## RECAP: THE COURSE SO FAR...

## First Lecture - Review

- Admin stuff
- Review - Why to scale, how to count and what to count
- How: O(...)


## Why to scale: c. 2001 (Banko \& Brill, ACL 2001)



Figure 1. Learning Curves for Confusion Set Disambiguation


Figure 2. Representation Size vs. Training Corpus Size

Task: distinguish pairs of easily-confused words ("affect" vs "effect") in context

## Numbers (Jeff Dean says) Everyone Should Know

| L1 cache reference | 0.5 ns |
| :--- | ---: |
| Branch mispredict | 5 ns |
| L2 cache reference | 7 ns |
| Mutex lock/unlock | 100 ns |
| Main memory reference | 100 ns |
| Compress 1K bytes with Zippy | $10,000 \mathrm{~ns}$ |
| Send 2K bytes over 1 Gbps network | $20,000 \mathrm{~ns}$ |
| Read 1 MB sequentially from memory | $250,000 \mathrm{~ns}$ |
| Round trip within same datacenter | $500,000 \mathrm{~ns}$ |
| Disk seek | $10,000,000 \mathrm{~ns}$ |
| Read 1 MB sequentially from network | $10,000,000 \mathrm{~ns}$ |
| Read 1 MB sequentially from disk | $30,000,000 \mathrm{~ns}$ |
| Send packet CA->Netherlands->CA | $150,000,000 \mathrm{~ns}$ |

## Update: Colin Scott, UCB

file:///Users/wcohen/Documents/code/interactive_latencies/ interactive_latency.html

This isn't on the web but is available thru GitHub


## What's Happening with Hardware?

- Clock speed: stuck at 3Ghz for $\sim 10$ years
- Net bandwidth doubles $\sim 2$ years
- Disk bandwidth doubles $\sim 2$ years
- SSD bandwidth doubles $\sim 3$ years
- Disk seek speed doubles $\sim 10$ years
- SSD latency nearly saturated

Historical Cost of Computer Memory and Storage


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## A typical CPU (not to scale)



## A typical CPU (not to scale)



## A typical CPU (not to scale)

K8 core in the AMD $\underline{\text { Athlon } 64}$ CPU


## A typical CPU (not to scale)



## A typical disk



## Numbers (Jeff Dean says) Everyone Should Know

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## What do we count?

- Compilers don't warn Jeff Dean. Jeff Dean warns compilers.
- Memory access/instructions are qualitatively different from disk access
- Seeks are qualitatively different from sequential reads on disk
- Cache, disk fetches, etc work best when you stream through data sequentially
- Best case for data processing: stream through the data once in sequential order, as it's found on disk.


## Other lessons -?

## Encoding Your Data



- CPUs are fast, memory/bandwidth are precious, ergo...
- Variable-length encodings
- Compression
- Compact in-memory representations
- Compression very important aspect of many systems
- inverted index posting list formats
- storage systems for persistent data
* but not important enough for this class's assignments....


## What to count



## First lecture: review

- Admin stuff
- Review - Why to scale, how to count and what to count
- What sort of computations do we want to do in (large-scale) machine learning programs?
- Probability


## PROBABILITY AND SCALABILITY: LEARNING AND COUNTING

Big ML c. 2001 (Banko \& Brill, "Scaling to Very Very Large...", ACL 2001)


Figure 1. Learning Curves for Confusion Set Disambiguation


Figure 2. Representation Size vs. Training Corpus Size

Task: distinguish pairs of easily-confused words (" affect" vs "effect") in context

## Why More Data Helps: A Demo

- Data:
- All 5-grams that appear >= 40 times in a corpus of 1M English books
- approx 80B words
- 5-grams: 30 Gb compressed, $250-300 \mathrm{~Gb}$ uncompressed
- Each 5-gram contains frequency distribution over years
- Wrote code to compute
- $\operatorname{Pr}(A, B, C, D, E \mid C=$ affect or $C=e f f e c t)$
- $\operatorname{Pr}($ any subset of $A, \ldots, E \mid$ any other fixed values of $A$, ..., E with C=affect V effect)
- Demo:
- /Users/wcohen/Documents/code/pyhack/bigml
- eg: python ngram-query.py data/aeffect-train.txt __B effect _ _


## Why More Data Helps

- Data:
- All 5-grams that appear >= 40 times in a corpus of 1M English books
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- 5-grams: 30Gb compressed, 250-300Gb uncompressed
- Each 5-gram contains frequency distribution over years
- Wrote code to compute
- $\operatorname{Pr}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E} \mid \mathrm{C}=$ affect or $\mathrm{C}=$ effect $)$
- $\operatorname{Pr}$ (any subset of $\mathrm{A}, \ldots, \mathrm{E} \mid$ any other fixed values of $\mathrm{A}, \ldots, \mathrm{E}$ with $\mathrm{C}=$ affect V effect)
- Observations [from playing with data]:
- Mostly effect not affect
- Most common word before affect is not
- After not effect most common word is a
- ...


