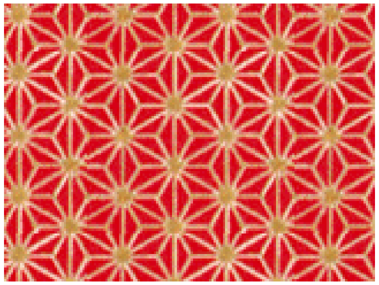

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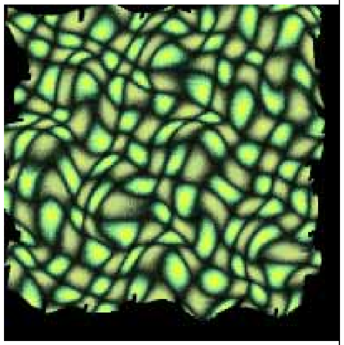
Computational Symmetry

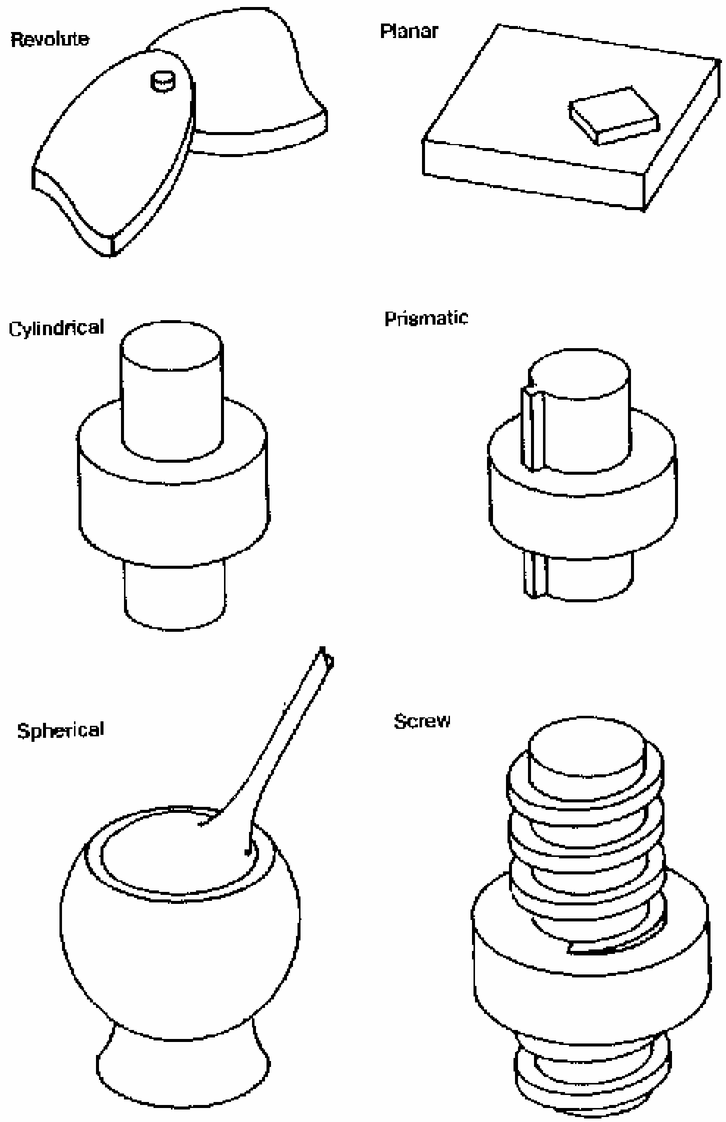


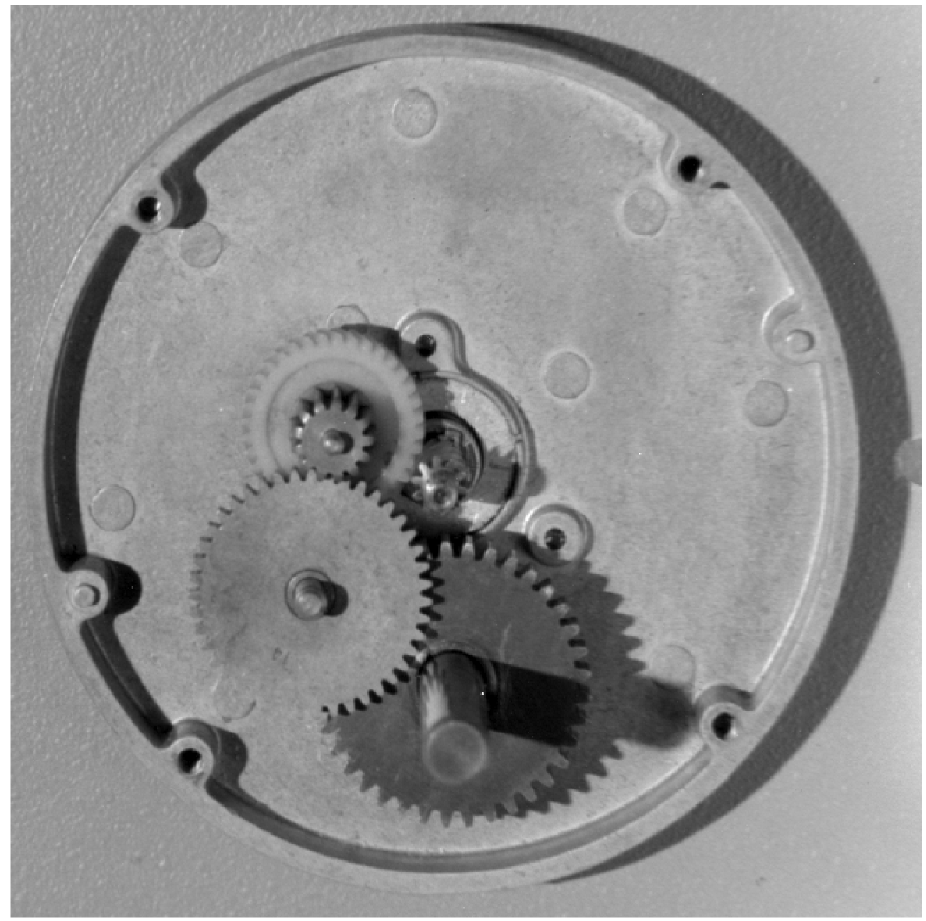
Today's Theme:
1. Problem Formalization
2. Computation of Symmetry Groups

Example I:
Symmetry in Contact Motions

Solids in Contact-Motion

Lower-pairs





The hierarchy of the subgroups of The Euclidean Group

O → orthogonal group
SO → special orthogonal group
T → translation group
D → dihedral group
C → cyclic group

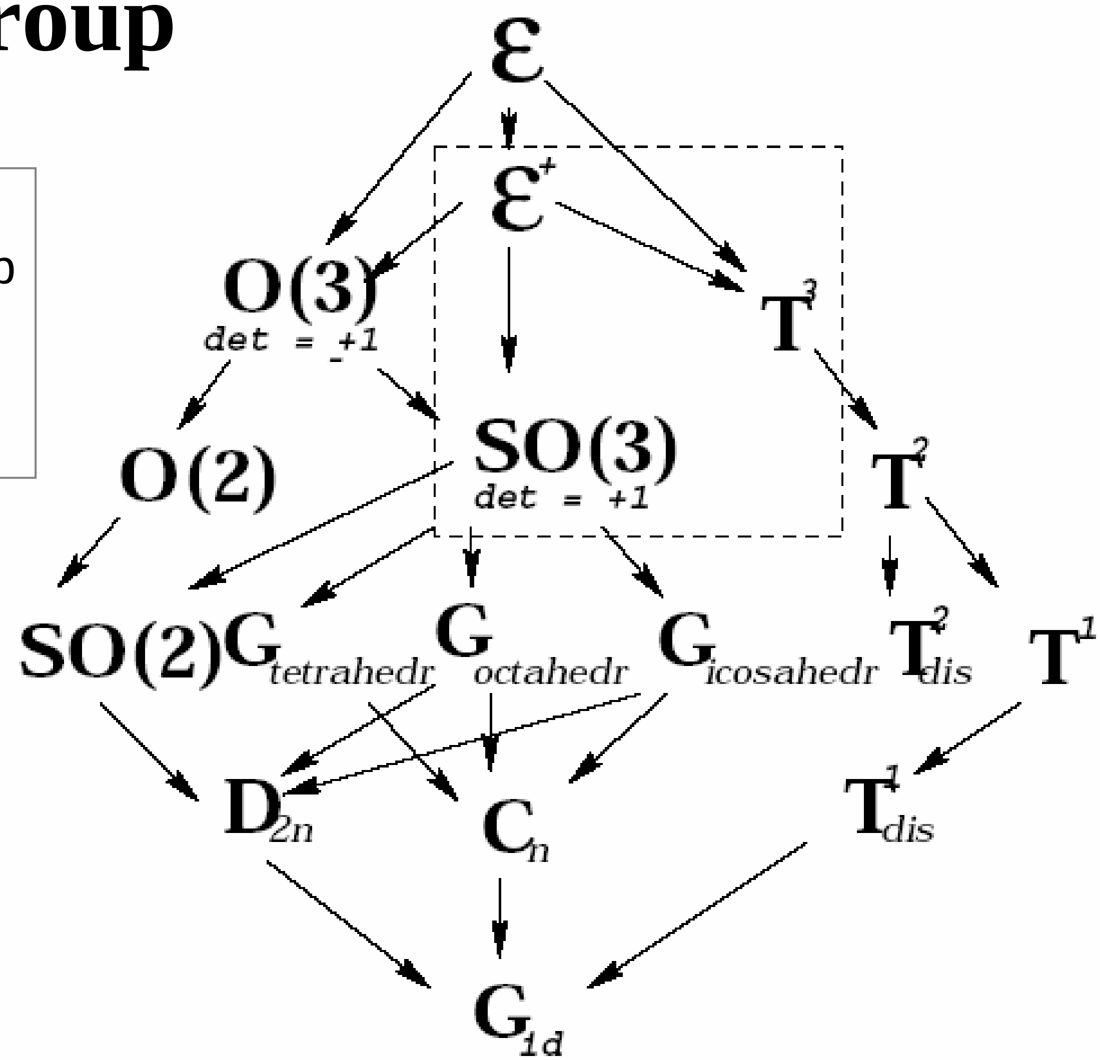
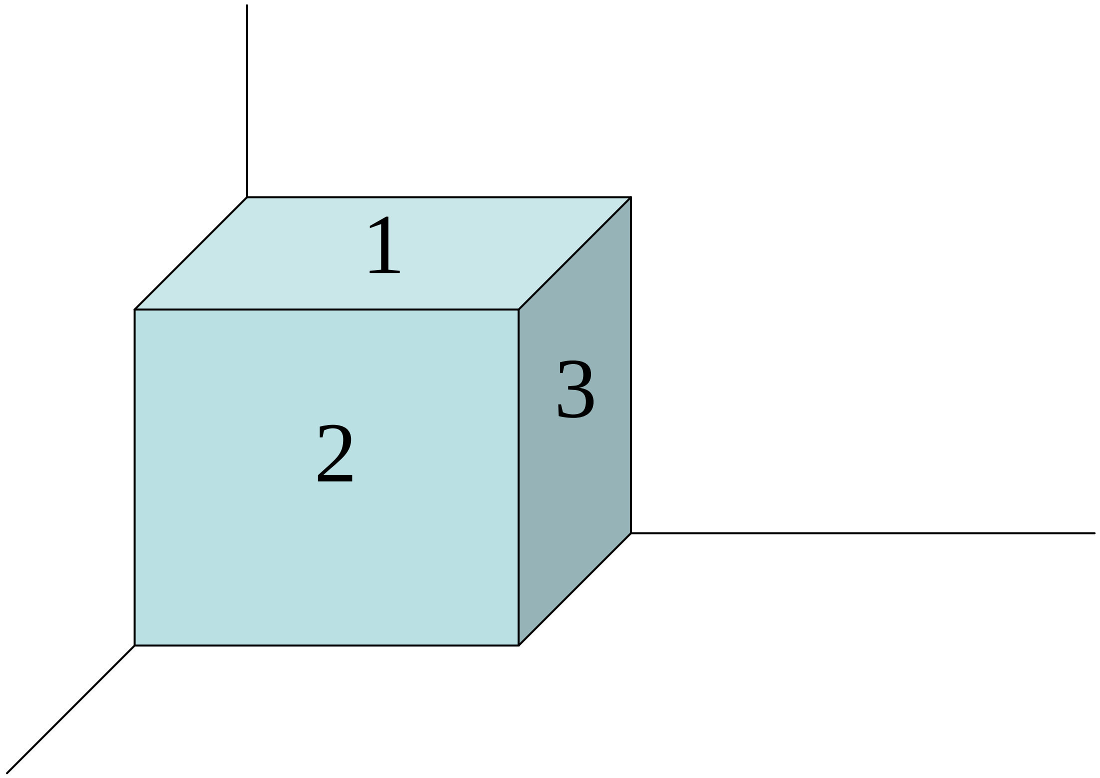


Figure 1.3: Here the arrows $A \to B$ means B is a *subgroup of* A.

Complete and unambiguous task specifications can be tedious for symmetrical objects



‘Put that cube in the corner with face 1 on top !’ (4 different ways)

‘Put that cube in the corner !’ (how many different ways?)

Constructive Solid Geometry (CSG) Representation

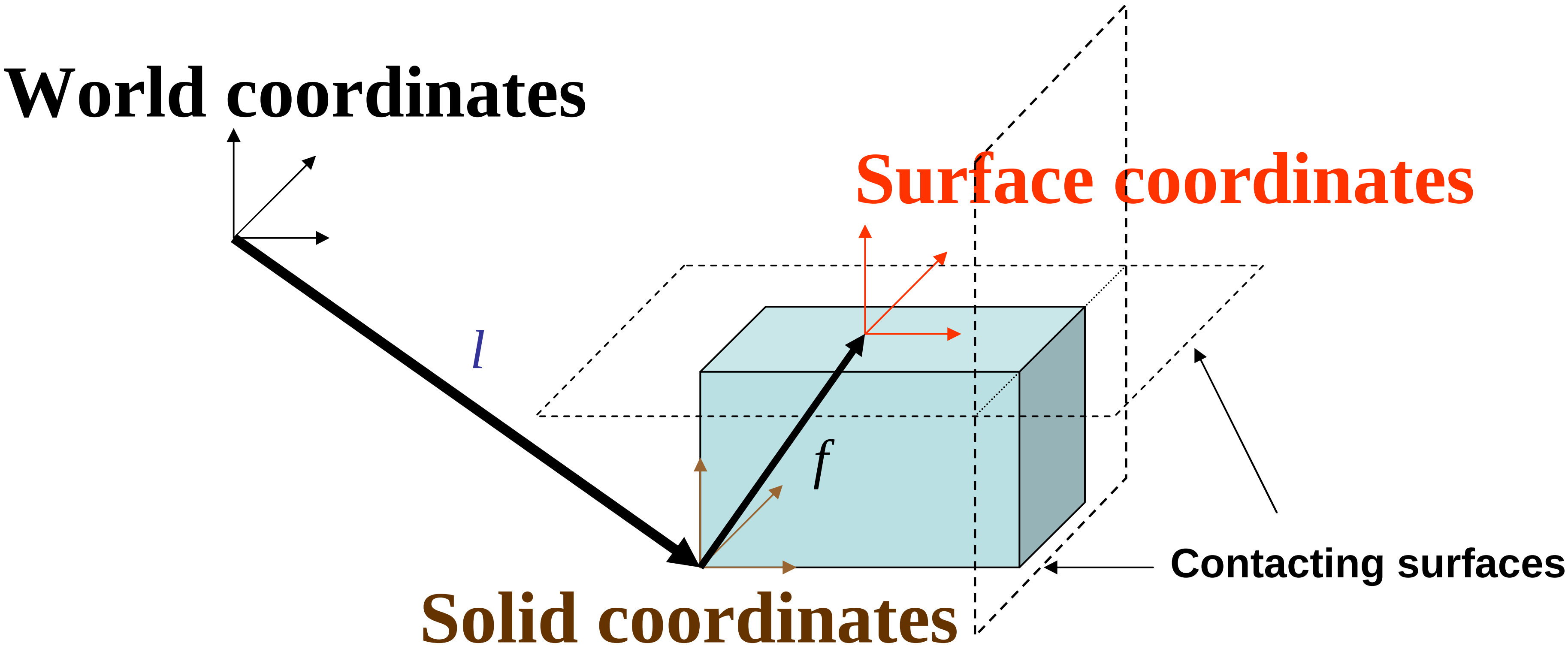
Different types of surfaces associated with different types of groups

The concept of "**conjugated symmetry groups**" are used here

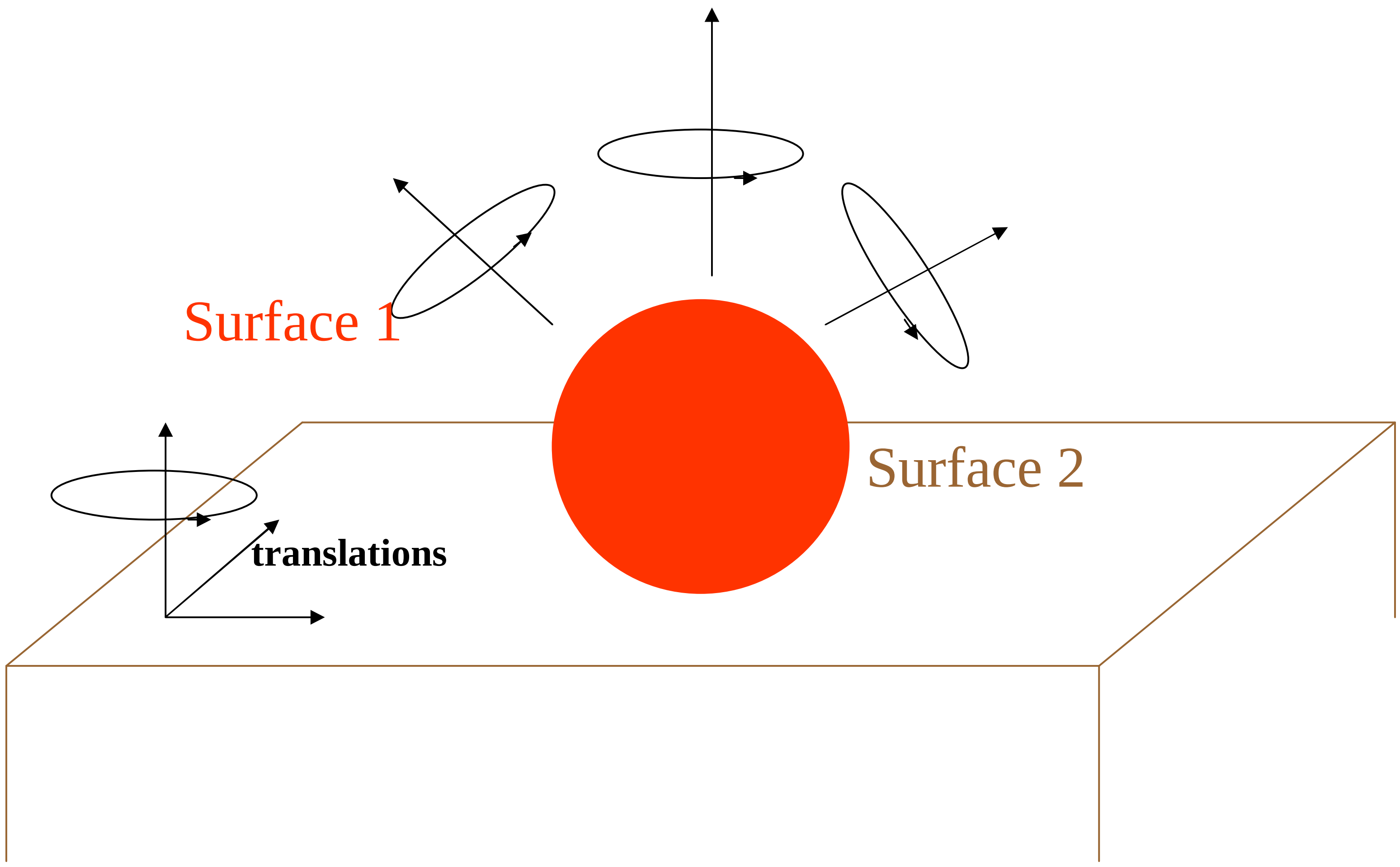
Different subgroups of the proper Euclidean Group

Canonical Groups	Definition
Identity Group	
G_{id}	$\{1\}$
Rotation Subgroups	
$SO(3)$	$\mathbf{gp}\{\mathbf{rot}(\mathbf{i},\theta)\mathbf{rot}(\mathbf{j},\sigma)\mathbf{rot}(\mathbf{k},\phi)\|\theta,\sigma,\phi\in R\}$
$O(2)$	$\mathbf{gp}\{\mathbf{rot}(\mathbf{k},\theta)\mathbf{rot}(\mathbf{i},n\pi)\|\theta\in R, n\in\mathcal{N}\}$
$SO(2)$	$\mathbf{gp}\{\mathbf{rot}(\mathbf{k},\theta)\|\theta\in R\}$
D_{2n}	$\mathbf{gp}\{\mathbf{rot}(\mathbf{k},2\pi/n)\mathbf{rot}(\mathbf{i},m\pi)\|m\in\mathcal{N}\}, n\in\mathcal{N}$
C_n	$\mathbf{gp}\{\mathbf{rot}(\mathbf{k},2\pi/n)\}, n\in\mathcal{N}$
Translation Subgroups	
$\mathcal{T}^1$	$\mathbf{gp}\{\mathbf{trans}(0,0,z)\|z\in R\}$
$\mathcal{T}^1_{dis}(t_0)$	$\mathbf{gp}\{\mathbf{trans}(0,0,t_0)\}, t_0\in R$
$\mathcal{T}^2$	$\mathbf{gp}\{\mathbf{trans}(x,y,0)\|x,y\in R\}$
$\mathcal{T}^3$	$\mathbf{gp}\{\mathbf{trans}(x,y,z)\|x,y,z\in R\}$
Mixed Subgroups	
G_{cyl}	$\mathbf{gp}\{\mathbf{trans}(0,0,z)\mathbf{rot}(\mathbf{k},\theta)\mathbf{rot}(\mathbf{i},n\pi)\|n\in\mathcal{N},\theta,z\in R\}$
G_{dir_cyl}	$\mathbf{gp}\{\mathbf{trans}(0,0,z)\mathbf{rot}(\mathbf{k},\theta)\|z,\theta\in R\}$
G_{plane}	$\mathbf{gp}\{\mathbf{trans}(x,y,0)\mathbf{rot}(\mathbf{k},\theta)\mathbf{rot}(\mathbf{i},n\pi)\|x,y,\theta\in R, n\in\mathcal{N}\}$
G_{dir_plane}	$\mathbf{gp}\{\mathbf{trans}(x,y,0)\mathbf{rot}(\mathbf{k},\theta)\|x,y,\theta\in R\}$
$G_{screw}(p)$	$\mathbf{gp}\{\mathbf{trans}(0,0,z)\mathbf{rot}(\mathbf{k},2z\pi/p)\|z\in R\}, p\in R$
$G_{T_1C_2}$	$\mathbf{gp}\{\mathbf{trans}(0,0,z)\mathbf{rot}(\mathbf{i},n\pi)\|n\in\mathcal{N}, z\in R\}$
$\mathcal{E}^+$	$\mathbf{gp}\{\mathbf{trans}(x,y,z)\mathbf{rot}(\mathbf{i},\theta)\mathbf{rot}(\mathbf{j},\sigma)\mathbf{rot}(\mathbf{k},\phi)\|x,y,z,\theta,\sigma,\phi\in\Re\}$

Each Algebraic Surface as one primitive feature associated with its own coordinates

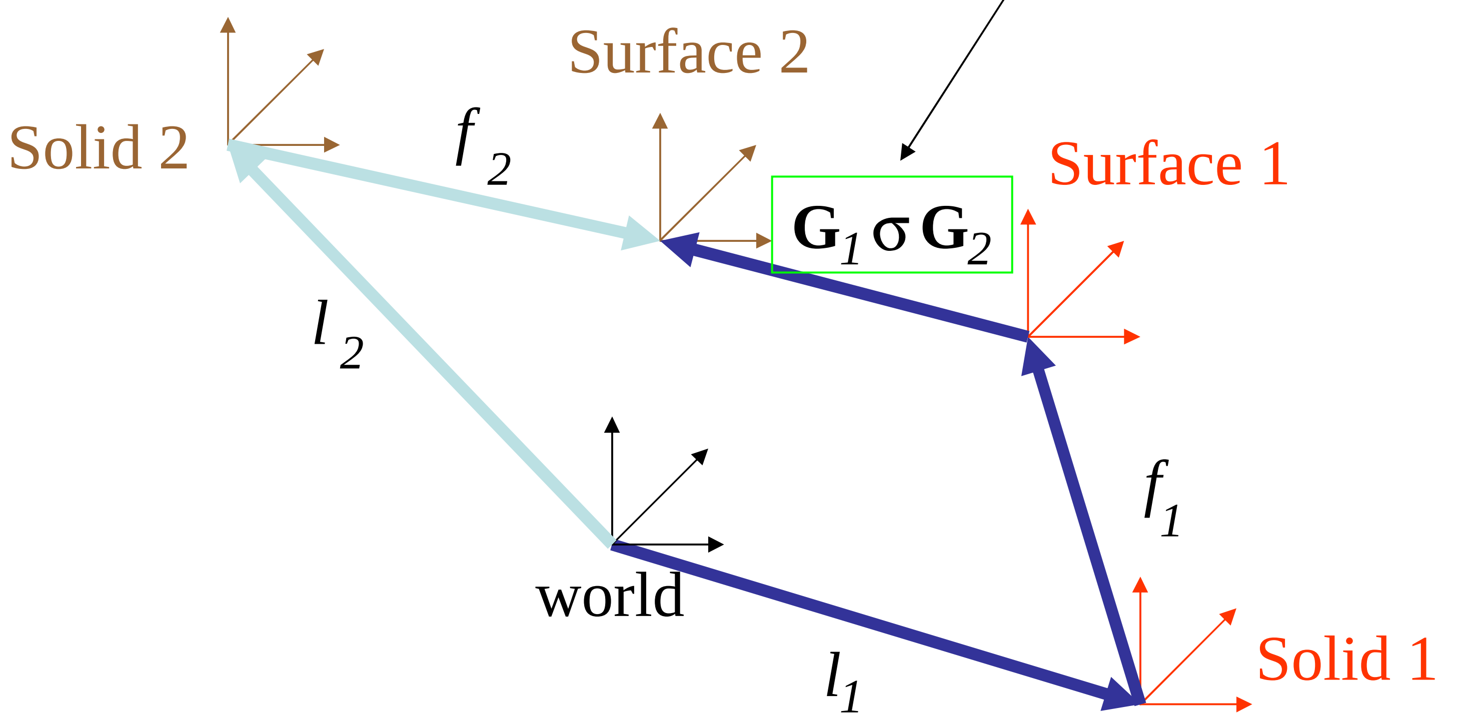


Relative Motion and Contacting Surface Symmetry



Relative Locations of Two Contacting Solids

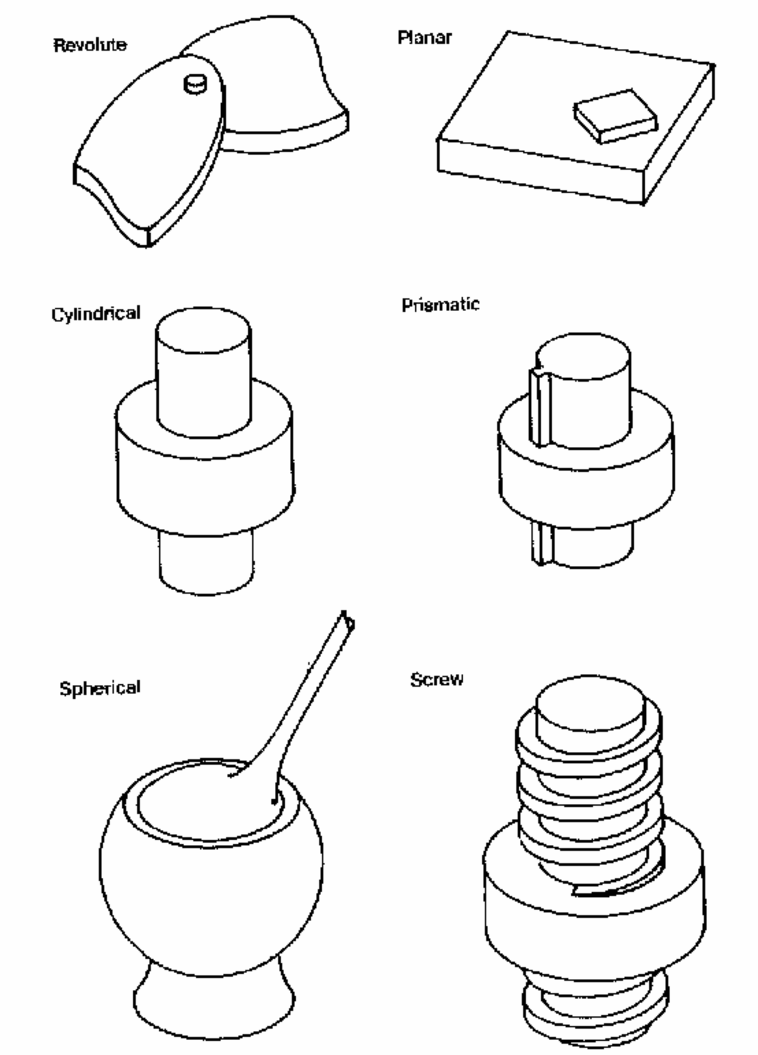
$$l_1^{-1}\ l_2 \sqsubset f_1\ \mathbf{G}_1\ \sigma\ \mathbf{G}_2\ f_2^{-1}$$



$\mathbf{G}_1$ -- symmetry group of surface 1

$\mathbf{G}_2$ -- symmetry group of surface 2

Symmetries at work: lower pairs



Insight:

The contacting surface pair from two different solids coincide, thus has the same symmetry group which determines their relative motions/locations

Relative locations of solids in terms of their contacting surface symmetry groups

General contact: $l_1^{-1}\, l_2 \subseteq f_1\, \mathbf{G}_1\, \sigma\, \mathbf{G}_2\, f_2^{-1}$

Relative locations of solids in terms of their contacting surface symmetry groups

Under surface contact: $l_1^{-1} \; l_2 \subset f_1 \mathbf{G} f_2^{-1}$

The specific surface contact can be expressed as a spatial relationship τ between solids B_1, B_2

$$\tau = \{(l_1, l_2) | l_1^{-1} l_2 \in f_1 G f_2^{-1}\}. \tag{1.3}$$

Relative locations of solids in terms of their contacting surface symmetry groups

Under **multiple** <u>*surface contacts:*</u>

$$l_1^{-1}\, l_2 \subseteq f_1\,(\mathbf{G}_1 \cap \mathbf{G}_2 \ldots)\, f_2^{-1}$$

Relative locations of solids in terms of their contacting surface symmetry groups

Under **multiple** ***general contacts:***

$$l_1^{-1} l_2 \in f_{11} G_{11} \sigma_1 G_{21} f_{21}^{-1} \cap f_{12} G_{12} \sigma_2 G_{22} f_{22}^{-1} \cap \ldots$$

$$\cap f_{1n} G_{1n} \sigma_n G_{2n} f_{2n}^{-1}$$

Relative locations of solids in terms of their contacting surface symmetry groups

m solids have a chaining general contact, the relative location of solid m with respect to solid 1:

$$l_1^{-1} l_m \in f_1 G_{12} \sigma_1 G_{21} f_{21}^{-1} f_2 G_{23} \sigma_2 G_{32} f_{32}^{-1} \ldots \tag{1.8}$$

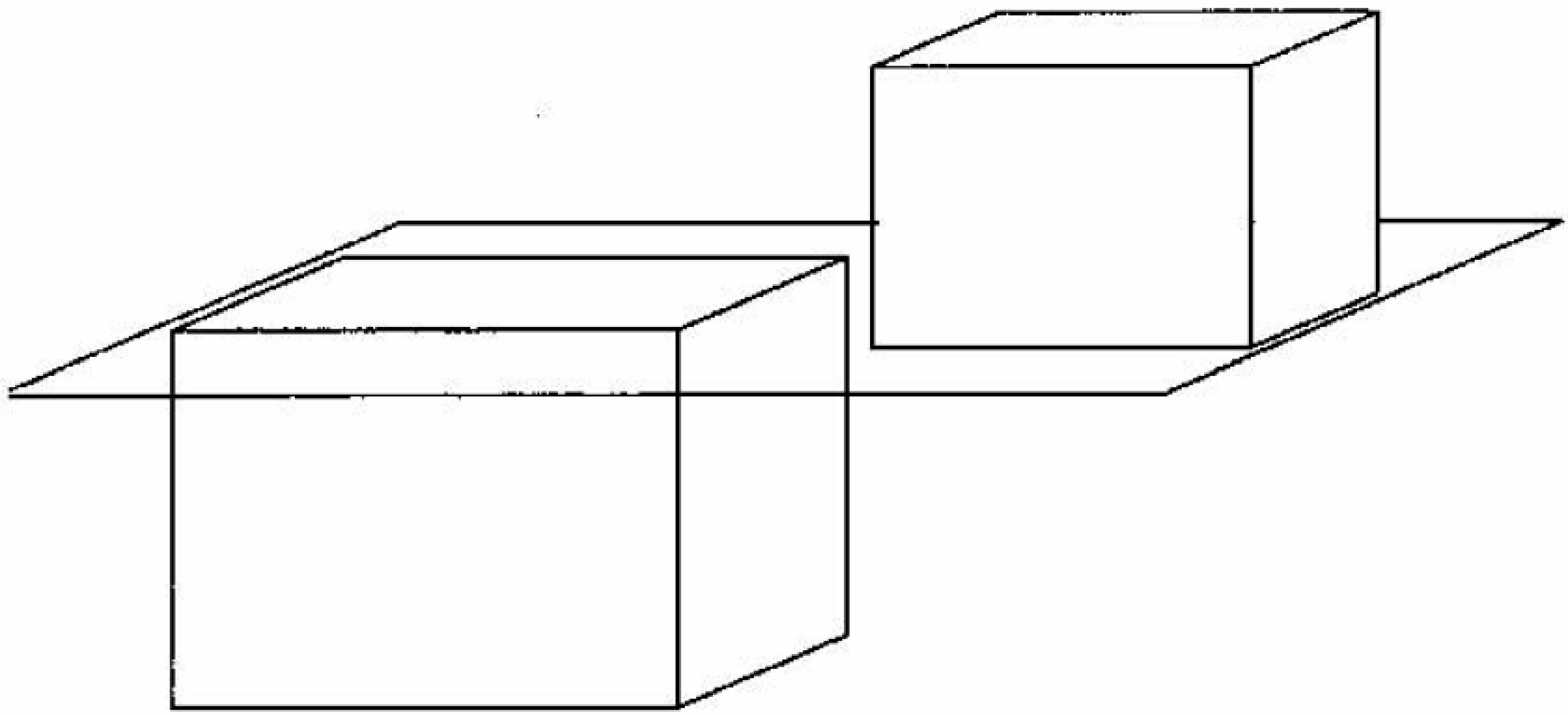
$$f_{m-1} G_{(m-1)m} \sigma_{m-1} G_{m(m-1)} f_{m(m-1)}^{-1}$$

where G_{ij} is the symmetry group of the surface on solid i in contact with solid j.

Primitive surface features:

finite or infinite?

To guarantee a physical contact, either it is considered as infinitesimal motion from a real contact, or additional constraints are needed

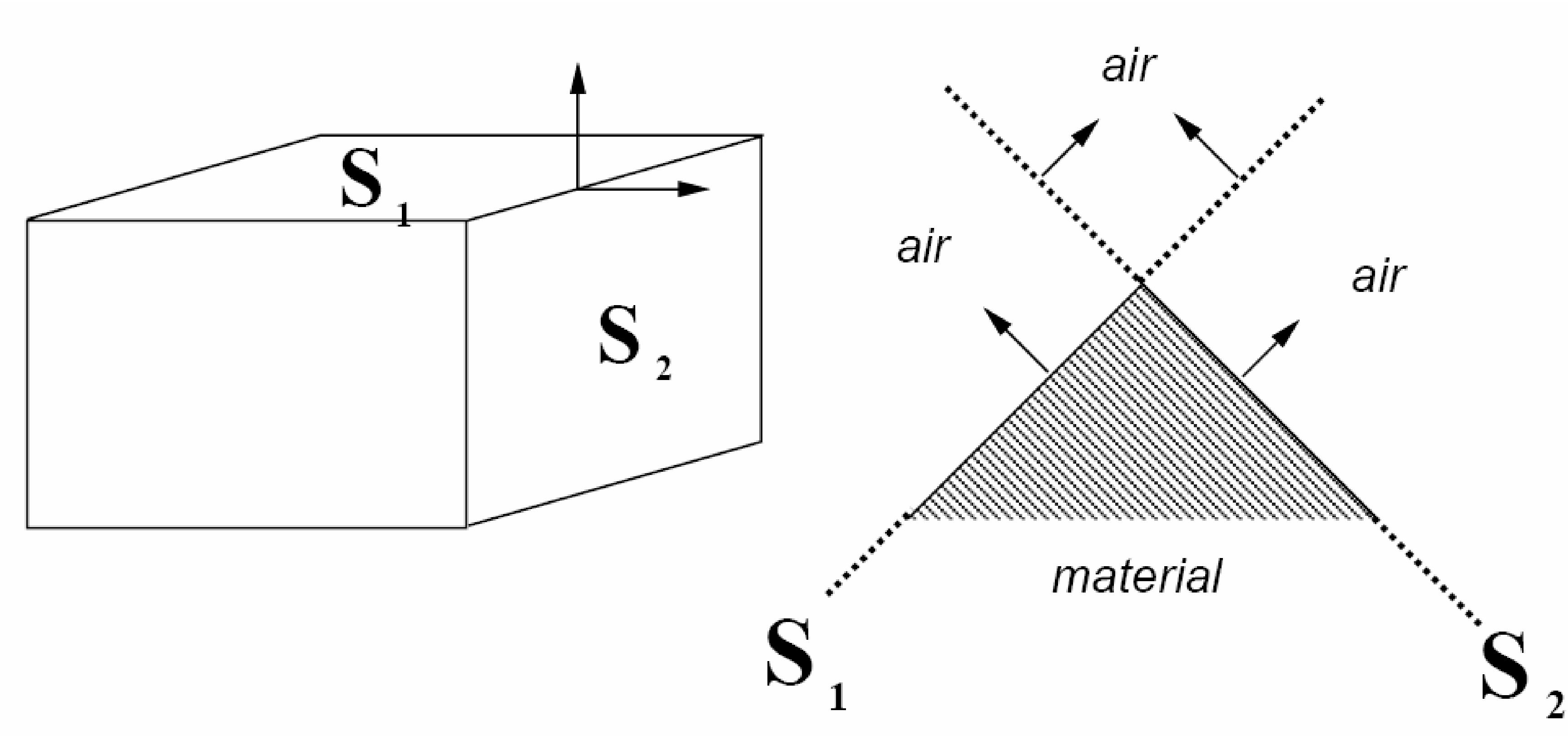


Primitive surface features:

‘hair’ or no ‘hair’?

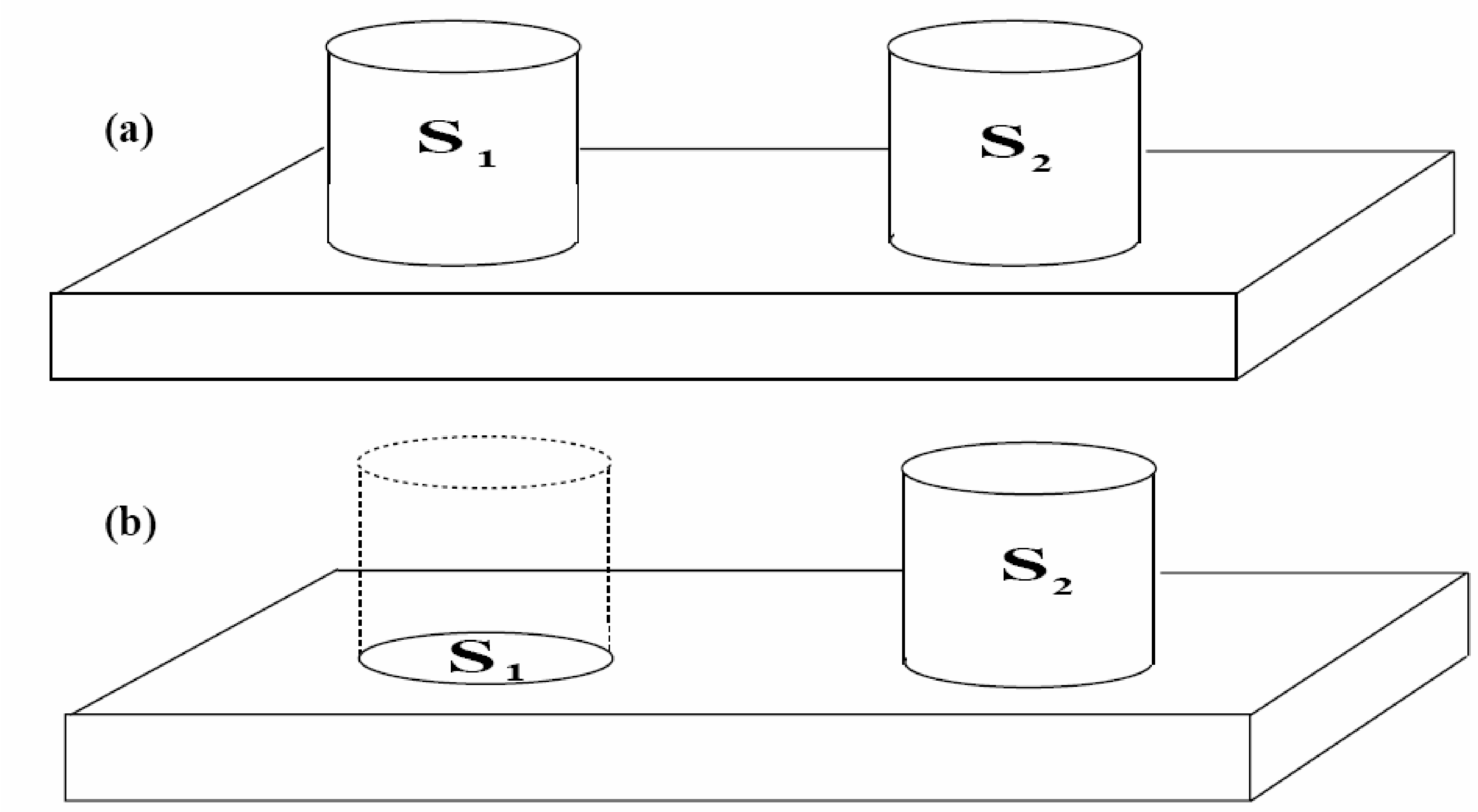
Symmetries for a Surface:

keeping the orientation invariant or not?



Symmetries for a Surface:

keeping the orientation invariant or not?



