Analysis of Social Media

MLD 10-802, LTI 11-772

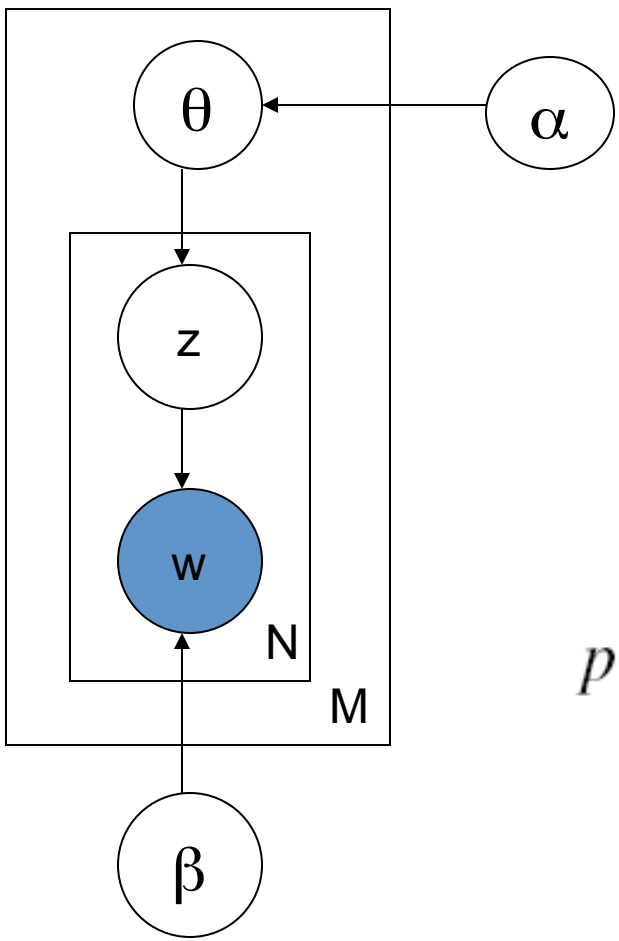
William Cohen

10-16-010

Review - LDA

"Mixed membership"

- Latent Dirichlet Allocation



- Randomly initialize each $z_{m,n}$
- Repeat for t=1,….
  - For each doc *m,* word *n*
    - Find Pr(z_{mn}=*k*|other z's)
    - Sample z_{mn} according to that distr.

$$p(z_i{=}k|\vec{z}_{\neg i}, \vec{w}) =$$

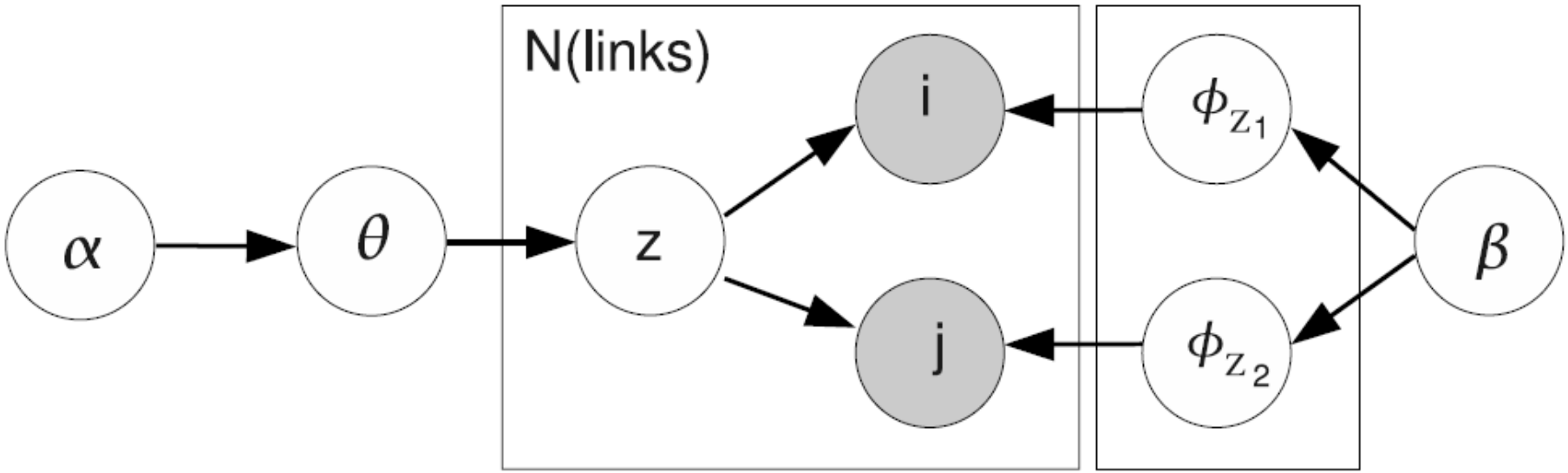
$$\propto \frac{n^{(t)}_{k,\neg i} + \beta_t}{\sum_{t=1}^{V} n^{(t)}_{k,\neg i} + \beta_t} \cdot \frac{n^{(t)}_{m,\neg i} + \alpha_k}{[\sum_{k=1}^{K} n^{(k)}_{m} + \alpha_k] - 1}$$