## The Joint Distribution

Example: Boolean variables $A$, B, C

Recipe for making a joint distribution of M variables:

1. Make a truth table listing all combinations of values of your variables (if there are M Boolean variables then the table will have $2^{\mathrm{M}}$ rows).
2. For each combination of values, say how probable it is.
3. If you subscribe to the axioms of probability, those numbers must sum to 1.

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | Prob |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0.30 |
| 0 | 0 | 1 | 0.05 |
| 0 | 1 | 0 | 0.10 |
| 0 | 1 | 1 | 0.05 |
| 1 | 0 | 0 | 0.05 |
| 1 | 0 | 1 | 0.10 |
| 1 | 1 | 0 | 0.25 |
| 1 | 1 | 1 | 0.10 |



## Some of the Joint Distribution

| A | B | C | D | E | p |
| :--- | :--- | :--- | :--- | :--- | :--- |
| is | the | effect | of | the | 0.00036 |
| is | the | effect | of | a | 0.00034 |
| . | The | effect | of | this | 0.00034 |
| to | this | effect | : | " | 0.00034 |
| be | the | effect | of | the | $\ldots$ |
|  | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| not | the | effect | of | any | 0.00024 |
|  | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| does | not | affect | the | general | 0.00020 |
| does | not | affect | the | question | 0.00020 |
| any | manner | affect | the | principle | 0.00018 |

## An experiment: how useful is the brute-force joint classifier?

- Extracted all affect/ effect 5-grams from an old Reuters corpus
- about 20k documents
- about 723 n-grams, 661 distinct
- Financial news, not novels or textbooks
- Tried to predict center word with:
$-\operatorname{Pr}(\mathrm{C} \mid \mathrm{A}=\mathrm{a}, \mathrm{B}=\mathrm{b}, \mathrm{D}=\mathrm{d}, \mathrm{E}=\mathrm{e})$
- then $\mathrm{P}(\mathrm{C} \mid \mathrm{A}, \mathrm{B}, \mathrm{D})$
- then $P(C \mid B, D)$
- then $P(C \mid B)$
- then $\mathrm{P}(\mathrm{C})$


## EXAMPLES

-"The cumulative _ of the" $\rightarrow$ effect (1.0)
-"Go into _ on January" $\rightarrow$ effect (1.0)
-"From cumulative _ of accounting" not present in train data

- Nor is ""From cumulative _ of _"
- But "_ cumulative _ of _" $\rightarrow$ effect (1.0)
-"Would not _ Finance Minister" not present
- But "_ not _ _ _" $\rightarrow$ affect (0.9625)


## Performance summary

\(\left.\begin{array}{l|c|c}\hline Pattern \& Used \& Errors <br>
\hline \mathrm{P}(\mathrm{C} \mid \mathrm{A}, \mathrm{B}, \mathrm{D}, \mathrm{E}) \& 101 \& 1 <br>
\mathrm{P}(\mathrm{C} \mid \mathrm{A}, \mathrm{B}, \mathrm{D}) \& 157 \& 6 <br>
\hline \mathrm{P}(\mathrm{C} \mid \mathrm{B}, \mathrm{D}) \& 163 \& 13 <br>

\mathrm{P}(\mathrm{C} \mid \mathrm{B}) \& 244 \& 78\end{array}\right] \quad\)| $3 \%$ error |
| :---: |
| $\mathrm{P}(\mathrm{C})$ |



Is this a useful density estimate?

