Dr. Perceptron

"Now, consider the following: You were admitted to this robot asylum. Therefore, you must be a robot. Diagnosis complete."
— Dr. Perceptron to Fry[source]

Dr. Perceptron is the head doctor at the Hal Institute for Criminally Insane Robots. He was destroyed briefly by Roberto during his escape from the Institute, but was apparently fixed/rebuilt and returned to work for Bender's second stay.

In 3008, Dr. Perceptron was damaged during a group therapy session, but like his encounter with Roberto, was quickly repaired to continue his duties at the Institute.

Appearances ↪ Edit
- Insane in the Mainframe
- Bender's Game

http://futurama.wikia.com/wiki/Dr._Perceptron
Where we are...

- Experiments with a hash-trick implementation of logistic regression
- Next question:
  - how do you parallelize SGD, or more generally, this kind of streaming algorithm?
  - each example affects the next prediction → order matters → parallelization changes the behavior
  - we will step back to perceptrons and then step forward to parallel perceptrons
  - then another nice parallel learning algorithm
  - then a midterm
Recap: perceptrons
The perceptron

Compute: \( y_i = \text{sign}(v_k \cdot x_i) \)

If mistake: \( v_{k+1} = v_k + y_i x_i \)

**Margin** \( \gamma \). A must provide examples that can be separated with some vector \( u \) with margin \( \gamma > 0 \), ie

\[
\exists u : \forall(x_i, y_i) \text{ given by } A, (u \cdot x)y_i > \gamma
\]

and furthermore, \( \|u\| = 1 \).

**Radius** \( R \). A must provide examples “near the origin”, ie

\[
\forall x_i \text{ given by } A, \|x\|^2 < R^2
\]
The perceptron

Compute: \( y_i = \text{sign}(v_k \cdot x_i) \)

If mistake: \( v_{k+1} = v_k + y_i x_i \)

Mistake bound:

\[
\Rightarrow \quad k \leq \frac{R^2}{\gamma^2} = \left( \frac{R}{\gamma} \right)^2
\]

A lot like SGD update for logistic regression!
\[ P(\text{error in } x) = \sum_k P(\text{error on } x | \text{picked } v_k) P(\text{picked } v_k) \]
\[ = \sum_k \frac{1}{m_k} \frac{m_k}{m} = \sum_k \frac{1}{m} = \frac{k}{m} \]

Imagine we run the on-line perceptron and see this result.

<table>
<thead>
<tr>
<th>(i)</th>
<th>guess</th>
<th>input</th>
<th>result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(v_0)</td>
<td>(x_1)</td>
<td>(X) (a mistake)</td>
</tr>
<tr>
<td>2</td>
<td>(v_1)</td>
<td>(x_2)</td>
<td>(\checkmark) (correct!)</td>
</tr>
<tr>
<td>3</td>
<td>(v_1)</td>
<td>(x_3)</td>
<td>(\checkmark)</td>
</tr>
<tr>
<td>4</td>
<td>(v_1)</td>
<td>(x_4)</td>
<td>(X) (a mistake)</td>
</tr>
<tr>
<td>5</td>
<td>(v_2)</td>
<td>(x_5)</td>
<td>(\checkmark)</td>
</tr>
<tr>
<td>6</td>
<td>(v_2)</td>
<td>(x_6)</td>
<td>(\checkmark)</td>
</tr>
<tr>
<td>7</td>
<td>(v_2)</td>
<td>(x_7)</td>
<td>(\checkmark)</td>
</tr>
<tr>
<td>8</td>
<td>(v_2)</td>
<td>(x_8)</td>
<td>(X)</td>
</tr>
<tr>
<td>9</td>
<td>(v_3)</td>
<td>(x_9)</td>
<td>(\checkmark)</td>
</tr>
<tr>
<td>10</td>
<td>(v_3)</td>
<td>(x_{10})</td>
<td>(X)</td>
</tr>
</tbody>
</table>

\(m_1 = 3\)
\(m_2 = 4\)
\(m = 10\)

1. Pick a \(v_k\) at random according to \(m_k/m\), the fraction of examples it was used for.
2. Predict using the \(v_k\) you just picked.
3. (Actually, use some sort of deterministic approximation to this).
predict using $\text{sign}(v^*. x)$

\[ v_* = \sum_k \left( \frac{m_k}{m} v_k \right) \]

Imagine we run the on-line perceptron and see this result.