Outline

- Stochastic block models & inference question
- Review of text models
  - Mixture of multinomials & EM
  - LDA and Gibbs (or variational EM)
- **Block models and inference**
- Mixed-membership block models
- Multinomial block models and inference w/ Gibbs
- Beastiary of other probabilistic graph models
  - Latent-space models, exchangeable graphs, p1, ERGM

Parkkinen et al paper

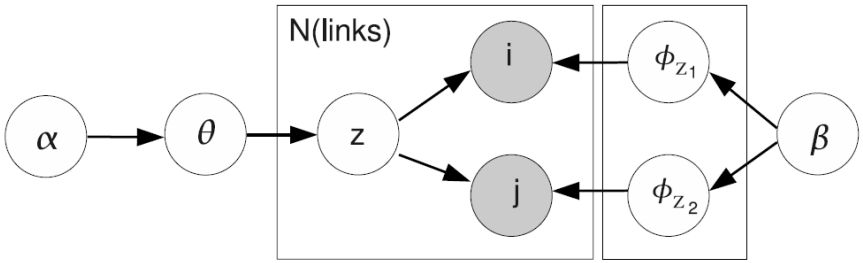
Another mixed membership block model



$$p(z_l|\{z\}^{\neg l}, \{(i,j)\}^{\neg l}, \alpha, \beta) \propto$$

$$(n_z^{\neg l} + \alpha) \cdot \frac{(q_{z_1 i}^{\neg l} + \beta)(q_{z_2 j}^{\neg l} + \beta)}{(q_{z_1 \cdot}^{\neg l} + M\beta)(q_{z_2 \cdot}^{\neg l} + M\beta + \delta_z)},$$

Another mixed membership block model



z=(zi,zj) is a pair of block ids

n_z = #pairs *z*

$q_{z1,i}$ = #links to *i* from block z1

$q_{z1,\cdot}$ = #outlinks in block z1

δ = indicator for diagonal

M = #nodes

$$p(z_l|\{z\}^{\neg l}, \{(i,j)\}^{\neg l}, \alpha, \beta) \propto$$

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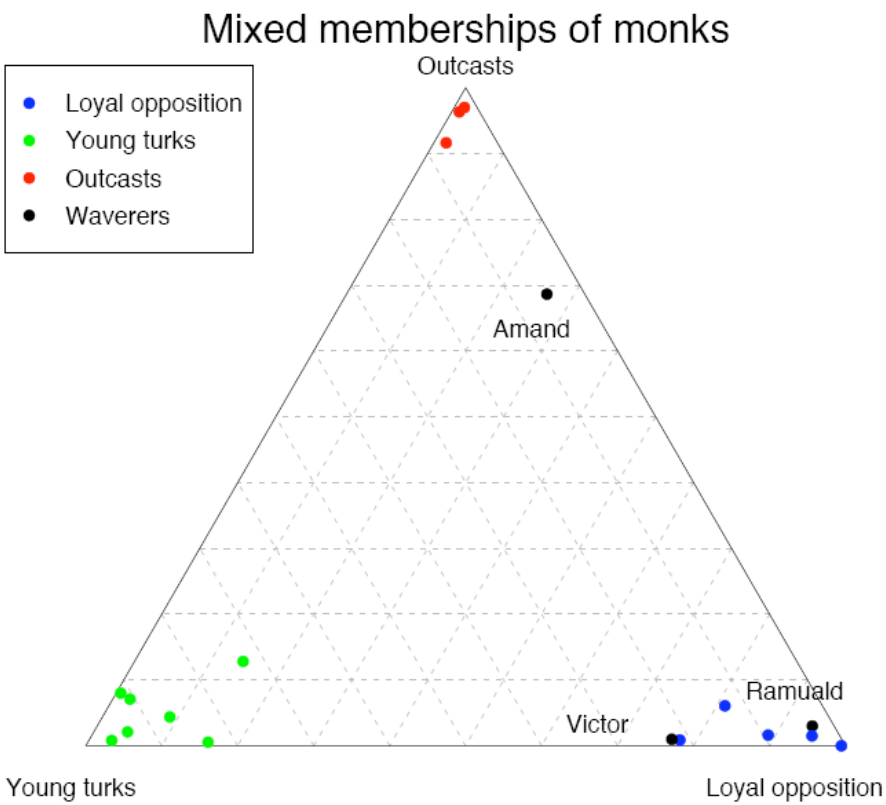

