10-405: Randomized Algorithms 2/2

```
int getRandomNumber()
{
    return 4;  // chosen by fair dice roll.
               // guaranteed to be random.
}
```

BLOOM FILTERS/COUNTMIN SKETCHES - RECAP

Bloom filters

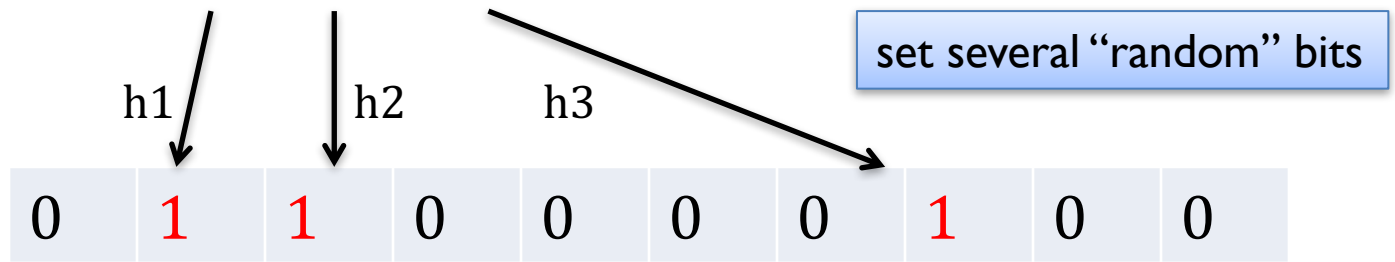
- Interface to a Bloom filter
  - BloomFilter(int maxSize, double p);
  - void bf.add(String s); // insert s
  - bool bd.contains(String s);
    - // If s was added return true;
    - // else with probability at least *1-p* return false;
    - // else with probability at most *p* return true;

  - I.e., a noisy "set" where you can test membership (and that's it)

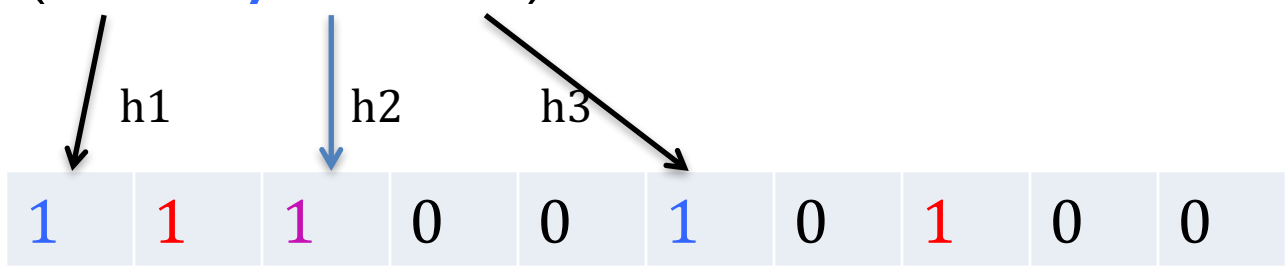
Bloom filters

0	0	0	0	0	0	0	0	0	0

bf.add("fred flintstone"):



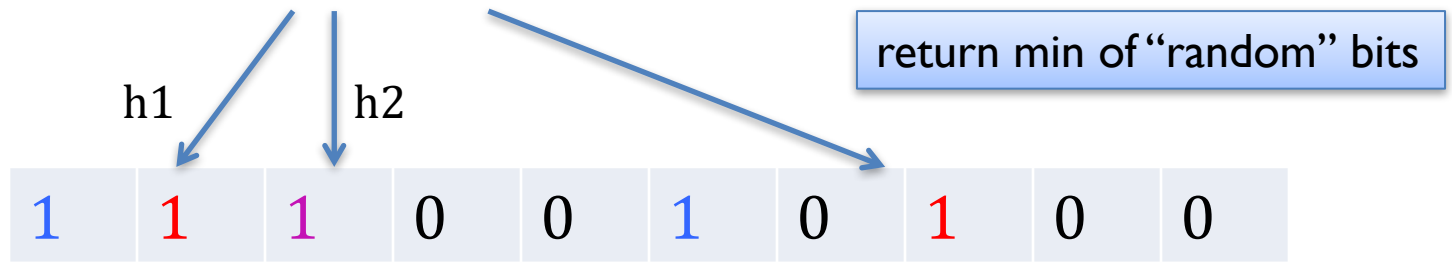
bf.add("barney rubble"):



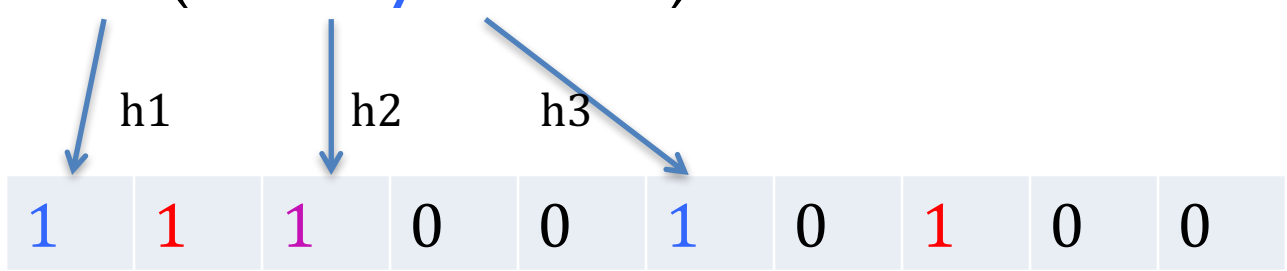
Bloom filters

1	1	1	0	0	1	0	1	0	0

bf.contains ("fred flintstone"):



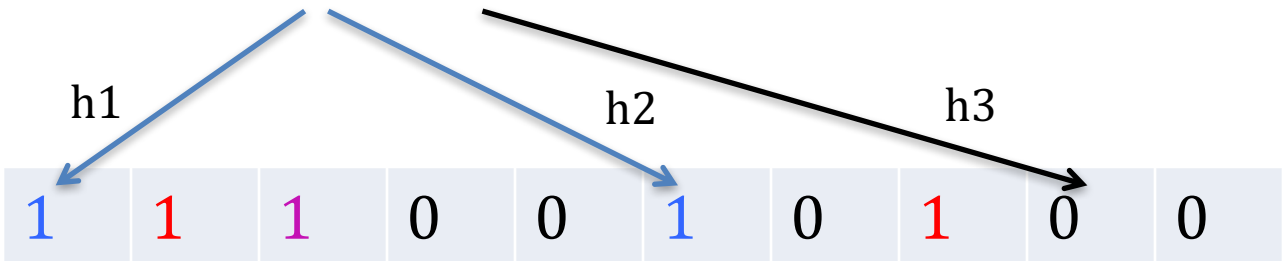
bf.contains("barney rubble"):



Bloom filters

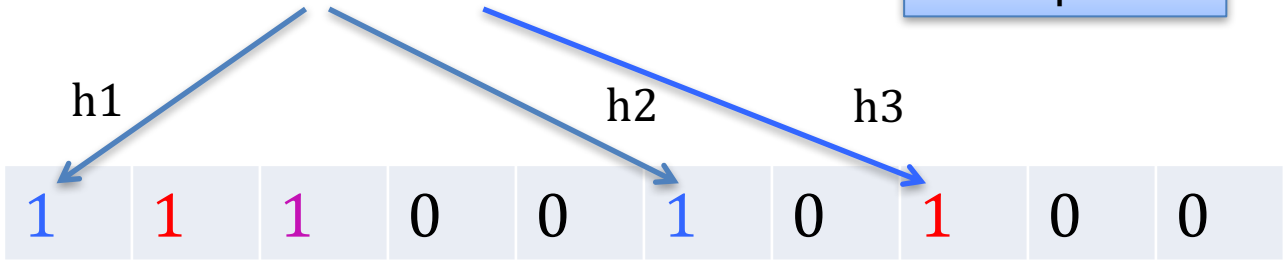
1	1	1	0	0	1	0	1	0	0

bf.contains("wilma flintstone"):



bf.contains("wilma flintstone"):

a false positive



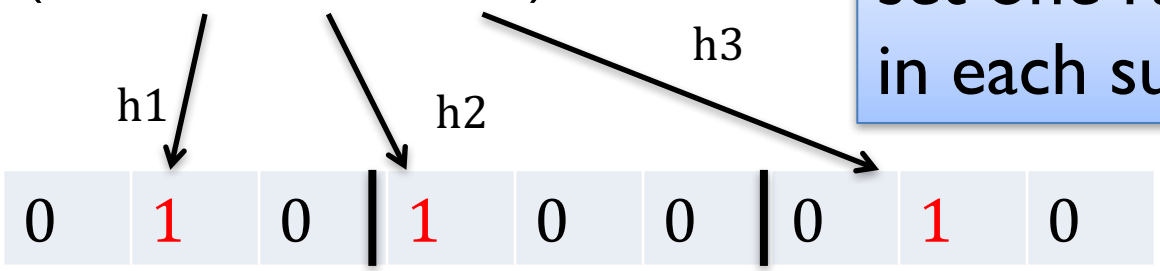
BLOOM FILTERS VS COUNT-MIN SKETCHES

Bloom filters – a variant

split the bit vector into k ranges, one for each hash function

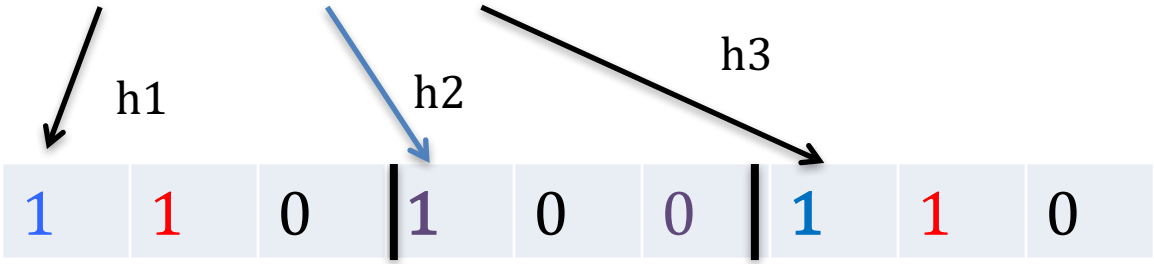
0	0	0	0	0	0	0	0	0

bf.add("fred flintstone"):



set one random bit in each subrange

bf.add("barney rubble"):



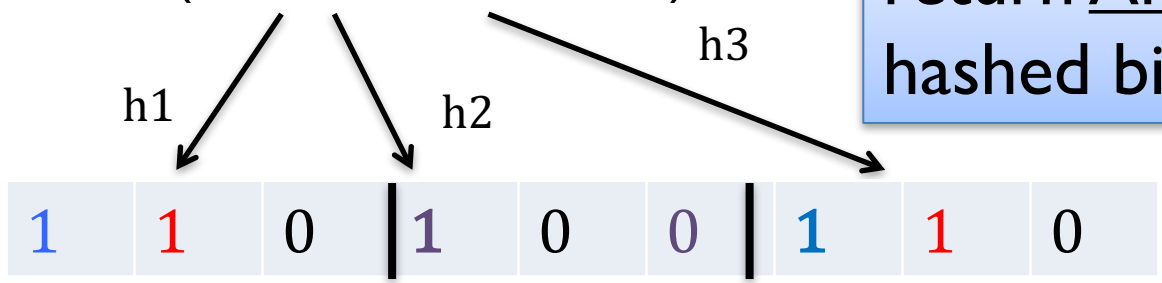
Bloom filters – a variant

split the bit vector into k ranges, one for each hash function

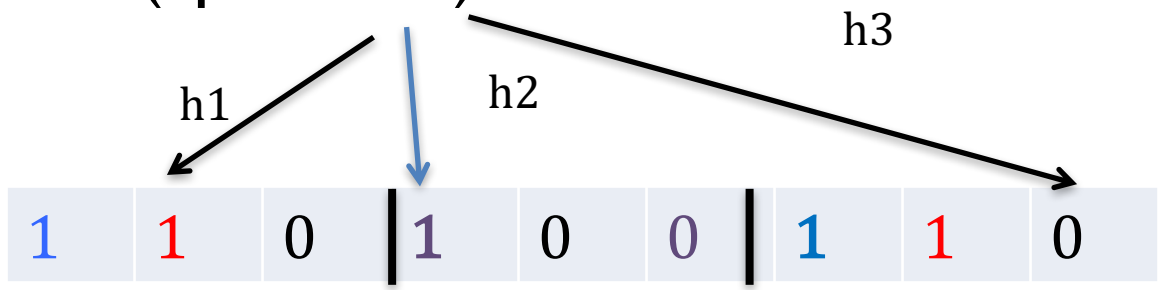
0	0	0	0	0	0	0	0	0

bf.contains("fred flintstore"):

return AND of all hashed bits



bf.contains("pebbles"):



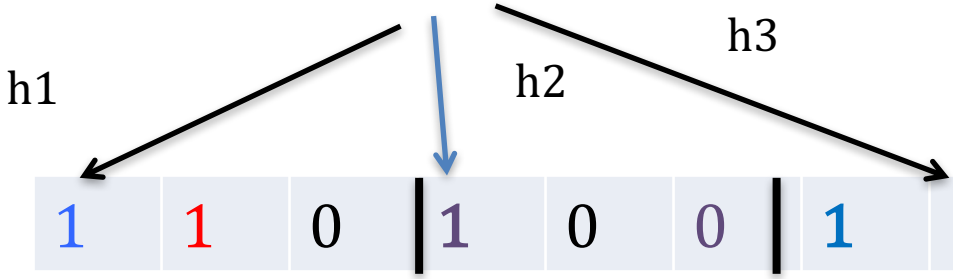
Bloom filters – a variant

split the bit vector into k ranges, one for each hash function

0	0	0	0	0	0	0	0	0

a false positive!

bf.contains("pebbles"):



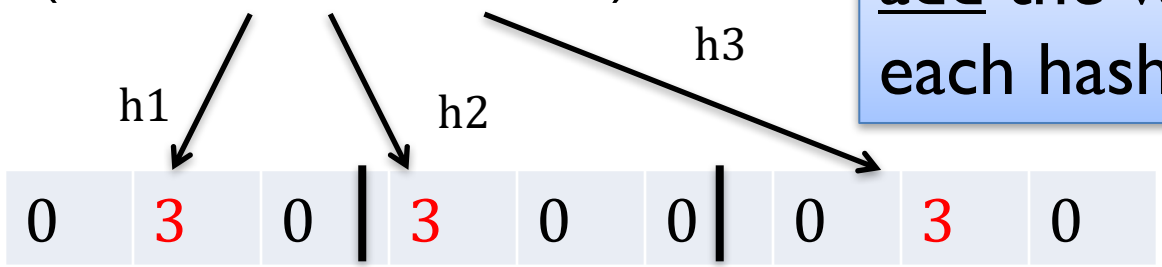
Count-min sketches

split a <u>real</u> vector into k ranges, one for each hash function

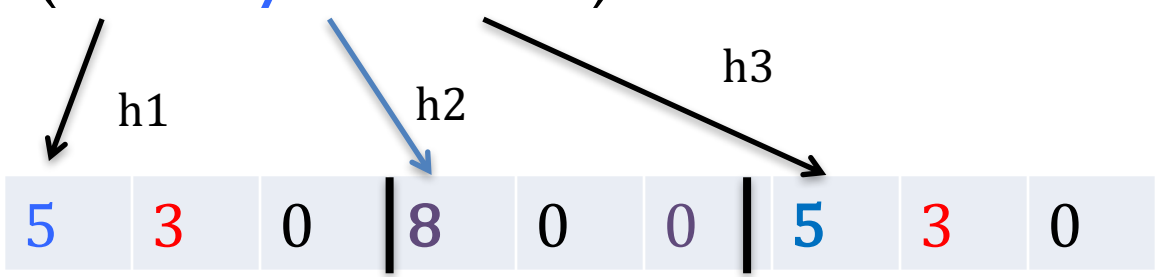
0	0	0	0	0	0	0	0	0

cm.inc("fred flintstone", 3):

