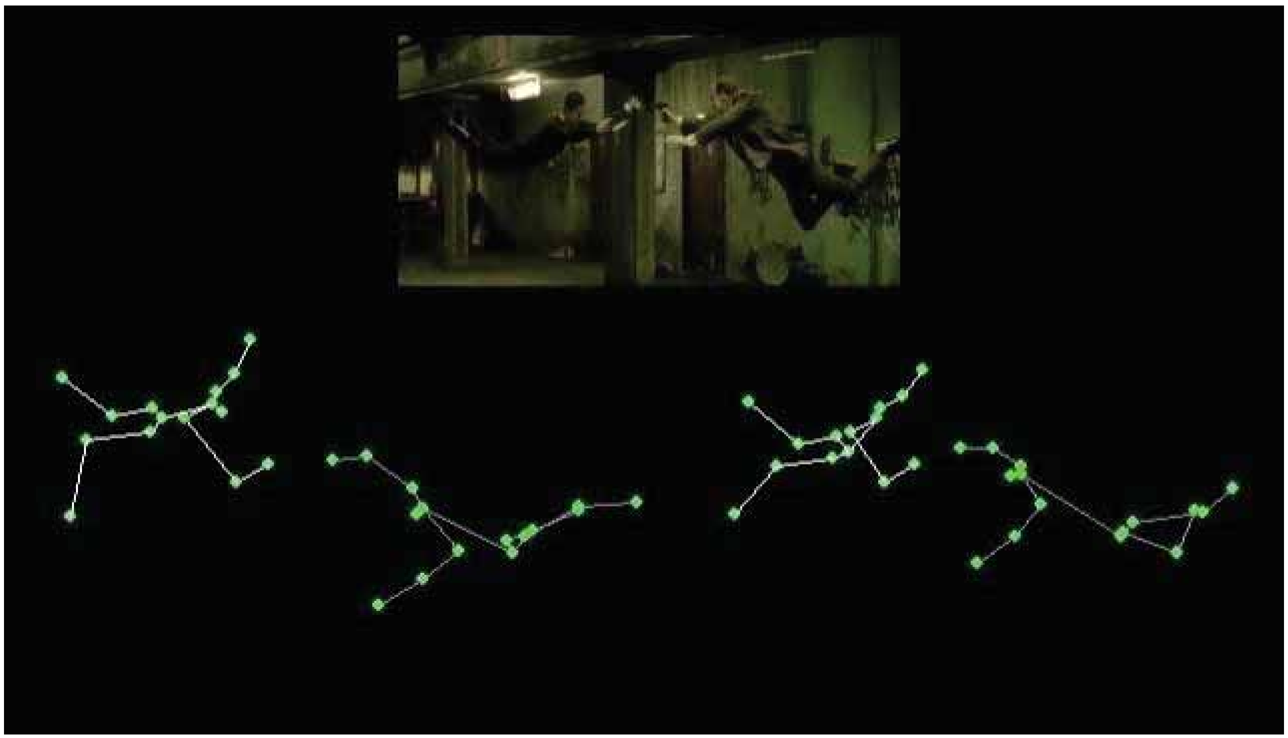


Two views of the reconstruction

RESULTS ON REAL VIDEOS

MATRIX SEQUENCE

30 points, 93 frames, K=3

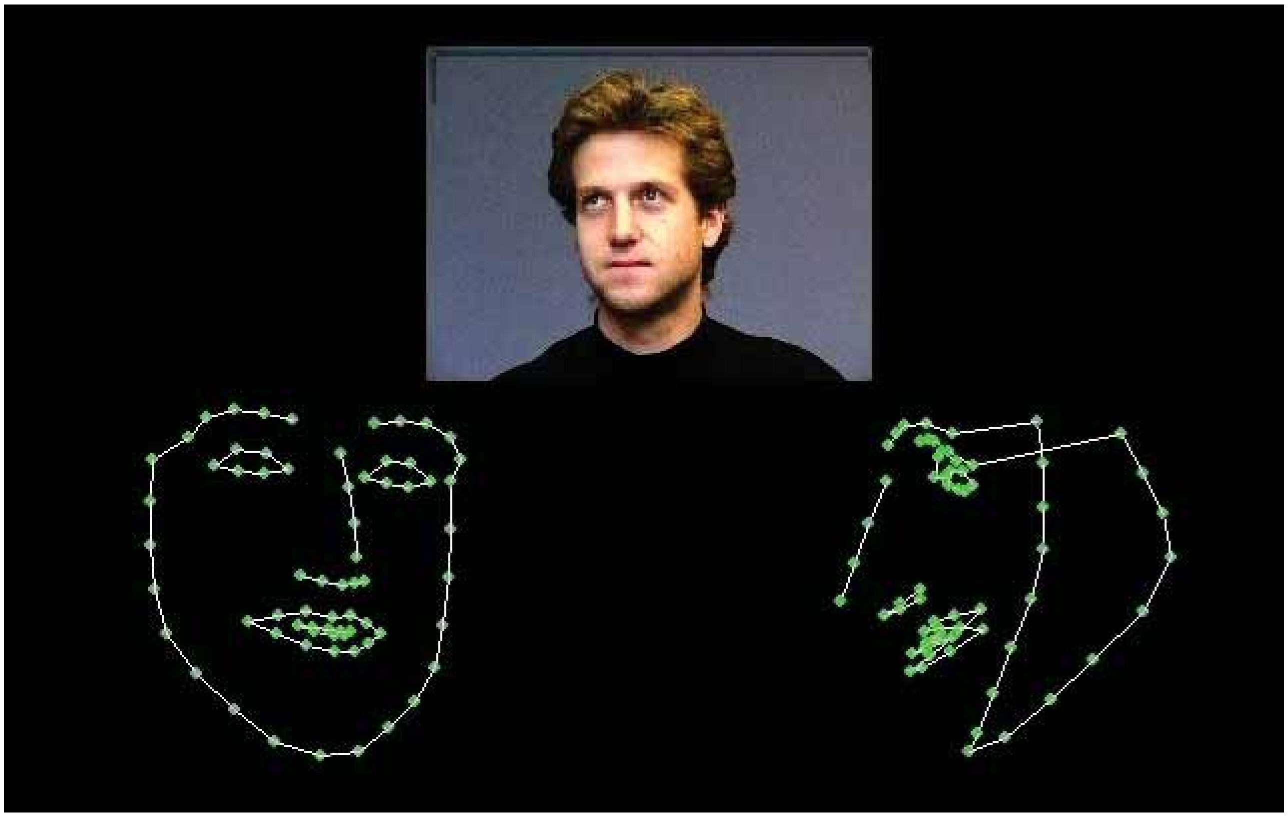


Two views of the reconstruction

RESULTS ON REAL VIDEOS

PIE DATASET

68 points, 240 frames, K=2

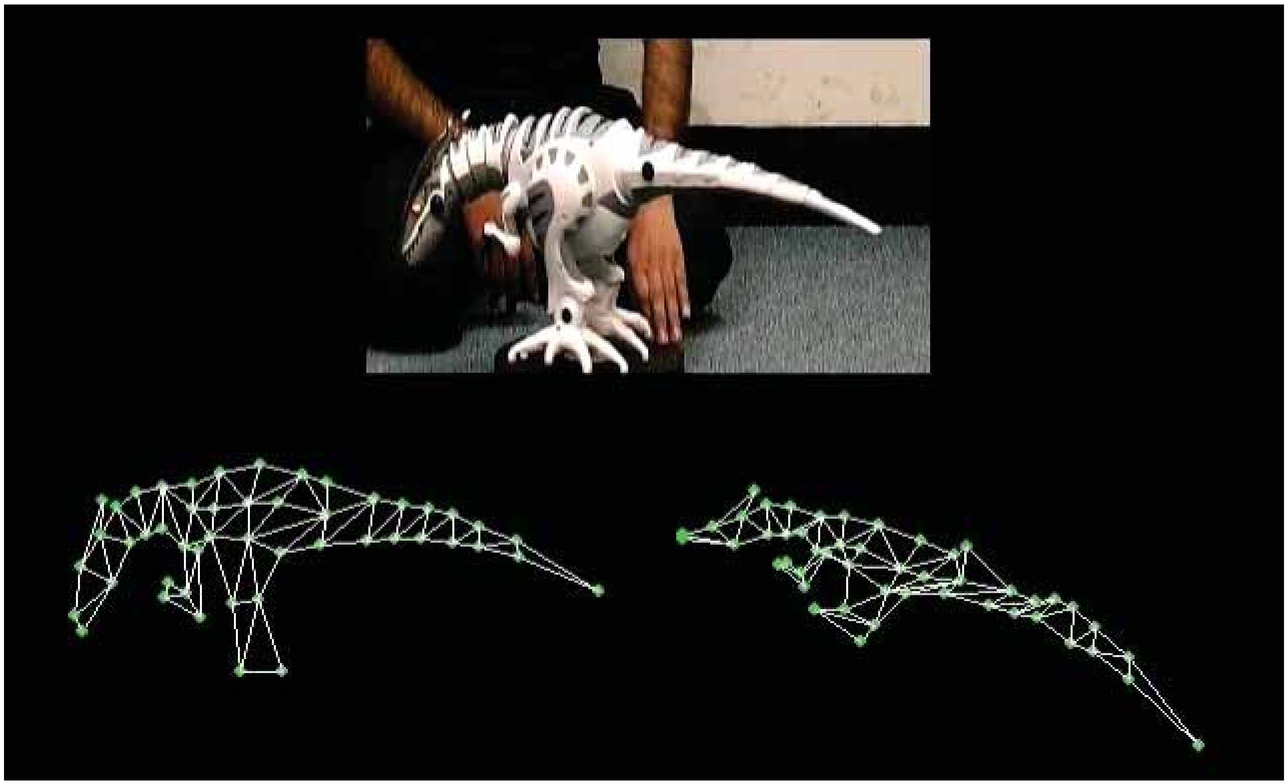


Two views of the reconstruction

RESULTS ON REAL VIDEOS

DINOSAUR SEQUENCE

49 points, 231 frames, K=12

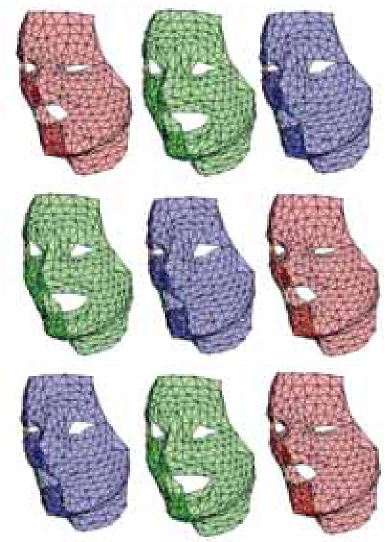


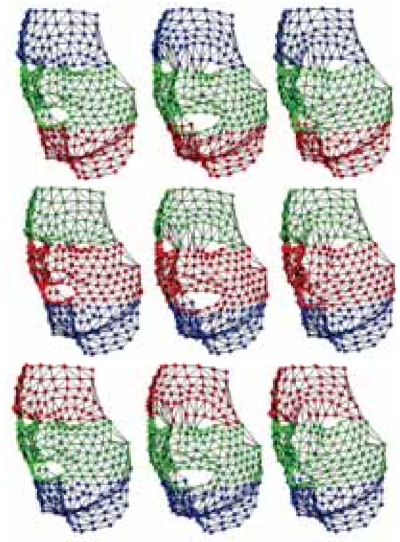
Two views of the reconstruction

RECONSTRUCTION STABILITY INCREASES
AS CAMERA MOTION INCREASES
AS OBJECT MOTION DECREASES

SHAPE MODEL VS. TRAJECTORY MODEL

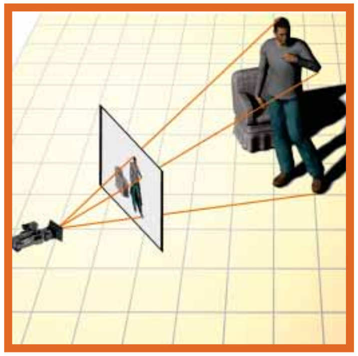
	Shape	Trajectory
Model	Can be learnt	Hard to specialize
Specificity	Object dependent	Generalize
Ordering of frames	Irrelevant	Exploited
Ordering of points	Exploited	Irrelevant