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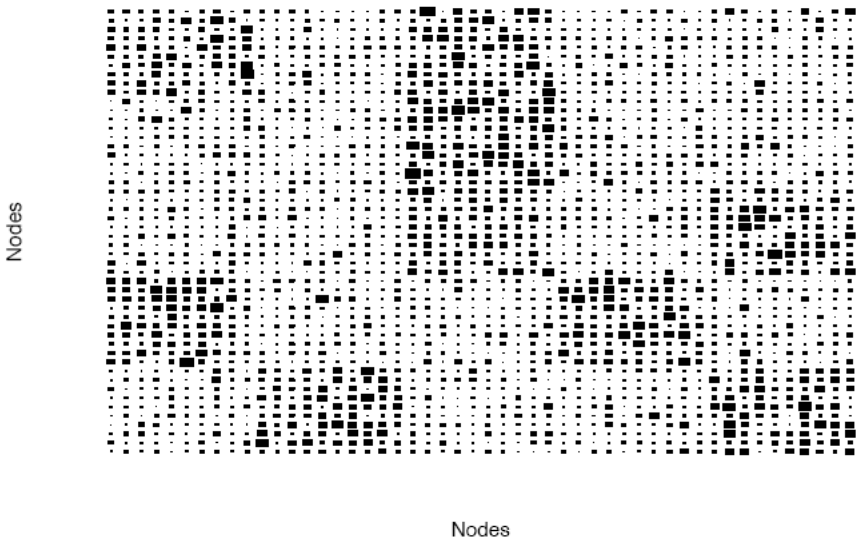