## What Have We Learned?

- Counting's not enough -?
- Counting goes a long way with big data -?
- Big data can sometimes be made small
-For a specific task, like this one
-It's all in the data preparation -?
- Often density estimation is more important than classification
- Counts are a good? density estimator


## Density Estimation

- Our Joint Distribution learner is our first example of something called Density Estimation
- A Density Estimator learns a mapping from a set of attributes values to a Probability



## Density Estimation

- Compare it against the two other major kinds of models:



## Density Estimation $\rightarrow$ Classification



To classify $\mathbf{x}$

1. Use your estimator to compute $\hat{\mathrm{P}}(\mathrm{x}, \mathrm{y} 1), \ldots ., \hat{\mathrm{P}}(\mathrm{x}, \mathrm{yk})$
2. Return the class $y^{*}$ with the highest predicted probability

Binary case: predict POS if $\hat{P}(\mathbf{x})>0.5$

Ideally is correct with $\widehat{P}\left(x, y^{*}\right)=\widehat{P}\left(x, y^{*}\right) /(\hat{P}(x, y 1)+\ldots .+\hat{P}(x, y k))$

## Classification vs Density Estimation

Classification
Density Estimation



## Classification vs density estimation



## PROBABILITY AND SCALABILITY: NAÏVE BAYES

Second most scalable learning method in the world?

## Performance ...

| Pattern | Used | Errors |
| :--- | :---: | :---: |
| $\mathrm{P}(\mathrm{C} \mid \mathrm{A}, \mathrm{B}, \mathrm{D}, \mathrm{E})$ | 101 | 1 |
| $\mathrm{P}(\mathrm{C} \mid \mathrm{A}, \mathrm{B}, \mathrm{D})$ | 157 | 6 |
| $\mathrm{P}(\mathrm{C} \mid \mathrm{B}, \mathrm{D})$ | 163 | 13 |
| $\mathrm{P}(\mathrm{C} \mid \mathrm{B})$ | 244 | 78 |
| $\mathrm{P}(\mathrm{C})$ | 58 | 31 |

- Is this good performance?
- If we care about recall, what should we do?


## Naïve Density Estimation

What's an alternative to the joint distribution?

The naïve model generalizes strongly:
Assume that each attribute is distributed independently of any of the other attributes.

## Using the Naïve Distribution

- Once you have a Naïve Distribution you can easily compute any row of the joint distribution.
- Suppose $A, B, C$ and $D$ are independently distributed. What is $P\left(A^{\wedge} \sim B^{\wedge} C^{\wedge} \sim D\right)$ ?


## Using the Naïve Distribution

- Once you have a Naïve Distribution you can easily compute any row of the joint distribution.
- Suppose A, B, C and D are independently distributed. What is $\mathrm{P}\left(\mathrm{A}^{\wedge} \sim \mathrm{B}^{\wedge} \mathrm{C}^{\wedge} \sim \mathrm{D}\right)$ ?
$\mathrm{P}(\mathrm{A}) \mathrm{P}(\sim \mathrm{B}) \mathrm{P}(\mathrm{C}) \mathrm{P}(\sim \mathrm{D})$


## Naïve Distribution General Case

- Suppose $X_{1}, X_{2}, \ldots, X_{d}$ are independently distributed.

$$
\operatorname{Pr}\left(X_{1}=x_{1}, \ldots, X_{d}=x_{d}\right)=\operatorname{Pr}\left(X_{1}=x_{1}\right) \cdot \ldots \cdot \operatorname{Pr}\left(X_{d}=x_{d}\right)
$$

- So if we have a Naïve Distribution we can construct any row of the implied Joint Distribution on demand.
- How do we learn this?


## Learning a Naïve Density Estimator

$$
\begin{gathered}
P\left(X_{i}=x_{i}\right)=\frac{\# \text { records with } X_{i}=x_{i}}{\# \text { records }} \quad \text { MLE } \\
P\left(X_{i}=x_{i}\right)=\frac{\text { \#records with } X_{i}=x_{i}+m q}{\# \text { records }+m} \quad \text { Dirichlet (MAP) }
\end{gathered}
$$

## Another trivial learning algorithm!

## Is this an interesting learning algorithm?

- For n-grams, what is $\mathrm{P}(\mathrm{C}=$ effect $\mid \mathrm{A}=$ will $)$ ?
- In joint: $\hat{P}(C=$ effect $\mid A=$ will $)=0.38$
- In naïve: $P(C=$ effect $\mid A=$ will $)=P(C=$ effect $)=\#[C=$ effect $] /$ \#totalNgrams = 0.94 (!)
- What is $P(C=$ effect $\mid B=n o)$ ?
- In joint: $\hat{P}(C=$ effect $\mid B=n o)=0.999$
- In naïve: $\hat{P}(C=$ effect $\mid B=$ no $)=\hat{P}(C=$ effect $)=0.94$