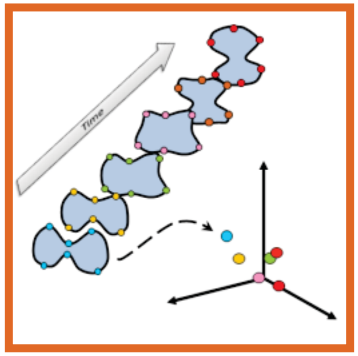


NONRIGID STRUCTURE FROM MOTION

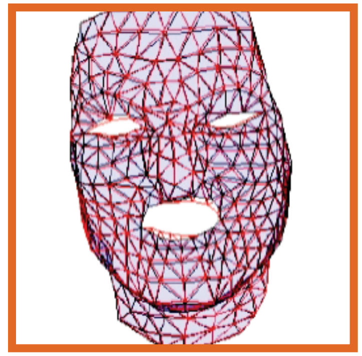
Tutorial Outline



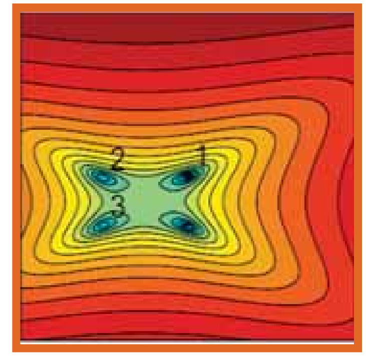
Introduction to Nonrigid SFM



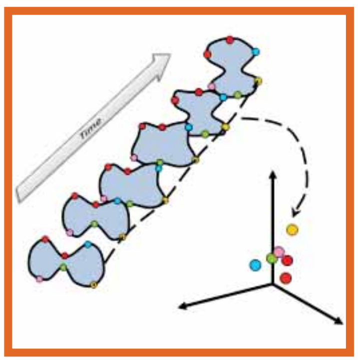
Shape Representation



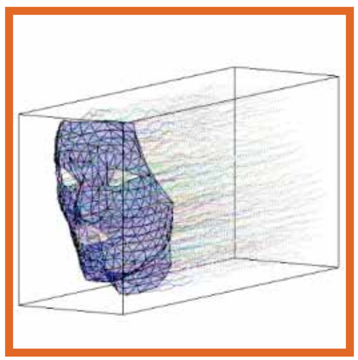
Shape Estimation



Ambiguity of Orthogonality Constraints

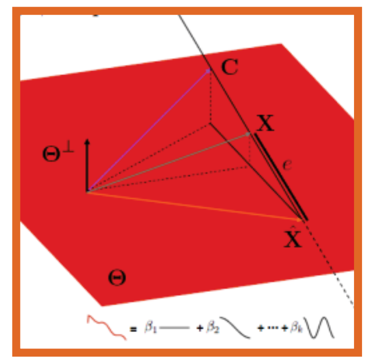


Trajectory Representation



Shape-Trajectory Duality

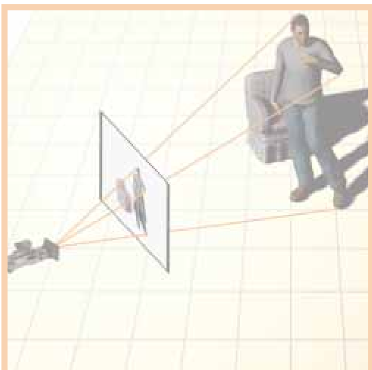
Trajectory Estimation



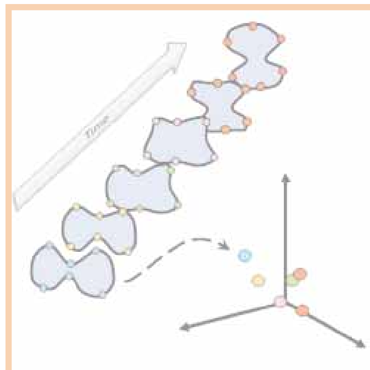
Reconstructibility and limitations

NONRIGID STRUCTURE FROM MOTION

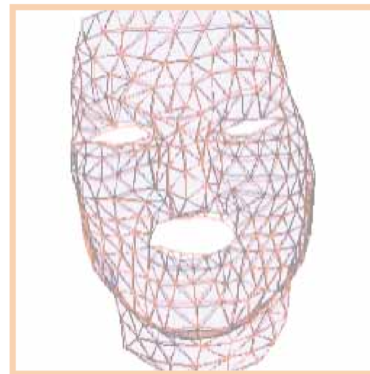
Tutorial Outline



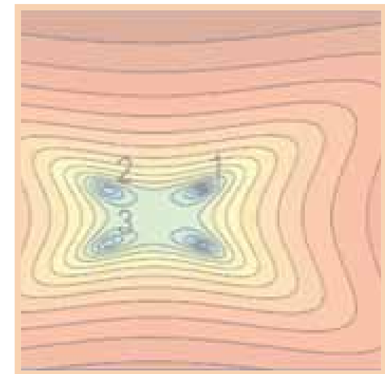
Introduction to
Nonrigid SFM



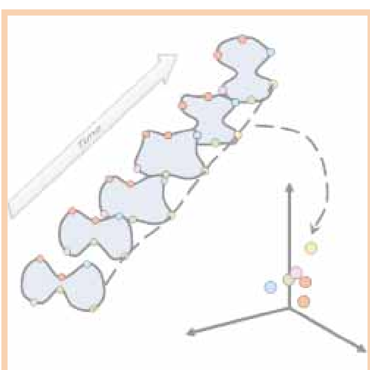
Shape
Representation



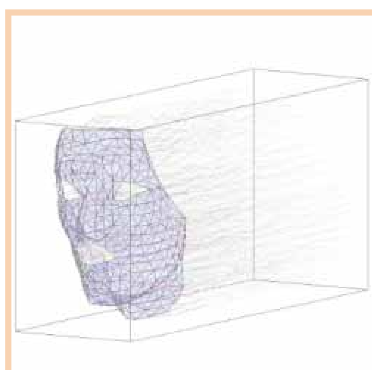
Shape
Estimation



Ambiguity of
Orthogonality
Constraints

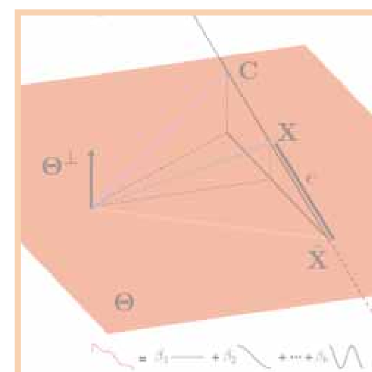


Trajectory
Representation

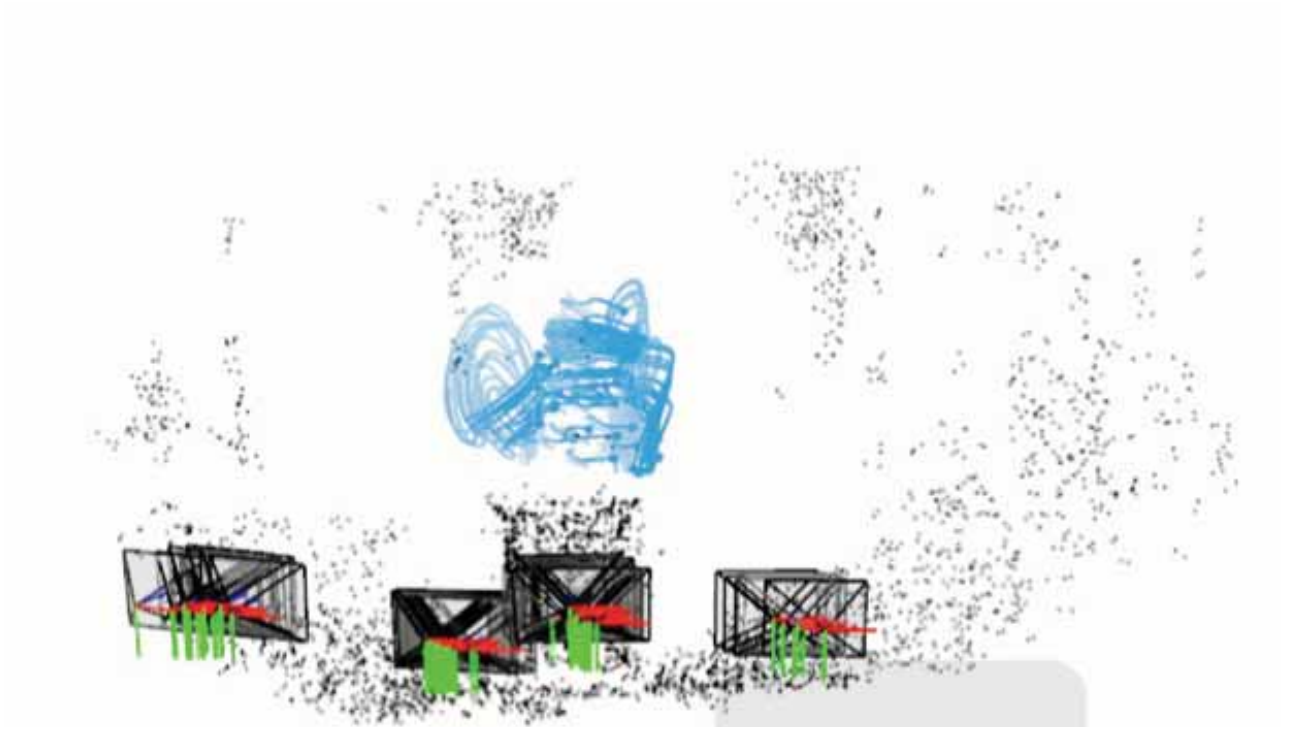


Shape-Trajectory
Duality

Trajectory
Estimation



Reconstructibility
and Limitations



3D TRAJECTORY ESTIMATION

ECCV 2010

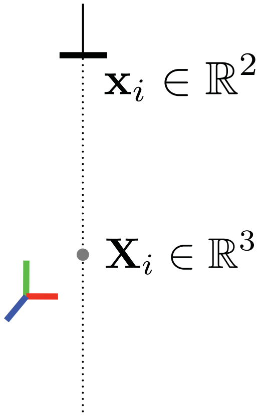
CHALLENGE

TRILINEAR ESTIMATION

$$\begin{bmatrix} \mathbf{R}_1 & & & \\ & \mathbf{R}_2 & & \\ & & \ddots & \\ & & & \mathbf{R}_F \end{bmatrix} \begin{bmatrix} \omega_{11} & \cdots & \omega_{1k} \\ \omega_{21} & & \omega_{2k} \\ \vdots & & \vdots \\ \omega_{F1} & \cdots & \omega_{Fk} \end{bmatrix} \begin{bmatrix} -\mathbf{b_1}- \\ -\mathbf{b_2}- \\ \vdots \\ -\mathbf{b_k}- \end{bmatrix}$$

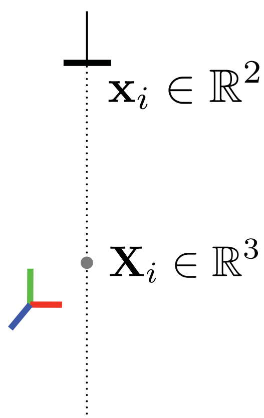
$$\mathbf{W} = \mathbf{R}\mathbf{\Omega}\mathbf{B}$$

SINGLE VIEW RECONSTRUCTION



Single View
Reconstruction

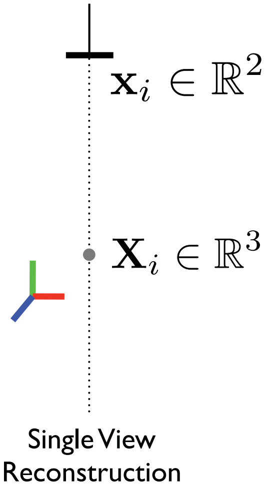
SINGLE VIEW RECONSTRUCTION



Single View Reconstruction

$$s_i \begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} = \mathbf{P}_i \begin{bmatrix} \mathbf{X}_i \\ 1 \end{bmatrix}$$

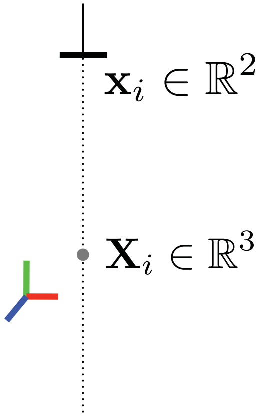
SINGLE VIEW RECONSTRUCTION



$$s_i \begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} = \mathbf{P}_i \begin{bmatrix} \mathbf{X}_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} \times \mathbf{P}_i \begin{bmatrix} \mathbf{X}_i \\ 1 \end{bmatrix} = 0$$

SINGLE VIEW RECONSTRUCTION

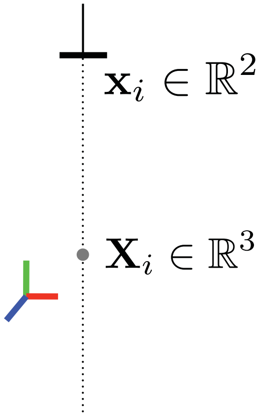


$$s_i \begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} = \mathbf{P}_i \begin{bmatrix} \mathbf{X}_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} \times \mathbf{P}_i \begin{bmatrix} \mathbf{X}_i \\ 1 \end{bmatrix} = 0$$

$$\mathbf{Q}_i \mathbf{X}_i = -\mathbf{q}_i$$

SINGLE VIEW RECONSTRUCTION

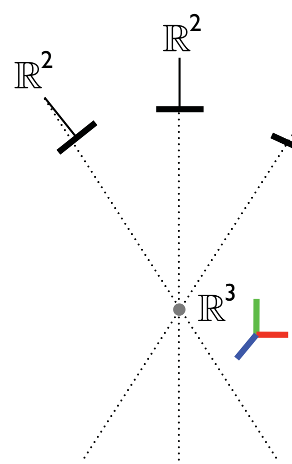


$$s_i \begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} = \mathbf{P}_i \begin{bmatrix} \mathbf{X}_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} \times \mathbf{P}_i \begin{bmatrix} \mathbf{X}_i \\ 1 \end{bmatrix} = 0$$

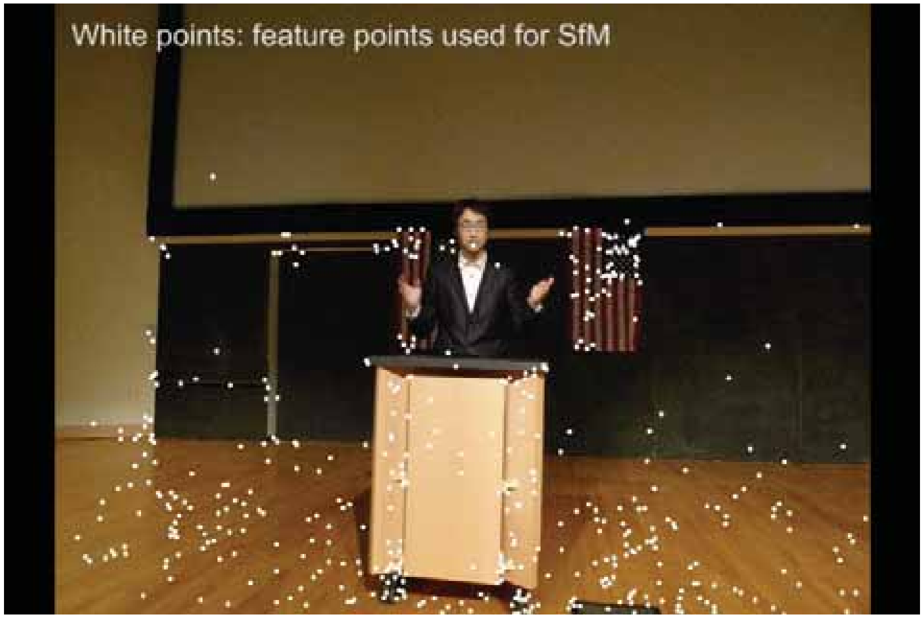
$$\underset{2\text{x}3}{\underline{\mathbf{Q}_i}} \underset{3\text{x}1}{\underline{\mathbf{X}_i}} = -\underset{2\text{x}1}{\underline{\mathbf{q}_i}}$$

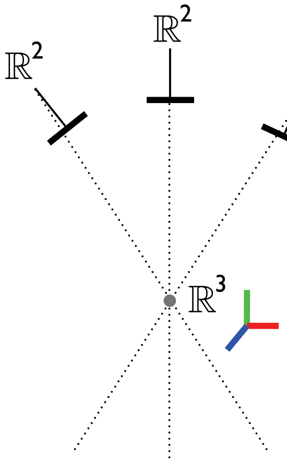
STRUCTURE FROM MOTION



$$\begin{bmatrix} \mathbf{Q}_1 \\ \vdots \\ \mathbf{Q}_F \end{bmatrix} \mathbf{X} = \begin{bmatrix} \mathbf{q}_1 \\ \vdots \\ \mathbf{q}_F \end{bmatrix}$$

STRUCTURE FROM MOTION





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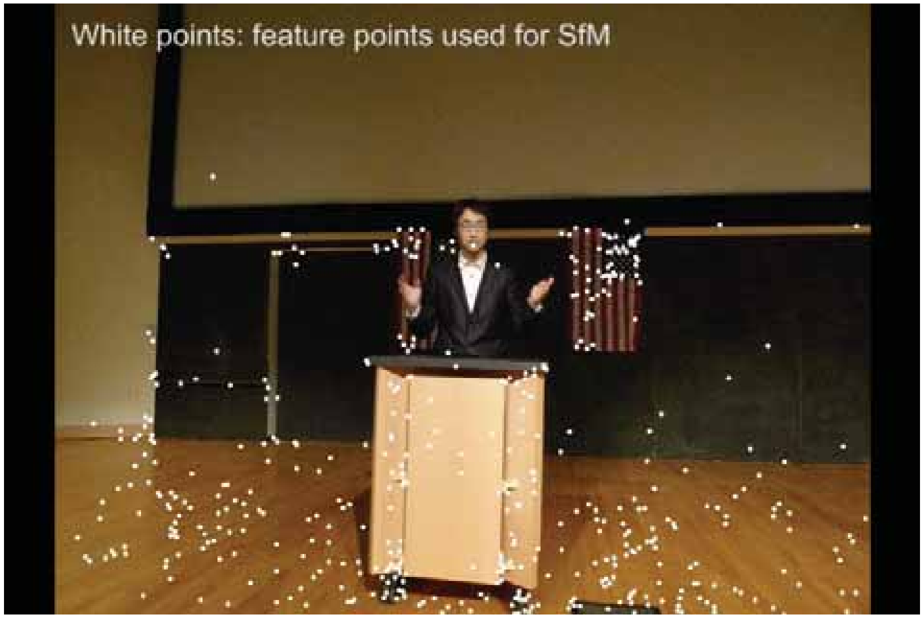
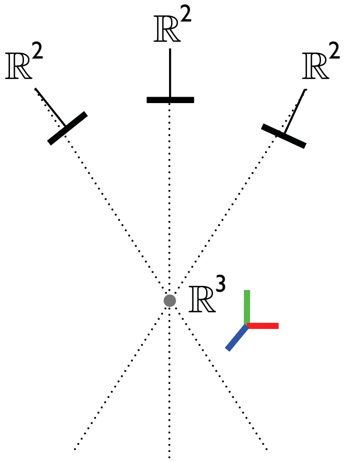
STRUCTURE FROM MOTION





$$\begin{bmatrix} \mathbf{Q}_1 \\ \vdots \\ \mathbf{Q}_F \end{bmatrix} \mathbf{X} = \begin{bmatrix} \mathbf{q}_1 \\ \vdots \\ \mathbf{q}_F \end{bmatrix}$$