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1. Pick a $v_k$ at random according to $m_k/m$, the fraction of examples it was used for.

2. Predict using the $v_k$ you just picked.

3. (Actually, use some sort of deterministic approximation to this).
predict using \( \text{sign}(v^*, x) \)

\[
v_* = \sum_k \left( \frac{m_k}{m} v_k \right)
\]

Also: there’s a sparsification trick that makes learning the averaged perceptron fast.

Last perceptron

Averaging/voting
KERNELS AND PERCEPTRONS
The kernel perceptron

Compute: \( y_i = \hat{v}_k \cdot x_i \)

If mistake: \( \hat{v}_{k+1} = \hat{v}_k + y_i x_i \)

If false positive (too high) mistake: add \( x_i \) to FP
If false positive (too low) mistake: add \( x_i \) to FN

Mathematically the same as before … but allows use of the kernel trick
The kernel perceptron

Mathematically the same as before ... but allows use of the “kernel trick”

Other kernel methods (SVM, Gaussian processes) aren’t constrained to limited set (+1/-1/0) of weights on the $K(x,v)$ values.
Some common kernels

• Linear kernel:
  \[ K(x, x') \equiv x \cdot x' \]

• Polynomial kernel:
  \[ K(x, x') \equiv (x \cdot x' + 1)^d \]

• Gaussian kernel:
  \[ K(x, x') \equiv e^{-\frac{||x-x'||^2}{\sigma}} \]
Some common kernels

- Polynomial kernel:
  \[ K(x, x') \equiv (x \cdot x' + 1)^d \]

- for \( d=2 \)
  \[
  \left( \langle x_1, x_2 \rangle \cdot \langle x'_1, x'_2 \rangle + 1 \right)^2
  = (x_1 x'_1 + x_2 x'_2 + 1)^2
  = (x_1 x'_1 + x_2 x'_2 + 1)(x_1 x'_1 + x_2 x'_2 + 1)
  = (x_1 x'_1)^2 + 2(x_1 x'_1 x_2 x'_2) + 2(x_1 x'_1) + (x_2 x'_2)^2 + 2(x_2 x'_2) + 1
  \approx \langle 1, x_1, x_2, x_1 x_2, x_1^2, x_2^2 \rangle \cdot \langle 1, x'_1, x'_2, x'_1 x'_2, x'_1^2, x'_2^2 \rangle
  = \langle 1, \sqrt{2}x_1, \sqrt{2}x_2, \sqrt{2}x_1 x_2, x_1^2, x_2^2 \rangle \cdot \langle 1, \sqrt{2}x'_1, \sqrt{2}x'_2, \sqrt{2}x'_1 x'_2, x'_1^2, x'_2^2 \rangle
  \]
Some common kernels

- Polynomial kernel:
  \[ K(x, x') \equiv (x \cdot x' + 1)^d \]

  for \( d = 2 \)
  \[
  (\langle x_1, x_2 \rangle \cdot \langle x'_1, x'_2 \rangle + 1)^2
  
  = \langle 1, \sqrt{2}x_1, \sqrt{2}x_2, \sqrt{2}x_1x_2, x_1^2, x_2^2 \rangle \cdot \langle 1, \sqrt{2}x'_1, \sqrt{2}x'_2, \sqrt{2}x'_1x'_2, x'_1^2, x'_2^2 \rangle
  
  \]

  Similarity with the kernel on \( x \) is equivalent to dot-product similarity on a \textbf{transformed} feature vector \( \phi(x) \).
Kernels 101

- **Duality:** two ways to look at this

\[
\hat{y} = \mathbf{x} \cdot \mathbf{w} = K(\mathbf{x}, \mathbf{w})
\]

\[
\mathbf{w} = \sum_{x_{k+} \in FN} x_{k+} - \sum_{x_{k-} \in FP} x_{k-}
\]