<u>add</u> the value to each hash location



cm.inc("barney rubble",5):



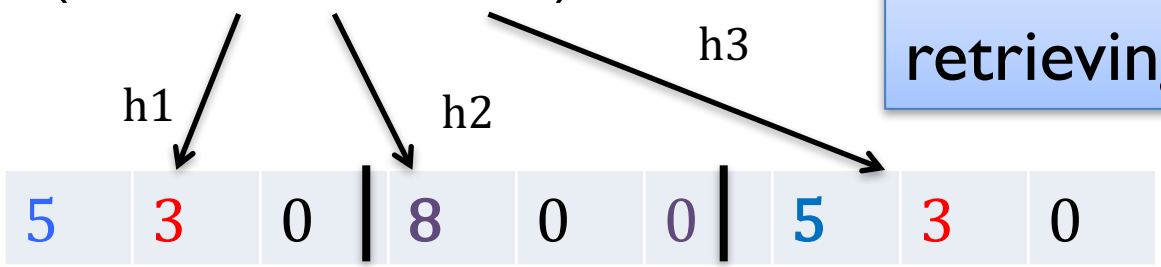
Count-min sketches

split a real vector into k ranges, one for each hash function

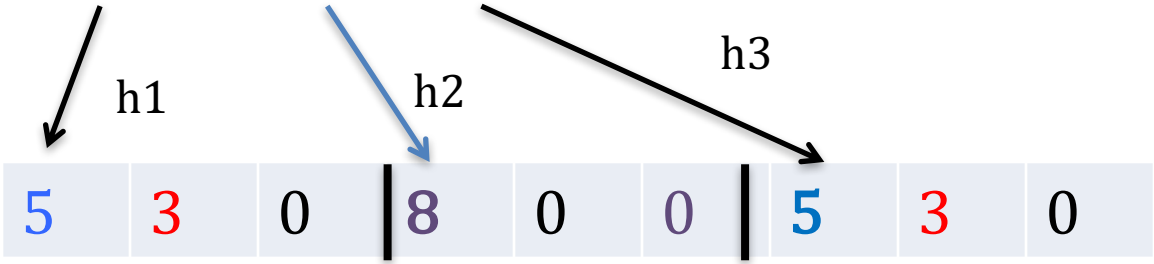
0	0	0	0	0	0	0	0	0

cm.get("fred flintstone"): 3

take min when retrieving a value



cm.get("barney rubble): 5

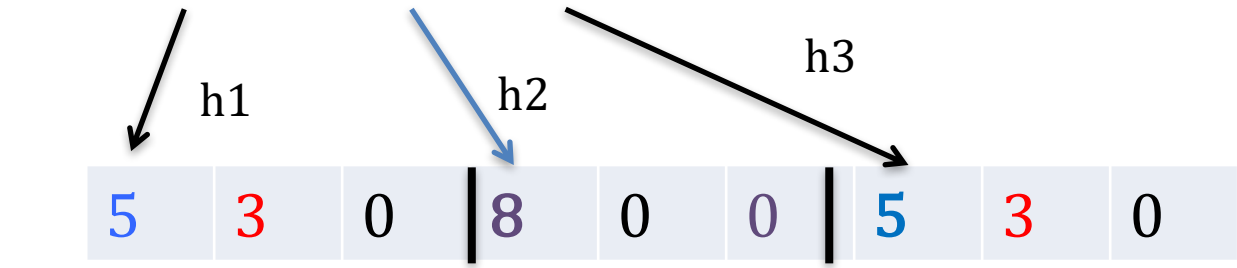


Count-min sketches

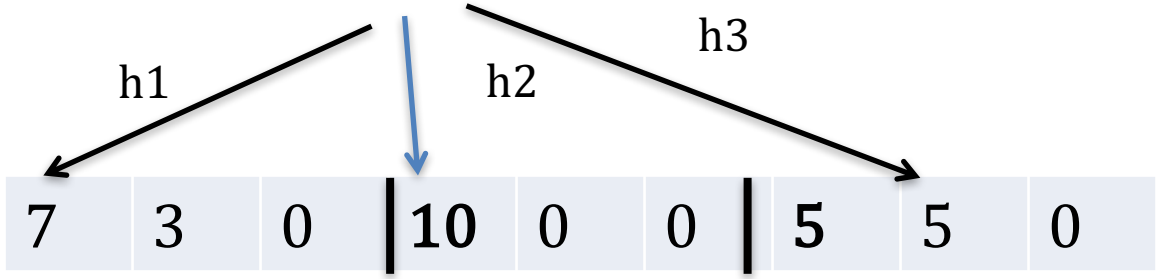
split a <u>real</u> vector into k ranges, one for each hash function

0	0	0	0	0	0	0	0	0

cm.get("barney rubble): 5



cm.add("pebbles", 2):



Count-min sketches

Equivalently, use a <u>matrix</u>, and each hash leads to a different row

cm.inc("fred flintstone", 3):

0	3	0
3	0	0
0	3	0

cm.inc("barney rubble",5):

5	3	0
8	0	0
5	3	0

Applications of Sketches

Applications of count-min sketches

- Keeping lots of lots of running totals
  - naïve Bayes event counters
  - word co-occurrence counters for semantic orientation, highly related words, ...
  - anywhere, in place of a large sparse feature vector

Applications of Bloom filters

- Replace a sparse set input to a deep-learning model
- Replace a sparse set output to a deep-learning model
- …

Some uses of Bloom filters

- An example application
  - Finding items in “sharded” data
    - Easy if you know the sharding rule
    - Harder if you don’t (like Google n-grams)

```
furter:google_ngram wcohen$ ls -alh *2gram* | tail
-rw-rw-rw-  1 13527  _lpoperator   264M Sep 17  2011 googlebooks-eng-all-2gram-20090715-90.csv.zip
-rw-rw-rw-  1 13527  _lpoperator   264M Sep 17  2011 googlebooks-eng-all-2gram-20090715-91.csv.zip
-rw-rw-rw-  1 13527  _lpoperator   264M Sep 17  2011 googlebooks-eng-all-2gram-20090715-92.csv.zip
-rw-rw-rw-  1 13527  _lpoperator   264M Sep 17  2011 googlebooks-eng-all-2gram-20090715-93.csv.zip
-rw-rw-rw-  1 13527  _lpoperator   264M Sep 17  2011 googlebooks-eng-all-2gram-20090715-94.csv.zip
-rw-rw-rw-  1 13527  _lpoperator   263M Sep 17  2011 googlebooks-eng-all-2gram-20090715-95.csv.zip
-rw-rw-rw-  1 13527  _lpoperator   264M Sep 17  2011 googlebooks-eng-all-2gram-20090715-96.csv.zip
-rw-rw-rw-  1 13527  _lpoperator   264M Sep 17  2011 googlebooks-eng-all-2gram-20090715-97.csv.zip
-rw-rw-rw-  1 13527  _lpoperator   264M Sep 17  2011 googlebooks-eng-all-2gram-20090715-98.csv.zip
-rw-rw-rw-  1 13527  _lpoperator   264M Sep 17  2011 googlebooks-eng-all-2gram-20090715-99.csv.zip
```

Some Uses of Bloom filters

- An example application
  - Finding items in "sharded" data
    - Easy if you know the sharding rule
    - Harder if you don't (like Google n-grams)
- Simple idea:
  - Build a BF of the contents of each shard
  - To look for *key,* load in the BF's one by one, and search only the shards that probably contain *key*
  - Analysis: you won't miss anything, you might look in some extra shards
  - You'll hit O(1) extra shards if you set p=1/#shards

Some Uses of Bloom filters

- An example application
  - discarding singleton features from a classifier
- Scan through data once and check each *w*:
  - if bf1.contains(*w*): bf2.add(*w*)
  - else bf1.add(*w*)
- Now:
  - bf1.contains(w) ⇔ w appears >= once
  - bf2.contains(w) ⇔ w appears >= 2x
- Then train, ignoring words not in bf2

Some Uses of Bloom filters

- An example application
  - discarding rare features from a classifier
  - seldom hurts much, can speed up experiments
- Scan through data once and check each *w*:
  - if bf1.contains(*w*):
    - if bf2.contains(w): bf3.add(*w*)
    - else bf2.add(w)
  - else bf1.add(*w*)
- Now:
  - bf2.contains(w) ⇔ w appears >= 2x
  - bf3.contains(w) ⇔ w appears >= 3x
- Then train, ignoring words not in bf3

Some Uses of Bloom filters

- An example application
  - discarding rare features from a classifier
  - seldom hurts much, can speed up experiments
- Scan through data once and check each *w*:
  - if bf1.contains(*w*):
    - if bf2.contains(w): bf3.add(*w*)
    - else bf2.add(w)
  - else bf1.add(*w*)
- Now:
  - bf2.contains(w) ⇔ w appears >= 2x
  - bf3.contains(w) ⇔ w appears >= 3x
- Then train, ignoring words not in bf3

LOCALITY SENSITIVE HASHING (LSH)

LSH: key ideas

- Goal:
  - map feature vector **x** to bit vector **bx**
  - ensure that **bx** preserves "similarity"

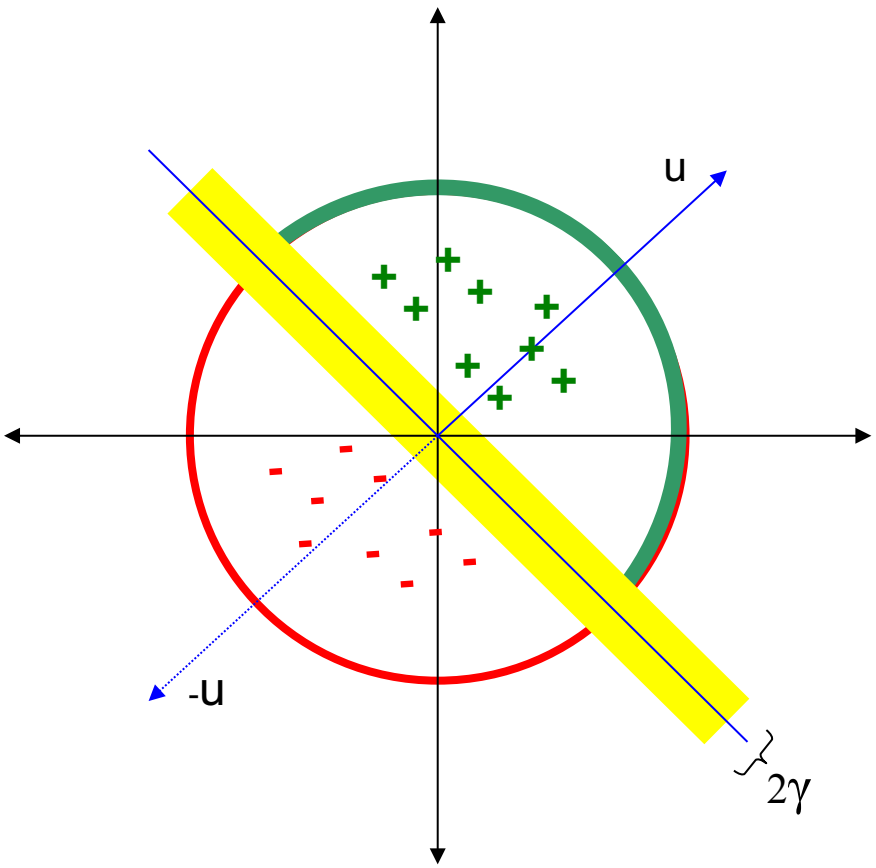
LSH: key ideas

- Goal:
  - map feature vector **x** to bit vector **bx**
  - ensure that **bx** preserves "similarity"

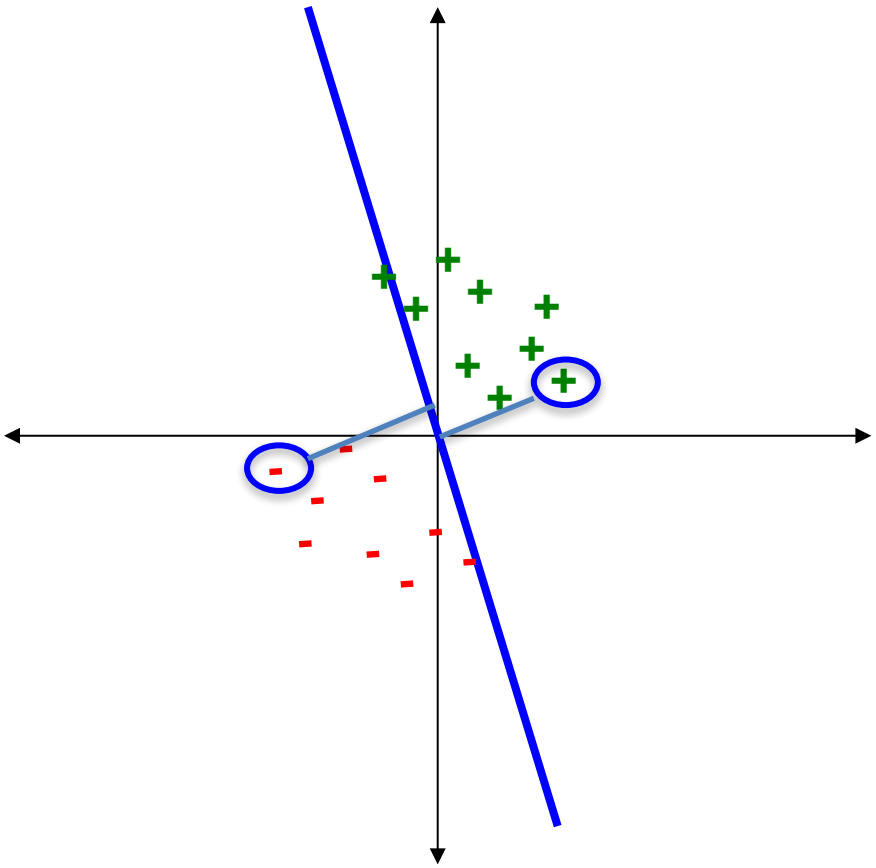
- Different similarities require different hash schemes
  - cosine distance: random projections
  - dot product: winner-take-all hash
  - Jaccard similarity: min-hash (Broder, 1997)
  - L2 distance: LSH with *p*-stable distributions
  - spherical LSH: …
  - …