IDEA: ESTIMATE CAMERA FROM RIGID PART

CHALLENGE

TRILINEAR ESTIMATION

$$\begin{bmatrix} \mathbf{R}_1 & & & \\ & \mathbf{R}_2 & & \\ & & \ddots & \\ & & & \mathbf{R}_F \end{bmatrix} \begin{bmatrix} \omega_{11} & \cdots & \omega_{1k} \\ \omega_{21} & & \omega_{2k} \\ \vdots & & \vdots \\ \omega_{F1} & \cdots & \omega_{Fk} \end{bmatrix} \begin{bmatrix} -\mathbf{b_1}- \\ -\mathbf{b_2}- \\ \vdots \\ -\mathbf{b_k}- \end{bmatrix}$$

$$\mathbf{W} = \mathbf{R}\mathbf{\Omega}\mathbf{B}$$

CHALLENGE

TRILINEAR ESTIMATION

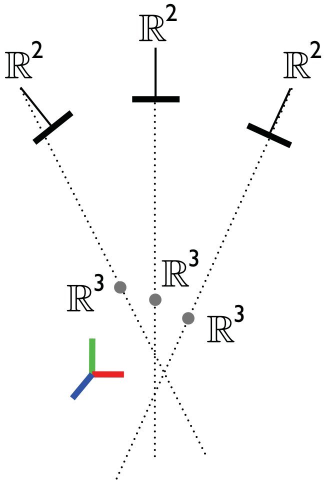
$$\begin{bmatrix} \mathbf{R}_1 & & & \\ & \mathbf{R}_2 & & \\ & & \ddots & \\ & & & \mathbf{R}_F \end{bmatrix} \begin{bmatrix} \omega_{11} & \cdots & \omega_{1k} \\ \omega_{21} & & \omega_{2k} \\ \vdots & & \vdots \\ \omega_{F1} & \cdots & \omega_{Fk} \end{bmatrix} \begin{bmatrix} -\mathbf{b_1}- \\ -\mathbf{b_2}- \\ \vdots \\ -\mathbf{b_k}- \end{bmatrix}$$

$$\mathbf{W} = \mathbf{R}\mathbf{\Omega}\mathbf{B}$$

RECONSTRUCTION EVENTS

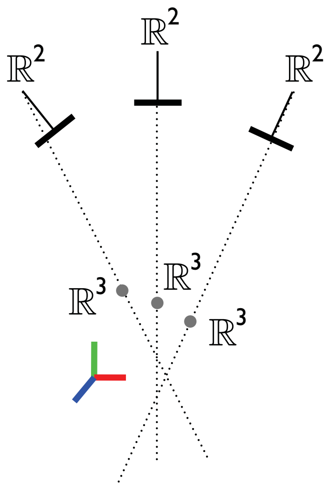
Trajectory
Reconstruction

RECONSTRUCTION EVENTS



Trajectory
Reconstruction

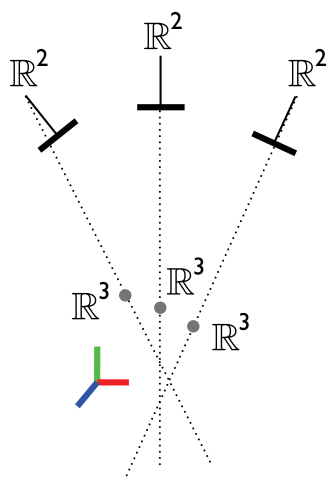
RECONSTRUCTION EVENTS



Trajectory Reconstruction

$$s_i \begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} = \mathbf{P}_i \begin{bmatrix} \mathbf{X}_i \\ 1 \end{bmatrix}$$

RECONSTRUCTION EVENTS



Trajectory Reconstruction

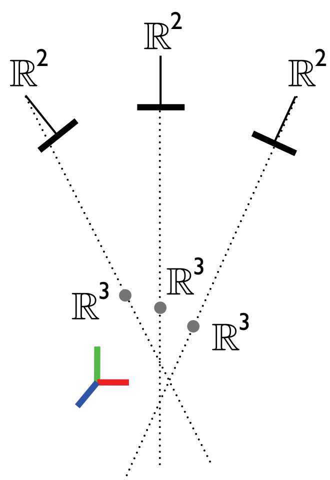
$$s_i \begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} = \mathbf{P}_i \begin{bmatrix} \mathbf{X}_i \\ 1 \end{bmatrix}$$

Structure from Motion

$$s_i \begin{bmatrix} \mathbf{x}_i \\ 1 \end{bmatrix} = \mathbf{P}_i \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \mathbf{Q}_1 \\ \vdots \\ \mathbf{Q}_F \end{bmatrix}}_{2F\text{x}3} \underbrace{\mathbf{X}}_{3\text{x}1} = \underbrace{\begin{bmatrix} \mathbf{q}_1 \\ \vdots \\ \mathbf{q}_F \end{bmatrix}}_{2F\text{x}1}$$

RECONSTRUCTION EVENTS



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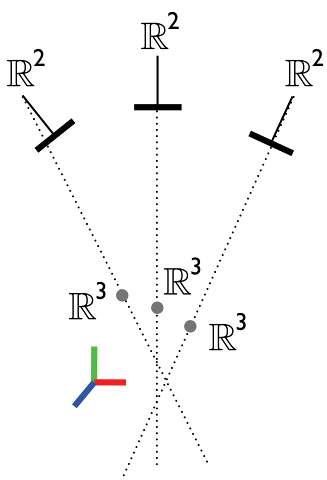
RECONSTRUCTION EVENTS



Trajectory
Reconstruction

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$$\underbrace{\begin{bmatrix} \mathbf{Q}_1 & & \\ & \ddots & \\ & & \mathbf{Q}_F \end{bmatrix}}_{2F\text{x}3F} \underbrace{\begin{bmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_F \end{bmatrix}}_{3F\text{x}1} = \underbrace{\begin{bmatrix} \mathbf{q}_1 \\ \vdots \\ \mathbf{q}_F \end{bmatrix}}_{2F\text{x}1}$$

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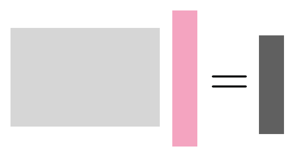
$$\begin{bmatrix} \mathbf{Q}_1 & & \\ & \ddots & \\ & & \mathbf{Q}_F \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_F \end{bmatrix} = \begin{bmatrix} \mathbf{q}_1 \\ \vdots \\ \mathbf{q}_F \end{bmatrix}$$

$$\mathbf{Q}\mathbf{X} = \mathbf{q}$$

$$
\begin{bmatrix} \mathbf{Q}_1 & & \\ & \ddots & \\ & & \mathbf{Q}_F \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_F \end{bmatrix} = \begin{bmatrix} \mathbf{q}_1 \\ \vdots \\ \mathbf{q}_F \end{bmatrix}
$$

$$
\mathbf{QX} = \mathbf{q}
$$



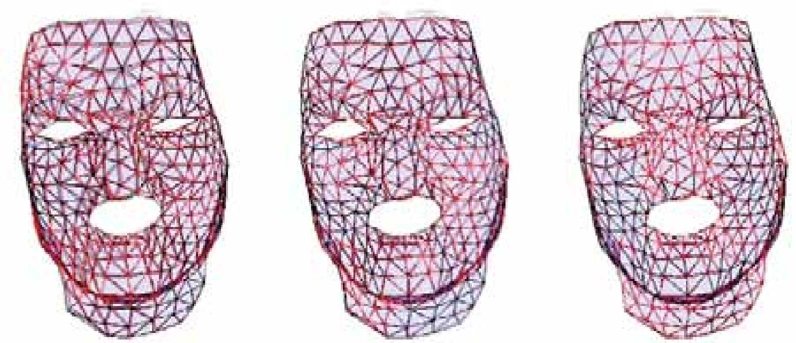
Trajectory Reconstruction

$$\begin{bmatrix} \mathbf{X}_{11} \\ \mathbf{X}_{12} \\ \vdots \\ \mathbf{X}_{1F} \end{bmatrix}_{3F\times 1} = \begin{bmatrix} | & | & & | \\ \theta_1 & \theta_2 & \cdots & \theta_k \\ | & | & & | \end{bmatrix}_{3F\times k} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix}_{k\times 1} = \Theta\beta$$

35 Basis 50 Basis 65 Basis

Trajectory Reconstruction

$$\begin{bmatrix} \mathbf{X}_{11} \\ \mathbf{X}_{12} \\ \vdots \\ \mathbf{X}_{1F} \end{bmatrix}_{3F\times 1} = \begin{bmatrix} | & | & & | \\ \theta_1 & \theta_2 & \cdots & \theta_k \\ | & | & & | \end{bmatrix}_{3F\times k} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix}_{k\times 1} = \Theta\beta$$



35 Basis 50 Basis 65 Basis

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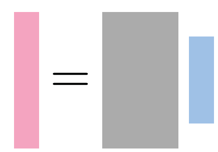
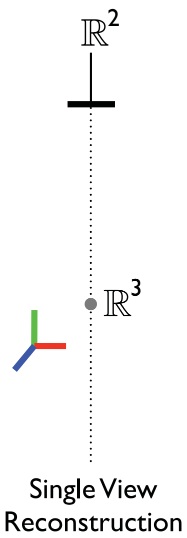
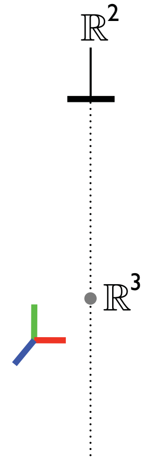
$$\mathbf{X} = \Theta\beta$$



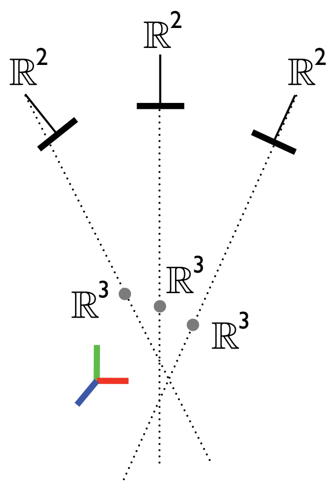
Trajectory Reconstruction



Trajectory Reconstruction

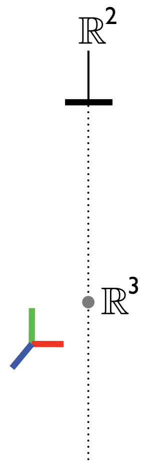


Single View
Reconstruction

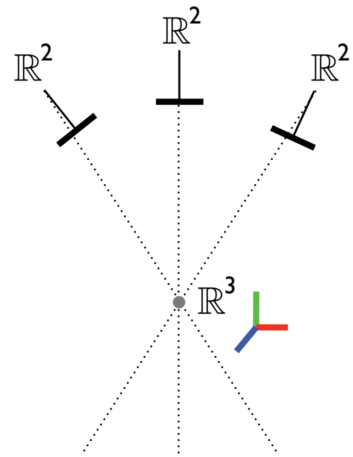


Trajectory
Reconstruction

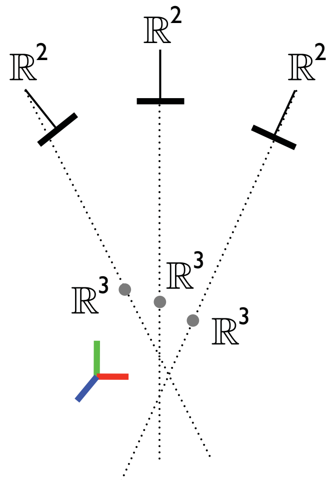
Trajectory Reconstruction



Single View
Reconstruction

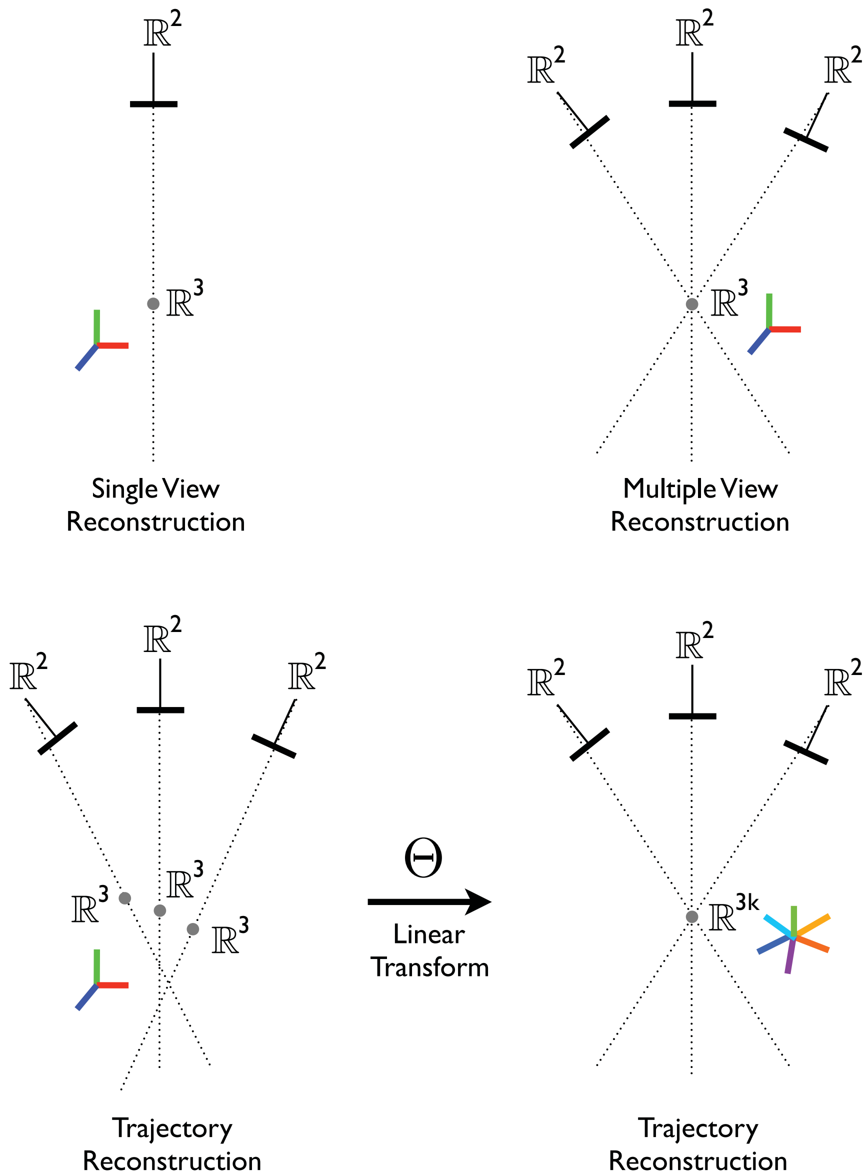


Multiple View
Reconstruction



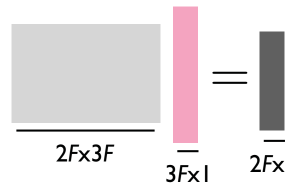
Trajectory
Reconstruction

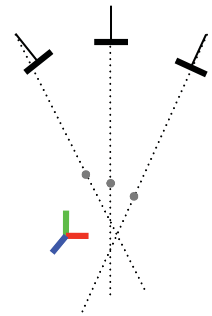
Trajectory Reconstruction



LINEAR SOLUTION

$$\mathbf{Q}\mathbf{X} = \mathbf{q}$$





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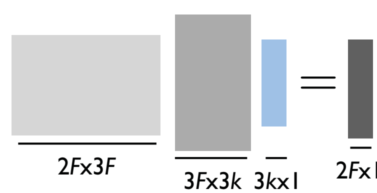

