HASHING AWAY DOT-PRODUCTS

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MOTIVATION
Dot products

• Used for computing similarity between vectors

• Object Detection in images
  – many dot products (~10^5) per image
  – High dimensional (3100) vectors
  – Bottleneck ...
Revision: Locality Sensitive Hashing

• Similar vectors => Similar hash values
The Power of Comparative Reasoning
Jay Yagnik, Dennis Strelow, David A. Ross, Ruei-sung Lin
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Comparative Reasoning

Figure 1. An ordinal measurement $A \geq B \geq C \geq E \geq D$ is robust to variations in individual filter values (top row). Ordinal measures of similarity capture filter response differences not reflected in linear measures (dot product) of similarity (bottom row).
Intuition

- Small differences in absolute value of filter don’t matter as long as “nature of filter” is the same.
Intuition

• Capture “nature of the filter” using a set of inequalities
  – $A \geq B$
  – $B \geq C$
Intuition

• Comparing two signals is comparing their set of inequalities
  – Expensive
WTA Hashing

Input $N$ – dim vector

Choose first $K=3$ elements

Permute elements $P=[6,5,1,2,4,3]$ 

Choose first $K=3$ elements

Set index of max=1, rest 0

Hash of vector
WTA Hashing – another view

• Take random projection of the vector

• Compute index of the dimension with max magnitude
Window Parameter

• Effect of window parameter $K$
  – $K = \text{length of signal}$
  – We are just storing the index of global max
    • Exactly equal to “Max-hash”

• In general, smaller $K$ captures more pair-wise ordering

• Larger $K$ captures a more “global max”
So if two hashes match?

• If two hashes of length $K$ match
  – $K-1$ inequalities in both signals have matched

• By computing and matching hash in $O(K)$ we can determine $K-1$ inequalities

• Notice that hash is sparse and thus allows for efficient storage/computation
• One hash isn’t enough

• Need more “random projections”
Multi-band WTA

Input Signal

$P$ permutations

WTA Hashes

Band $W$ hashes

$M$ hash bands

Normal hash

(C++ unordered_map)
Multi-band WTA

- $P = 2400$ (number of permutations)
- $K = 16$ (window size)
- $M = 600$ (bands)
- $W = N/M = 4$ (number of WTA-hashes banded together)
- Each hash takes $W \log_2 K$ bits = 16 bits to store in memory => C++ unordered_map
OBJECT DETECTION
Why Object Detection?

• Face recognition/detection

• Generic object (e.g. cat, dog, bottle)
Revision: 2D Convolution

Input Signal

Template/Filter
Revision: 2D Convolution
Revision: 2D Convolution
Revision: 2D Convolution
Revision: 2D Convolution
Revision: 2D Convolution
Revision: 2D Convolution
Revision: 2D Convolution
Revision: 2D Convolution
Template based object detection
Template based object detection
Template based object detection

• It works ... sort of
Template based object detection

• Number of templates $\geq$ number of classes

• Each template – $\sim 10^5$ dot products per image
Why can’t we scale object detection?

• Detecting one class for a regular (640x480 image) requires ~1-2 seconds

• 1000 classes => 1000x
  – ~1000 seconds per image
  – Imagine video, which has 30 frames per second
  – Not even close to real-time

• Even though embarrassingly parallel, latency is too high for real-time applications
Why can’t we scale object detection?

• Each class template is a high dimensional (~4000) vector of single/double precision values

• More classes, more memory to store templates

• Scarce main memory (on-board computers)
Paper – Scaling up object detection

Fast, Accurate Detection of 100,000 Object Classes on a Single Machine

Thomas Dean    Mark A. Ruzon    Mark Segal
Jonathon Shlens    Sudheendra Vijayanarasimhan    Jay Yagnik†
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Best Paper

Follow up: Uses similar LSH based techniques to scale detection for CNN based detectors -Vijayanarasimhan et al., arXiv 2014
What’s the bottleneck?