**Observation about perceptron**

\[
\hat{y} = \phi(\mathbf{x}) \cdot \mathbf{w}
\]

\[
\mathbf{w} = \sum_{x_{k+} \in FN} \phi(x_{k+}) - \sum_{x_{k-} \in FP} \phi(x_{k-})
\]

**Generalization of perceptron**

\[
\hat{y} = \sum_{x_{k+} \in FN} K(x_i, x_{k+}) - \sum_{x_{k-} \in FP} K(x_i, x_{k-})
\]

\[
K(\mathbf{x}, \mathbf{x}_k) \equiv \phi(\mathbf{x}) \cdot \phi(\mathbf{x}_k)
\]

**Generalization:** add weights to the sums for \( \mathbf{w} \)

*Explicitly* map from \( \mathbf{x} \) to \( \phi(\mathbf{x}) \) – i.e. to the point corresponding to \( \mathbf{x} \) in the Hilbert space (RKHS)

*Implicitly* map from \( \mathbf{x} \) to \( \phi(\mathbf{x}) \) by changing the kernel function \( K \)

*same behavior but compute time/space are different*
### Kernels 101

- **Duality**
- **Gram matrix:** $K_{ij} = K(x_i, x_j)$

$K(x, x') = K(x', x) \Rightarrow$ Gram matrix is *symmetric*

$K(x, x) > 0 \Rightarrow$ diagonal of $K$ is positive $\Rightarrow$ $K$ is “positive semi-definite” $\Rightarrow$ $z^T K z \geq 0$ for all $z$
A FAMILIAR KERNEL
Learning as optimization for regularized logistic regression + hashes

• Algorithm: 
  \[ w^j = w^j + \lambda (y - p) x^j - \lambda 2\mu w^j \]

• Initialize arrays \( W, A \) of size \( R \) and set \( k=0 \)

• For each iteration \( t=1,\ldots,T \)
  – For each example \((x_i, y_i)\)
    • \( V \) is a hash table
    • For \( j : x_j > 0 \) increment \( V[h[j]] \) by \( x_j \)
    • \( p_i = \ldots ; k++ \)
  
• For each hash value \( h: V[h] > 0 \):
  \[
  \begin{align*}
  & \Rightarrow W[h] \quad * = (1 - \lambda 2\mu)^{k-A[h]} \\
  & \Rightarrow W[h] = W[h] + \lambda (y_i - p^i) V[h] \\
  & \Rightarrow A[h] = k
  \end{align*}
  \]
Hash Kernels

Qinfeng Shi, James Petterson
Australian National University and NICTA,
Canberra, Australia

John Langford, Alex Smola, Alex Strehl
Yahoo! Research
New York, NY and Santa Clara, CA, USA

Gideon Dror
Department of Computer Science
Academic College of Tel-Aviv-Yaffo, Israel

Vishy Vishwanathan
Department of Statistics
Purdue University, IN, USA
Some details

Slightly different hash to avoid systematic bias

\[
V[h] = \sum_{j: \text{hash}(j) \% R == h} x_i^j
\]

\[
\varphi[h] = \sum_{j: \text{hash}(j) \% m == h} \xi(j) x_i^j, \quad \text{where } \xi(j) \in \{-1, +1\}
\]

\(m\) is the number of buckets you hash into (\(R\) in my discussion)
Some details

Slightly different hash to avoid systematic bias

\[ \varphi[h] = \sum_{j: \text{hash}(j) \% m == h} \xi(j)x_i^j, \quad \text{where } \xi(j) \in \{-1,+1\} \]

Lemma 2 The hash kernel is unbiased, that is

\[ E_\varphi[\langle x, x' \rangle_\varphi] = \langle x, x' \rangle. \quad \text{Moreover, the variance is } \]

\[ \sigma_{x,x'}^2 = \frac{1}{m} \left( \sum_{i \neq j} x_i^2 x'_j^2 + x_i x'_i x_j x'_j \right), \quad \text{and thus, for } \]

\[ \|x\|_2 = \|x'\|_2 = 1, \quad \sigma_{x,x'}^2 = O \left( \frac{1}{m} \right). \]