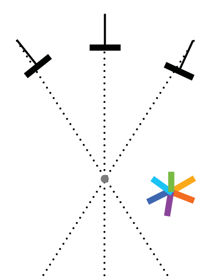

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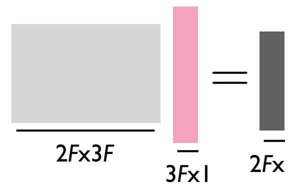
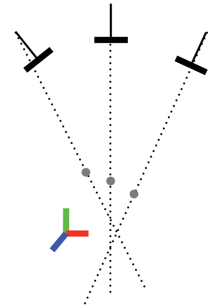
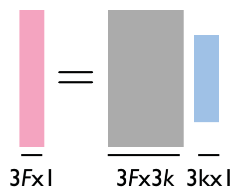
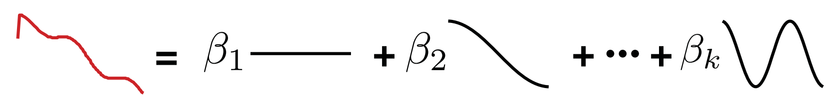




$$\mathbf{X} = \Theta\beta$$





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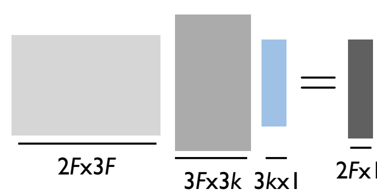

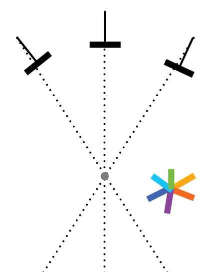

ALGORITHM

ALGORITHM

- GIVEN POINT CORRESPONDENCES AND EXIF DATA

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  - ESTIMATE THE CAMERA MATRICES USING RANSAC

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  - USING CAMERA MATRICES AND DYNAMIC POINT CORRESPONDENCES:
    - CREATE OVERLOADED LINEAR SYSTEM USING DCT BASIS

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        - SOLVE LINEAR SYSTEM FOR DCT COEFFICIENTS

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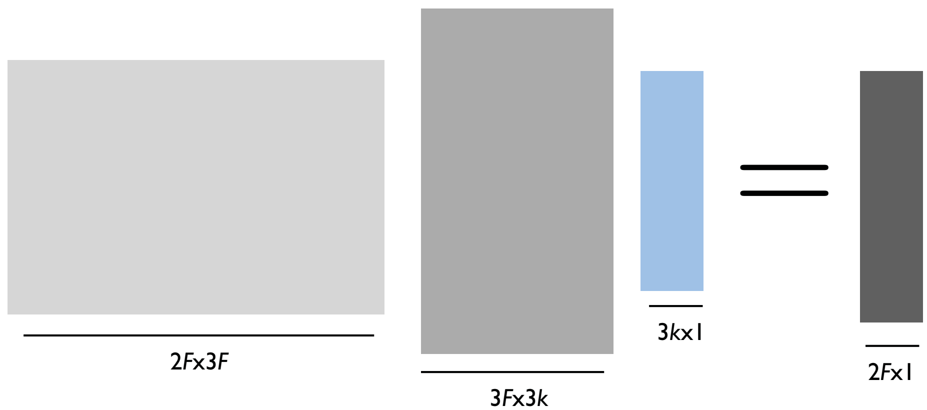
- GIVEN POINT CORRESPONDENCES AND EXIF DATA
  - ESTIMATE THE CAMERA MATRICES USING RANSAC
  - USING CAMERA MATRICES AND DYNAMIC POINT CORRESPONDENCES:
    - CREATE OVERLOADED LINEAR SYSTEM USING DCT BASIS
    - SOLVE LINEAR SYSTEM FOR DCT COEFFICIENTS
  - BUNDLE ADJUSTMENT

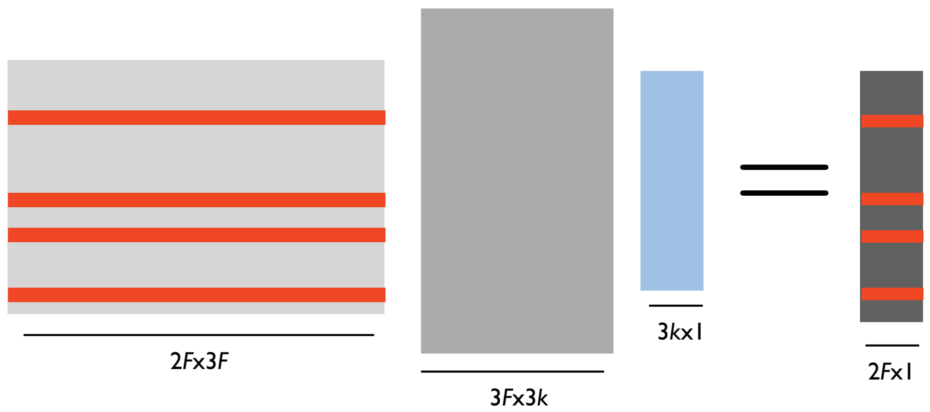
MISSING DATA

$$\mathbf{Q}\Theta\beta = \mathbf{q}$$



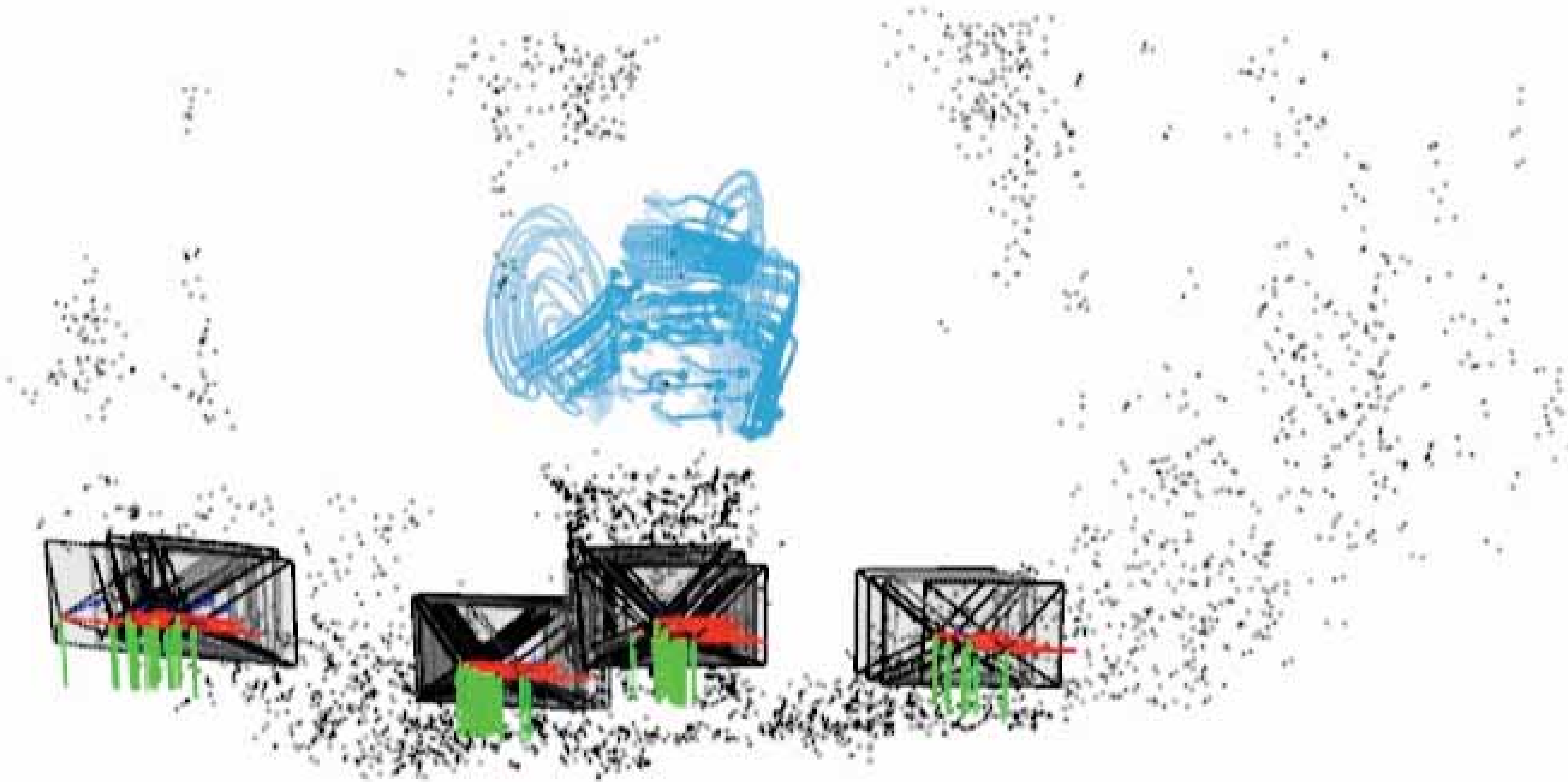
MISSING DATA

$$\mathbf{Q}\Theta\beta = \mathbf{q}$$



PARK ET AL., ECCV 2010

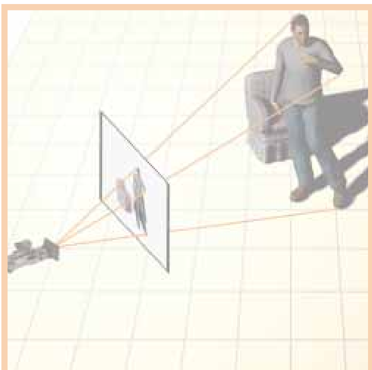
3D Reconstruction of a Moving Point from a Series of 2D Projections



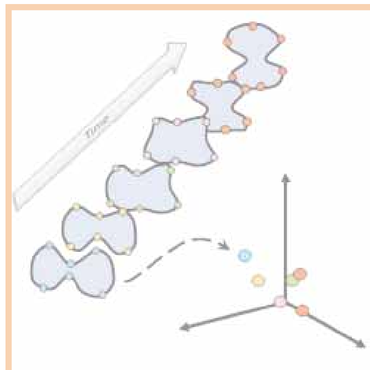
Result of 3D trajectory reconstruction

NONRIGID STRUCTURE FROM MOTION

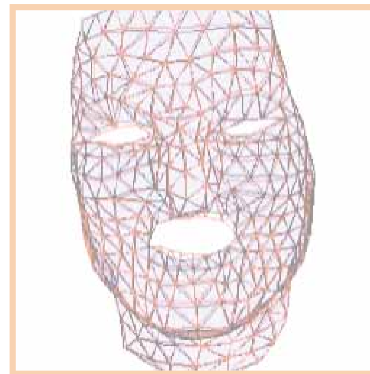
Tutorial Outline



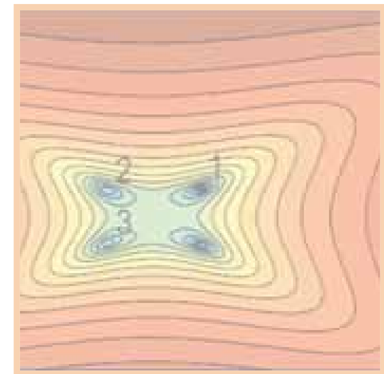
Introduction to Nonrigid SFM



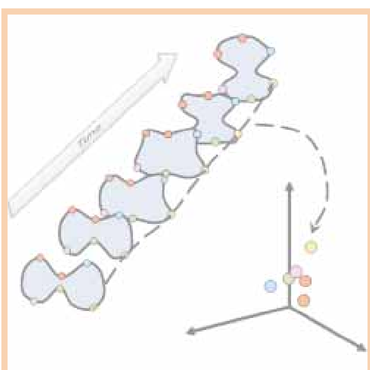
Shape Representation



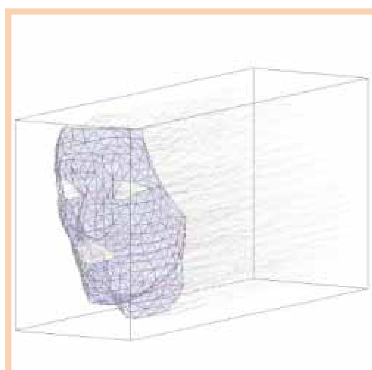
Shape Estimation



Ambiguity of Orthogonality Constraints

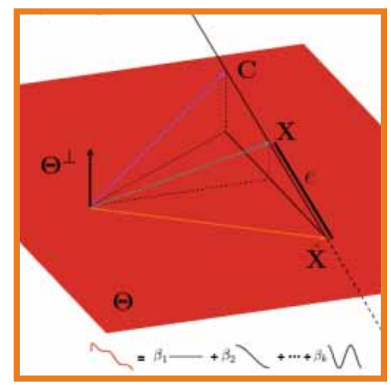


Trajectory Representation



Shape-Trajectory Duality

Trajectory Estimation



Reconstructibility and Limitations

AMBIGUITY

AMBIGUITY

THEOREM 1: Trajectory reconstruction using any linear trajectory basis is impossible if *corr*(X,C) = ± 1

AMBIGUITY

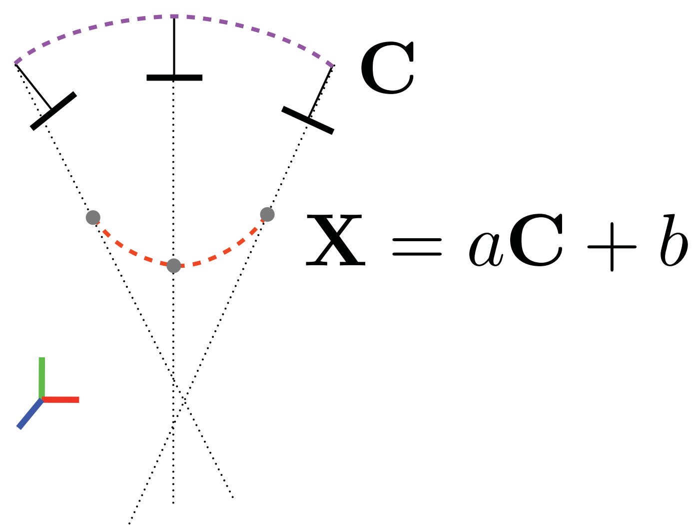
THEOREM 1: Trajectory reconstruction using any linear trajectory basis is impossible if *corr*(X,C) = ± 1

THEOREM 2: $\lim_{\eta \to \inf} \beta = \hat{\beta}$

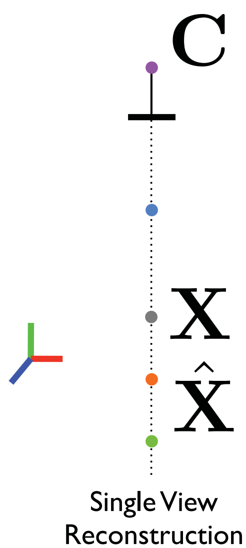
$$\eta = \frac{\|\Theta^{\perp} \beta_{\mathbf{C}}^{\perp}\|}{\|\Theta^{\perp} \beta_{\mathbf{X}}^{\perp}\|}$$

CORRELATED **X** and **C**

<u>THEOREM 1</u>: Trajectory reconstruction using any linear trajectory basis is impossible if *corr*(X,C) = ± 1



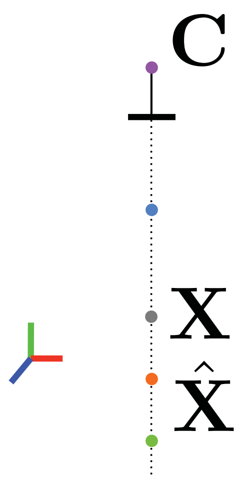
HYPERPLANE OF SOLUTIONS



$$\hat{\mathbf{X}} = a\mathbf{X} + (1-a)\mathbf{C}$$

SINGLE VIEW
3D RECONSTRUCTION

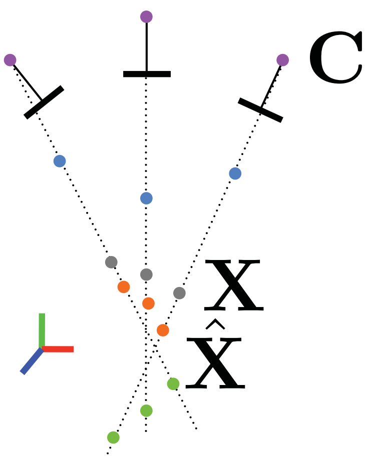
HYPERPLANE OF SOLUTIONS



Single View
Reconstruction

$$\hat{\mathbf{X}} = a\mathbf{X} + (1 - a)\mathbf{C}$$

SINGLE VIEW
3D RECONSTRUCTION



Trajectory
Reconstruction

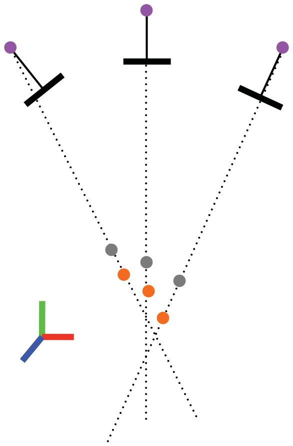
$$\hat{\mathbf{X}} = \mathbf{A}\mathbf{X} + (1 - \mathbf{A})\mathbf{C}$$