Each step is a dot product of two high dimensional (3100) vectors.
What’s the bottleneck?

Each step is a dot product of two high dimensional (3100) vectors $C$ detectors (one per class) => $O(C)$ expensive convolutions
Computation bottleneck

- Feature computation: 28.34%
- Convolution: 71.26%
- Others: 0.40%

Misra et al., 2014
Main idea

• Reduce number of dot products

• Replace dot products by hash lookup

• All detectors will not give high “dot product” response

• Use hashing to find “high response” detectors. Dot product for only these detectors
Main idea

Detectors

Input image

Image region

Hash Table
Main idea

Input image → Detectors 1, 2, 3 → Hash Table → Closest detectors
Main idea

2

Compute dot product only for closest detector
What should the hash do?

• The hash should make sure that two vectors with “similar filter response” should be close.

• WTA Hash 😊
After Training, hash all templates

- Input Signal
- Permutations
- WTA Hashes
- Band W hashes
- Normal hash (C++ unordered_map)
- M hash bands
Test time

Detectors

Multi-band WTA

Closest detectors

Image features
DATASETS
Pascal VOC2007

- Object detection benchmark
- Consists of images from Flickr
- 20 diverse object classes
  - Bottle, dog, human, car, aeroplane, bus, plant etc.
Pascal VOC 2007
RESULTS
Results (Accuracy vs. Speed)

20 classes on PASCAL VOC 2007
5 seconds per image
Speed-up of 20x over standard DPM implementation (*not the cascade version*)

Table 1. Comparison of the hashing-based and baseline algorithms on the PASAL VOC 2007 dataset
Results – 100,000 detectors

<table>
<thead>
<tr>
<th>time</th>
<th>t = 8</th>
<th>t = 28</th>
<th>t = 76</th>
</tr>
</thead>
<tbody>
<tr>
<td>mAP</td>
<td>0.07</td>
<td>0.11</td>
<td>0.16</td>
</tr>
<tr>
<td>speed-up</td>
<td>62,500×</td>
<td>17,857×</td>
<td>6,580×</td>
</tr>
</tbody>
</table>

Train them using 5000 machines
Test them on a single machine with 20GB RAM ($P=2400$, $K=16$, $M=600$)

Paper: Fast Accurate Detection of 100,000 Object Classes on a Single Machine, Dean et al., 2013
Bonus: Results

• Training a deep net with millions of classes as output

Figure 7: Accuracy of the various models on the Sports 1M evaluation set as a function of the number of training steps.

Figure 8: Accuracy of the various models on the Sports 1M evaluation set as a function of the total training time.

Paper: Deep Networks With Large Output Spaces, Vijayanarasimhan et al. 2014
THANK YOU!
Optimized Implementation

![Graph showing optimized and original implementations of number of detectors vs time (ms).]

- Optimized
- Original

Time (ms)

Number of detectors

Misra et al., 2014
METRICS
Correct detection?
Average Precision (AP)

- Area under precision/recall curve
Results – Fixed compute budget

Figure 7. Increasing the number of objects gracefully degrades prediction accuracy on PASCAL VOC 2007 for a fixed computational budget.
Test time

- Test image feature
- $P$ permutations
- WTA Hashes
- Band $W$ hashes
- Normal hash (C++ unordered_map)
- Hash lookup
- $M$ hash bands
WTA Hashing

• Winner Takes All Hashing
  – Input Signal of length $L = [5,10, 2, 8]$
  – Parameter: Window size $K = 2$
  – $P$ is a permutation of indices 1 to $N = [4,3,2,1]$
  – Permute signal according to $P => [8,2,10,5]$
  – Look at first $K$ elements => [8,2]
  – Find index of the maximum element
  – Hash = all zeros except at max; $hash = [1 0]$

If $K = N$, then this is exactly Min-Hash
Paper – Scaling up object detection

Fast Accurate Detection of 100,000 Object Classes on a Single Machine – Thomas Dean et al., CVPR 2013