Random Projections

Random projections

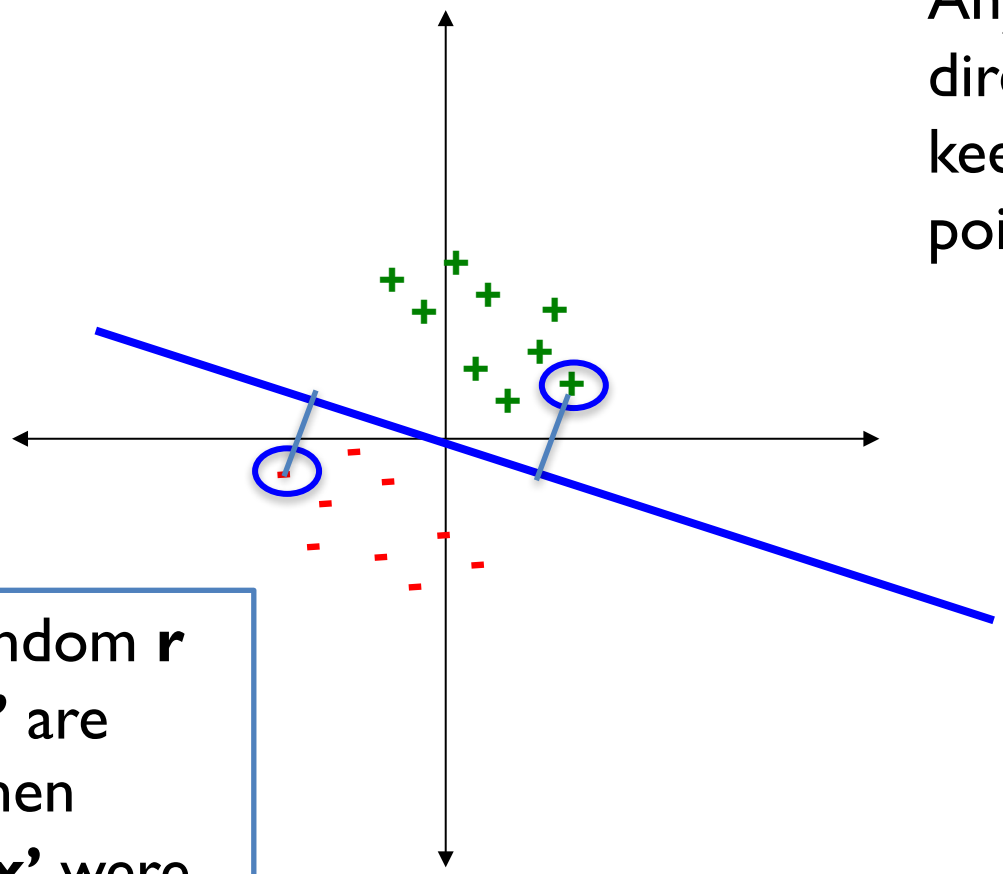


Random projections



To make those points "close" we need to project to a direction orthogonal to the line between them

Random projections

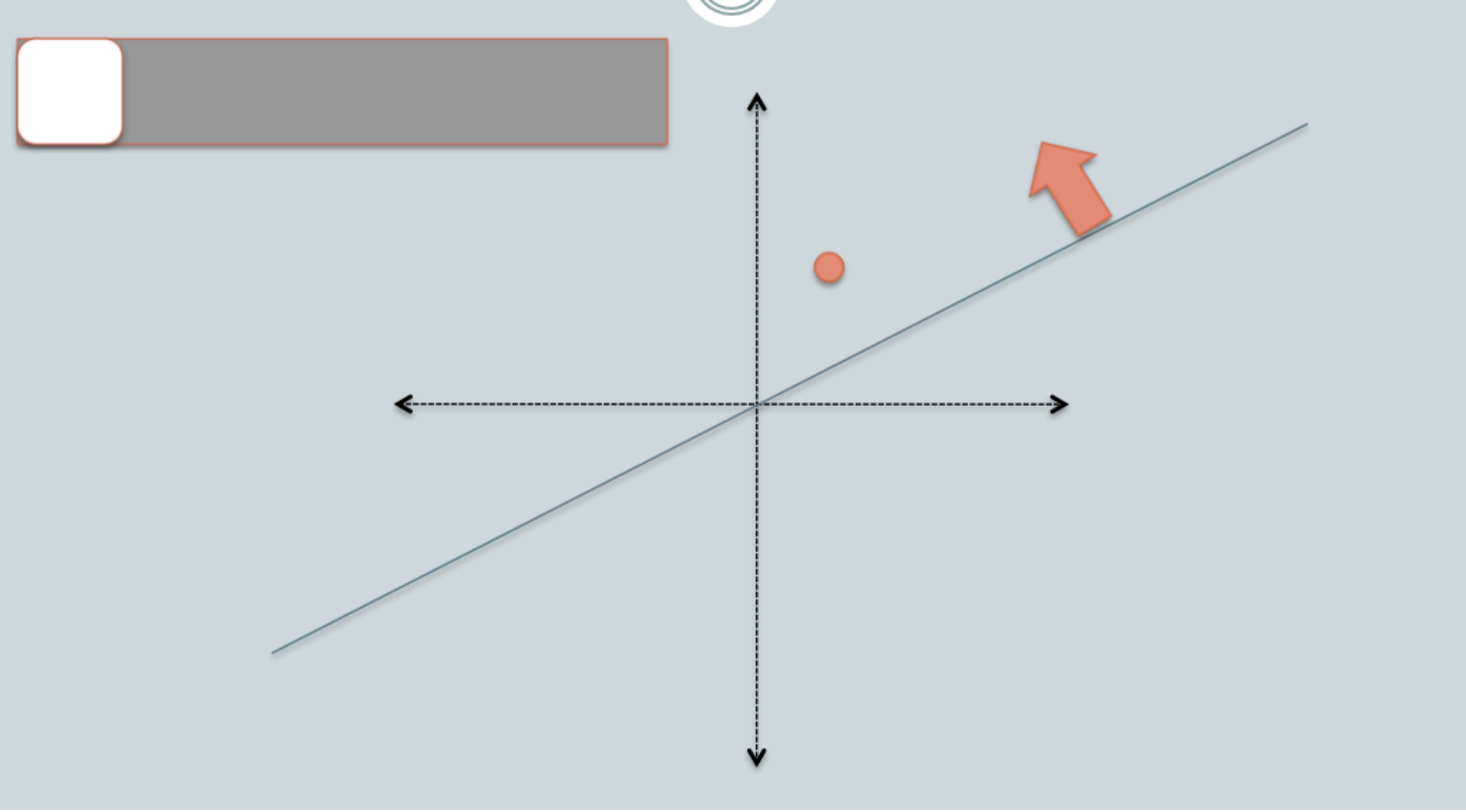


Any other direction will keep the distant points distant.

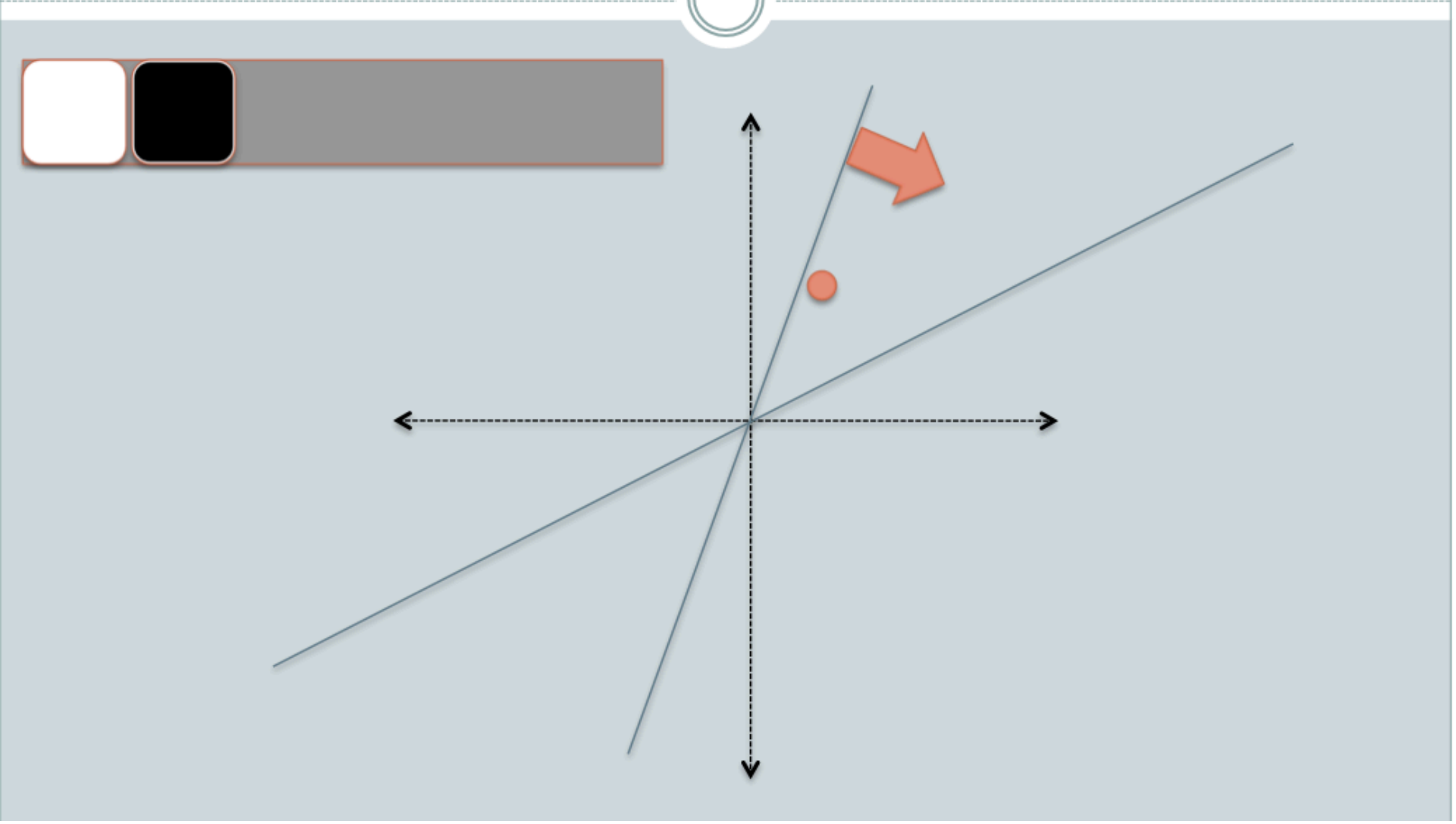
So if I pick a random **r** and **r.x** and **r.x'** are closer than γ then *probably* **x** and **x'** were close to start with.

LSH: key ideas

- Goal:
  - map feature vector **x** to bit vector **bx**
  - ensure that **bx** preserves "similarity"
- Basic idea: use *random projections* of **x**
  - Repeat many times:
    - Pick a random hyperplane **r** by picking random weights for each feature (say from a Gaussian)
    - Compute the inner product of **r** with **x**
    - Record if **x** is "close to" **r** (**r.x**>=0)
      - the next bit in **bx**
    - Theory says that is **x'** and **x** have small cosine distance then **bx** and **bx'** will have small Hamming distance



[Slides: Ben van Durme]



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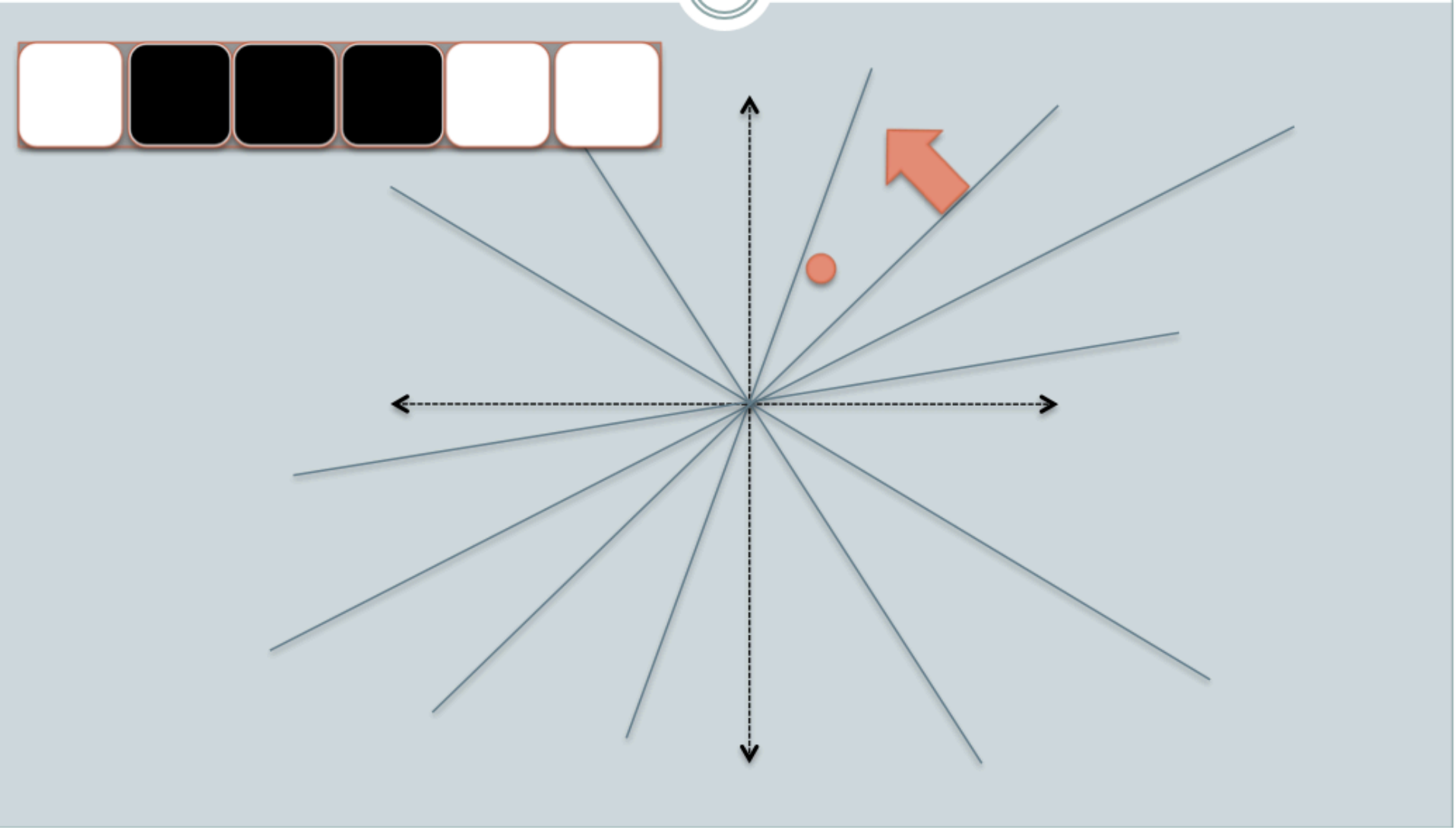