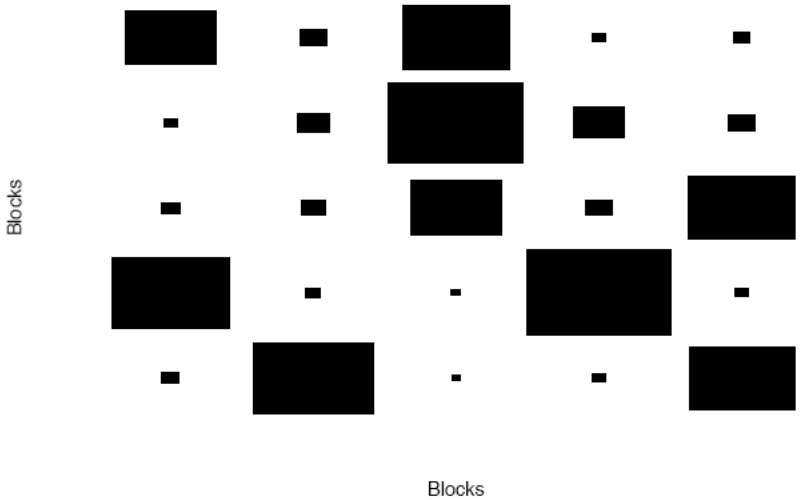

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