Model Reduction for Verification of Hybrid Systems

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**Objective**

**Verification of Embedded Control Systems**

- **Control Loop**
  - Embedded Controller
  - Actuator
  - Plant
  - Sensor

**Verification Methods**

- Computing reachable sets in continuous state space (difficult for systems with order > 7)
- Verifying safety properties using conservative approximation
- Using counterexamples to guide the refinement procedure

**Model Reduction**

- Approximating a high order component with a lower order one
- The error of approximation is bounded\textsuperscript{**} for Linear Time Invariant Systems (LTI)

\textsuperscript{*}Using oriented rectangular hull approximation. B. H. Krogh and O. Strusberg, On efficient representation and computation of reachable sets for hybrid systems, in Hybrid Systems: Computation and Control (HSCC’03). Lecture Notes in Computer Science (LNCS), Springer


**Applying Model Reduction in Verification**

- Reachable sets are restricted to the states of interests (output states + states used in control loop)
- Computing reachable sets in reduced state space, then projecting to the states of interests
- Including the error introduced by model reduction in the results

**Case Study**

**Electrical Throttle Control (ETC) System**

- Sliding-Mode Controller
- PD Controller
- Actuator/Plant
- Filter
- Sensor

2 inputs: set-point, sliding mode signal
2 outputs: throttle angle, sliding surface

**Conservative Flow-pipes for different reduced models**

(Using balanced truncation\textsuperscript{***} method)

- 7\textsuperscript{th} order
- 6\textsuperscript{th} order
- 5\textsuperscript{th} order
- 4\textsuperscript{th} order


**Future Work**

- Implementation as subroutines in CheckMate/VTB
- Applying Model Reduction in Counterexample-Guided Verification scheme
- Composition of reduced models