Tales of MADCAP SCIENCE

BEYOND NAIVE BAYES
where NOBODY can HEAR you STREAM!

There was NO WARNING of their ARRIVAL!
They had NO MERCY! They gave NO QUARTER!

CREATED WITH PULP-O-MIZER COVER MAKER
RECAP
Parallel NB Training

Key Points:
• The “full” event counts are a sum of the “local” counts
• Easy to combine independently computed local counts
Parallel Rocchiclo

Key Points:
• We need shared read access to DF’s, but not write access.
• The “full classifier” is a weighted average of the “local” classifiers – still easy!
Parallel Perceptron Learning?

Like DFs or event counts, size is $O(|V|)$

Key Points:
- The “full classifier” is a weighted average of the “local” classifiers.
- Obvious solution requires read/write access to a shared classifier.
Key Point: We need shared write access to the classifier – not just read access. So we only need to not copy the information but synchronize it. Question: How much extra communication is there?

Answer: Depends on how the learner behaves…
…how many weights get updated with each example … (in Naïve Bayes and Rocchio, only weights for features with non-zero weight in $x$ are updated when scanning $x$)
…how often it needs to update weight … (how many mistakes it makes)
The perceptron game

\(\mathbf{x}\) is a vector
\(\mathbf{y}\) is -1 or +1

Compute: \(y_i = \text{sign}(\mathbf{v}_k \cdot \mathbf{x}_i)\)

If mistake: \(\mathbf{v}_{k+1} = \mathbf{v}_k + y_i \mathbf{x}_i\)

\(\text{mistake bound: } k \leq \left(\frac{R}{\gamma}\right)^2\)

**Margin** \(\gamma\). A must provide examples that can be separated with some vector \(\mathbf{u}\) with margin \(\gamma > 0\), ie

\[\exists \mathbf{u} : \forall (x_i, y_i) \text{ given by } A, (\mathbf{u} \cdot \mathbf{x})y_i > \gamma\]

and furthermore, \(\|\mathbf{u}\| = 1\).

**Radius** \(R\). A must provide examples “near the origin”, ie

\[\forall x_i \text{ given by } A, \|x_i\|^2 < R^2\]
STRUCTURED PERCEPTRONS…
Distributed Training Strategies for the Structured Perceptron

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Parallel Structured Perceptrons

• Simplest idea:
  – Split data into S “shards”
  – Train a perceptron on each shard independently
    • weight vectors are \( w^{(1)}, w^{(2)}, \ldots \)
  – Produce some weighted average of the \( w^{(i)} \)'s as the final result

\[
\text{PerceptronParamMix}(T = \{(x_t, y_t)\}_{t=1}^{\mid T \mid})
\]
1. Shard \( T \) into \( S \) pieces \( T = \{T_1, \ldots, T_S\} \)
2. \( w^{(i)} = \text{Perceptron}(T_i) \) \( \uparrow \)
3. \( w = \sum_i \mu_i w^{(i)} \) \( \downarrow \)
4. return \( w \)

Figure 2: Distributed perceptron using a parameter mixing strategy. \( \uparrow \) Each \( w^{(i)} \) is computed in parallel. \( \downarrow \) \( \mu = \{\mu_1, \ldots, \mu_S\}, \forall \mu_i \in \mu: \mu_i \geq 0 \) and \( \sum_i \mu_i = 1. \)
Parallelizing perceptrons

Instances/labels

Instances/labels – 1
vk -1

Instances/labels – 2
vk - 2

Instances/labels – 3
vk-3

Split into example subsets

Compute vk’s on subsets

Combine by some sort of weighted averaging
Parallel Perceptrons

• Simplest idea:
  – Split data into $S$ “shards”
  – Train a perceptron on each shard independently
    • weight vectors are $w^{(1)}$, $w^{(2)}$, …
  – Produce some weighted average of the $w^{(i)}$’s as the final result

• Theorem: this doesn’t always work.
• Proof: by constructing an example where you can converge on every shard, and still have the averaged vector not separate the full training set – no matter how you average the components.
Parallel Perceptrons – take 2

Idea: do the simplest possible thing iteratively.