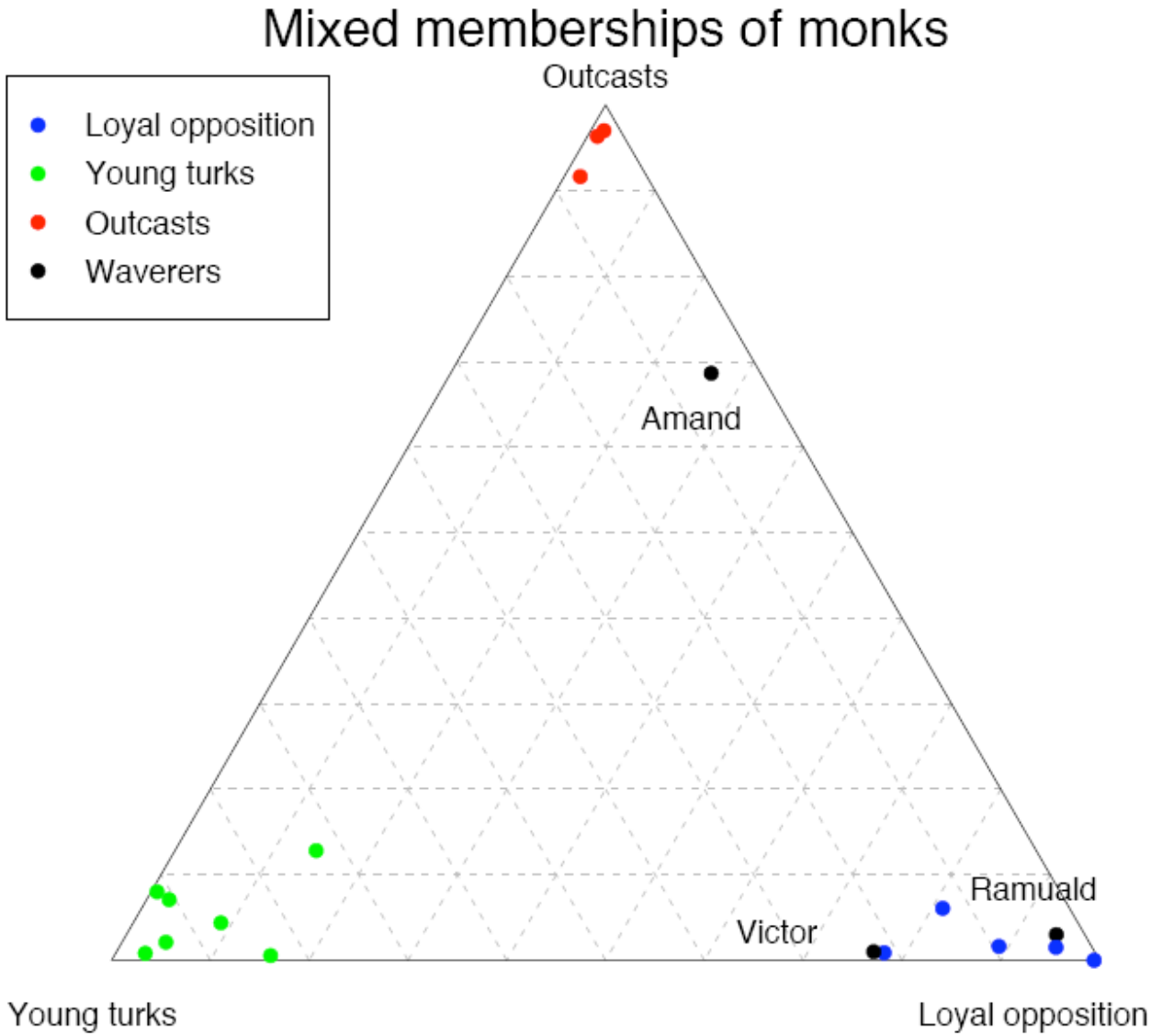
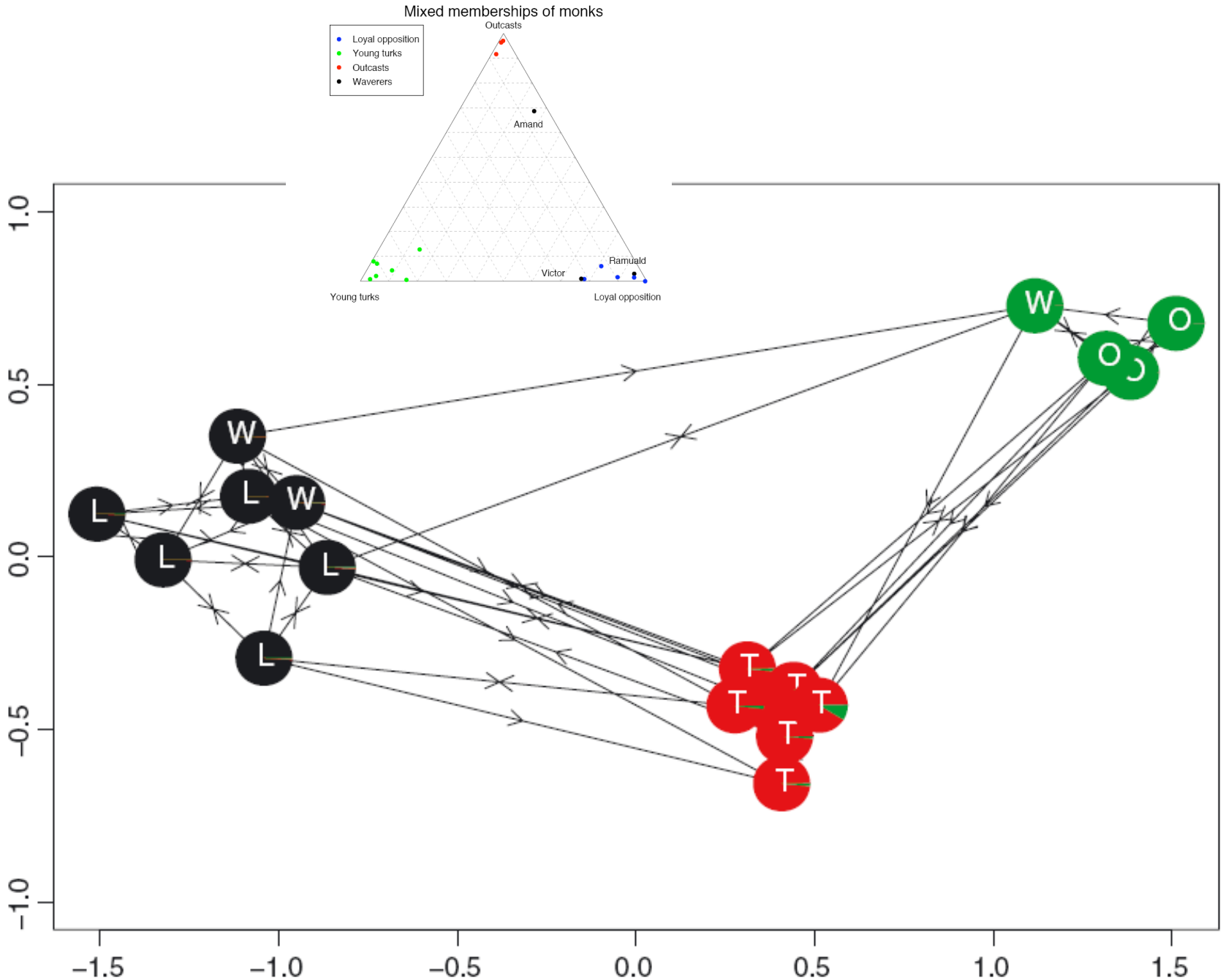
Latent Space Model