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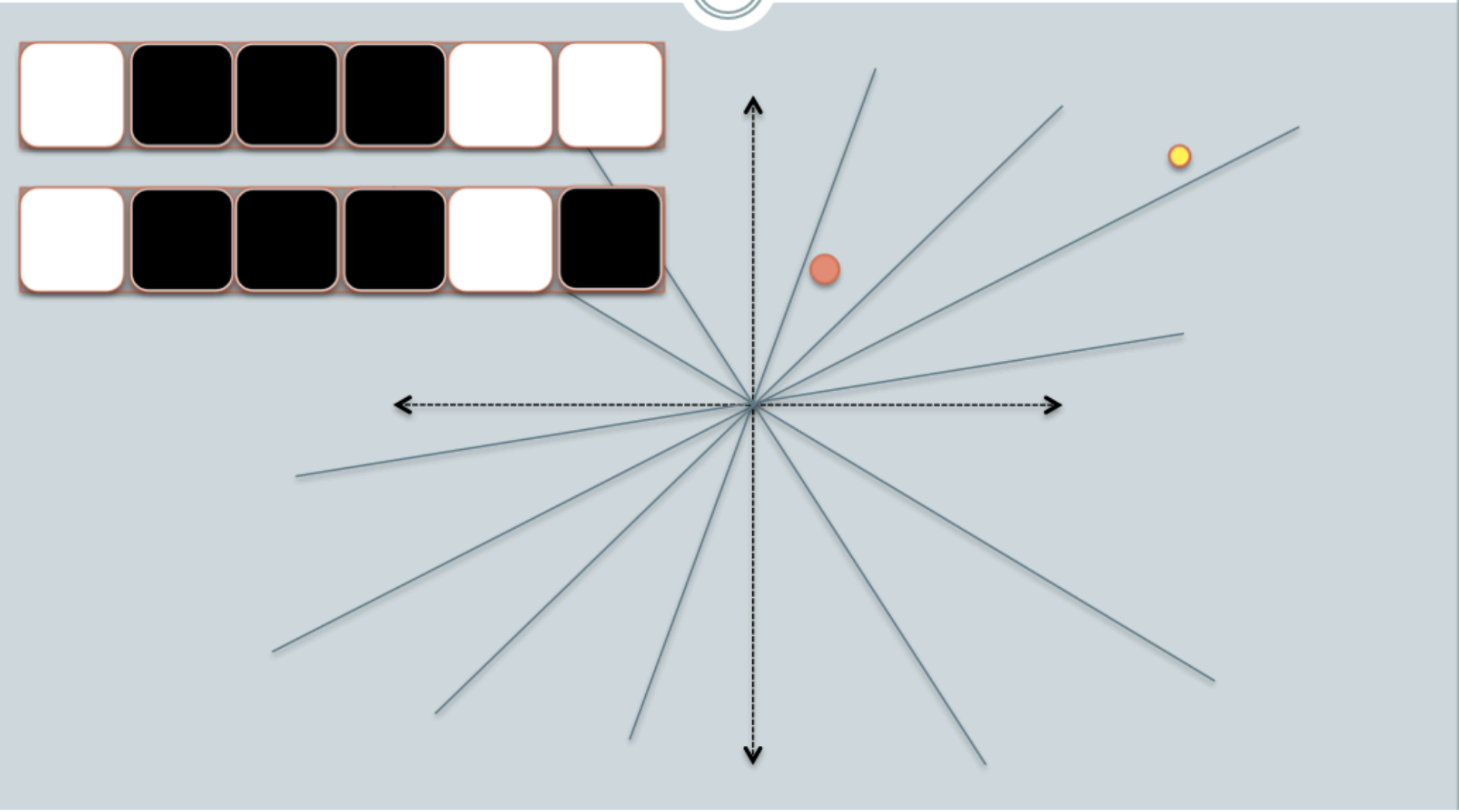

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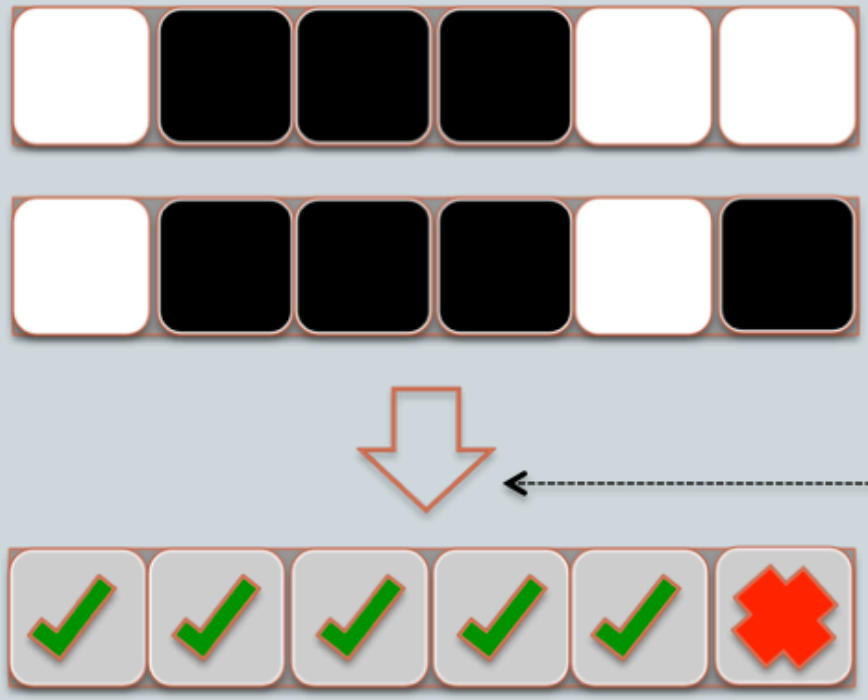
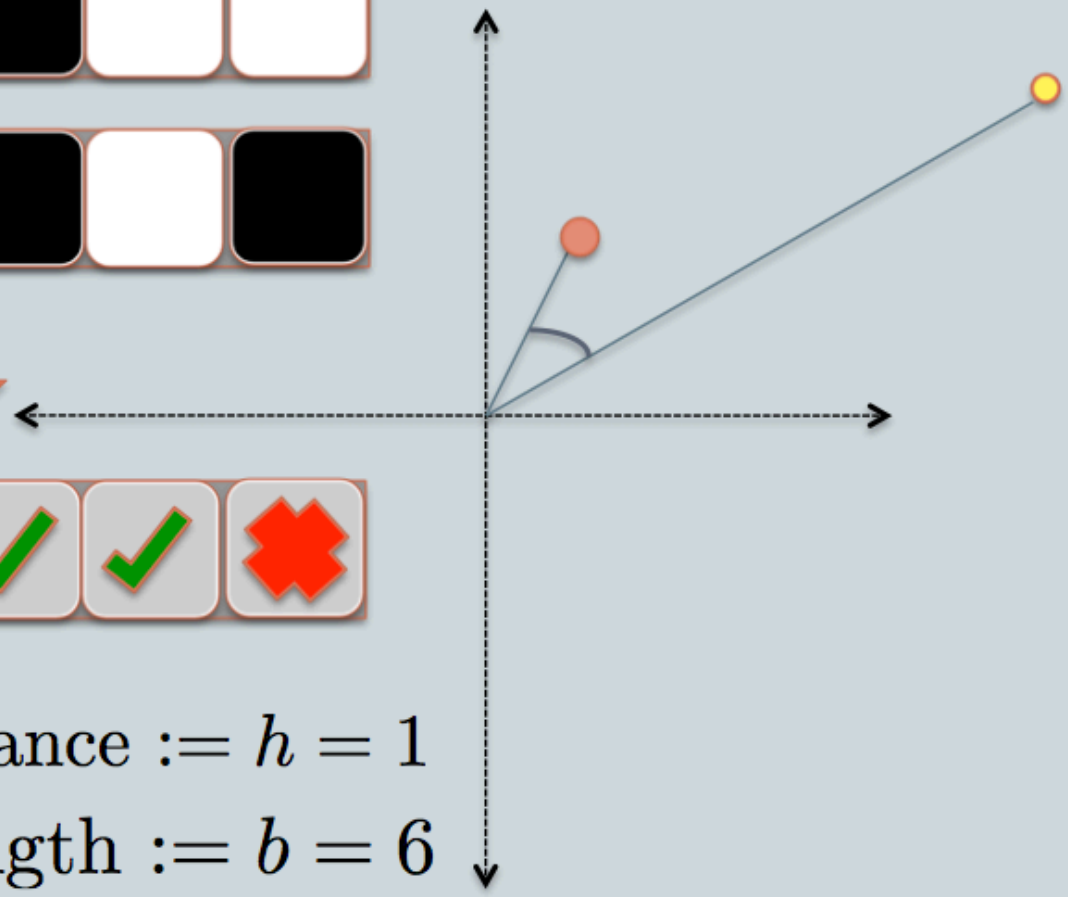




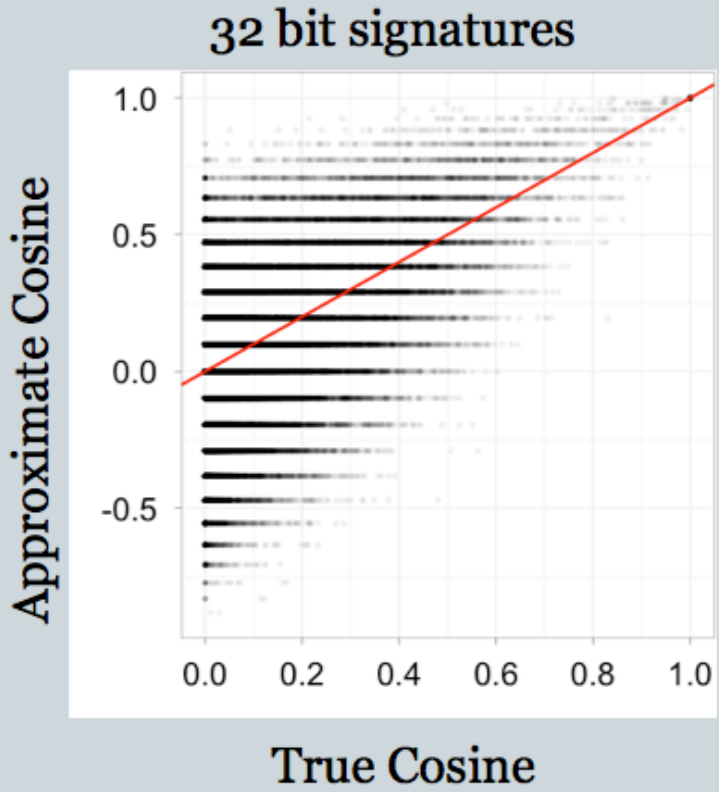
Hamming Distance $:= h = 1$

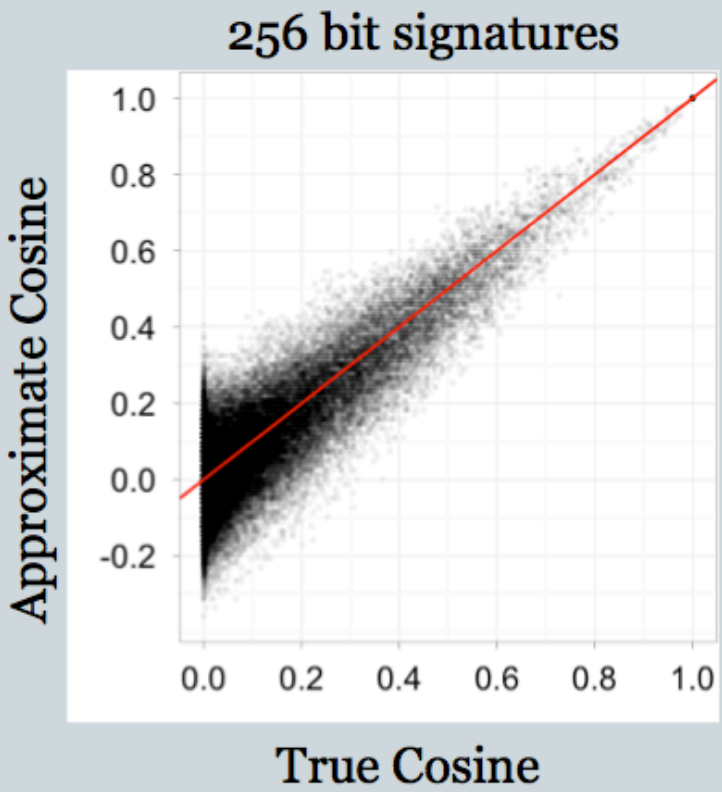
Signature Length $:= b = 6$

$$\cos(\theta) \approx \cos\left(\frac{h}{b}\pi\right)$$

$$= \cos\left(\frac{1}{6}\pi\right)$$

[Slides: Ben van Durme]





Cheap

Accurate

[Slides: Ben van Durme]

LSH applications

- Compact storage of data
  - and we can still compute similarities
- LSH also gives very fast approximations:
  - approx nearest neighbor method
    - just look at other items with **bx'=bx**
    - also very fast nearest-neighbor methods for Hamming distance
  - very fast clustering
    - cluster = all things with same **bx** vector

Online LSH and Pooling

Online Generation of Locality Sensitive Hash Signatures

Benjamin Van Durme and Ashwin Lall

human language technology
center of excellence
JOHNS HOPKINS
UNIVERSITY

DENISON
UNIVERSITY

LSH algorithm

- Naïve algorithm:
  - Initialization:
    - For i=1 to outputBits:
      - For each feature *f:*
        - » Draw r(f,i) ~ Normal(0,1)
  - Given an instance **x**
    - For i=1 to outputBits:

      LSH[i] =

      sum(**x**[*f*]*r[i,*f*] for *f* with non-zero weight in **x**) > 0 ? 1 : 0
    - Return the bit-vector LSH

LSH algorithm

- But: storing the *k classifiers* is expensive in high dimensions
  - For each of 256 bits, a dense vector of weights for every feature in the vocabulary
- Storing seeds and random number generators:
  - Possible but somewhat fragile

LSH: "pooling" (van Durme)