• Split the data into shards
• Let $w = 0$
• For $n=1,…$
  • Train a perceptron on each shard with one pass starting with $w$
  • Average the weight vectors (somehow) and let $w$ be that average

Extra communication cost:
• redistributing the weight vectors
• done less frequently than if fully synchronized, more frequently than if fully parallelized

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Figure 3: Distributed perceptron using an iterative parameter mixing strategy. † Each $w^{(i,n)}$ is computed in parallel. ‡ $\mu_n = \{\mu_{1,n}, \ldots, \mu_{S,n}\}$, $\forall \mu_{i,n} \in \mu_n$: $\mu_{i,n} \geq 0$ and $\forall n: \sum_i \mu_{i,n} = 1.$

PerceptronIterParamMix($T = \{(x_t, y_t)\}_{t=1}^{|T|}$)
1. Shard $T$ into $S$ pieces $T = \{T_1, \ldots, T_S\}$
2. $w = 0$
3. For $n : 1..N$
4. $w^{(i,n)} = \text{OneEpochPerceptron}(T_i, w)$ †
5. $w = \sum_i \mu_{i,n} w^{(i,n)}$ ‡
6. return $w$

OneEpochPerceptron($T$, $w^*$)
1. $w^{(0)} = w^*$; $k = 0$
2. For $t : 1..|T|
3. Let $y' = \arg \max_{y'} w^{(k)} \cdot f(x_t, y')$
4. if $y' \neq y_t$
5. $w^{(k+1)} = w^{(k)} + f(x_t, y_t) - f(x_t, y')$
6. $k = k + 1$
7. return $w^{(k)}$
Parallelizing perceptrons – take 2

1. Split into example subsets
2. Compute local vk's
3. Combine by some sort of weighted averaging

w (previous)
A theorem

**Theorem 3.** Assume a training set $\mathcal{T}$ is separable by margin $\gamma$. Let $k_{i,n}$ be the number of mistakes that occurred on shard $i$ during the $n$th epoch of training. For any $N$, when training the perceptron with iterative parameter mixing (Figure 3),

$$
\sum_{n=1}^{N} \sum_{i=1}^{S} \mu_{i,n} k_{i,n} \leq \frac{R^2}{\gamma^2}
$$

**Corollary:** if we weight the vectors uniformly, then the number of mistakes is still bounded.

**I.e.,** this is “enough communication” to guarantee convergence.
What we know and don’t know

Theorem 3. Assume a training set \( T \) is separable by margin \( \gamma \). Let \( k_{i,n} \) be the number of mistakes that occurred on shard \( i \) during the \( n \)th epoch of training. For any \( N \), when training the perceptron with iterative parameter mixing (Figure 3),

\[
\sum_{n=1}^{N} \sum_{i=1}^{S} \mu_{i,n} k_{i,n} \leq \frac{R^2}{\gamma^2} \quad \implies \quad \sum_{n=1}^{N} \sum_{i=1}^{S} k_{i,n} \leq S \times \frac{R^2}{\gamma^2}
\]

uniform mixing… \( \mu = 1/S \)

could we lose our speedup-from-
parallelizing to slower convergence?

speedup by factor of \( S \) is cancelled by slower convergence by factor of \( S \)
Results on NER

Reg. Perceptron F-measure | Avg. Perceptron F-measure
---|---
Serial (All Data) | 85.8 | 88.2
Serial (Sub Sampling) | 75.3 | 76.6
Parallel (Parameter Mix) | 81.5 | 81.6
Parallel (Iterative Parameter Mix) | 87.9 | 88.1
Results on parsing

<table>
<thead>
<tr>
<th></th>
<th>Reg. Perceptron</th>
<th>Averaged Perceptron</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unlabeled Attachment Score</td>
<td>Unlabeled Attachment Score</td>
</tr>
<tr>
<td>Serial (All Data)</td>
<td>81.3</td>
<td>84.7</td>
</tr>
<tr>
<td>Serial (Sub Sampling)</td>
<td>77.2</td>
<td>80.1</td>
</tr>
<tr>
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</tbody>
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The theorem...

**Theorem 3.** Assume a training set $\mathcal{T}$ is separable by margin $\gamma$. Let $k_{i,n}$ be the number of mistakes that occurred on shard $i$ during the $n$th epoch of training. For any $N$, when training the perceptron with iterative parameter mixing (Figure 3),

$$
\sum_{n=1}^{N} \sum_{i=1}^{S} \mu_{i,n} k_{i,n} \leq \frac{R^2}{\gamma^2}
$$

$$
\|w^{(i,n)}\|^2 = \|w^{([i,n]-1)}\|^2 + \|f(x_t, y_t) - f(x_t, y')\|^2 + 2w^{([i,n]-1)}(f(x_t, y_t) - f(x_t, y'))
$$