MULTIPLE VIEW
DYNAMIC 3D
RECONSTRUCTION

GEOMETRY OF **C** AND **X**

$\mathbb{R}^{3F}$

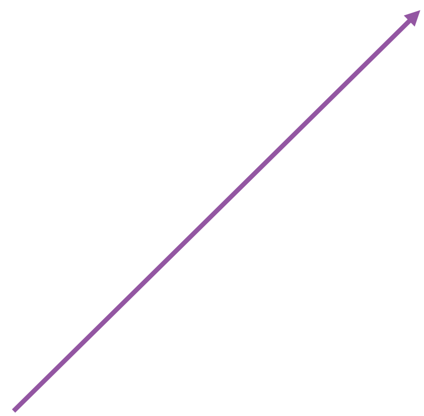
$$\mathbf{Q}\boldsymbol{\Theta}\beta = \mathbf{q}$$



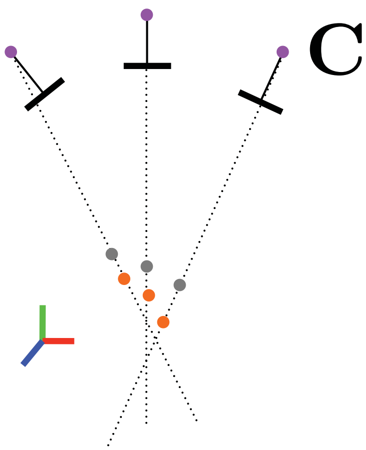
GEOMETRY OF **C** AND **X**

$\mathbb{R}^{3F}$

$$\mathbf{Q}\boldsymbol{\Theta}\beta = \mathbf{q}$$



$\mathbf{C}$

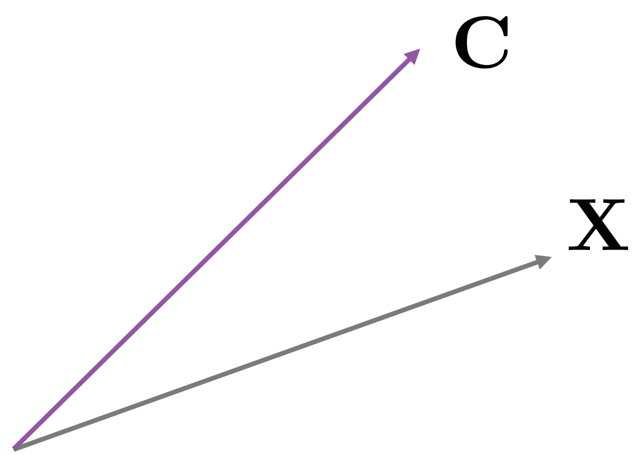


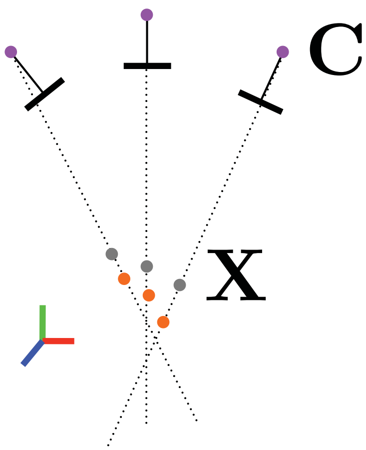
Trajectory
Reconstruction

GEOMETRY OF **C** AND **X**

$\mathbb{R}^{3F}$

$$\mathbf{Q}\Theta\beta = \mathbf{q}$$





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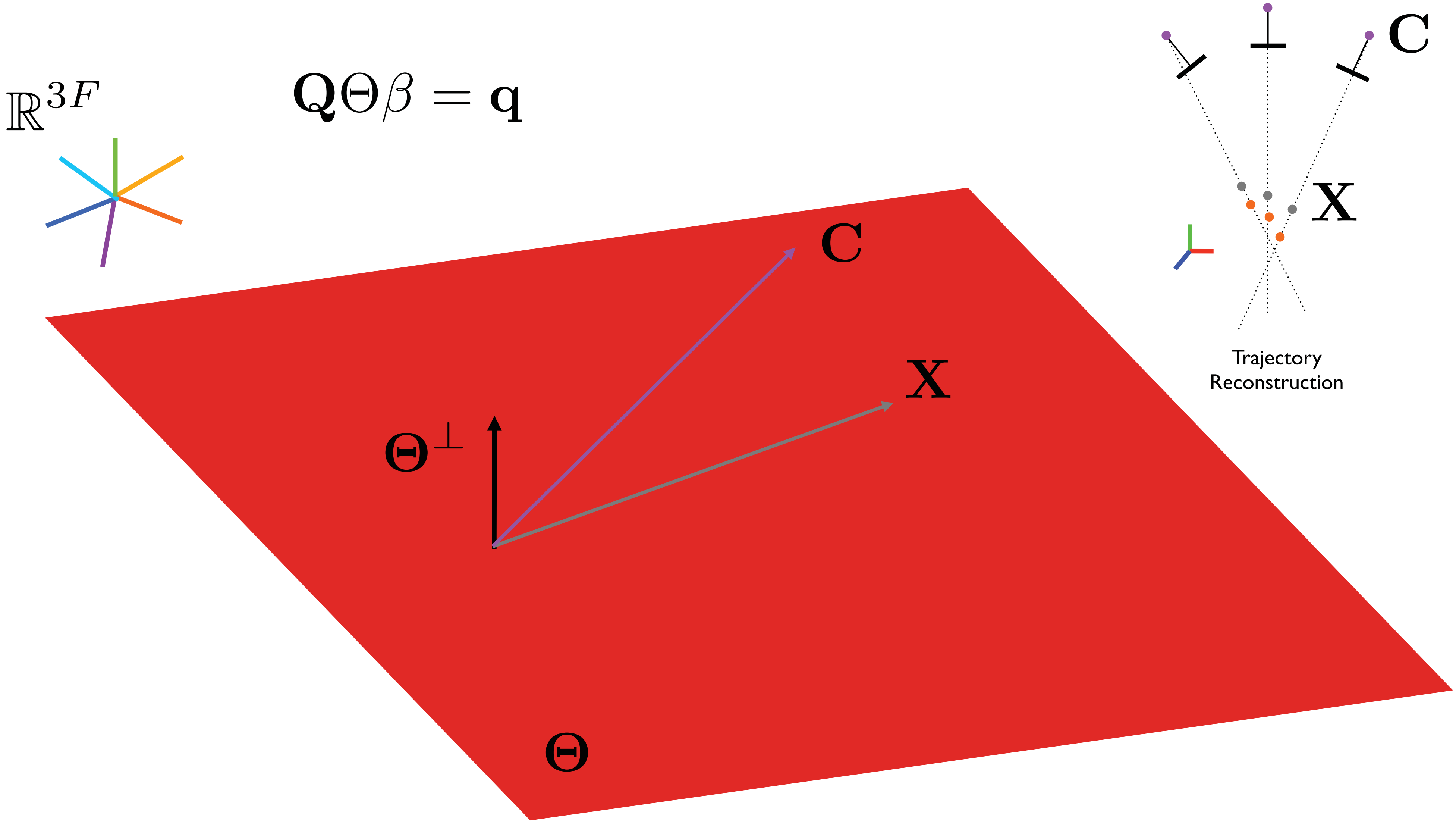

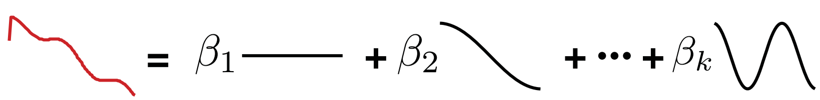

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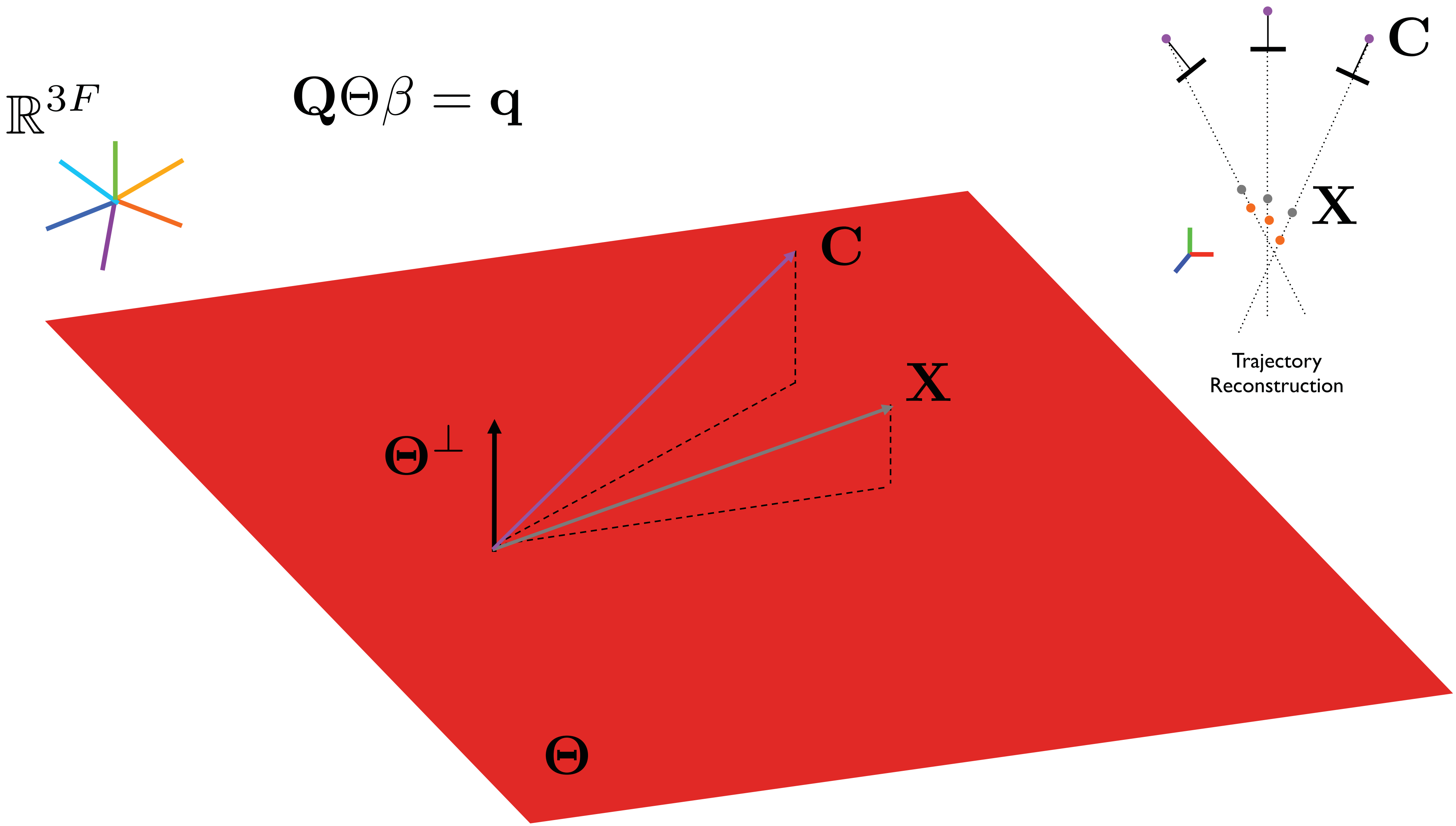

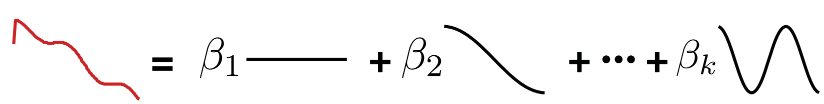

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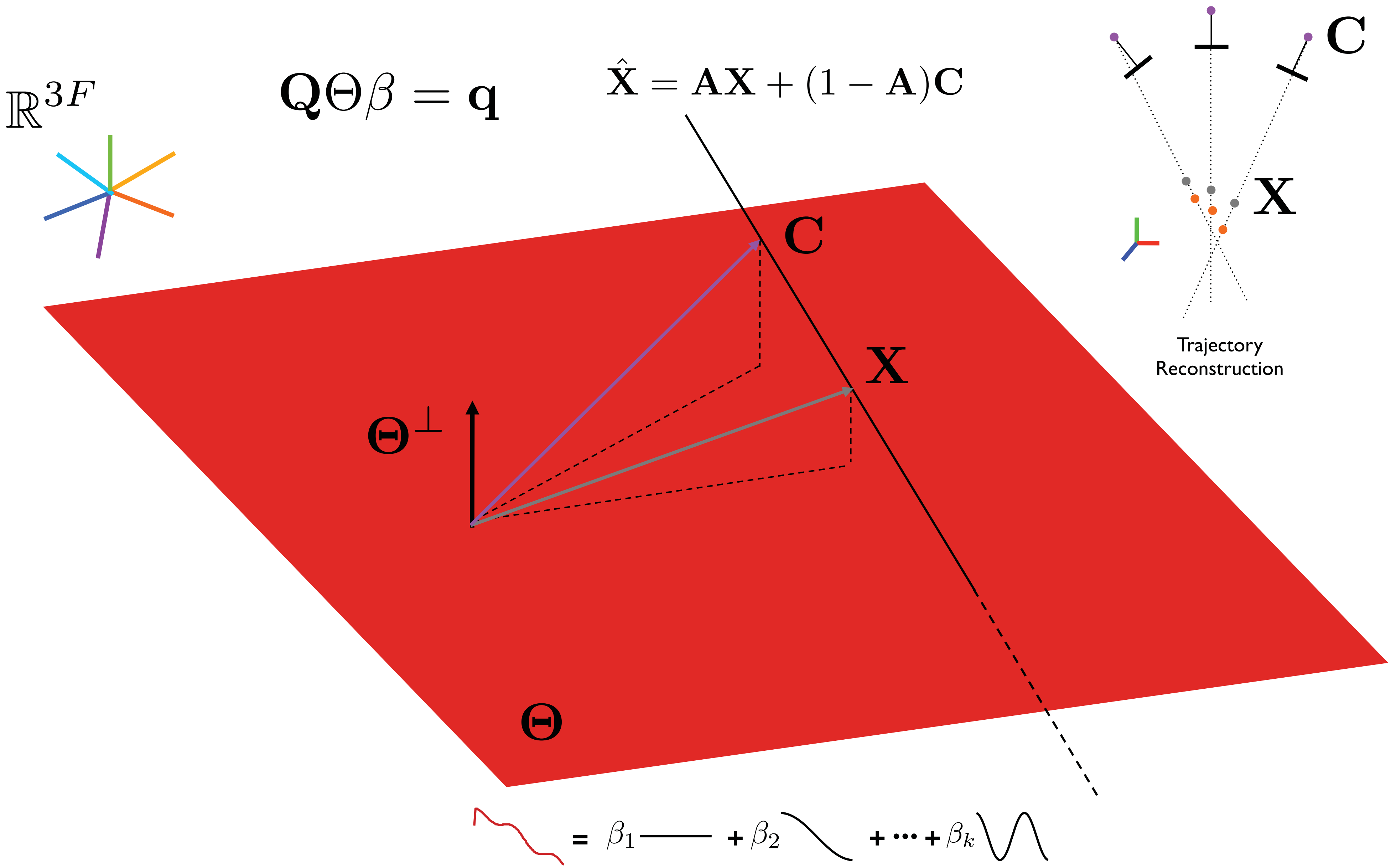