- Each node *i* has a latent position in Euclidean space, *z(i)*
- *z(i)'s* drawn from a mixture of Gaussians
- Probability of interaction between *i* and *j* depend on the *distance* between *z(i)* and *z(j)*
- Inference is a little more complicated… [Handcock & Raftery, 2007]

Airoldi's MMSBM




Mixed memberships of monks
Loyal opposition
Young turks
Outcasts
Waverers
Outcasts
Amand
Ramuald
Victor
Young turks
Loyal opposition
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
W
O
O
O
W
L
L
W
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T
T
T
T

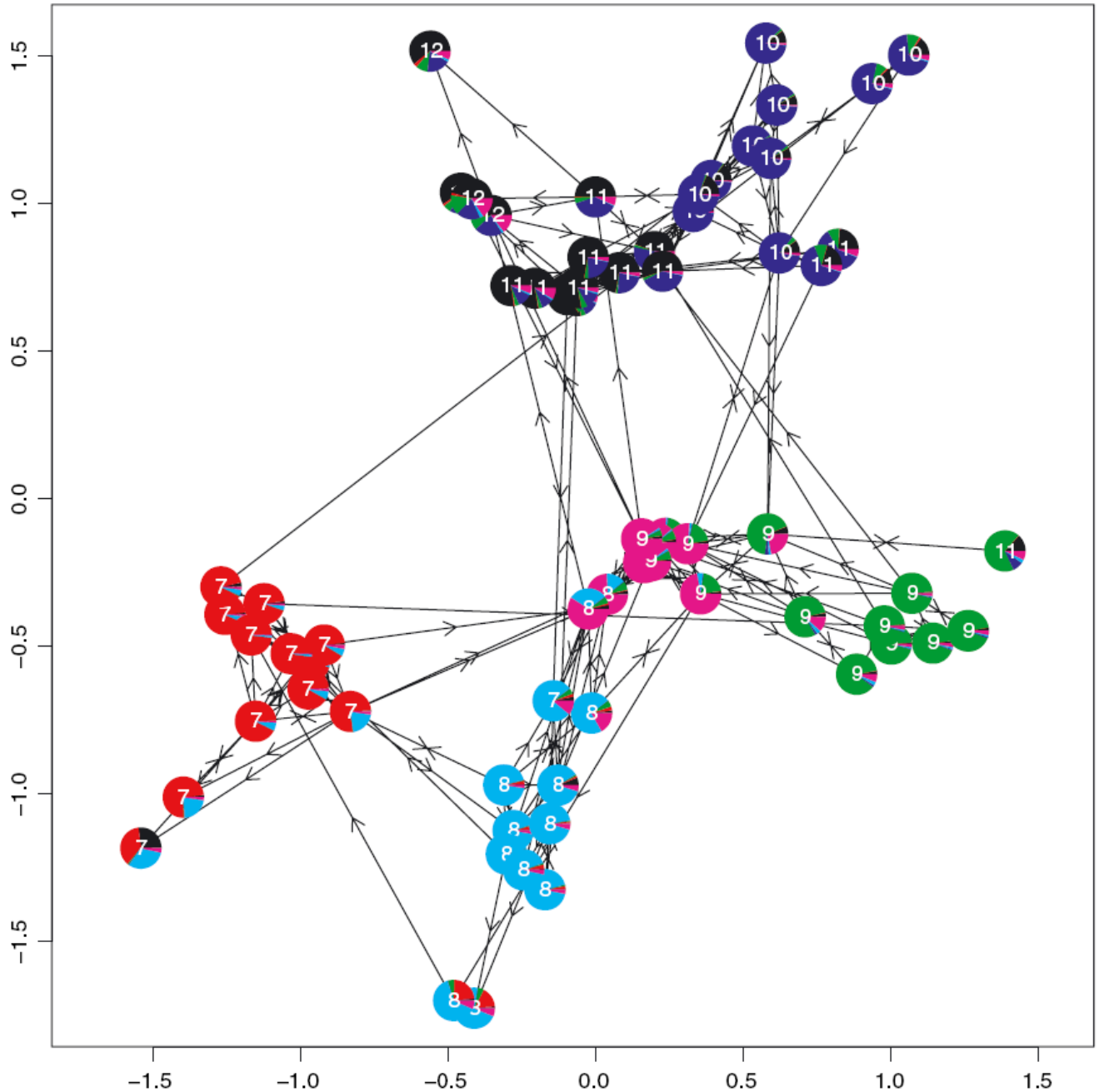

Fig. 8. Pie charts for posterior probabilities of cluster assignment for each actor, at the Bayesian estimates of posterior latent positions for the friendship network in the adolescent health school: the students' grades are shown as numbers