## Can we make this interesting? Yes!

- Key ideas:
- Pick the class variable Y
- Instead of estimating $\mathrm{P}\left(\mathrm{X}_{1}, \ldots, \mathrm{X}_{\mathrm{n}}, \mathrm{Y}\right)=\mathrm{P}\left(\mathrm{X}_{1}\right)^{*} \ldots{ }^{*} \mathrm{P}\left(\mathrm{X}_{\mathrm{n}}\right)^{*} \mathrm{Y}$, estimate $\mathrm{P}\left(\mathrm{X}_{1}, \ldots, \mathrm{X}_{\mathrm{n}} \mid \mathrm{Y}\right)=\mathrm{P}\left(\mathrm{X}_{1} \mid \mathrm{Y}\right)^{*} \ldots{ }^{*} \mathrm{P}\left(\mathrm{X}_{\mathrm{n}} \mid \mathrm{Y}\right)$
- Or, assume $P\left(X_{i} \mid Y\right)=\operatorname{Pr}\left(X_{i} \mid X_{1}, \ldots, X_{i-1}, X_{i+1}, \ldots X_{n}, Y\right)$
- Or, that $X_{i}$ is conditionally independent of every $X_{j}, j!=i$, given Y .
- How to estimate?


## The Naïve Bayes classifier - v1

- Dataset: each example has
- A unique id id
- Why? For debugging the feature extractor
- $d$ attributes $X_{1}, \ldots, X_{d}$
- Each $X_{i}$ takes a discrete value in $\operatorname{dom}\left(X_{i}\right)$
- One class label $Y$ in $\operatorname{dom}(Y)$
- You have a train dataset and a test dataset
- Assume:
- the dataset doesn't fit in memory
- the model does
stream through it


## The Naïve Bayes classifier - v0

- You have a train dataset and a test dataset
- Initialize an "event counter" (hashtable) C
- For each example id, $y, x_{1}, \ldots, x_{d}$ in train:
- C(" $Y=A N Y$ ") $++; \quad C\left(" Y=y^{\prime \prime}\right)++$
- For $j$ in 1..d:
- C(" $\left.Y=y^{\wedge} X_{j}=x_{j}^{\prime \prime}\right)++$
- For each example id, $y, x_{1}, \ldots, x_{d}$ in test:
- For each $y^{\prime}$ in $\operatorname{dom}(Y)$ :
- Compute $\operatorname{Pr}\left(y^{\prime}, x_{1}, \ldots, x_{d}\right)=\left(\prod_{j=1}^{d} \operatorname{Pr}\left(X_{j}=x_{j} \mid Y=y^{\prime}\right)\right) \operatorname{Pr}\left(Y=y^{\prime}\right)$

$$
=\left(\prod_{j=1}^{d}=\frac{\operatorname{Pr}\left(X_{j}=x_{j}, Y=y^{\prime}\right)}{\operatorname{Pr}\left(Y=y^{\prime}\right)}\right) \operatorname{Pr}\left(Y=y^{\prime}\right)
$$

- Return the best $y^{\prime}$


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- For each example $i d, y, x_{1}, \ldots, x_{d}$ in test:
- For each $y^{\prime}$ in $\operatorname{dom}(Y)$ :
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$=\left(\prod_{j=1}^{d} \frac{C\left(X_{j}=x_{j} \wedge Y=y^{\prime}\right)}{C\left(Y=y^{\prime}\right)}\right) \frac{C\left(Y=y^{\prime}\right)}{C(Y=A N Y)}$
- Return the best $y^{\prime}$


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- You have a train dataset and a test dataset
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$=\left(\prod_{j=1}^{d} \frac{C\left(X_{j}=x_{j} \wedge Y=y^{\prime}\right)}{C\left(Y=y^{\prime}\right)}\right) \frac{C\left(Y=y^{\prime}\right)}{C(Y=A N Y)} \quad$ This may overfit, so $\ldots$
- Return the best $y^{\prime}$