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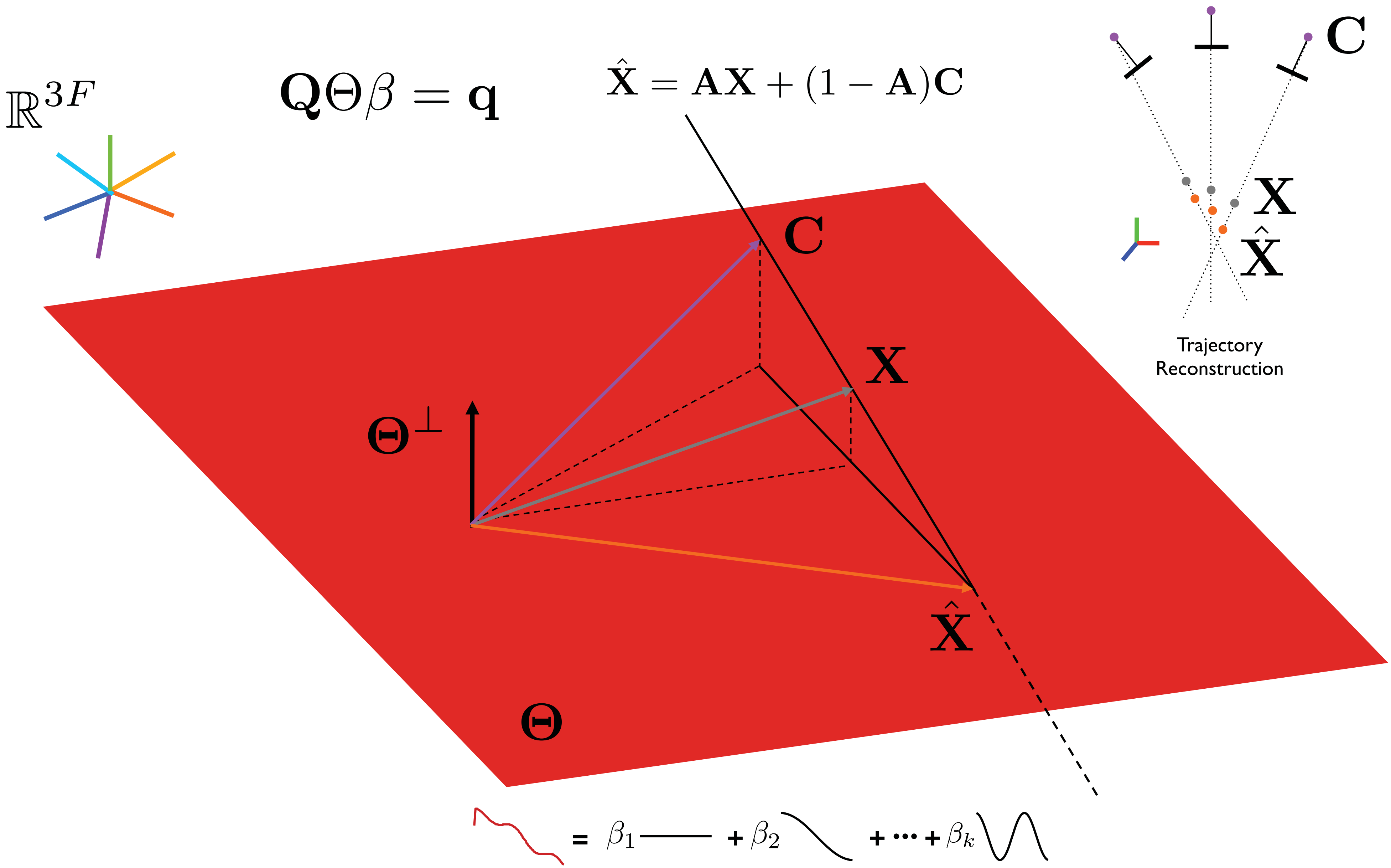

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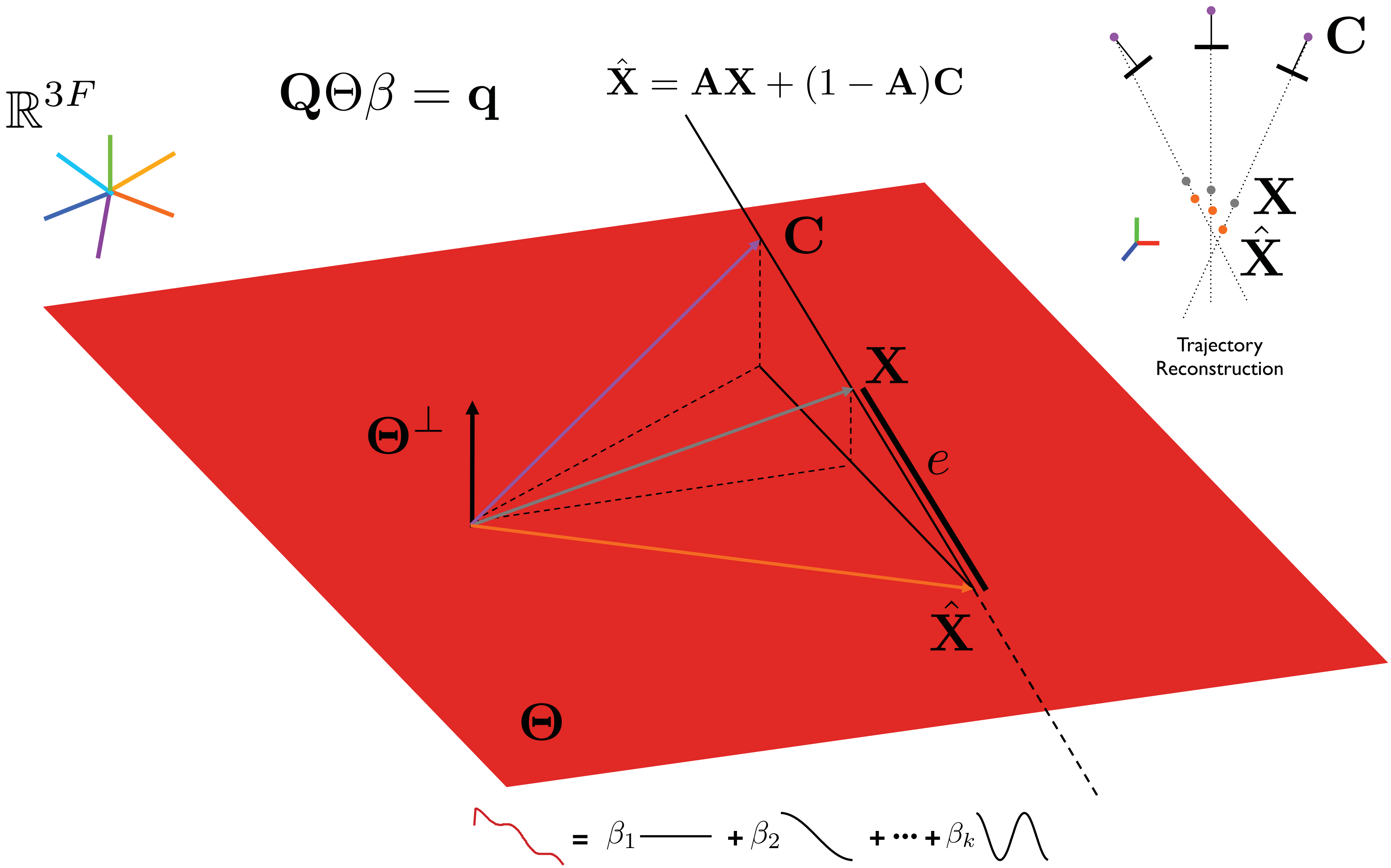
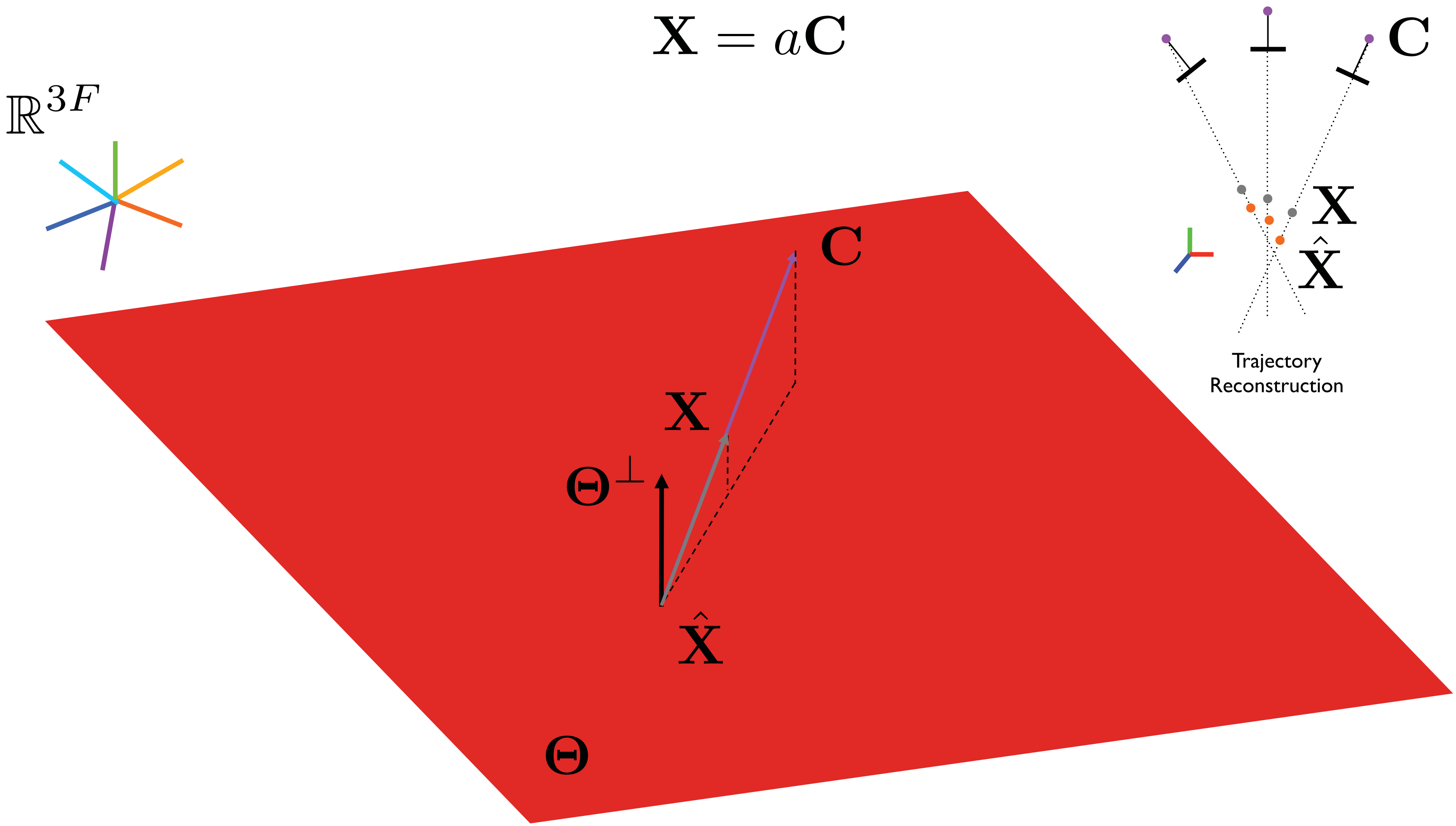


THEOREM 1: CORRELATED **C** AND **X**

$$\mathbf{X} = a\mathbf{C}$$



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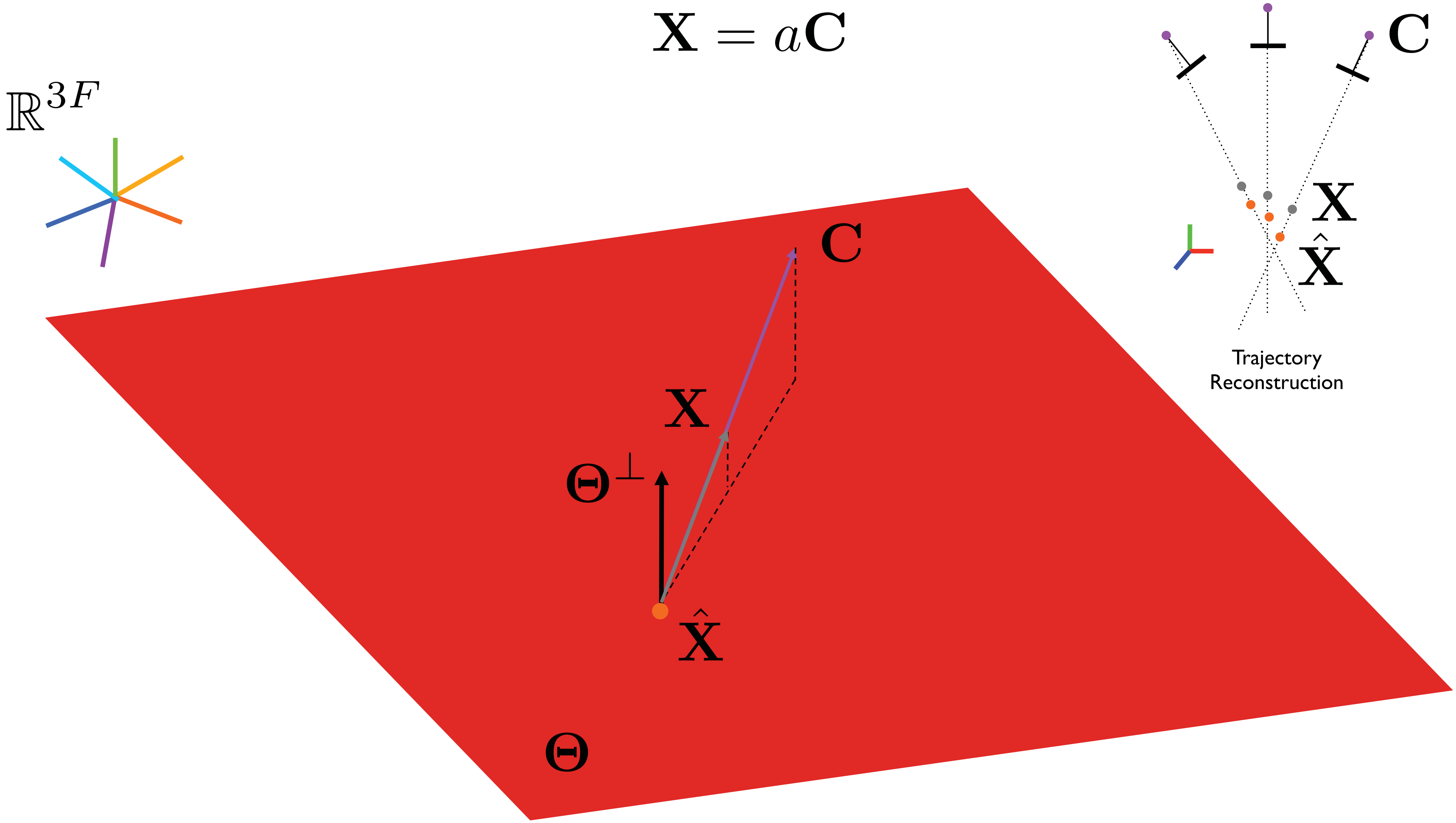