$$
\leq \|w^{([i,n]-1)}\|^2 + R^2
$$

$$
\leq \|w^{([i,n]-2)}\|^2 + 2R^2
$$

$$
\ldots \leq \|w^{(\text{avg},n-1)}\|^2 + k_{i,n}R^2 \quad (A2)
$$

Perceptron($\mathcal{T} = \{(x_t, y_t)\}_{t=1}^{|\mathcal{T}|}$)
1. $w^{(0)} = 0$; $k = 0$
2. for $n : 1..N$
3. for $t : 1..T$
4. Let $y' = \arg \max_y w^{(k)} \cdot f(x_t, y')$
5. if $y' \neq y_t$
6. $w^{(k+1)} = w^{(k)} + f(x_t, y_t) - f(x_t, y')$
7. $k = k + 1$
8. return $w^{(k)}$

This is not new....
\((3a)\) The guess \(v_2\) after the two positive examples: \(v_2 = v_1 + x_2\)

\[(3b)\] The guess \(v_2\) after the one positive and one negative example: \(v_2 = v_1 - x_2\)

Lemma 2 \(\forall k, \|v_k\|^2 \leq kR\). In other words, the norm of \(v_k\) grows "slowly", at a rate depending on \(R\).

Proof:

\[
v_{k+1} \cdot v_{k+1} = (v_k + y_i x_i) \cdot (v_k + y_i x_i)
\]
\[
\Rightarrow \quad \|v_{k+1}\|^2 = \|v_{k+1}\|^2 + 2y_i x_i \cdot v_k + y_i^2 \|x\|^2
\]
\[
\Rightarrow \quad \|v_{k+1}\|^2 = \|v_{k+1}\|^2 + [\text{something negative}] + 1 \|x\|^2
\]
\[
\Rightarrow \quad \|v_{k+1}\|^2 \leq \|v_{k+1}\|^2 + \|x\|^2
\]
\[
\Rightarrow \quad \|v_{k+1}\|^2 \leq \|v_{k+1}\|^2 + R^2
\]
\[
\Rightarrow \quad \|v_k\|^2 \leq kR^2
\]

\text{If mistake: } y_i x_i v_k < 0

\(\forall x_i\) given by \(A\), \(\|x\|^2 < R^2\)
Using A1/A2 we prove two inductive hypotheses:

\[ u \cdot w^{(\text{avg},N)} \geq \sum_{n=1}^{N} \sum_{i=1}^{S} \mu_{i,n} k_{i,n} \gamma \]  
(IH1)

\[ \|w^{(\text{avg},N)}\|^2 \leq \sum_{n=1}^{N} \sum_{i=1}^{S} \mu_{i,n} k_{i,n} R^2 \]  
(IH2)

The base case is \( w^{(\text{avg},1)} \), where we can observe:

\[ u \cdot w^{\text{avg},1} = \sum_{i=1}^{S} \mu_{i,1} u \cdot w^{(i,1)} \geq \sum_{i=1}^{S} \mu_{i,1} k_{i,1} \gamma \]

Follows from: \( u \cdot w^{(i,1)} \geq k_{1,i} \gamma \)

This is new .... We’ve never considered averaging operations before
IH1 inductive case:

\[ u \cdot w^{(\text{avg},N)} = \sum_{i=1}^{S} \mu_{i,N} \left( u \cdot w^{(i,N)} \right) \]

\[ \geq \sum_{i=1}^{S} \mu_{i,N} \left( u \cdot w^{(\text{avg},N-1)} + k_{i,N} \gamma \right) \]

\[ \sum_{i} \mu_{i,N} = u \cdot w^{(\text{avg},N-1)} + \sum_{i=1}^{S} \mu_{i,N} k_{i,N} \gamma \]

\[ \geq \left[ \sum_{n=1}^{N-1} \sum_{i=1}^{S} \mu_{i,n} k_{i,n} \gamma \right] + \sum_{i=1}^{S} \mu_{i,N} k_{i,N} \gamma \]

\[ = \sum_{n=1}^{N} \sum_{i=1}^{S} \mu_{i,n} k_{i,n} \gamma \]

\[ u \cdot w^{(\text{avg},N)} \geq \sum_{n=1}^{N} \sum_{i=1}^{S} \mu_{i,n} k_{i,n} \gamma \quad \text{(IH1)} \]