## The Naïve Bayes classifier - v1

- You have a train dataset and a test dataset
- Initialize an "event counter" (hashtable) C
- For each example $i d, y, x_{1}, \ldots, x_{d}$ in train:
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- For each example id, $y, x_{1}, \ldots, x_{d}$ in test:
- For each $y^{\prime}$ in $\operatorname{dom}(Y)$ :
- Compute $\operatorname{Pr}\left(y^{\prime}, x_{1}, \ldots, x_{d}\right)=\left(\prod_{j=1}^{d} \operatorname{Pr}\left(X_{j}=x_{j} \mid Y=y^{\prime}\right)\right) \operatorname{Pr}\left(Y=y^{\prime}\right)$

$$
=\left(\prod_{j=1}^{d} \frac{C\left(X_{j}=x_{j} \wedge Y=y^{\prime}\right)+m q_{x}}{C\left(Y=y^{\prime}\right)+m}\right) \frac{C\left(Y=y^{\prime}\right)+m q_{y}}{C(Y=A N Y)+m} \quad \begin{aligned}
& \text { where: } \\
& \begin{array}{l}
q_{j}=1 /\left|\operatorname{dom}\left(X_{j}\right)\right| \\
q_{y}=1 /|\operatorname{dom}(Y)| \\
m q_{x}=1
\end{array}
\end{aligned}
$$

- Return the best $y^{\prime}$

This may underflow, so ...

## The Naïve Bayes classifier - v1

- You have a train dataset and a test dataset
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- For each example id, $y_{1}, x_{1}, \ldots, x_{d}$ in test:
- For each $y^{\prime}$ in $\operatorname{dom}(Y)$ :
where:
$q_{j}=1 /\left|\operatorname{dom}\left(X_{j}\right)\right|$
$q_{y}=1 /|\operatorname{dom}(Y)|$
$m q_{x}=1$
- Compute $\log \operatorname{Pr}\left(y^{\prime}, x_{1}, \ldots, x_{d}\right)=$

$$
=\left(\sum_{j} \log \frac{C\left(X_{j}=x_{j} \wedge Y=y^{\prime}\right)+m q_{j}}{C\left(Y=y^{\prime}\right)+m}\right)+\log \frac{C\left(Y=y^{\prime}\right)+m q_{j}}{C(Y=A N Y)+m}
$$

- Return the best $y^{\prime}$


## The Naïve Bayes classifier - v2

- For text documents, what features do you use?
- One common choice:
$-X_{1}=$ first word in the document
$-X_{2}=$ second word in the document
$-X_{3}=$ third $\ldots$
$-X_{4}=\ldots$
- But: $\operatorname{Pr}\left(X_{13}=h o c k e y \mid Y=\right.$ sports $)$ is probably not that different from $\operatorname{Pr}\left(X_{11}=\right.$ hockey $\mid Y=$ sports $)$...so instead of treating them as different variables, treat them as different copies of the same variable


## The Naïve Bayes classifier - v1

- You have a train dataset and a test dataset
- Initialize an "event counter" (hashtable) C
- For each example id, $y, x_{1}, \ldots, x_{d}$ in train:
- C(" $Y=A N Y$ ") $++; \quad C\left(" Y=y^{\prime \prime}\right)++$
- For $j$ in 1..d:
- C(" $\left.Y=y^{\wedge} X_{j}=x_{j}^{\prime \prime}\right)++$
- For each example id, $y, x_{1}, \ldots, x_{d}$ in test:
- For each $y^{\prime}$ in $\operatorname{dom}(Y)$ :
- Compute $\operatorname{Pr}\left(y^{\prime}, x_{1}, \ldots, x_{d}\right)=\left(\prod_{j=1}^{d} \operatorname{Pr}\left(X_{j}=x_{j} \mid Y=y^{\prime}\right)\right) \operatorname{Pr}\left(Y=y^{\prime}\right)$

$$
=\left(\prod_{j=1}^{d}=\frac{\operatorname{Pr}\left(X_{j}=x_{j}, Y=y^{\prime}\right)}{\operatorname{Pr}\left(Y=y^{\prime}\right)}\right) \operatorname{Pr}\left(Y=y^{\prime}\right)
$$

- Return the best $y^{\prime}$


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=\left(\prod_{j=1}^{d} \frac{\operatorname{Pr}\left(X_{j}=x_{j}, Y=y^{\prime}\right)}{\operatorname{Pr}\left(Y=y^{\prime}\right)}\right) \operatorname{Pr}\left(Y=y^{\prime}\right)
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- For each example id, $y, x_{1}, \ldots, x_{d}$ in test:
- For each $y^{\prime}$ in $\operatorname{dom}(Y)$ :
where:
$q_{j}=1 /|V|$
$q_{y}=1 /|\operatorname{dom}(Y)|$
$m q_{x}=1$
- Compute $\log \operatorname{Pr}\left(y^{\prime}, x_{1}, \ldots, x_{d}\right)=$
$=\left(\sum_{j} \log \frac{C\left(X=x_{j} \wedge Y=y^{\prime}\right)+m q_{x}}{C\left(X=A N Y \wedge Y=y^{\prime}\right)+m}\right)+\log \frac{C\left(Y=y^{\prime}\right)+m q_{y}}{C(Y=A N Y)+m}$
- Return the best $y^{\prime}$