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Exchangeable Graph Model

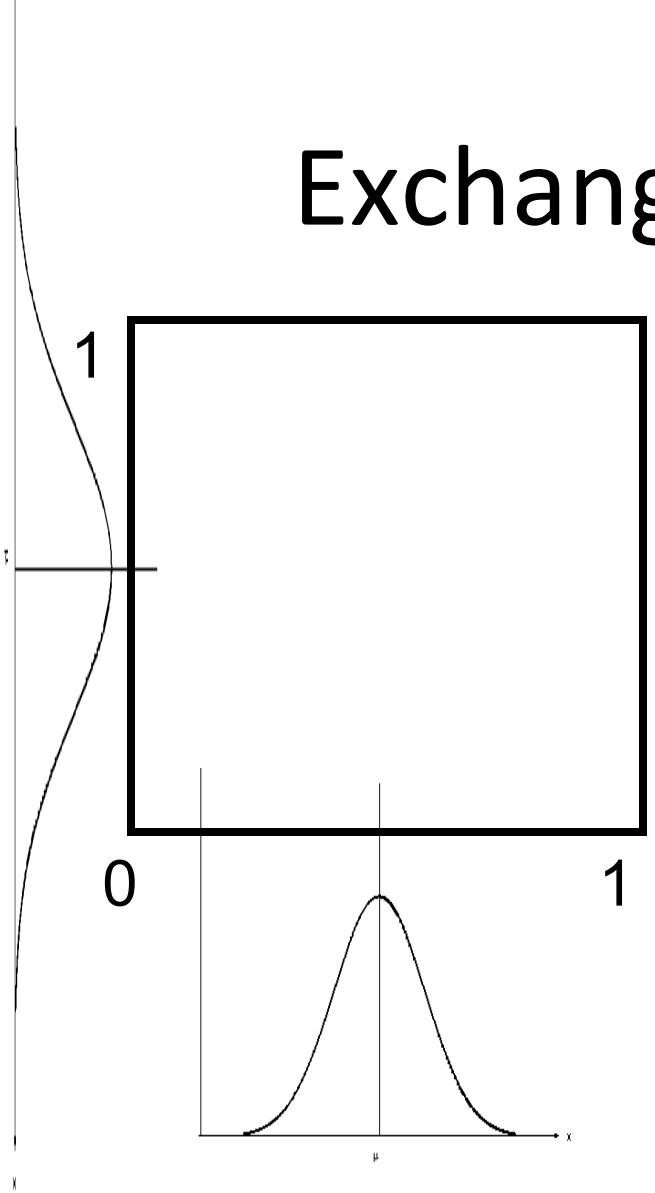
- Defined by a 2^k x 2^k table $q(b_1,b_2)$
- Draw a length-k bit string $b(n)$ like 01101 for each node n from a ~~*uniform*~~ distribution.

complicated

- For each pair o
  - Flip a coin wit
  - If it's heads co

- Pick k-dimensional vector u from a multivariate normal w/ variance α and covariance β – so u_i's are correlated.
- Pass each u_i thru a sigmoid so it's in [0,1] – call that p_i
- Pick b_i using p_i

Exchangeable Graph Model



If α is big then *ux,uy* are really big (or small) so *px,py* will end up in a corner.

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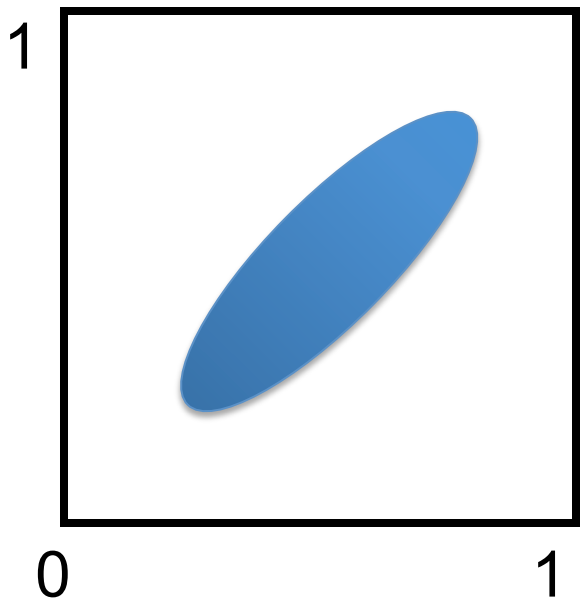
Exchangeable Graph Model



If α is big then *ux,uy* are really big (or small) so *px,py* will end up in a corner.

- Pick *k*-dimensional vector *u* from a multivariate normal w/ variance α and covariance β – so u_i's are **correlated**.
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The p_1 model for a directed graph

- Parameters, per node *i:*
  - Θ: background edge probability
  - α_i: "expansiveness" – how extroverted is *i?*
  - β_i: "popularity" – how much do others want to be with *i?*
  - ρ_{ij}: "reciprocation" – how likely is *i* to respond to an incomping link with an outgoing one?

$$\log \Pr(i \ldots j) = \lambda_{ij}$$

$$\log \Pr(i \rightarrow j) = \lambda_{ij} + \boxed{\alpha_i + \beta_j + \theta}$$

$$\log \Pr(i \leftarrow j) = \lambda_{ij} + \boxed{\alpha_j + \beta_i + \theta}$$

$$\log \Pr(i \leftrightarrow j) = \lambda_{ij} + \boxed{} + \boxed{} + \rho_{ij}$$