The first inequality uses A1, the second step \( \sum_{i} \mu_{i,N} = 1 \) and the second inequality the inductive hypothesis IH1.
Using A1/A2 we prove two inductive hypotheses:

\[ \mathbf{u} \cdot \mathbf{w}^{(\text{avg}, N)} \geq \sum_{n=1}^{N} \sum_{i=1}^{S} \mu_{i,n} k_{i,n} \gamma \]  
(\text{IH1})

\[ \|\mathbf{w}^{(\text{avg}, N)}\|^2 \leq \sum_{n=1}^{N} \sum_{i=1}^{S} \mu_{i,n} k_{i,n} R^2 \]  
(\text{IH2})

IH1 implies \( \|\mathbf{w}^{(\text{avg}, N)}\| \geq \sum_{n=1}^{N} \sum_{i=1}^{S} \mu_{i,n} k_{i,n} \gamma \) since \( \mathbf{u} \cdot \mathbf{w} \leq \|\mathbf{u}\| \|\mathbf{w}\| \) and \( \|\mathbf{u}\| = 1 \).

IH2 proof is similar

IH1, IH2 together imply the bound (as in the usual perceptron case)

**Theorem 3.** Assume a training set \( \mathcal{T} \) is separable by margin \( \gamma \). Let \( k_{i,n} \) be the number of mistakes that occurred on shard \( i \) during the \( n \)th epoch of training. For any \( N \), when training the perceptron with iterative parameter mixing (Figure 3),

\[ \sum_{n=1}^{N} \sum_{i=1}^{S} \mu_{i,n} k_{i,n} \leq \frac{R^2}{\gamma^2} \]
Review/outline

• Streaming learning algorithms … and beyond
  – Naïve Bayes
  – Rocchio’s algorithm
• Similarities & differences
  – Probabilistic vs vector space models
  – Computationally similar
  – Parallelizing Naïve Bayes and Rocchio
• Alternative:
  – Adding up contributions for every example vs conservatively updating a linear classifier
  – On-line learning model: mistake-bounds
    • some theory
    • a mistake bound for perceptron
  – Parallelizing the perceptron
What we know and don’t know

Theorem 3. Assume a training set $\mathcal{T}$ is separable by margin $\gamma$. Let $k_{i,n}$ be the number of mistakes that occurred on shard $i$ during the $n$th epoch of training. For any $N$, when training the perceptron with iterative parameter mixing (Figure 3),

$$\sum_{n=1}^{N} \sum_{i=1}^{S} \mu_{i,n} k_{i,n} \leq \frac{R^2}{\gamma^2} \quad \Rightarrow \quad \sum_{n=1}^{N} \sum_{i=1}^{S} k_{i,n} \leq S \times \frac{R^2}{\gamma^2}$$

uniform mixing…

could we lose our speedup-from-parallelizing to slower convergence?
What we know and don’t know

Figure 6: Training errors per epoch for different shard size and parameter mixing strategies.
What we know and don’t know

Thus, for cases where training errors are uniformly distributed across shards, it is possible that, in the worst-case, convergence may slow proportional the the number of shards. This implies a trade-off between slower convergence and quicker epochs when selecting a large number of shards. In fact, we observed a tipping point for our experiments in which increasing the number of shards began to have an adverse effect on training times, which for the named-entity experiments occurred around 25-50 shards. This is both due to reasons described in this section as well as the added overhead of maintaining and summing multiple high-dimensional weight vectors after each distributed epoch.
What we know and don’t know

In this paper we have investigated distributing the structured perceptron via simple parameter mixing strategies. Our analysis shows that an iterative parameter mixing strategy is both guaranteed to separate the data (if possible) and significantly reduces the time required to train high accuracy classifiers. However, there is a trade-off between increasing training times through distributed computation and slower convergence relative to the number of shards.
Review/outline

• Streaming learning algorithms … and beyond
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Where we are…

• Summary of course so far:
  – Math tools: complexity, probability, on-line learning
  – Algorithms: Naïve Bayes, Rocchio, Perceptron, Phrase-finding as BLRT/pointwise KL comparisons, …
  – Design patterns: stream and sort, messages
    • How to write scanning algorithms that scale linearly on large data (memory does not depend on input size)
  – Beyond scanning: parallel algorithms for ML
  – Formal issues involved in parallelizing
    • Naïve Bayes, Rocchio, … easy?
    • Conservative on-line methods (e.g., perceptron) … hard?
• Next: practical issues in parallelizing
  – details on Hadoop