## The Naïve Bayes classifier - v2

- You have a train dataset and a test dataset
- To classify documents, these might be:
- http://wcohen.com academic,FacultyHome William W. Cohen Research Professor Machine Learning Department Carnegie Mellon University Member of the Language Technology Institute the joint CMU-Pitt Program in Computational Biology the Lane Center for Computational Biology and the Center for Bioimage Informatics Director of the Undergraduate Minor in Machine Learning Bio Teaching Projects Publications recent all Software Datasets Talks Students Colleagues Blog Contact Info Other Stuff ...
- http://google.com commercial Search Images Videos ....
- How about for n-grams?


## The Naïve Bayes classifier - v2

- You have a train dataset and a test dataset
- To do C-S spelling correction these might be - ng1223 effect a_the b_main d_of e_the - ng1224 affect a_shows b_not d_mice e_in
- I.e., encode event $X_{i}=w$ with another event $X=i \_w$
- Question: are there any differences in behavior from using A,B,C,D ?

Assume hashtable holding all counts fits in memory

## Complexity of Naïve Bayes

- You have a train dataset and a test dataset
- Initialize an "event counter" (hashtable) C
- For each example $i d, y, x_{1}, \ldots, x_{d}$ in train:
- C(" $Y=A N Y$ ") $++; \quad C\left(" Y=y^{\prime \prime}\right)++$
- For $j$ in 1..d:
- $\mathrm{C}\left(\right.$ " $\mathrm{Y}=y^{\wedge} \mathrm{X}=x_{j}$ ") ++
- For each example id, $y, x_{1}, \ldots, x_{d}$ in test:
- For each $y^{\prime}$ in $\operatorname{dom}(Y)$ :
- Compute $\log \operatorname{Pr}\left(y^{\prime}, x_{1}, \ldots, x_{d}\right)=$

Complexity: $\mathrm{O}(n)$, $n=$ size of train

## Complexity of Naïve Bayes

- You have a train dataset and a test dataset
- Process:
- Count events in the train dataset
- $\mathrm{O}(n)$, where $n$ is total size of train
- Write the counts to disk
- $\mathrm{O}\left(\min \left(|\operatorname{dom}(X)|^{*}|\operatorname{dom}(Y)|, n\right)\right.$
- $\mathrm{O}(|V|)$, if V is vocabulary and $\operatorname{dom}(Y)$ is small
- Classify the test dataset
- $\mathrm{O}\left(|V|+n^{\prime}\right)$
- Worst-case memory usage:
- $\mathrm{O}\left(\min \left(|\operatorname{dom}(X)|^{*}|\operatorname{dom}(Y)|, n\right)\right.$


## Naïve Bayes v2

- This is one example of a streaming classifier
- Each example is only read only once
- You can create a classifier and perform classifications at any point
- Memory is minimal ( $\ll \mathrm{O}(\mathrm{n})$ )
- Ideally it would be constant
- Traditionally less than $\mathrm{O}(\operatorname{sqrt}(\mathrm{N}))$
- Order doesn't matter
- Nice because we may not control the order of examples in real life
- This is a hard one to get a learning system to have!
- There are few competitive learning methods that as stream-y as naïve Bayes...


## Rocchio's Algorithm

## Motivation

- Naïve Bayes is unusual as a learner:
-Only one pass through data
-Order doesn't matter


## Rocchio's algorithm

- Relevance Feedback in Information Retrieval, SMART Retrieval System Experiments in Automatic Document Processing, 1971, Prentice Hall Inc.