I.e., for large feature sets the variance should be low
Some details

**Theorem 3** Let $\epsilon < 1$ be a fixed constant and $x$ be a given instance. Let $\eta = \frac{\|x\|_\infty}{\|x\|_2}$. Under the assumptions above, the hash kernel satisfies the following inequality

$$\Pr \left\{ \left| \frac{\|x\|_\phi^2 - \|x\|_2^2}{\|x\|_2^2} \right| \geq \sqrt{2} \sigma_{x,x} + \epsilon \right\} \leq \exp \left( -\frac{\sqrt{\epsilon}}{4\eta} \right).$$

I.e. – a hashed vector is probably close to the original vector
Some details

Corollary 4  For two vectors $x$ and $x'$, let us define

$$\sigma := \max(\sigma_{x,x}, \sigma_{x',x'}, \sigma_{x-x',x-x'})$$

$$\eta := \min\left(\frac{\|x\|_\infty}{\|x\|_2}, \frac{\|x'\|_\infty}{\|x'\|_2}, \frac{\|x-x'\|_\infty}{\|x-x'\|_2}\right).$$

Also let $\Delta = \|x\|^2 + \|x'\|^2 + \|x-x'\|^2$. Under the assumptions above, we have that

$$\Pr\left[|\langle x, x'\rangle_\phi - \langle x, x'\rangle| > (\sqrt{2}\sigma + \epsilon)\Delta/2\right] < 3e^{-\frac{\sqrt{\epsilon}}{4\eta}}.$$

I.e. the inner products between $x$ and $x'$ are probably not changed too much by the hash function: a classifier will probably still work.
The Voted Perceptron for Ranking and Structured Classification

William Cohen
The voted perceptron for ranking

\[ y_i = \hat{v}_k \cdot x_i \]

Return: the index \( b^* \) of the “best” \( x_i \)

If mistake:
\[ v_{k+1} = v_k + x_b - x_{b^*} \]

Margin \( \gamma \). \( A \) must provide examples that can be correctly ranked with some vector \( u \) with margin \( \gamma > 0 \), i.e.

\[ \exists u : \forall x_{i,1}, \ldots, x_{i,n_i}, \ell \text{ given by } A, \forall j \neq \ell, \ u \cdot x_\ell - u \cdot x_j > \gamma \]

and furthermore, \( \|u\|^2 = 1 \).

Radius \( R \). \( A \) must provide examples “near the origin”, i.e.

\[ \forall x_i \text{ given by } A, \|x\|^2 < R^2 \]
Ranking some $x$'s with the target vector $u$.
Ranking some $x$’s with some guess vector $v$ – part 1
Ranking some $x$’s with some guess vector $v$ – part 2.

The purple-circled $x$ is $x_{b^*}$ - the one the learner has chosen to rank highest. The green circled $x$ is $x_b$, the right answer.
Correcting \( \mathbf{v} \) by adding \( x_b - x_{b^*} \)
Correcting $\mathbf{v}$ by adding $x_b - x_{b^*}$

(part 2)
(3a) The guess $v_2$ after the two positive examples: $v_2 = v_1 + x_2$

Lemma 1 \( \forall k, \ v_k \cdot u \geq k\gamma \). In other words, the dot product between \( v_k \) and \( u \) increases with each mistake, at a rate depending on the margin \( \gamma \).

