Improved Multi-Heuristic A* for searching with uncalibrated heuristics

Venkatraman Narayanan, Sandip Aine, and Maxim Likhachev

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Admissible Heuristics
Admissible Heuristics

Solution quality
Admissible Heuristics

Solution quality

Combining multiple heuristics
Admissible Heuristics

Solution quality

Combining multiple heuristics

Planning time
Admissible Heuristics

Solution quality

Combining multiple heuristics

Inadmissible Heuristics

Planning time
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- Solution quality
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Combining multiple heuristics

MHA*: [Aine, Swaminathan, Narayanan, Hwang and Likhachev, RSS 2014]
MH-GBFS: [Roger and Helmert, ICAPS 2010]
Multi-heuristic Search: [Isto, ISIR 1996]
**Admissible Heuristics**

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Calibration: $g$ and $h$ different units

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Admissible Heuristics

- Solution quality
- Combining multiple heuristics
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Inadmissible Heuristics

- Planning time
- Solution quality
- Combining multiple heuristics
- Calibration: g and h different units

This work: Improved Multi-Heuristic A*

MHA*: [Aine, Swaminathan, Narayanan, Hwang and Likhachev, RSS 2014]
MH-GBFS: [Roger and Helmert, ICAPS 2010]
Multi-heuristic Search: [Isto, ISIR 1996]
12 DoF Full-body Motion Planning
Calibration Problem
Calibration Problem

\[ g: \text{ execution time (sec)} \]
\[ h_1: \text{ base distance (m)} \]
\[ h_2: \text{ vertical orientation (rad)} \]
\[ h_3: \text{ horizontal orientation (rad)} \]
Calibration Problem

\[ g(s) + w \cdot h_1(s) \]
\[ g(s) + w \cdot h_2(s) \]
\[ g(s) + w \cdot h_3(s) \]

\( g \): execution time (sec)
\( h_1 \): base distance (m)
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Calibration Problem

\[ g(s) + w \cdot h_1(s) \quad g(s) + w \cdot h_2(s) \quad g(s) + w \cdot h_3(s) \]

Combining \( g \) and \( h \) additively not meaningful
Related Work
Related Work

Multi-Heuristic Greedy Best-First Search [Roger and Helmert, ICAPS 2010]
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Multi-Heuristic Greedy Best-First Search [Roger and Helmert, ICAPS 2010]

pick s with minimum $h_1(s)$

pick s with minimum $h_2(s)$

pick s with minimum $h_3(s)$
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No calibration problem
Multi-Heuristic Greedy Best-First Search [Roger and Helmert, ICAPS 2010]

Related Work

No calibration problem

No bounds on solution quality
Related Work

Multi-Heuristic Greedy Best-First Search [Roger and Helmert, ICAPS 2010]

- Pick s with minimum $h_1(s)$
- Pick s with minimum $h_2(s)$
- Pick s with minimum $h_3(s)$

No calibration problem

No bounds on solution quality

Multi-Heuristic A* [Aine, Swaminathan, Narayanan, Hwang and Likhachev, RSS 2014]

- Anchor Search
  - Consistent Heuristic $h_0$
  - Pick s with minimum $g + w_1 \cdot h_0(s)$

- Pick s with minimum $g + w_1 \cdot h_1(s)$
- Pick s with minimum $g + w_1 \cdot h_2(s)$
- Pick s with minimum $g + w_1 \cdot h_3(s)$
Related Work

Multi-Heuristic Greedy Best-First Search [Roger and Helmert, ICAPS 2010]

pick s with minimum $h_1(s)$

pick s with minimum $h_2(s)$

pick s with minimum $h_3(s)$

No calibration problem

No bounds on solution quality

Multi-Heuristic A* [Aine, Swaminathan, Narayanan, Hwang and Likhachev, RSS 2014]

Consistent Heuristic $h_0$

Anchor Search

pick s with minimum $g + w_1 \cdot h_0(s)$

while not done
for i in 1 to 3
if min.key($\text{OPEN}_i$) $\leq w_2 \cdot \min\text{.key}(\text{OPEN}_0)$
expand from $\text{OPEN}_i$
else
expand from $\text{OPEN}_0$

pick s with minimum $g + w_1 \cdot h_1(s)$

pick s with minimum $g + w_1 \cdot h_2(s)$

pick s with minimum $g + w_1 \cdot h_3(s)$
Related Work

Multi-Heuristic Greedy Best-First Search [Roger and Helmert, ICAPS 2010]

![Diagram of Multi-Heuristic Greedy Best-First Search]

- pick s with minimum $h_1(s)$
- pick s with minimum $h_2(s)$
- pick s with minimum $h_3(s)$

No calibration problem

Multi-Heuristic A* [Aine, Swaminathan, Narayanan, Hwang and Likhachev, RSS 2014]

- Anchor Search
  - Consistent Heuristic $h_0$
    - pick s with minimum $g + w_1 \cdot h_0(s)$

- while not done
  - for i in 1 to 3
    - if min.key(OPEN$_i$) $\leq w_2 \cdot$ min.key(OPEN$_0$)
      - expand from OPEN$_i$
    - else
      - expand from OPEN$_0$

Bounds on solution quality, re-expansions

No bounds on solution quality
Related Work

Multi-Heuristic Greedy Best-First Search [Roger and Helmert, ICAPS 2010]

- Pick $s$ with minimum $h_1(s)$
- Pick $s$ with minimum $h_2(s)$
- Pick $s$ with minimum $h_3(s)$

No calibration problem

Multi-Heuristic A* [Aine, Swaminathan, Narayanan, