Solution to Bonus Problem

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Proof. Given a CFG L over single alphabet a, we will show it is also a regular language. Suppose the pumping length of langauge L is l; then for any string $z \in L$ of length larger than l, we know we can write into $uvwxy$ such that uv^iwx^iy is also in the language for every i. Notice that $uv^{i+1}wx^{i+1}y$ is just adding $za^{i(|v|+|x|)}$. Also Notice that $|vwx| \leq l$, therefore $(|v| + |x|)$ is divisible by $(l!)$. Therefore, we know the following property of language L:

for any $z \in L$, if $|z| > l$, then $z(a^{l!})^*$ is also in L.

Then we can divide the string in L with length larger than l in to at most $l!$ classes.

For $1 \leq i \leq (l!)$, let z_i be the shortest string in L such that

$$|z_i| \geq l, \text{and } |z_i| - l \equiv i \mod (l!).$$

Notice that for some i, there might not exists some z_i that satisfy above equation. We use S to denote the set that contains all the i such that z_i exists.

Now we can write L by the following regular expression,

$$L = \{x \mid |x| \leq l, x \in L\} \cup \bigcup_{i \in S} z_i \big(a^{(q!)}\big)^*.$$

$\square$