Proof:

\[
\begin{align*}
v_{k+1} \cdot u &= (v_k + y_i x_i) \cdot u \\
\Rightarrow \quad v_{k+1} \cdot u &= (v_k \cdot u) + y_i (x_i \cdot u) \\
\Rightarrow \quad v_{k+1} \cdot u &\geq v_k \cdot u + \gamma \\
\Rightarrow \quad v_k \cdot u &\geq k\gamma
\end{align*}
\]
(3a) The guess $v_2$ after the two positive examples: $v_2 = v_1 + x_2$

\[ u - u \]

**Lemma 3** \( \forall k, \; v_k \cdot u \geq k\gamma \). In other words, the dot product between $v_k$ and $u$ increases with each mistake, at a rate depending on the margin $\gamma$.

\[
\begin{align*}
v_{k+1} \cdot u &= (v_k + y_i x_i) \cdot u \\
\implies v_{k+1} \cdot u &= (v_k \cdot u) + y_i (x_i \cdot u) \\
\implies v_{k+1} \cdot u &\geq v_k \cdot u + \gamma \\
\implies v_k \cdot u &\geq k\gamma \\
\end{align*}
\]

\[
\begin{align*}
v_{k+1} \cdot u &= (v_k + x_{i,\ell} - x_{i,\ell}) \cdot u \\
\implies v_{k+1} \cdot u &= v_k \cdot u + x_{i,\ell} \cdot u - x_{i,\ell} \cdot u \\
\implies v_{k+1} \cdot u &\geq v_k \cdot u + \gamma \\
\implies v_k \cdot u &\geq k\gamma \\
\end{align*}
\]
The guess \( v_2 \) after the two positive examples: 

\[
v_2 = v_1 + x_2 - u
\]

**Lemma 3** \( \forall k, v_k \cdot u \geq k \gamma \). In other words, the dot product between \( v_k \) and \( u \) increases with each mistake, at a rate depending on the margin \( \gamma \).
Notice this doesn’t depend at all on the number of $x$’s being ranked

(3a) The guess $v_2$ after the two positive examples: $v_2 = v_1 + x_2$

Lemma 4 $\forall k, \|v_k\|^2 \leq 2kR$.

Theorem 2 Under the rules of the ranking perceptron game, it is always the case that $k < 2R/\gamma^2$.

Neither proof depends on the dimension of the $x$’s.
Ranking perceptrons ➔ structured perceptrons

• The API:
  – A sends B a (maybe huge) set of items to rank
  – B finds the single best one according to the current weight vector
  – A tells B which one was actually best

• Structured classification on a sequence
  – Input: list of words: \( x = (w_1, \ldots, w_n) \)
  – Output: list of labels: \( y = (y_1, \ldots, y_n) \)
  – If there are \( K \) classes, there are \( K^n \) labels possible for \( x \)
Borkar et al’s: HMMs for segmentation

- Example: Addresses, bib records
- Problem: some DBs may split records up differently (eg no “mail stop” field, combine address and apt #, …) or not at all
- Solution: Learn to segment textual form of records

IE with Hidden Markov Models

Transition probabilities

| Author | Smith | 0.01 |
|        | Cohen | 0.05 |
|        | Jordan | 0.3 |
| Year   | dddd | 0.8 |
|        | dd | 0.2 |

Emission probabilities

| Journal | Learning | 0.06 |
|         | Convex | 0.03 |
|         | ... | .. |
|         | Comm. | 0.04 |
|         | Trans. | 0.02 |
|         | Chemical | 0.004 |
Inference for linear-chain CRFs

When will prof Cohen post the notes ...

Idea 1: features are properties of *two adjacent tokens*, and the *pair* of labels assigned to them (Begin,Inside,Outside)

- \((y(i)==B \text{ or } y(i)==I) \text{ and } (\text{token}(i) \text{ is capitalized})\)
- \((y(i)==I \text{ and } y(i-1)==B) \text{ and } (\text{token}(i) \text{ is hyphenated})\)
- \((y(i)==B \text{ and } y(i-1)==B)\)
  - eg “tell Rose William is on the way”

Idea 2: construct a graph where each *path* is a possible sequence labeling.
Inference for a linear-chain CRF

- Inference: find the highest-weight path given a weighting of features
- This can be done efficiently using dynamic programming (Viterbi)
• The API:
  – A sends B a (maybe huge) set of items to rank
  – B finds the single best one according to the current weight vector
  – A tells B which one was actually best