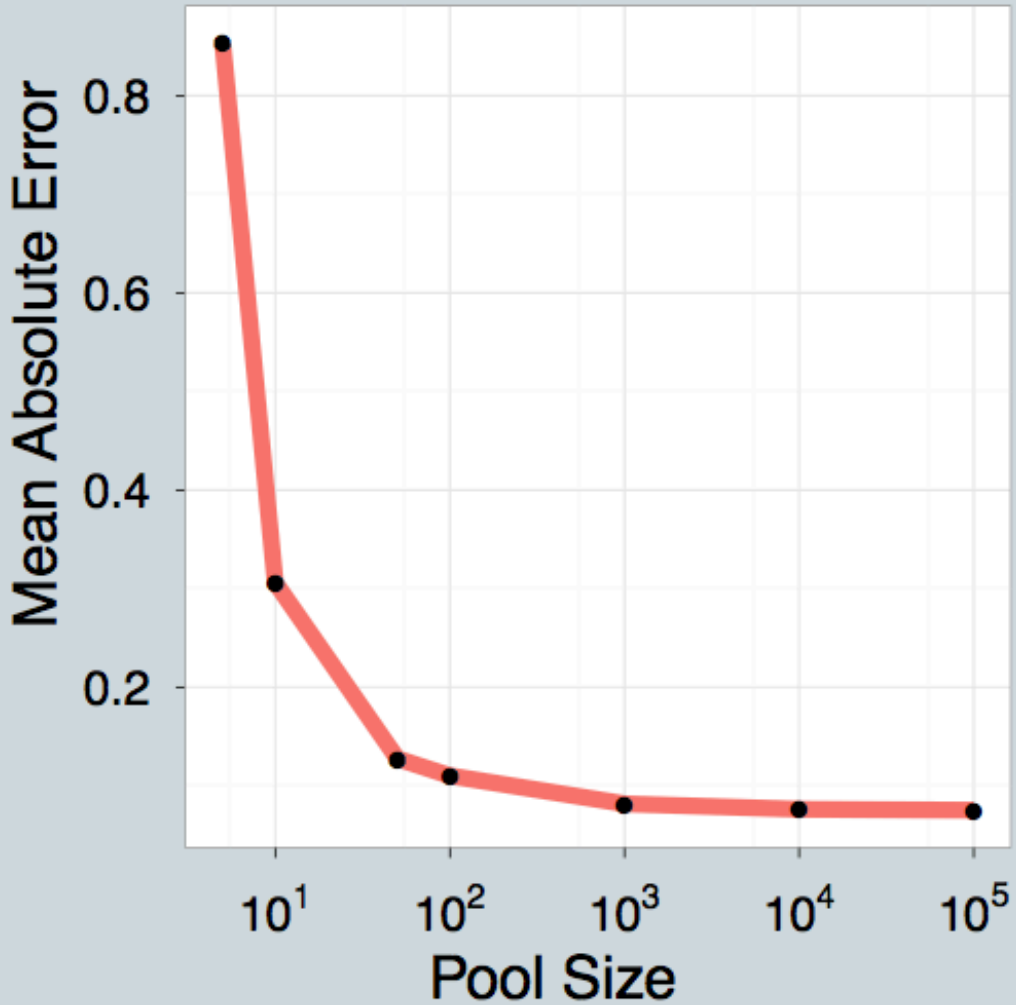
- Better algorithm:
  - Initialization:
    - Create a pool:
      - Pick a random seed *s*
      - For i=1 to poolSize:
        - » Draw pool[i] ~ Normal(0,1)
    - For i=1 to outputBits:
      - Devise a random hash function hash(i,*f*):
  - Given an instance **x**
    - For i=1 to outputBits:

      LSH[i] = sum(

      x[f] * pool[hash(i, *f*) % poolSize] for *f* in **x**) > 0 ?

      1 : 0
    - Return the bit-vector LSH

The Pooling Trick



LSH: key ideas: pooling

- Advantages:
  - with pooling, this is a compact re-encoding of the data
    - you don't need to store the **r**'s, just the pool

Locality Sensitive Hashing (LSH) in an On-line Setting

LSH: key ideas: online computation

- Common task: **distributional clustering**
  - for a word *w*, ***v****(w)* is sparse vector of words that co-occur with *w*
  - cluster the ***v****(w)'s*

…guards at Pentonville prison in North **London** discovered that an escape attempt…

An American Werewolf in **London** is to be remade by the son of the original director…

…UK pop up shop on Monmouth Street in **London** today and on Friday the brand…

v**(London):** Pentonville, prison, in, North, …. and, on Friday

LSH: key ideas: online computation

- Common task: distributional clustering
  - for a word *w*, ***v***(*w*) is sparse vector of words that co-occur with *w*
  - cluster the ***v***(*w*)'s – or here, compute similarities

London is similar to:

Milan$_{.97}$, **Madrid**$_{.96}$, **Stockholm**$_{.96}$, **Manila**$_{.95}$, **Moscow**$_{.95}$

in is similar to:

during$_{.99}$, **on**$_{.98}$, **beneath**$_{.98}$, **from**$_{.98}$, **onto**$_{.97}$

sold is similar to:

deployed$_{.84}$, **presented**$_{.83}$, **sacrificed**$_{.82}$, **held**$_{.82}$, **installed**$_{.82}$

LSH: key ideas: online computation

- Common task: distributional clustering
  - for a word *w*, ***v***(*w*) is **sparse** vector of words that co-occur with *w*
  - cluster the ***v***(*w*)'s

v(w) is very similar to a word embedding (eg, from word2vec or GloVE)

Levy, Omer, Yoav Goldberg, and Ido Dagan. "Improving distributional similarity with lessons learned from word embeddings." *Transactions of the Association for Computational Linguistics* 3 (2015): 211-225.

LSH: key ideas: online computation

- Construct LSH($\boldsymbol{v}(w)$) for each word w in the corpus
  - stream through the corpus *once*
  - use minimal amount of memory
  - ideally no more than size of output

…guards at Pentonville prison in North **London** discovered that an escape attempt…

An American Werewolf in **London** is to be remade by the son of the original director…

…UK pop up shop on Monmouth Street in **London** today and on Friday the brand…

v(London): Pentonville, prison, in, North, …. and, on Friday
LSH(v(London)): 0101101010111011101
LSH(v(Monmouth)): 0111101000110010111
…

$\vec{v} \in \mathbb{R}^d$ *v is context vector; d is vocab size*

$\vec{r}_i \sim N(0,1)^d$ *r_i is i-th random projection*

…guards at Pentonville prison in North **London** c

An American Werewolf in **London** is to be remad

…UK pop up shop on Monmouth Street in **Londo**

v**(London):** Pentonville, prison, in, North, …. and

$$h_i(\vec{v}) = \begin{cases} 1 & \text{if } \vec{v} \cdot \vec{r}_i \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

$h_i(v)$ i-th bit of LSH encoding

$$\text{if } \vec{v} = \Sigma_j \vec{v}_j$$

because context vector is sum of *mention contexts*

$$\text{then } \vec{v} \cdot \vec{r}_i = \Sigma_j \vec{v}_j \cdot \vec{r}_i$$

Break into local products

these come one by one as we stream thru the corpus

Online

$$h_{it}(\vec{v}) = \begin{cases} 1 & \text{if } \Sigma_j^t \vec{v}_j \cdot \vec{r}_i \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

keep this

Algorithm 1 STREAMING LSH ALGORITHM

Parameters:

m : size of pool

d : number of bits (size of resultant signature)

s : a random seed

$h_1, ..., h_d$: hash functions mapping $\langle s, f_i \rangle$ to $\{0, \ldots, m-1\}$

INITIALIZATION:

```
Initialize floating point array P[0, ..., m − 1]
Initialize H, a hashtable mapping words to floating point
    arrays of size d
for i := 0 ... m − 1 do
    P[i] := random sample from N(0, 1), using s as seed
```

ONLINE:

```
for each word w in the stream do
    for each feature f_i associated with w do
        for j := 1 ... d do
            H[w][j] := H[w][j] + P[h_j(s, f_i)]
```

SIGNATURECOMPUTATION:

```
for each w ∈ H do
    for i := 1 ... d do
        if H[w][i] > 0 then
            S[w][i] := 1
        else
            S[w][i] := 0
```

weight of f_i in r_j

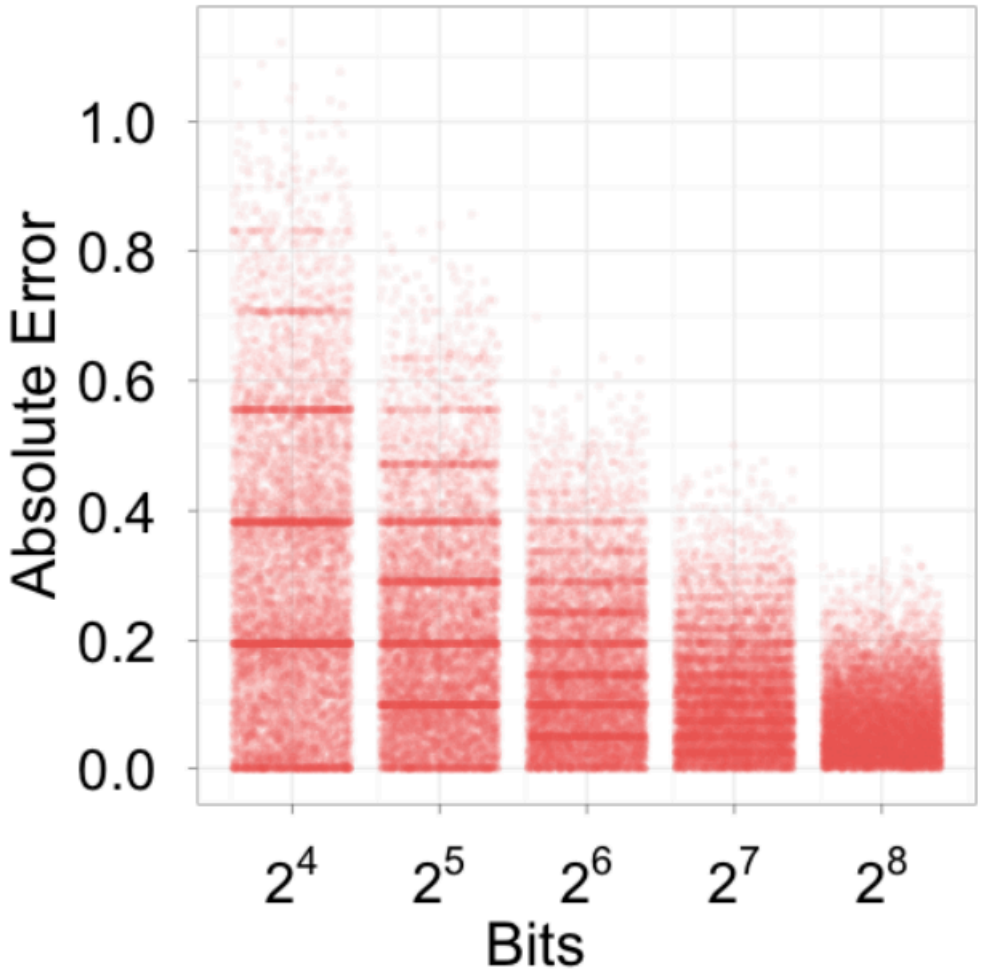
j-th hash of f_i

$$\sum_j^t \vec{v}_j \cdot \vec{r}_i$$

Experiment

- Corpus: 700M+ tokens, 1.1M distinct bigrams
- For each, build a feature vector of words that co-occur near it, using on-line LSH
- Check results with 50,000 actual vectors

Experiment



Closest based on true cosine

London

	Milan$_{.97}$, **Madrid**$_{.96}$, **Stockholm**$_{.96}$, **Manila**$_{.95}$, **Moscow**$_{.95}$
16	ASHER$_0$, Champaign$_0$, MANS$_0$, NOBLE$_0$, come$_0$
32	Prague$_1$, Vienna$_1$, suburban$_1$, synchronism$_1$, Copenhagen$_2$
64	Frankfurt$_4$, Prague$_4$, Taszar$_5$, Brussels$_6$, Copenhagen$_6$
128	Prague$_{12}$, Stockholm$_{12}$, Frankfurt$_{14}$, Madrid$_{14}$, Manila$_{14}$
256	Stockholm$_{20}$, Milan$_{22}$, Madrid$_{24}$, Taipei$_{24}$, Frankfurt$_{25}$

Points to review

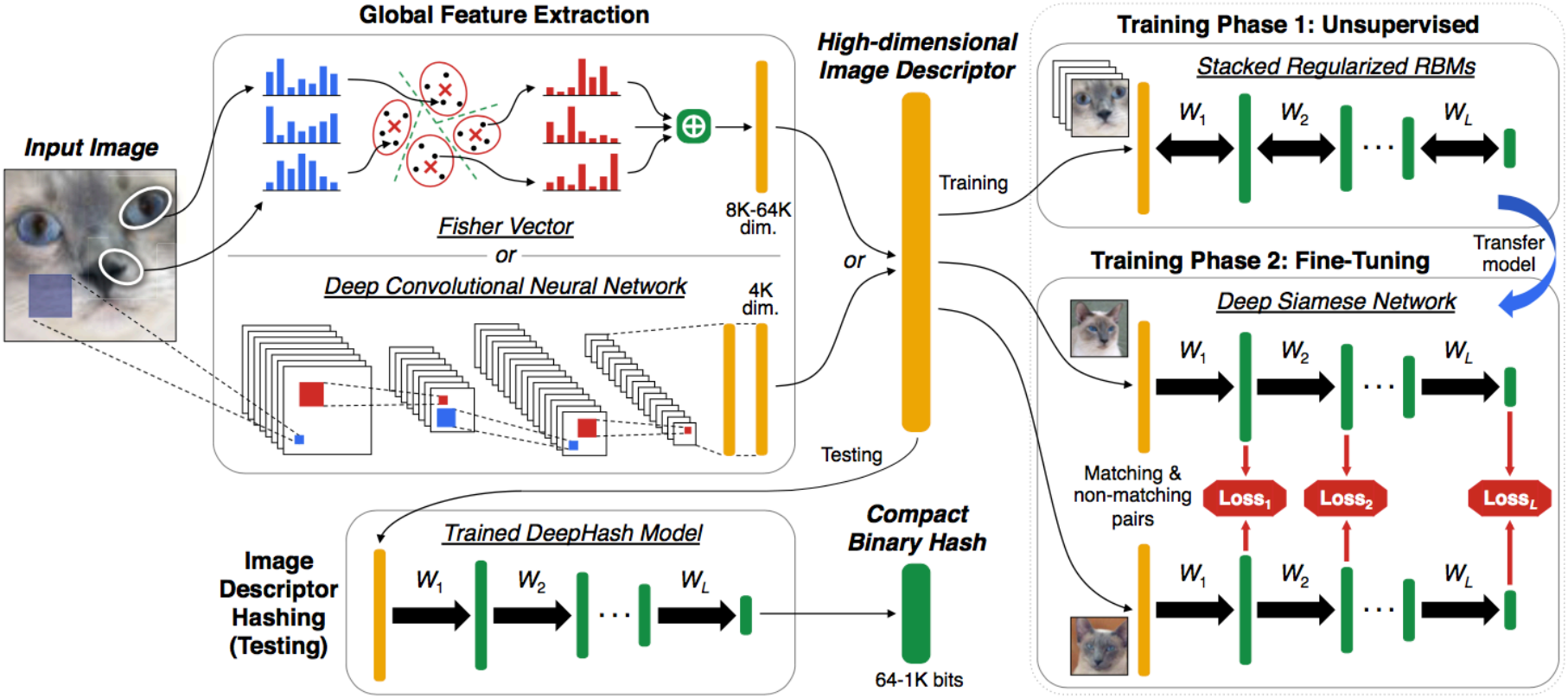
- APIs for:
  - Bloom filters, CM sketch, LSH
- Key applications of:
  - Very compact noisy sets
  - Efficient counters accurate for *large* counts
  - Fast approximate cosine distance
- Key ideas:
  - Uses of hashing that allow collisions
  - Random projection
  - Multiple hashes to control Pr(collision)
  - Pooling to compress a lot of random draws