## Rocchio's algorithm

$D F(w)=$ \# different docs $w$ occurs in
$T F(w, d)=$ \# different times $w$ occurs in $\operatorname{doc} d$

$$
\begin{aligned}
I D F(w) & =\frac{|D|}{D F(w)} \\
u(w, d) & =\log (T F(w, d)+1) \cdot \log (I D F(w))
\end{aligned}
$$

$$
\mathbf{u}(d)=\left\langle u\left(w_{1}, d\right), \ldots ., u\left(w_{\mathrm{IV}}, d\right)\right\rangle
$$

Store only non-zeros in $\mathbf{u}(d)$, so size is $\mathrm{O}(|d|)$

But size of $\mathbf{u}(y)$ is $\mathrm{O}\left(\left|n_{V}\right|\right)$

$$
\mathbf{u}(y)=\alpha \frac{1}{\left|C_{y}\right|} \sum_{d \in C_{y}} \frac{\mathbf{u}(d)}{\|\mathbf{u}(d)\|_{2}}-\beta \frac{1}{\left|D-C_{y}\right|} \sum_{d^{\prime} \in D-C_{y}} \frac{\mathbf{u}\left(d^{\prime}\right)}{\left\|\mathbf{u}\left(d^{\prime}\right)\right\|_{2}}
$$

$$
f(d)=\arg \max _{y} \frac{\mathbf{u}(d)}{\|\mathbf{u}(d)\|_{2}} \cdot \frac{\mathbf{u}(y)}{\|\mathbf{u}(y)\|_{2}}
$$

## Rocchio results...

Joacchim '98, "A Probabilistic Analysis of the Rocchio Algorithm..."

|  | PrTFIDF | BAYES | TFIDF |
| :--- | :---: | :---: | :---: |
| Newsgroups | 91.8 | 89.6 | 86.3 |
| "acq" | 88.9 | 88.5 | 84.5 |
| "wheat" | 93.9 | 94.8 | 90.9 |
| "crude" | 90.2 | 95.5 | 85.4 |
| "earn" | 90.5 | 90.9 | 90.6 |
| "cbond" | 91.9 | 90.9 | 87.7 |

Table 2: Maximum accuracy in percentages.


## Rocchio results...

Schapire, Singer, Singhal, "Boosting and Rocchio Applied to Text Filtering", SIGIR 98



Reuters 21578 - all classes (not just the frequent ones)

## A hidden agenda

- Part of machine learning is good grasp of theory
- Part of ML is a good grasp of what hacks tend to work
- These are not always the same
- Especially in big-data situations
- Catalog of useful tricks so far
- Brute-force estimation of a joint distribution
- Naive Bayes
- Stream-and-sort, request-and-answer patterns
- BLRT and KL-divergence (and when to use them)
- TF-IDF weighting - especially IDF
- it's often useful even when we don't understand why


## One more Rocchio observation

Rennie et al, ICML 2003, "Tackling the Poor Assumptions of Naïve Bayes Text Classifiers"

|  | MNB | TWCNB | SVM |
| :---: | :---: | :---: | :---: |
| Industry Sector | 0.582 | 0.923 | 0.934 |
| 20 Newsgroups | 0.848 | 0.861 | 0.862 |
| Reuters (micro) | 0.739 | 0.844 | 0.887 |
| Reuters (macro) | 0.270 | 0.647 | 0.694 |

NB + cascade of hacks

## One more Rocchio observation

Rennie et al, ICML 2003, "Tackling the Poor Assumptions of Naïve Bayes Text Classifiers"

- $\operatorname{TWCNB}(\vec{d}, \vec{y})$

1. $d_{i j}=\log \left(d_{i j}+1\right)(\mathrm{TF}$ transform § 4.1)
2. $d_{i j}=d_{i j} \log \frac{\sum_{k} 1}{\sum_{k} \delta_{i k}}$ (IDF transform § 4.2)
3. $d_{i j}=\frac{d_{i j}}{\sqrt{\sum_{k}\left(d_{k j}\right)^{2}}}$ (length norm. §4.3)
4. $\hat{\theta}_{c i}=\frac{\sum_{j: y_{j} \neq c} d_{i j}+\alpha_{i}}{\sum_{j: y_{j} \neq c} \sum_{k} d_{k j}+\alpha}$ (complement § 3.1)
5. $w_{c i}=\log \hat{\theta}_{c i}$
6. $w_{c i}=\frac{w_{c i}}{\sum_{i} w_{c i}}$ (weight normalization § 3.2)
7. Let $t=\left(t_{1}, \ldots, t_{n}\right)$ be a test document; let $t_{i}$ be the count of word $i$.
8. Label the document according to

$$
l(t)=\arg \min _{c} \sum_{i} t_{i} w_{c i}
$$