Symmetries for an oriented primitive feature F = (S,ρ)

- S is a connected, irreducible and continuous algebraic surface (a point set), bounding a finite solid M
- ρ $\subseteq$ S x unit sphere is a set of pairs
- For all s in S and (s,v) in ρ v poins away from M

Intuitively, now primitive feature F is composed of both 'skin' S and 'hair' ρ

Symmetries of primitive feature F

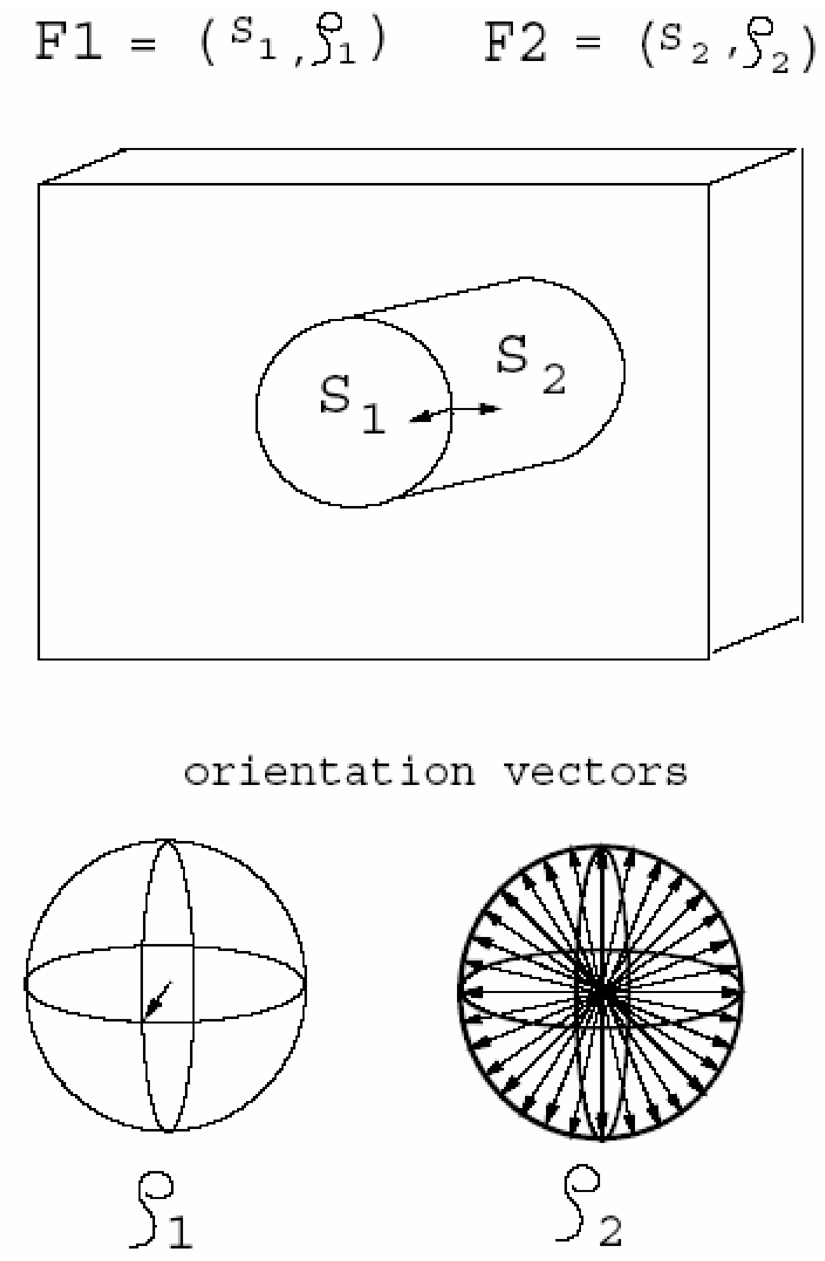
- A symmetry of F has to keep two sets of points invariant
- Symmetries of an oriented feature F form a group (proof the four properties of a group)

Consider the symmetry group of the contacting surfaces collectively

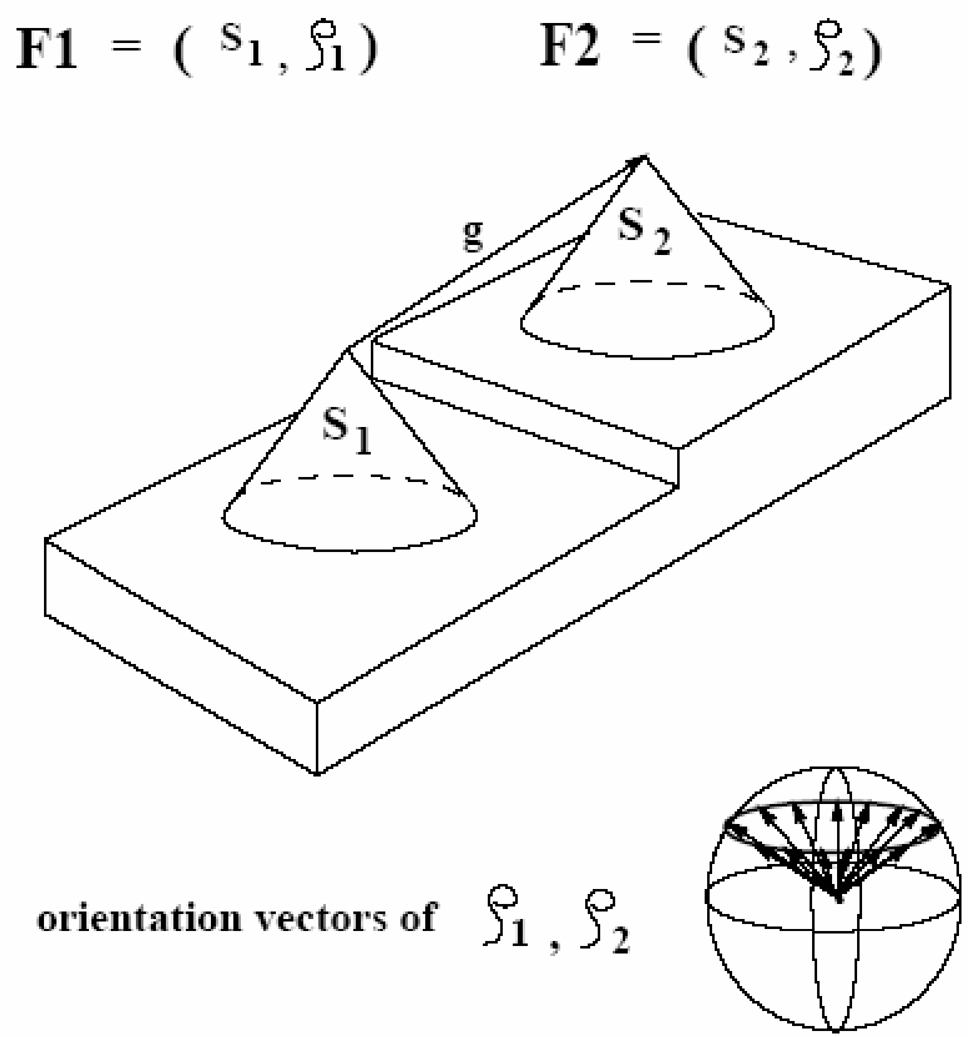
Definition 2.1.18 *A* **compound feature** $F = (S, \rho)$ *of primitive features* $F_1 = (S_1, \rho_1), ..., F_n = (S_n, \rho_n)$, *is defined to be*