Logistic-regression like procedure can be used to fit this to data from a graph

$$\log Pr_{p_1}(y) \propto y_{++}\theta + \sum_i y_{i+}\alpha_i + \sum_j y_{+j}\beta_j + \sum_{ij} y_{ij}y_{ji}\rho,$$

Exponential Random Graph Model

- Basic idea:
  - Define some features of the graph (e.g., number of edges, number of triangles, …)
  - Build a MaxEnt-style model based on these features
- General:
  - includes Erdos-Renyi, p_1, …
- Issues
  - Partition function is intractible
  - Alternative: model conditional pseudo-likelihood of a each edge (i.e., Pr(edge|rest of graph)

Kroneker product graphs

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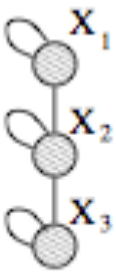

(a) Graph K_1

1	1	0
1	1	1
0	1	1

(d) Adjacency matrix of K_1

Kroneker product graphs

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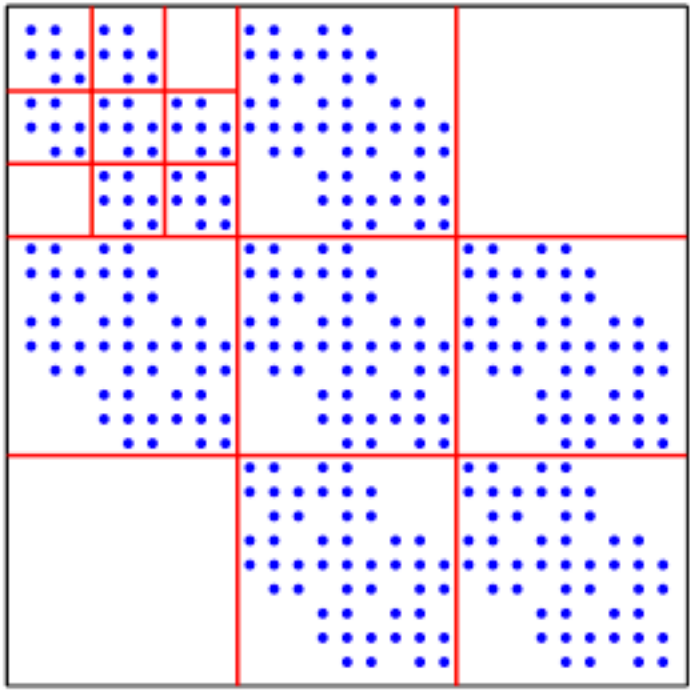
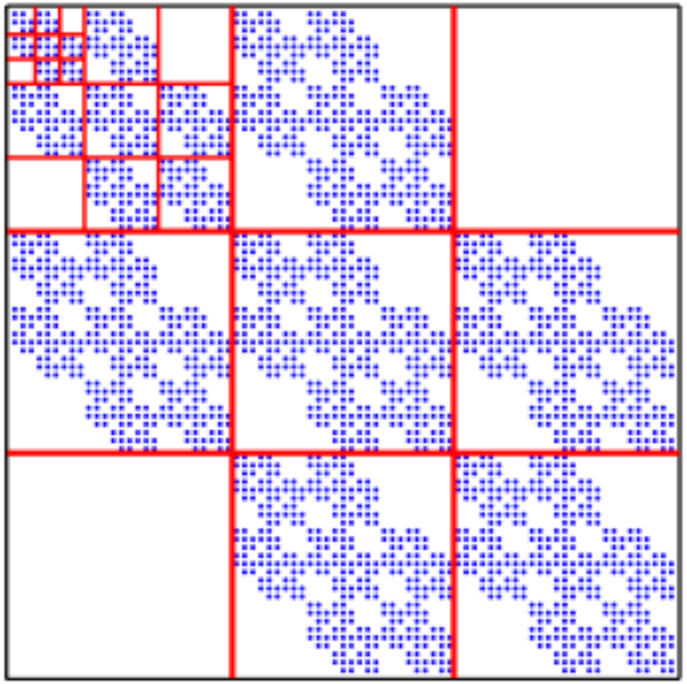
(d) Adjacency matrix of K_1



(a) K_3 adjacency matrix (27×27)



(b) K_4 adjacency matrix (81×81)

Kroneker product graphs

- Good fit to many commonly-observed network properties
  - scale-free degree distribution
  - diameter
  - …
- Gradient descent can be used to fit an "initiator matrix" to a real adjacency matrix