A DEEP-LEARNING VARIANT OF LSH

DeepHash: Getting Regularization, Depth and Fine-Tuning Right

Jie Lin*[,1,3], Olivier Morère*[,1,2,3], Vijay Chandrasekhar[1,3], Antoine Veillard[2,3], Hanlin Goh[1,3]
I2R[1], UPMC[2], IPAL[3]

ICMR 2017

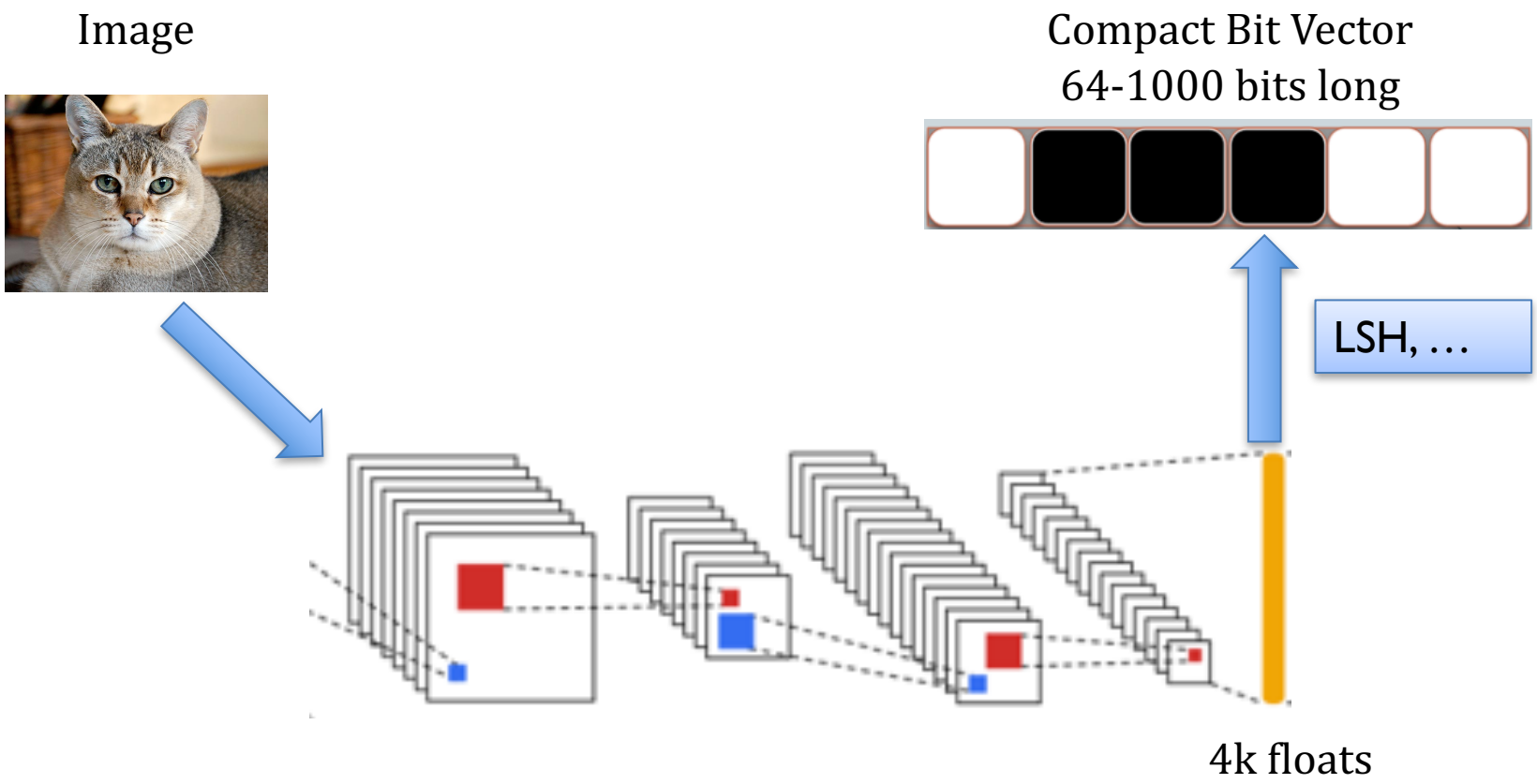


DeepHash

Image

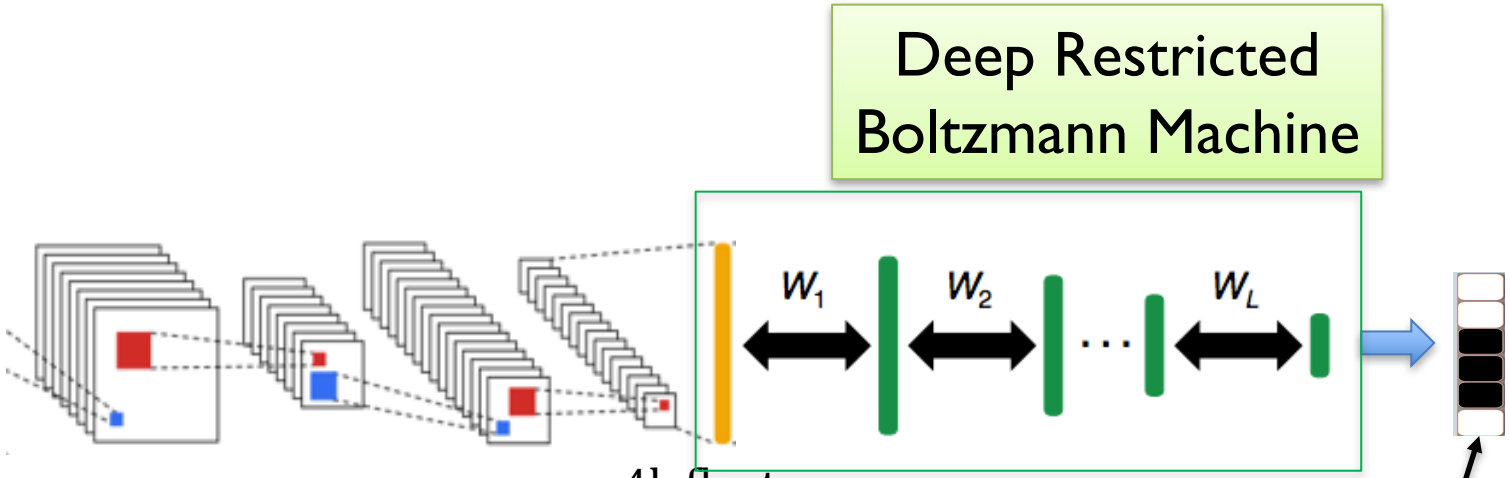
Compact Bit Vector
64-1000 bits long

DeepHash



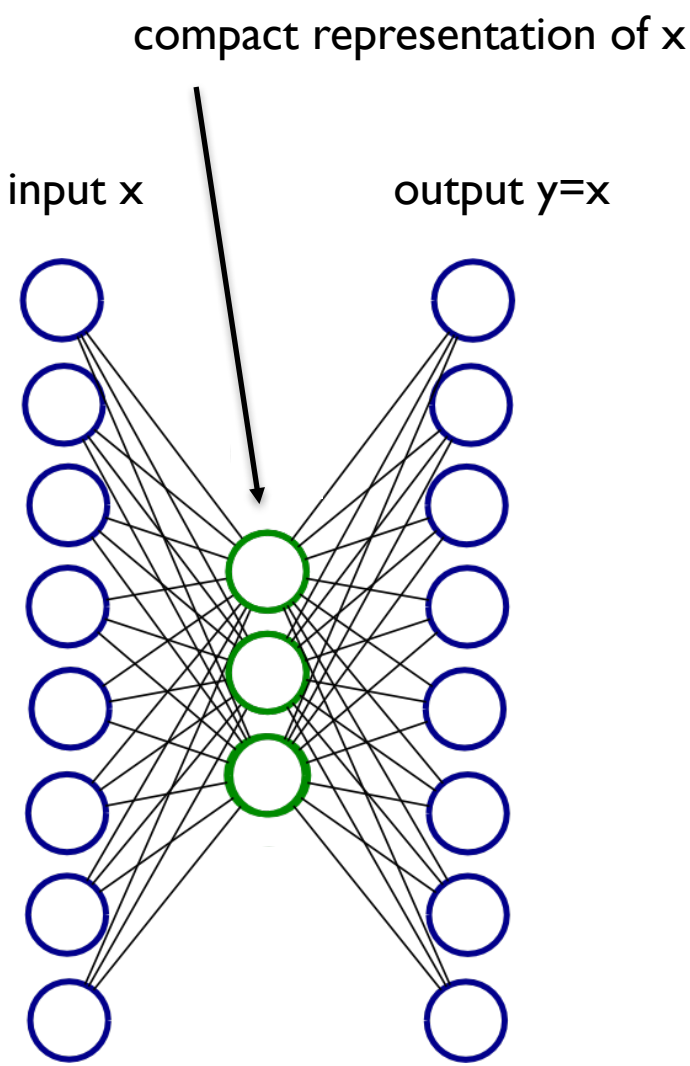
DeepHash

Image



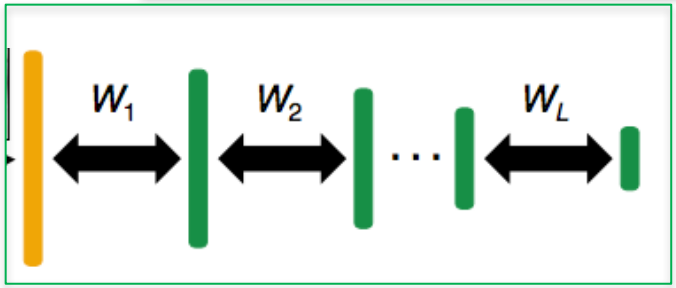
Compact Bit Vector
64-1000 bits long

DeepHash



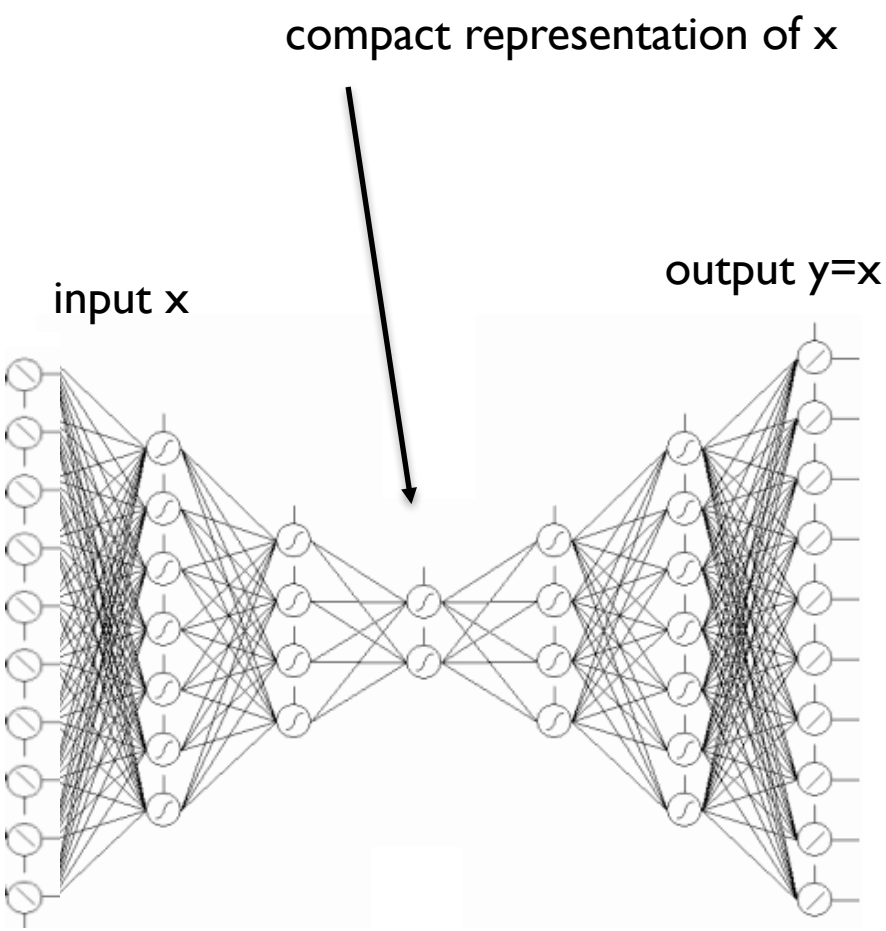
An autoencoder

Deep Restricted Boltzmann Machine



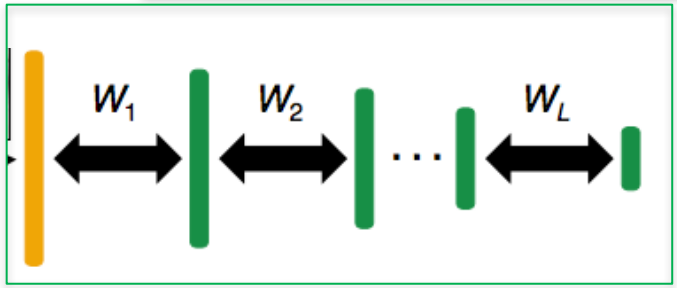
Restricted Boltzmann Machine is closely related to an autoencoder

DeepHash



A deeper autoencoder

Deep Restricted Boltzmann Machine



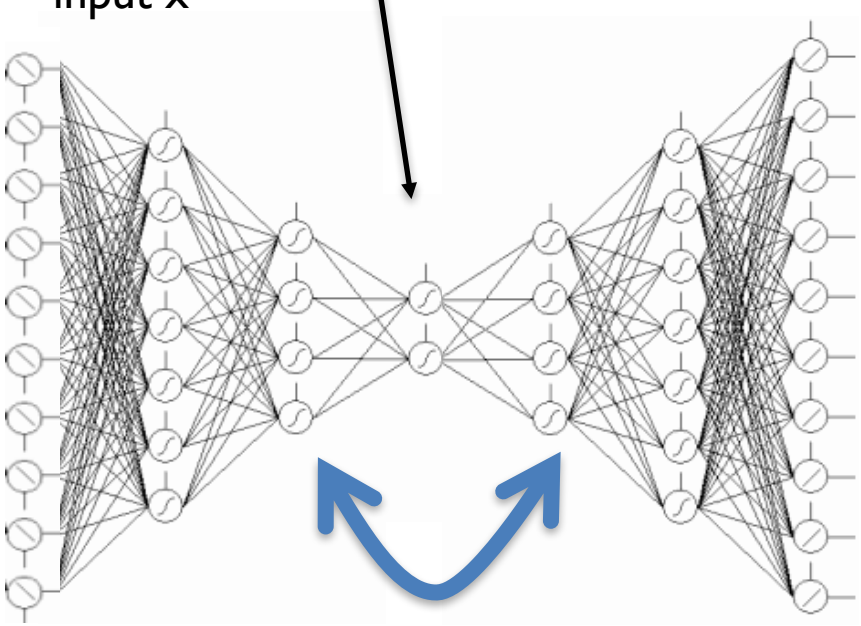
Deep Restricted Boltzmann Machine is closely related to a deep autoencoder

DeepHash

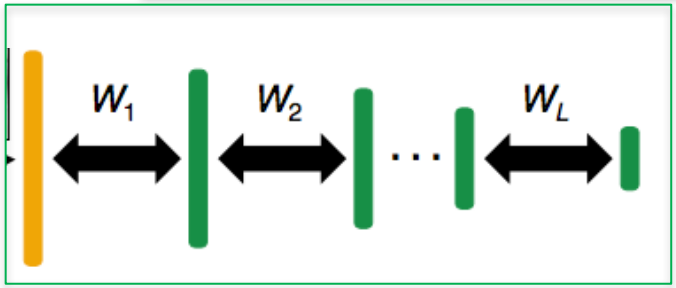
compact representation of x

input x

output y=x



Deep Restricted Boltzmann Machine



Deep Restricted Boltzmann Machine is

But the RBM is symmetric: weights W from layer *j* to *j+1* are the transpose of weights from *j+1* back to *j*

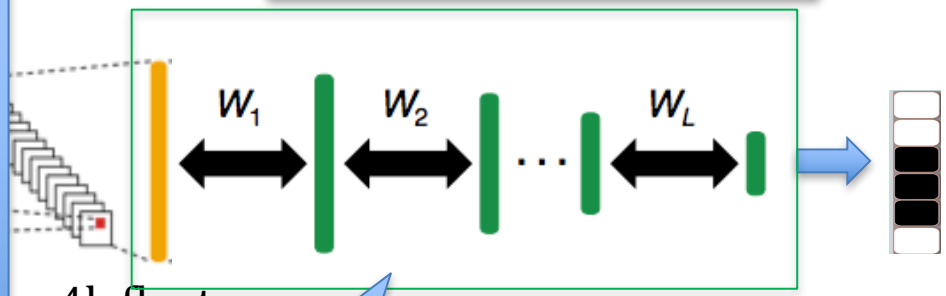
RBM is also stochastic: compute Pr(hidden|visible), **sample** from that distribution, compute Pr(visible|hidden), **sample**, ...