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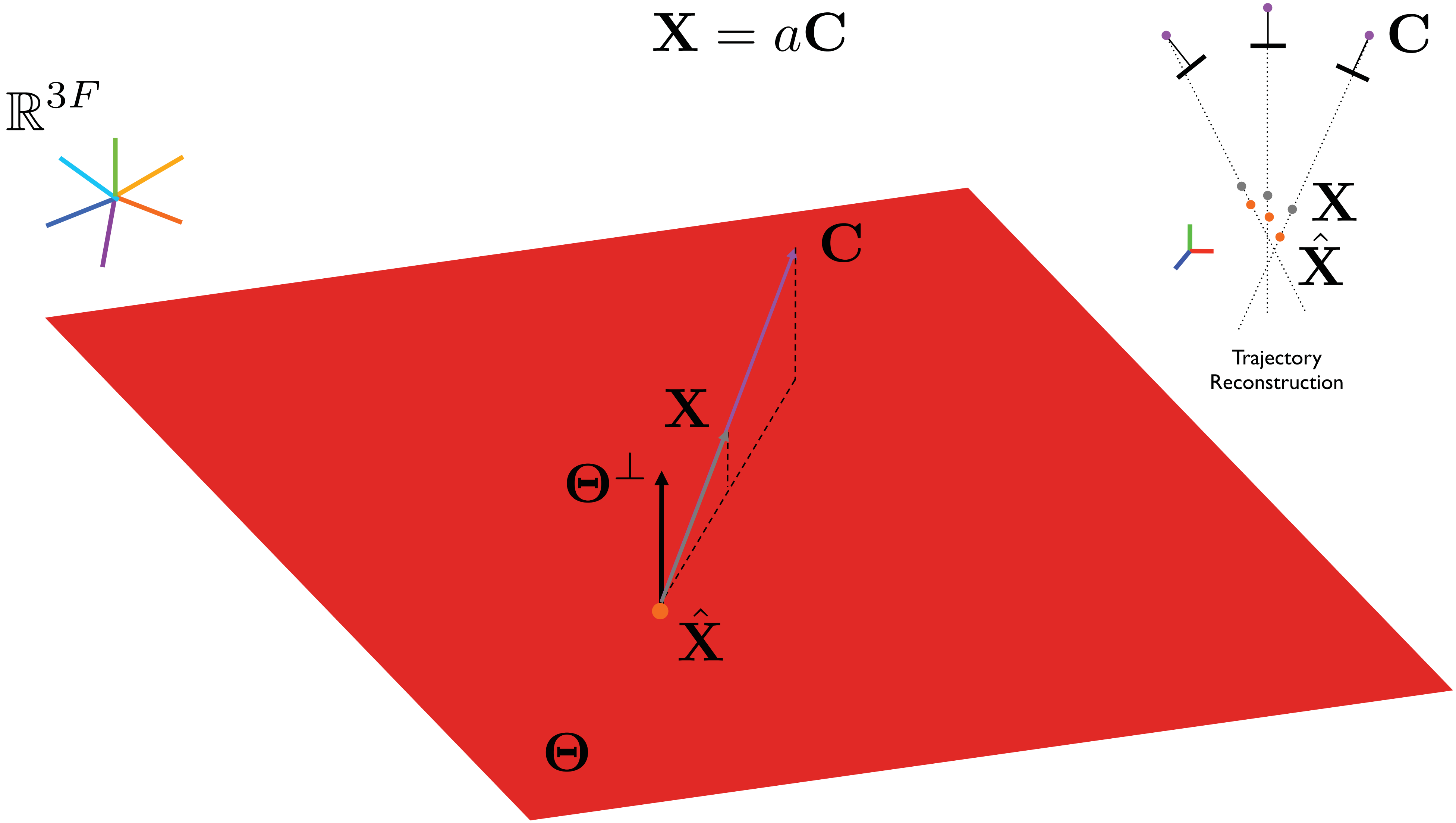

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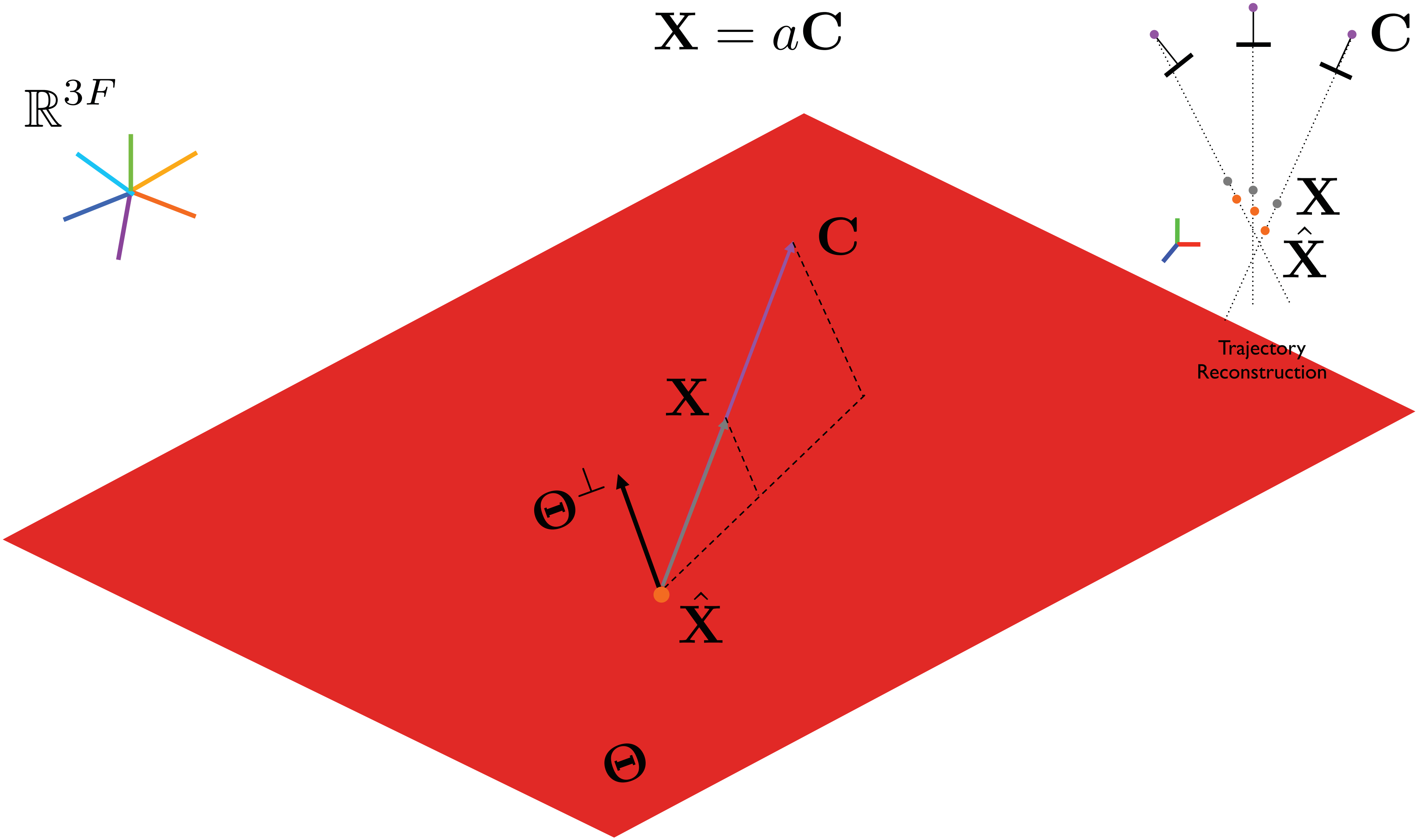
$\mathbb{R}^{3F}$



RECONSTRUCTIBILITY

THEOREM 2: $\lim_{\eta \to \inf} \beta = \hat{\beta}$

$$\eta = \frac{\|\Theta^{\perp} \beta_{\mathbf{C}}^{\perp}\|}{\|\Theta^{\perp} \beta_{\mathbf{X}}^{\perp}\|}$$

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$\eta \propto$ HOW POORLY THE BASIS DESCRIBES **C** $= \|\Theta^{\perp} \beta^{\perp}_{\mathbf{C}}\|$

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THEOREM 2: $\lim_{\eta \to \inf} \beta = \hat{\beta}$

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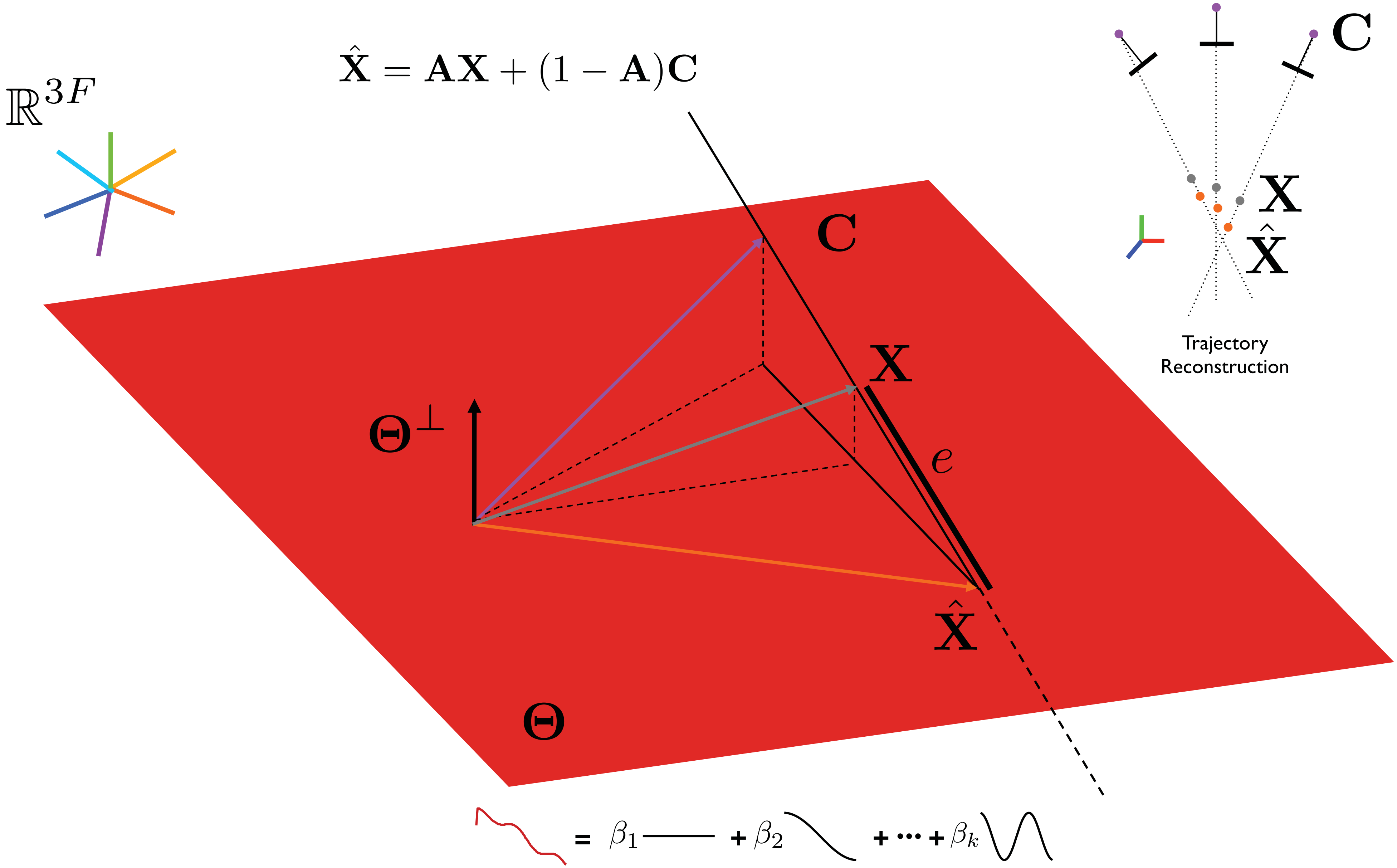
$\eta \propto$ HOW POORLY THE BASIS DESCRIBES **C** $= \|\Theta^{\perp} \beta_{\mathbf{C}}^{\perp}\|$

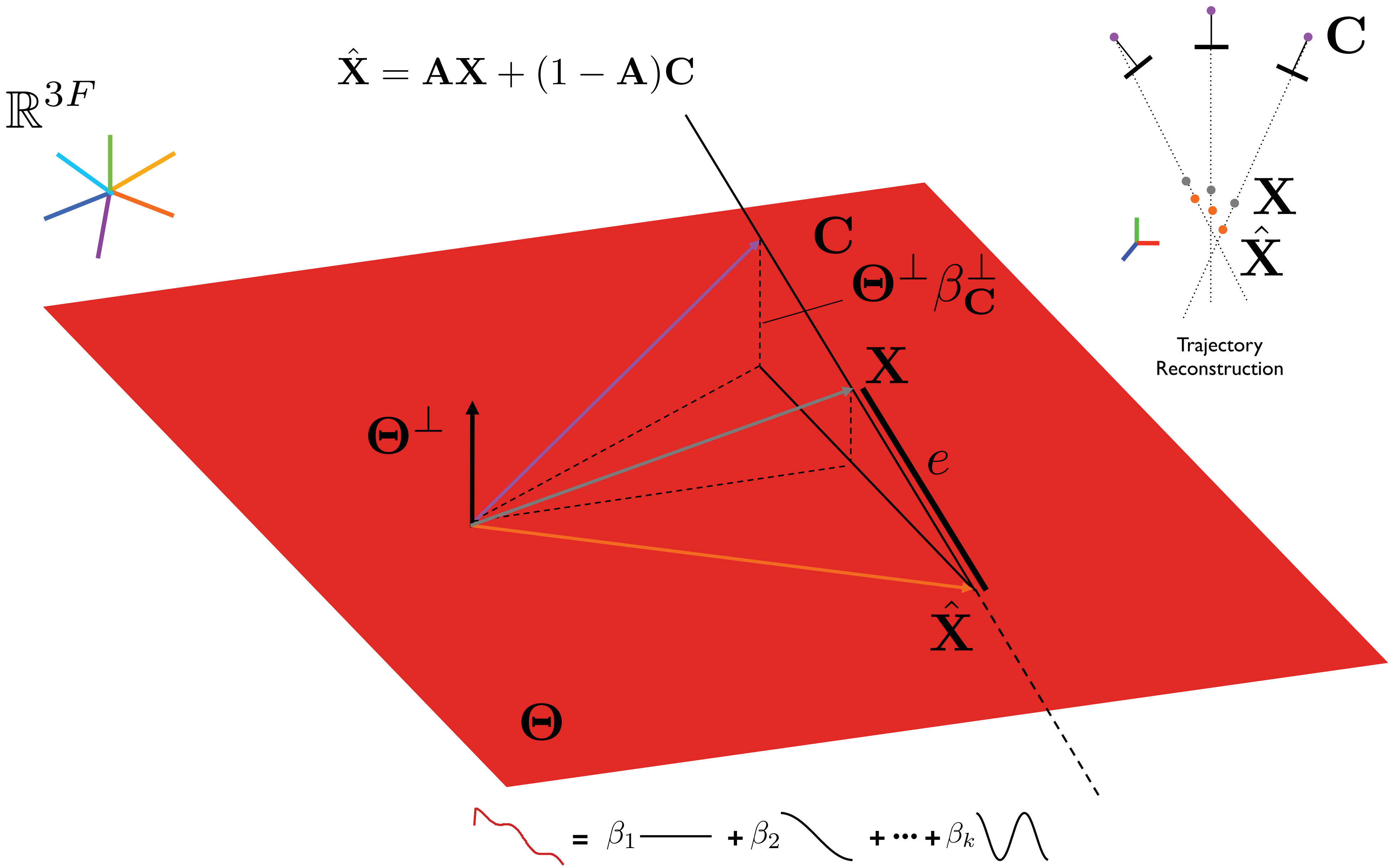
$\eta \propto$ HOW WELL THE BASIS DESCRIBES **X** $= \frac{1}{\|\Theta^{\perp} \beta_{\mathbf{X}}^{\perp}\|}$