Best Paper

Follow up: Uses similar LSH based techniques to scale detection for CNN based detectors - Vijayanarasimhan et al., arXiv 2014
Winner Takes All (WTA) Hashing

The Power of Comparative Reasoning – J. Yagnik, D. Strelow, D. Ross, R. Lin in ICCV 2011

The Power of Comparative Reasoning

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Abstract

Rank correlation measures are known for their resilience to perturbations in numeric values and are widely used in many evaluation metrics. Such ordinal measures have rarely been applied to measurement of numeric features as a representational transformation. We emphasize the benefits of ordinal representations of input features both theoretically and empirically. We present a family of algorithms for computing ordinal embeddings based on partial order statistics. Apart from having the stability benefits of ordinal measures, these embeddings are highly non-linear, giving rise to sparse feature spaces highly favored by several machine learning methods. These embeddings are deterministic, data independent and are useful in being based on partial order statistics, and another degree of resilience to noise. These machine learning-free methods when applied to the task of joint similarity search outperform state-of-the-art machine learning methods with complex optimization setups. For solving classification problems, the embeddings provide a nonlinear transformation resulting in sparse binary codes that are well-suited for a large class of machine learning algorithms. These methods show significant improvements on VOC 2010 using simple linear classifiers which can be trained quickly. Our method can be extended to the case of polynomial kernels, while preserving very efficient computation. Further, since the popular MIFHash algorithms is a special case of our method, we demonstrate an efficient scheme for computing MIFHash on approximations of binary features. The actual method can be implemented in about 10 lines of code in most languages (2 lines in MATLAB), and does not require any closed-form optimization.

1. Introduction

Rank correlation measures have been well regarded as robust measures in many performance evaluation schemes. Since these methods rely on relative ordering of elements, they are very resilient to noise and variations that do not affect the implicit order. We make the case of applying this thought to feature representation. High-dimensional feature sets are quite common across all disciplines of signal analysis and many other domains. Precise values of each feature dimension in such high-dimensional spaces are often not important. We argue for creating representations that are based solely on the relative ranking of feature dimensions. Such representations would enjoy all the benefits of rank correlation measures while being useful to generic discriminative features. Further, we base our embeddings on multiple partial order statistics rather than total orderings, giving rise to another degree of resilience to noise and giving rise to representations that have local support on feature dimensions useful for learning algorithms to distinguish "useless" dimensions (if needed).

The main contribution of our work is the Winner Take All (WTA) hash, a sparse embedding method that transforms the input feature space into binary codes such that Hamming distance in the resulting space closely correlates with rank similarity measures. Algorithm 1 gives the WTA hash and Figure 1 shows it operating on four example input vectors. In short, for each hash we permute the input feature vector with 0, take the first 8 components from the permuted vector, and output the index of the maximum component. Many hashes corresponding to different h can be combined into an output hash vector.

This simple and efficient method that results from it is very widely applicable. Our embedding method requires no data-driven optimization. While being training-free and easy to compute it outperforms state-of-the-art methods on several tasks. Our method gives rise to a space of sparse binary vectors. This makes it readily amenable for many problems including:

• Using sparse binary vectors as tokens for locality sensitive hashing schemes for fast similarity search. Here we show that our method outperforms several state-of-the-art machine learning methods [79, 15] for learning hash codes that are optimal for the specific problem of similarity search on LabelMe. The performance gap is significant, particularly when doing sub-linear (approximate) nearest neighbor search.

• Using our embeddings to induce a ranking metric on well known descriptors for similarity search. Here we show that our method improves on SIFT [21] and DAISY [26, 27] by about 11-12% (error rate for 95% recall) on standard benchmarks.

• Our embeddings act as a nonlinear feature-space transformation. When applied with linear classifiers they outperform linear and chi-square kernel classifiers on vocabulary histogram features. Furthermore we make the following algorithmic contributions:
  • We can compute rank embeddings of polynomial spaces of degree p in O(p).
  • Our method can bias the rank embeddings to be more sensitive to elements at the head of the rank list.