- $S = S_1 \cup ... \cup S_n$
- $\rho = \rho_1 \cup ... \cup \rho_n$

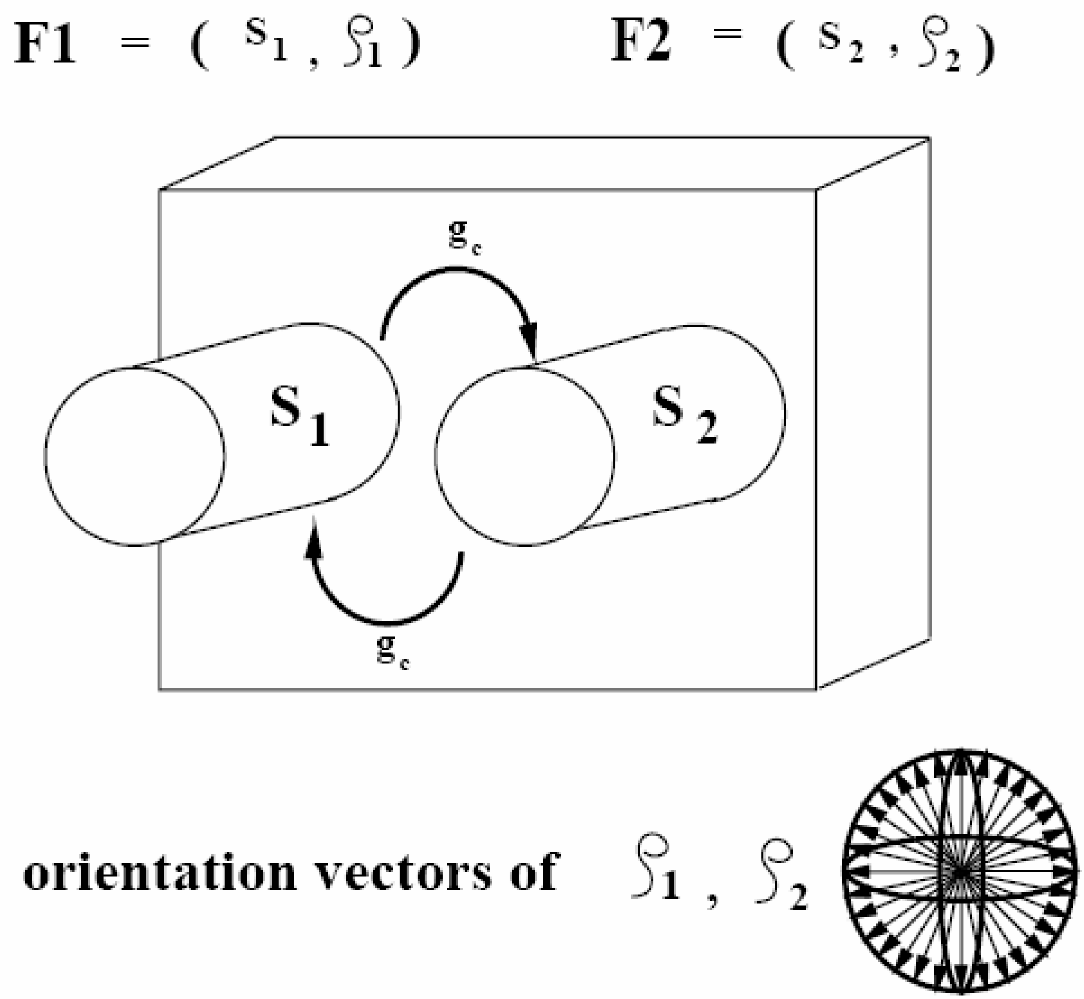
DISTINCT



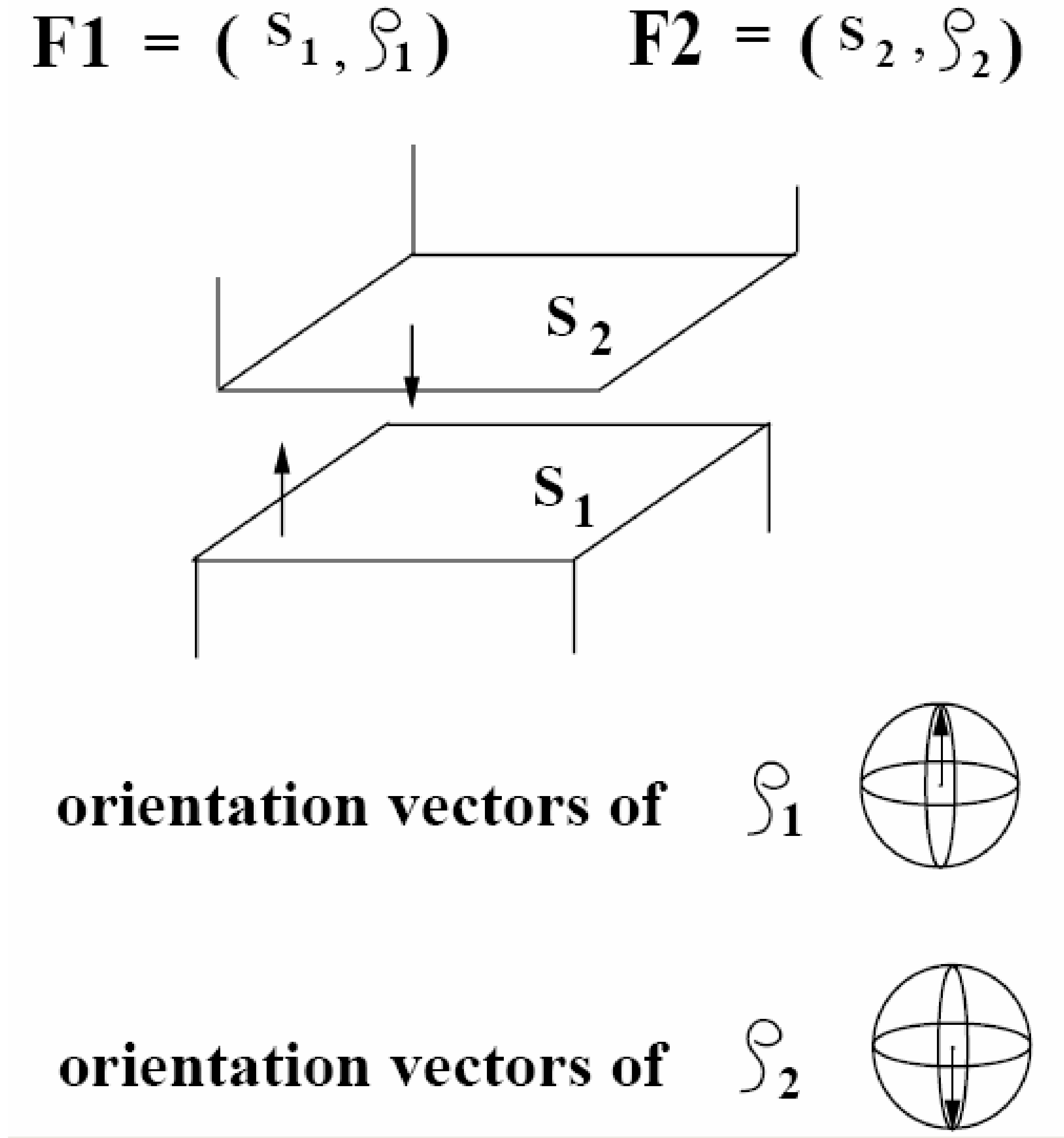
1-congruent



2-congruent



COMPLEMENT



F1 = (S1 , ℘1)
F2 = (S2 , ℘2)
S2
S1
orientation vectors of ℘1
orientation vectors of ℘2

Proposition 2.2.22

Distinct, 1-congruent, 2-congruent and complementary are the ONLY possible relationships between a pair of primitive features

Proposition 2.3.30

If a compound feature F is composed of pairwise distinct primitive features, the symmetry group G of F is the intersection of the symmetry groups of the primitive features

How to represent this diverse set of subgroups on computers?

How to do group intersection on computers?

Symmetry groups of contacting surfaces can be finite, infinite, discrete, non-discrete and continuous …