• Structured classification on a sequence
  – Input: list of words: $x=(w_1,\ldots,w_n)$
  – Output: list of labels: $y=(y_1,\ldots,y_n)$
  – If there are $K$ classes, there are $K^n$ labels possible for $x$
Ranking perceptrons ➔ structured perceptrons

- New API:
  - A sends B the word sequence $x$
  - B finds the single best $y$ according to the current weight vector using Viterbi
  - A tells B which $y$ was actually best
  - This is equivalent to ranking pairs $g=(x,y')$

- Structured classification on a sequence
  - Input: list of words: $x=(w_1,\ldots,w_n)$
  - Output: list of labels: $y=(y_1,\ldots,y_n)$
  - If there are $K$ classes, there are $K^n$ labels possible for $x$
The voted perceptron for ranking

Compute: \( y_i = \mathbf{v}_k \cdot \mathbf{x}_i \)

Return: the index \( b^* \) of the “best” \( \mathbf{x}_i \)

If mistake: \( \mathbf{v}_{k+1} = \mathbf{v}_k + \mathbf{x}_b - \mathbf{x}_{b^*} \)

Change number one is notation: replace \( \mathbf{x} \) with \( \mathbf{g} \)
The voted perceptron for structured classification tasks

\[
\text{Compute: } y_i = \hat{v}_k \cdot g_i
\]

Return: the index \(b^*\) of the “best” \(g_i\)

If mistake: \(v_{k+1} = v_k + g_b - g_{b^*}\)

1. A sends B feature functions, and instructions for creating the instances \(g\):
   - A sends a word vector \(x_i\). Then B could create the instances \(g_1 = F(x_i, y_1), g_2 = F(x_i, y_2), \ldots\)
   - but instead B just returns the \(y^*\) that gives the best score for the dot product \(v_k \cdot F(x_i, y^*)\) by using Viterbi.

2. A sends B the correct label sequence \(y_i\).

3. On errors, B sets \(v_{k+1} = v_k + g_b - g_{b^*} = v_k + F(x_i, y) - F(x_i, y^*)\)
Results from the original paper....


 Michael Collins
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 EMNLP 2002, Best paper
Collins’ Experiments

- POS tagging
- NP Chunking (words and POS tags from Brill’s tagger as features) and BIO output tags
- Compared logistic regression methods (MaxEnt) and “Voted Perceptron trained HMM’s”
  - With and w/o averaging
  - With and w/o feature selection (count>5)
Collins’ results

<table>
<thead>
<tr>
<th>NP Chunking Results</th>
<th>POS Tagging Results</th>
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</thead>
<tbody>
<tr>
<td><strong>Method</strong></td>
<td><strong>F-Measure</strong></td>
</tr>
<tr>
<td>Perc, avg, cc=0</td>
<td>93.53</td>
</tr>
<tr>
<td>Perc, noavg, cc=0</td>
<td>93.04</td>
</tr>
<tr>
<td>Perc, avg, cc=5</td>
<td>93.33</td>
</tr>
<tr>
<td>Perc, noavg, cc=5</td>
<td>91.88</td>
</tr>
<tr>
<td>ME, cc=0</td>
<td>92.34</td>
</tr>
<tr>
<td>ME, cc=5</td>
<td>92.65</td>
</tr>
</tbody>
</table>

Figure 4: Results for various methods on the part-of-speech tagging and chunking tasks on development data. All scores are error percentages. Numits is the number of training iterations at which the best score is achieved. Perc is the perceptron algorithm, ME is the maximum entropy method. Avg/noavg is the perceptron with or without averaged parameter vectors. cc=5 means only features occurring 5 times or more in training are included, cc=0 means all features in training are included.
Where we are…

- Experiments with a hash-trick implementation of logistic regression
- Next question:
  - how do you parallelize SGD, or more generally, this kind of streaming algorithm?
  - each example affects the next prediction \(\Rightarrow\) order matters \(\Rightarrow\) parallelization changes the behavior
  - we will step back to perceptrons and then step forward to parallel perceptrons
  - then another nice parallel learning algorithm
  - then a midterm