DeepHash

Deep Restricted Boltzmann Machine

Image

A trick: regularize so that the representations are dense (about 50-50 "on" and "off" for an image) and each bit has 50-50 chance of being "on"

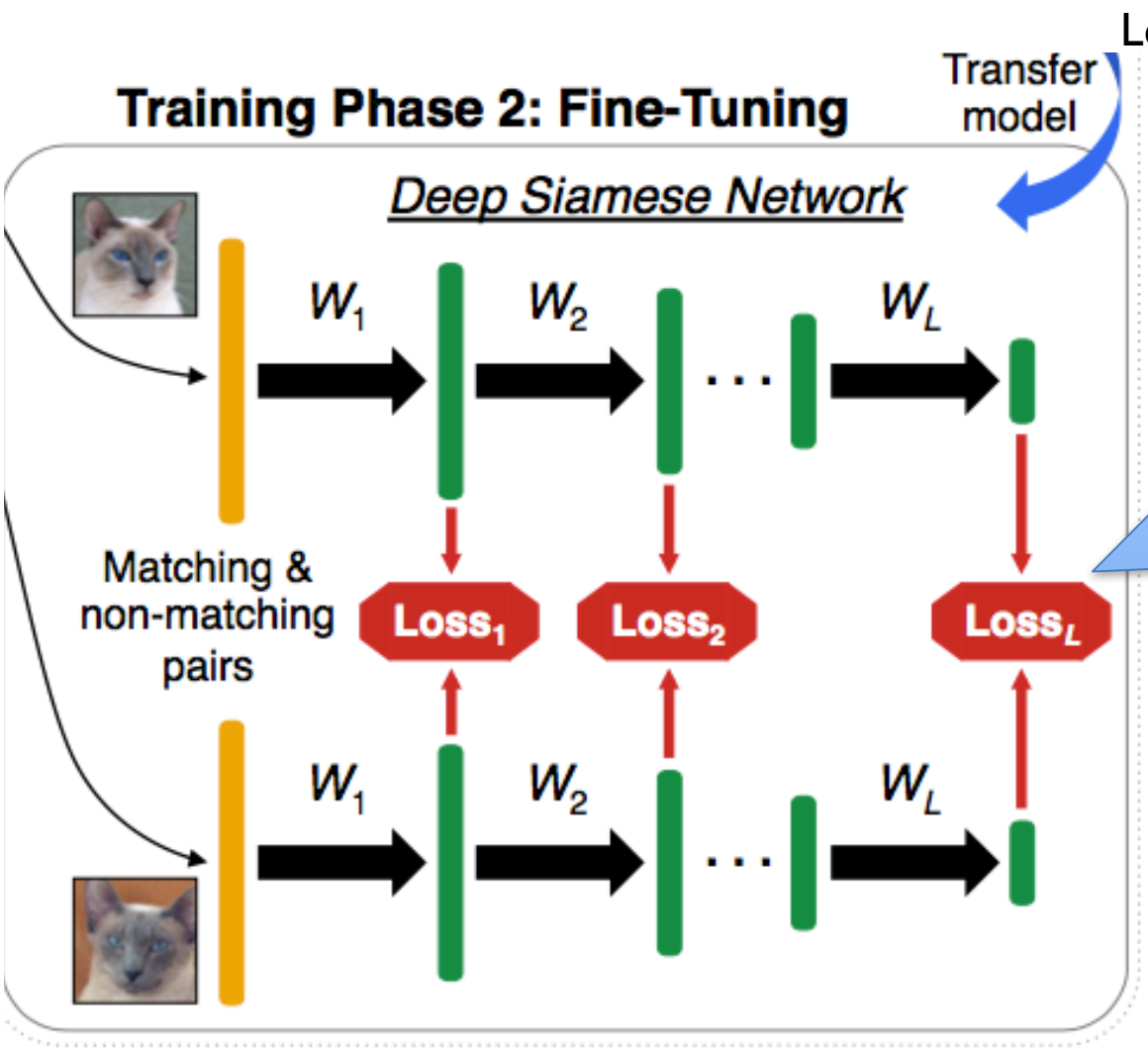


4k floats

This model is trained to compress image features, then reconstruct the image features from the reconstructions.

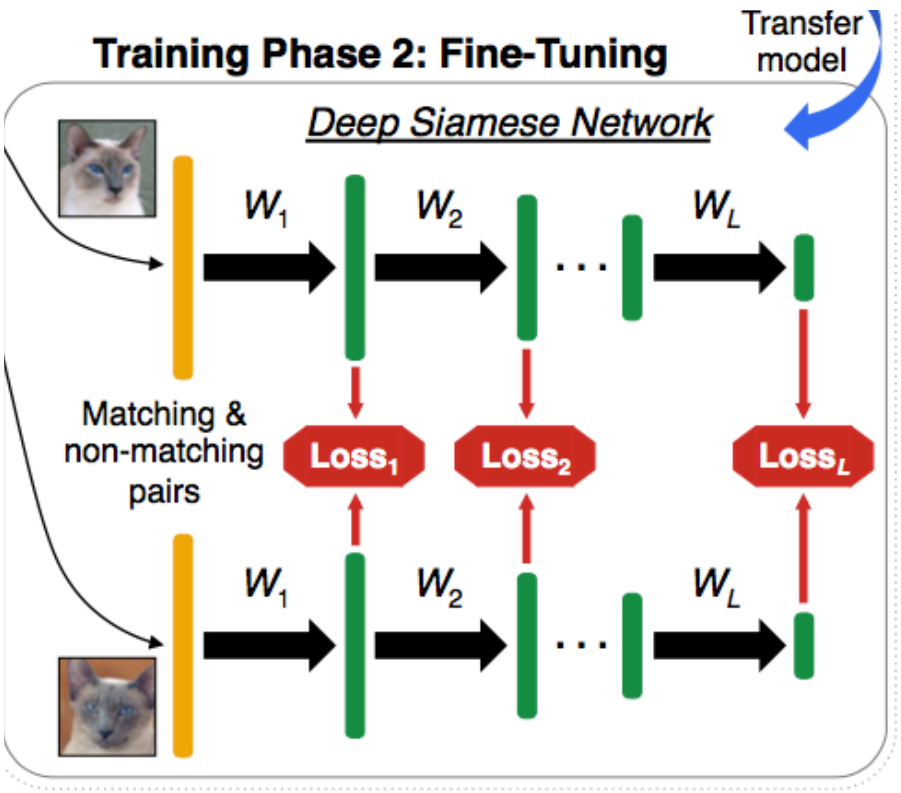
And then more training….

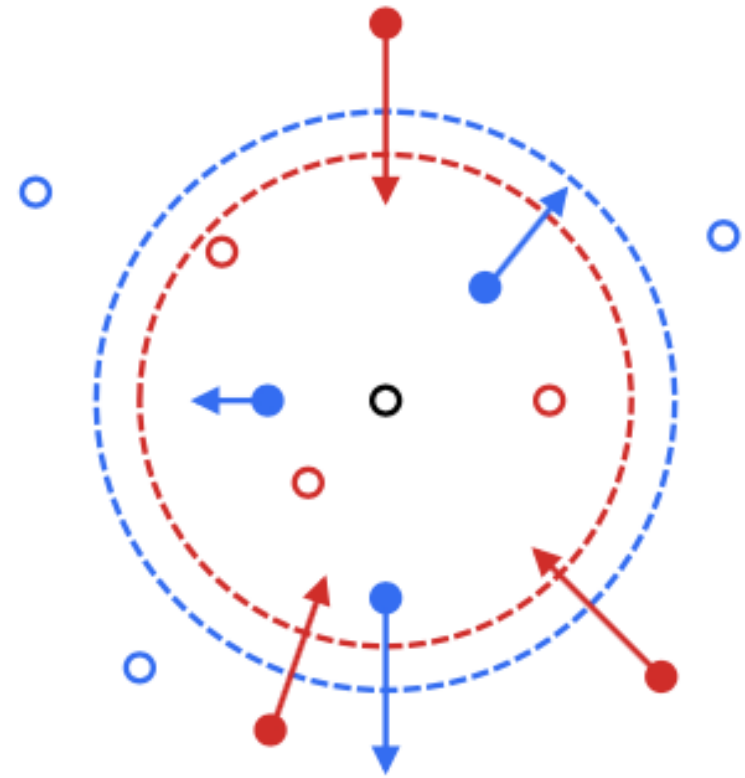
Training on matching vs non-matching pairs



Loss pushes the representation for "matching" pairs together and representation for non-matching pairs apart

Training on matching vs non-matching pairs

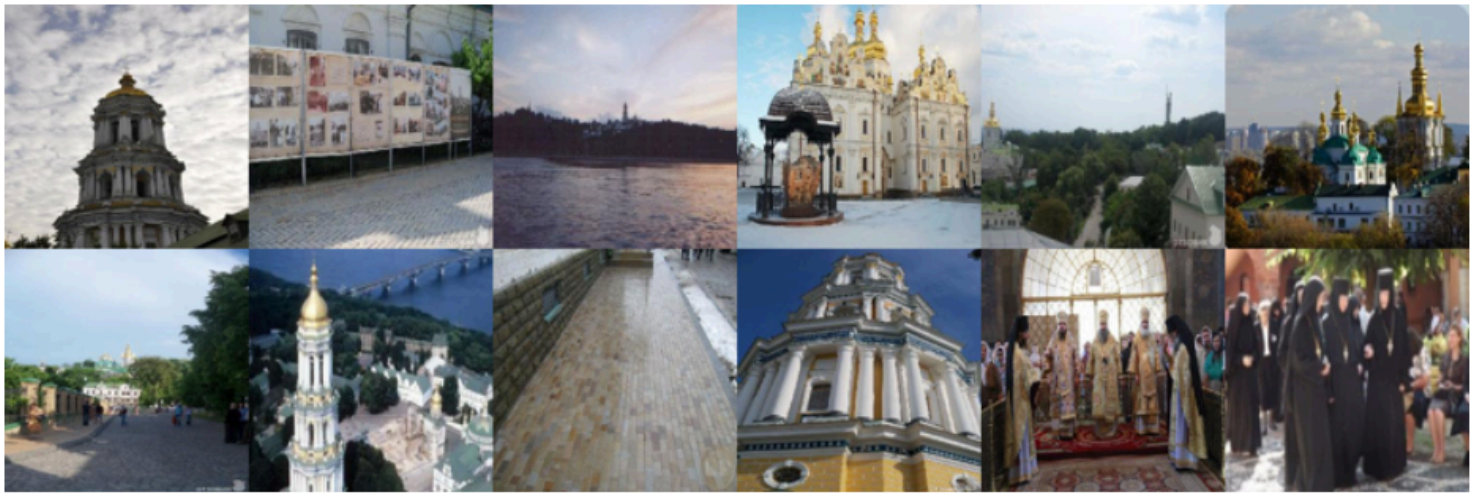




margin-based matching loss

Training on matching vs non-matching pairs

matching pairs were photos of the same "landmark"



DeepHash: Getting Regularization, Depth and Fine-Tuning Right

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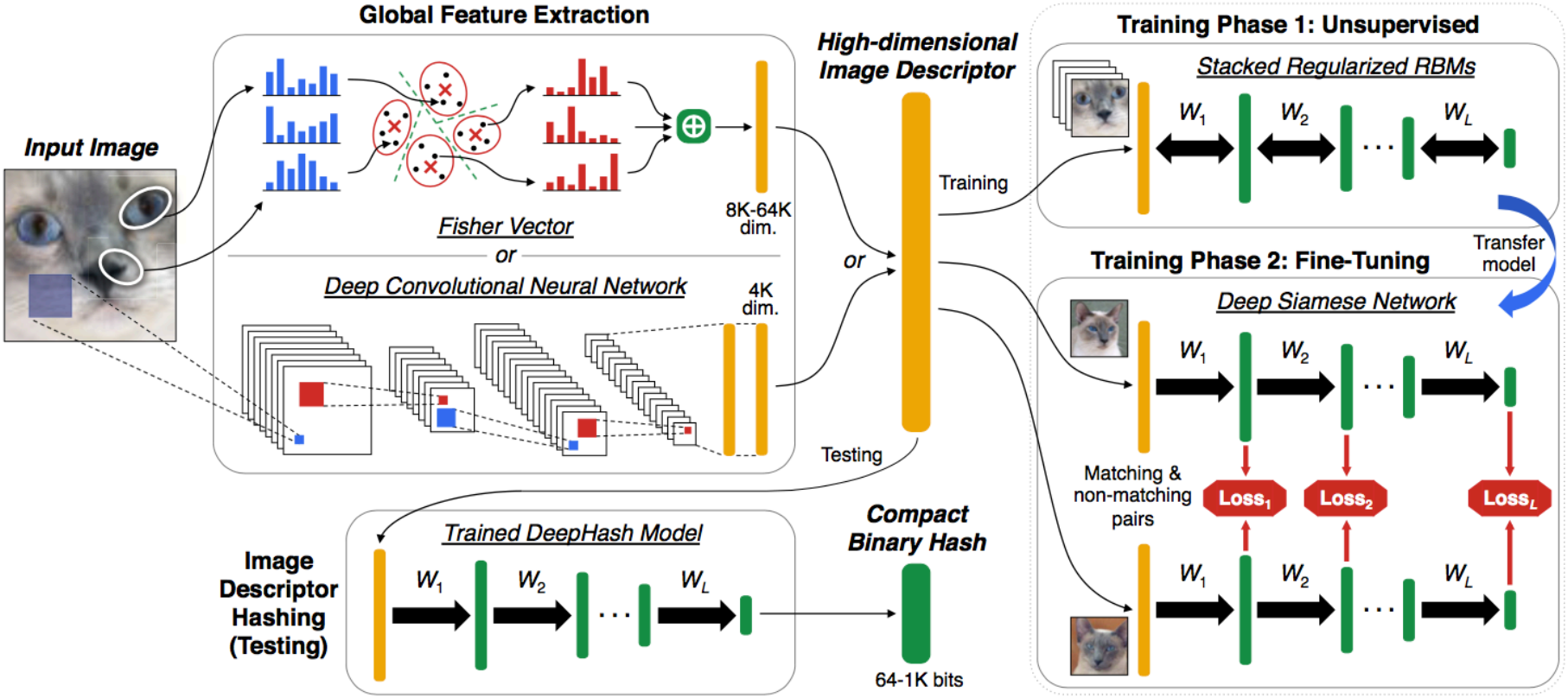
ICML 2017