GEOMETRY OF **C** AND **X**

$$\hat{\mathbf{X}} = \mathbf{A}\mathbf{X} + (1 - \mathbf{A})\mathbf{C}$$



GEOMETRY OF **C** AND **X**



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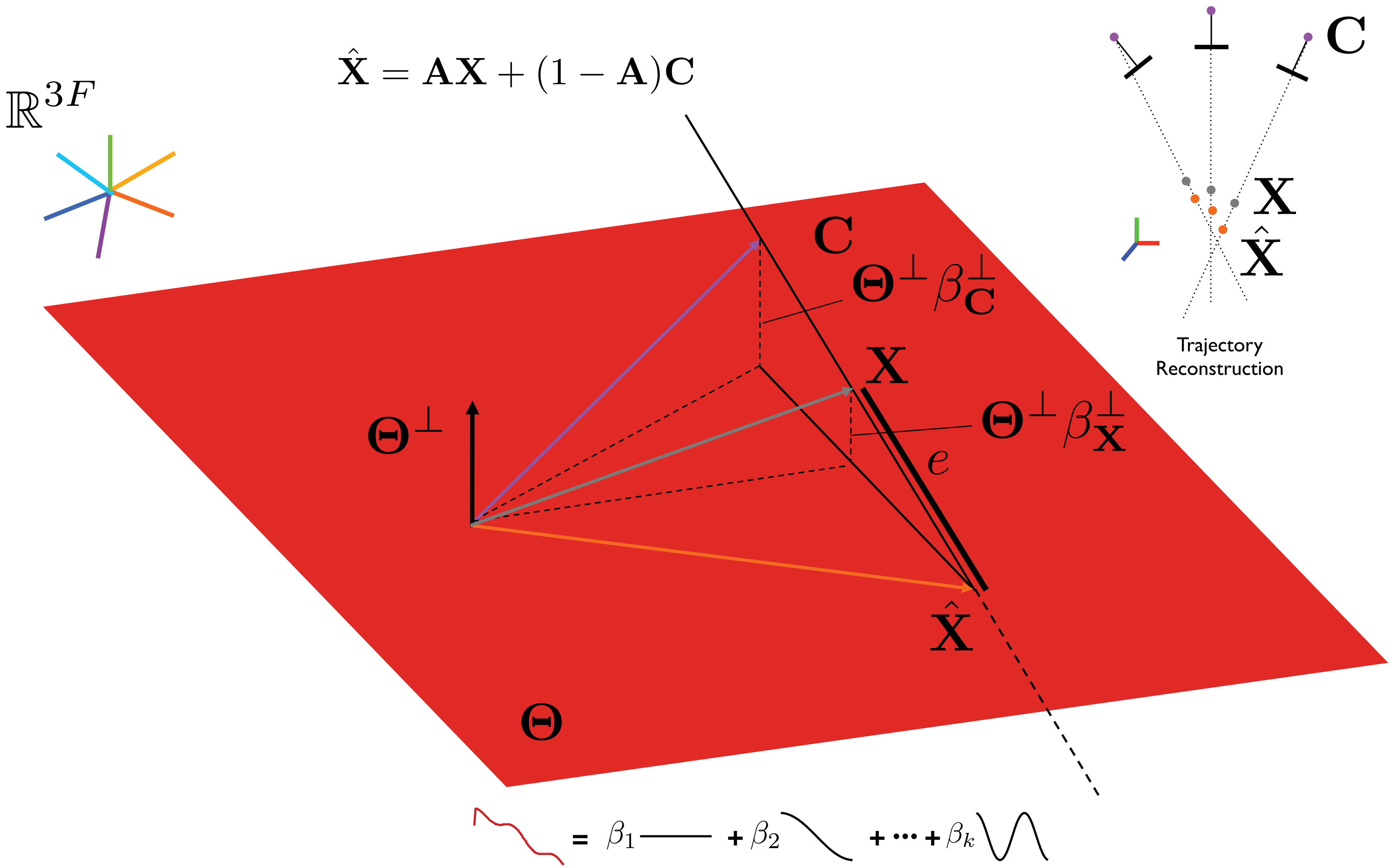
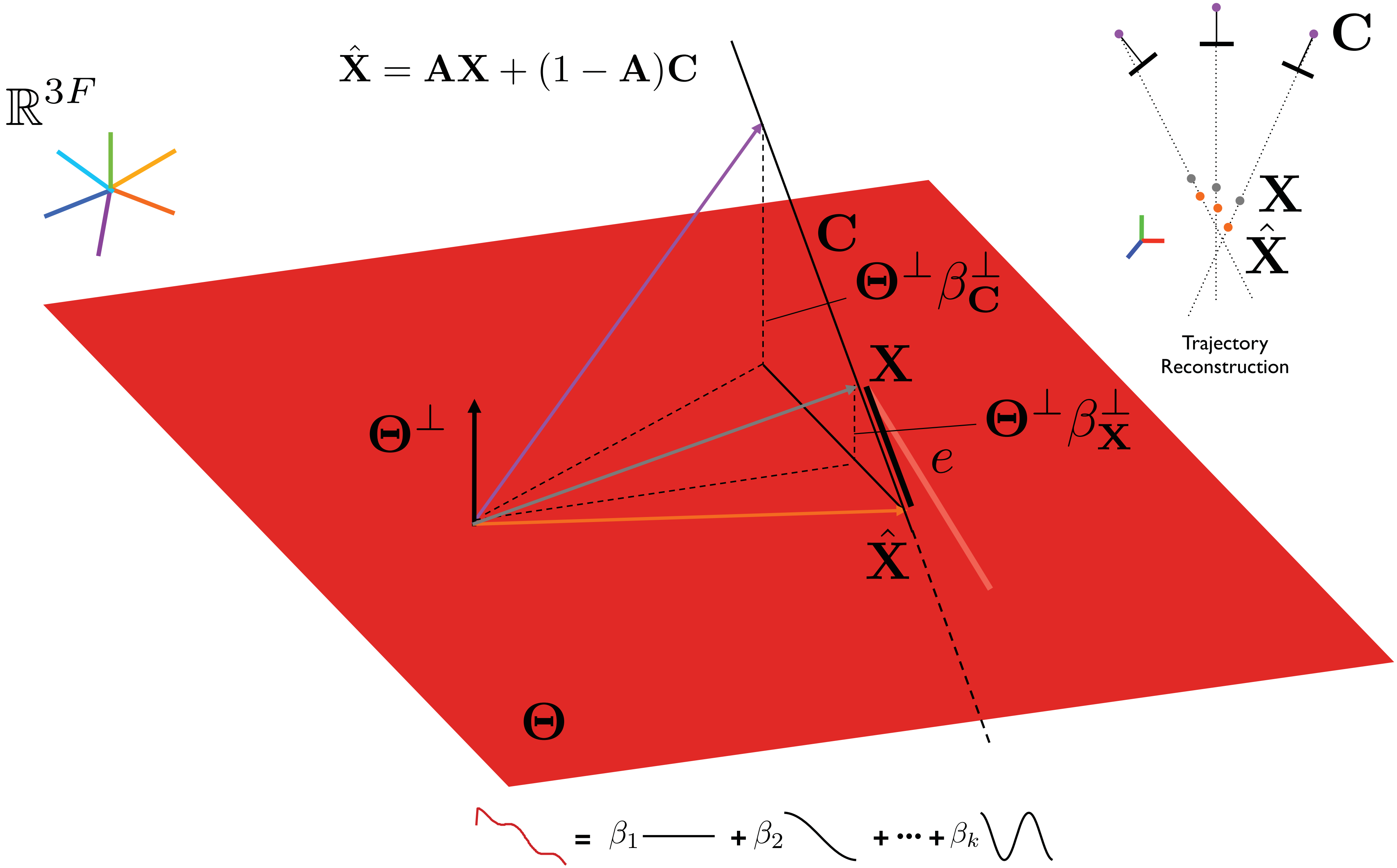


GEOMETRY OF **C** AND **X**



WHAT DOES THIS TELL US?

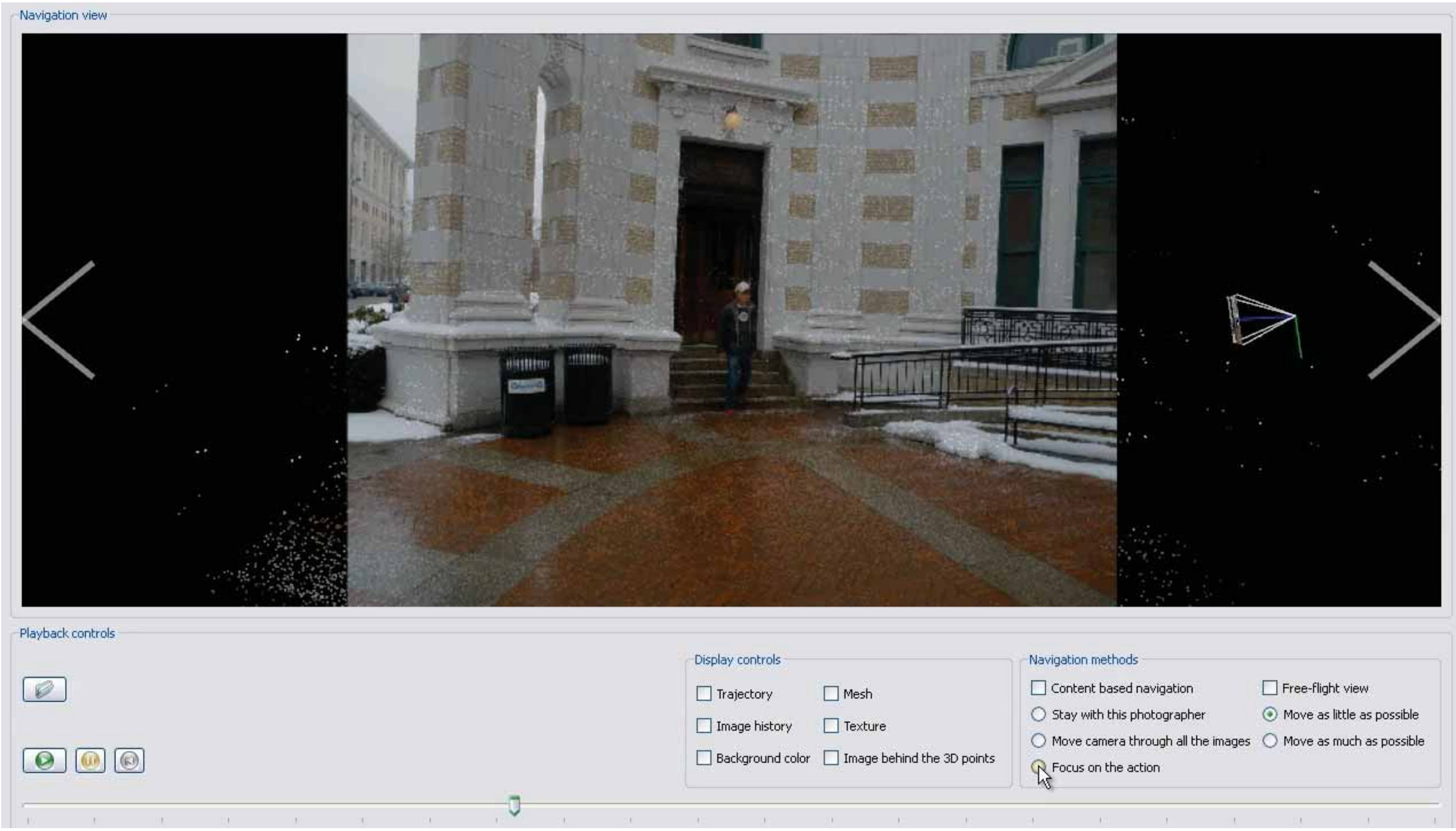
- DE-CORRELATE CAMERA AND OBJECT MOTION

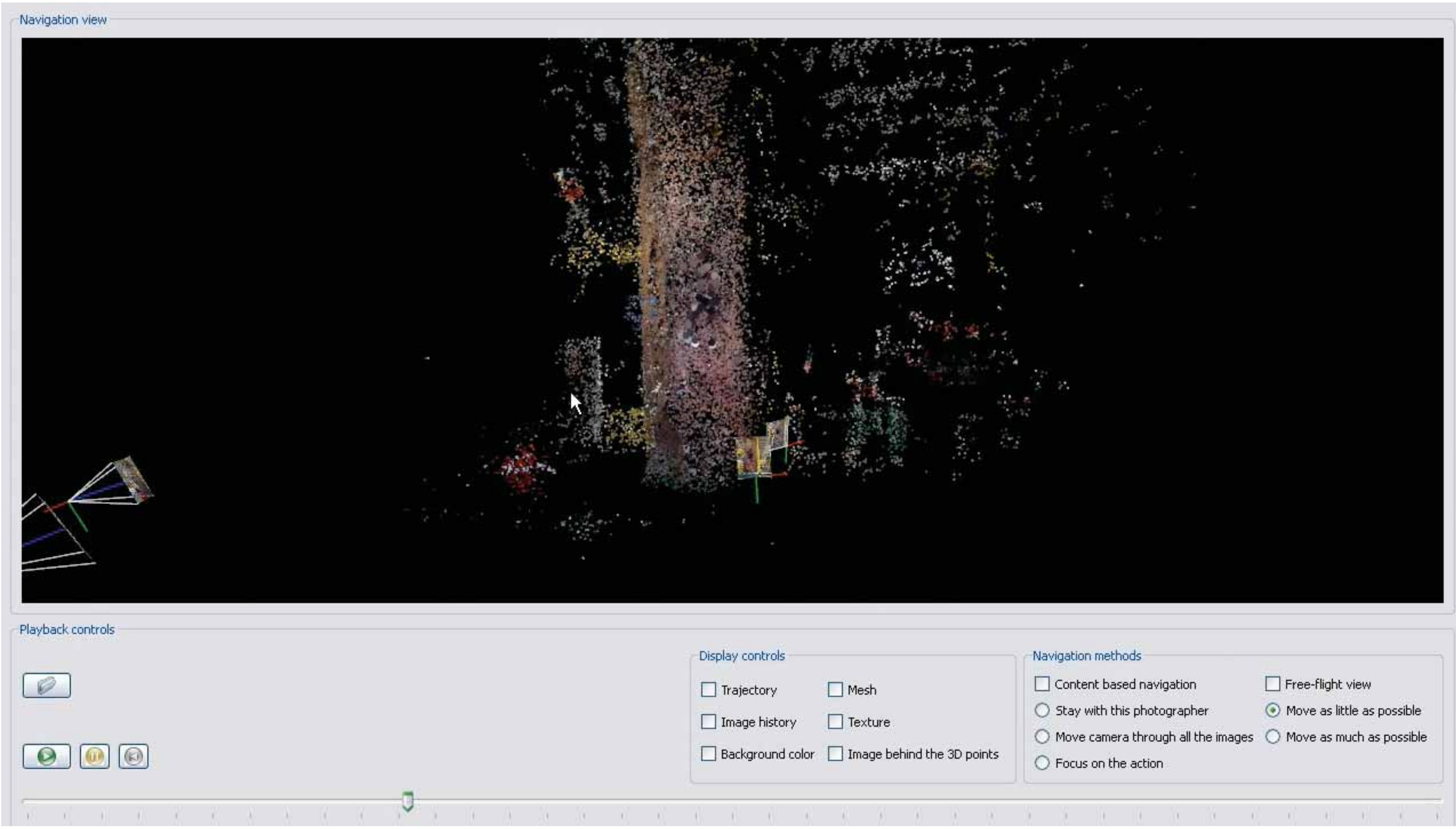
Hand Shake

Greeting


Dance

Rock Climbing


Navigation view
Playback controls
Display controls
Trajectory
Mesh
Image history
Texture
Background color
Image behind the 3D points
Navigation methods
Content based navigation
Free-flight view
Stay with this photographer
Move as little as possible
Move camera through all the images
Move as much as possible
Focus on the action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PARK ET AL. 2010

IN PERSPECTIVE

- **PROBLEMS SOLVED**:
  - PERSPECTIVE RECONSTRUCTION
  - HANDLES MISSING DATA
  - LINEAR SOLVE (FAST, GLOBAL OPTIMUM)
- **OPEN PROBLEMS**:
  - HANDLING SMOOTHLY MOVING CAMERAS
  - AUTOMATIC COMPUTATION OF *K*
  - EXPLOITING DEPENDENCIES *BETWEEN* TRAJECTORIES
  - "PHYSICS-AWARE" ESTIMATION

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