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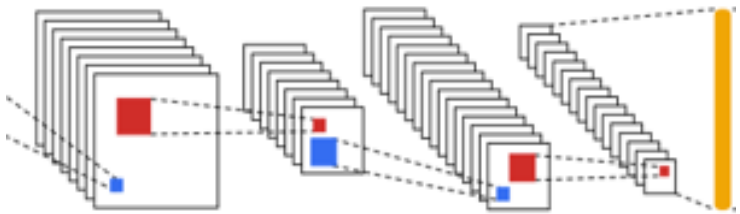
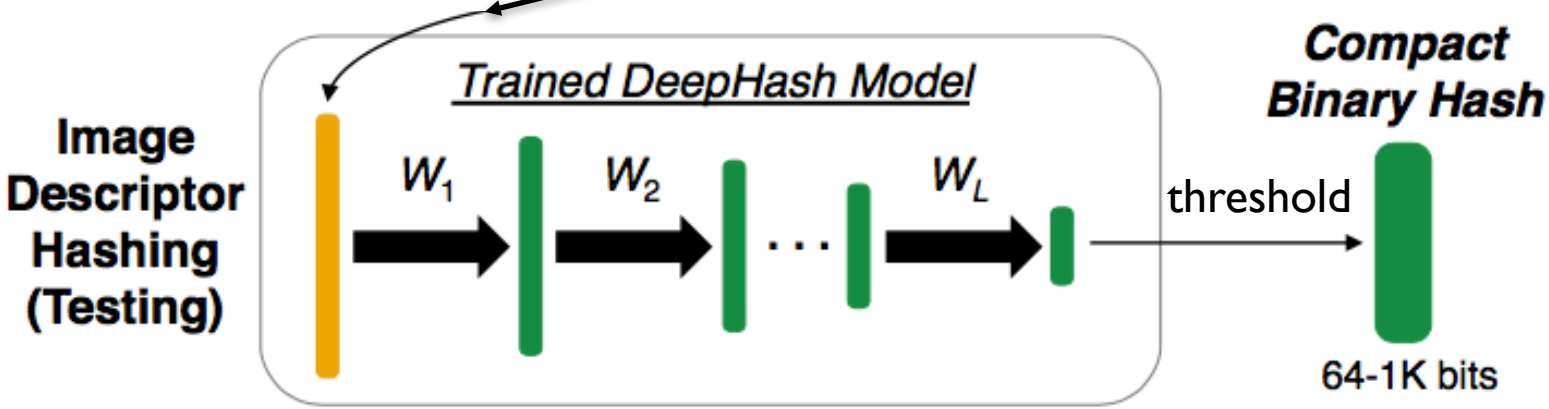
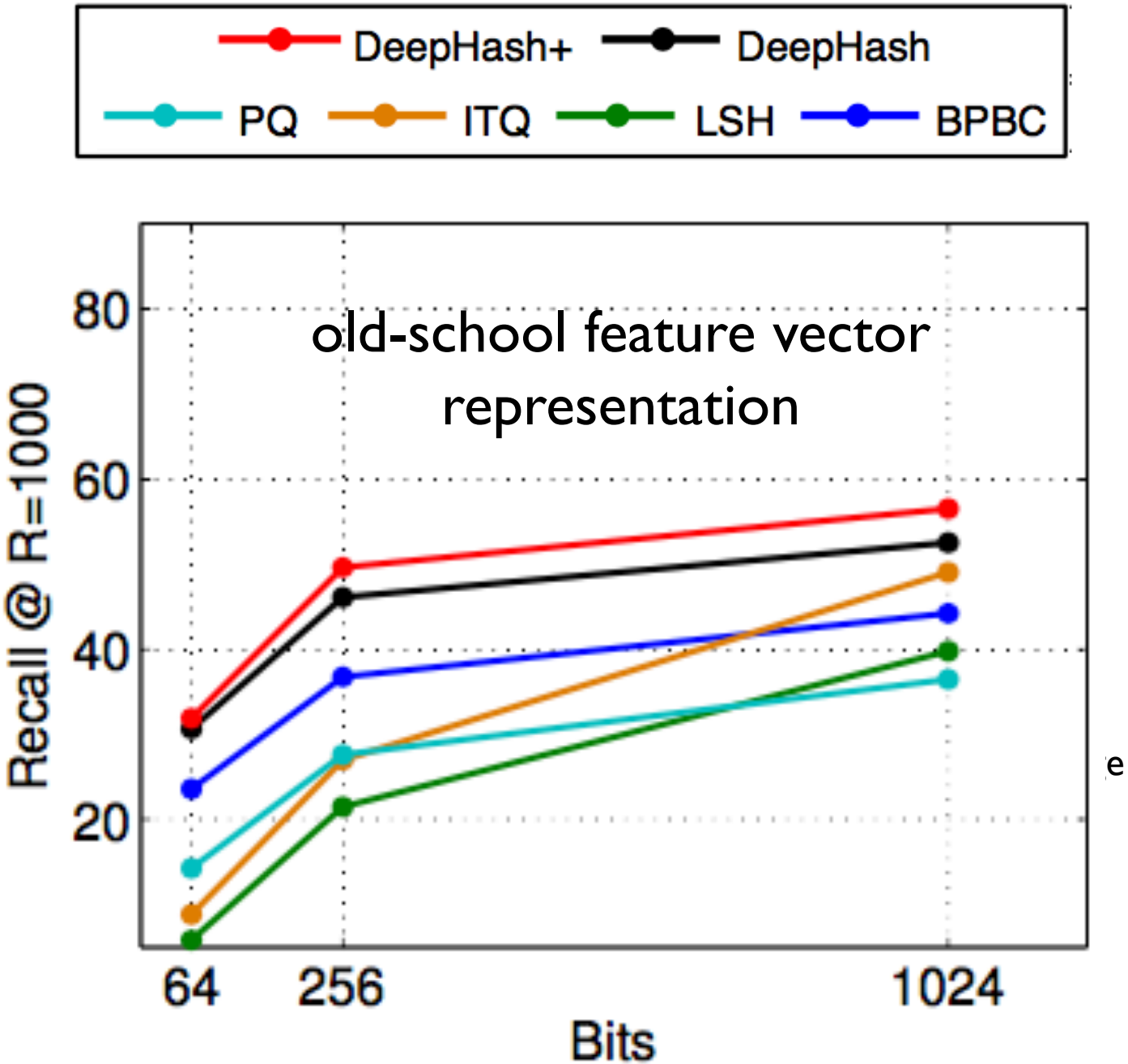
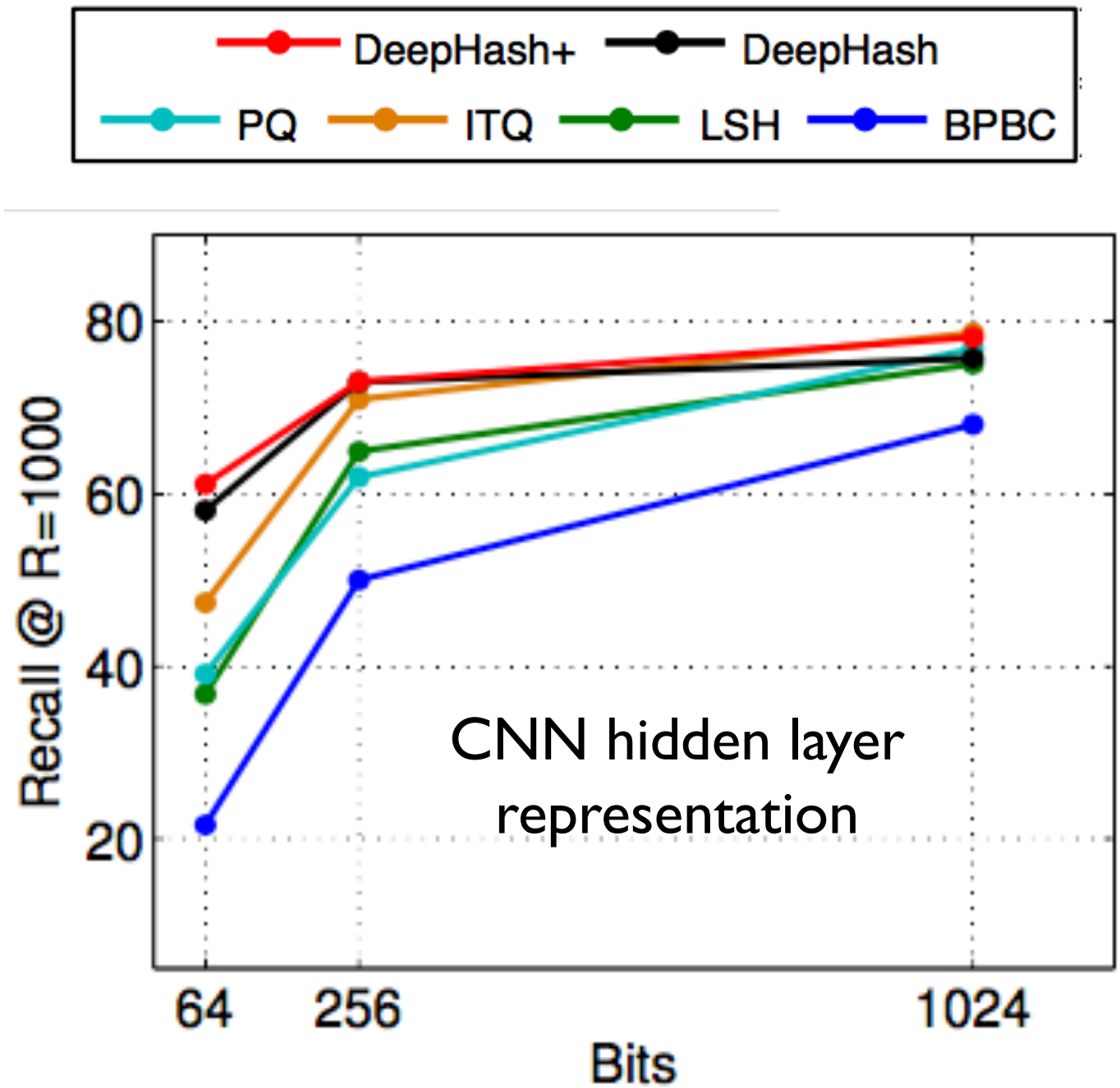





DeepHash+
DeepHash
PQ
ITQ
LSH
BPBC
old-school feature vector representation
Recall @ R=1000
80
60
40
20
64
256
1024
Bits


DeepHash+
DeepHash
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80
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Bits
CNN hidden layer representation

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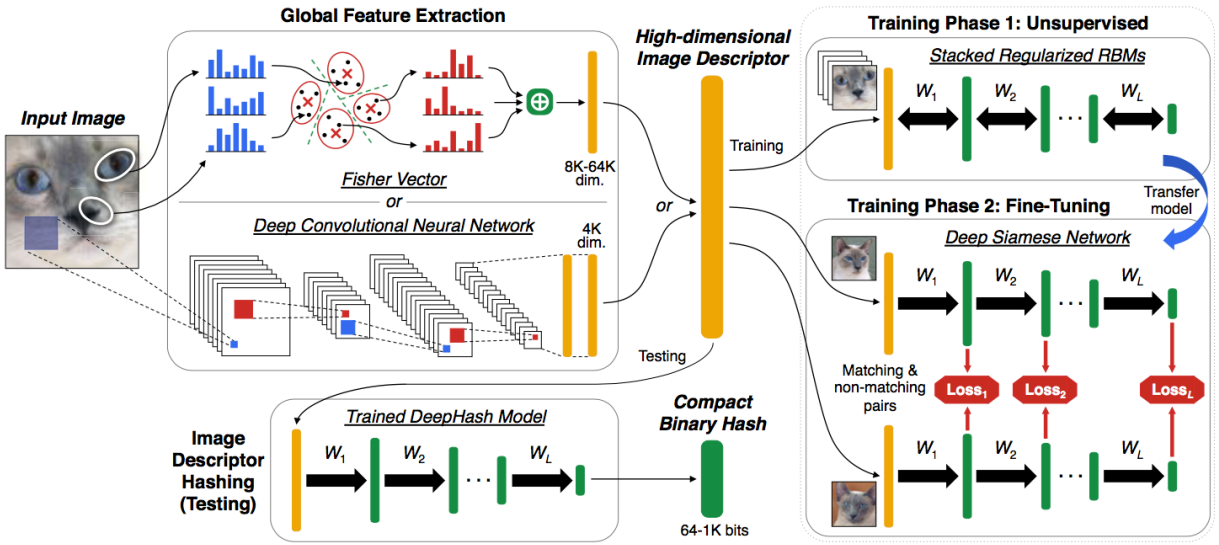

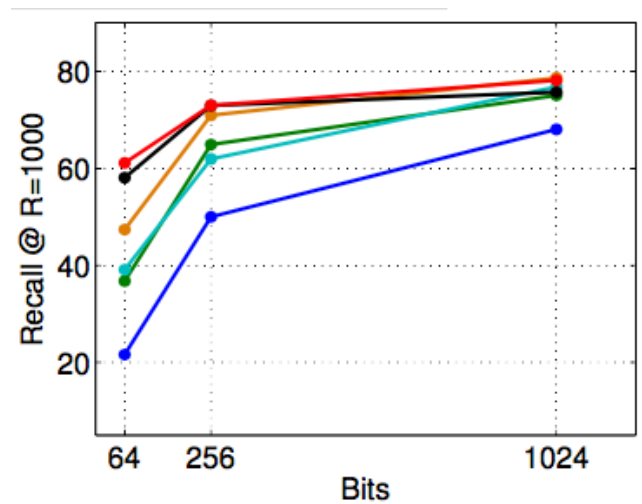

Picking random classifiers (with LSH) is surprisingly close in performance to DeepHash --- when the similarity metric you start with is good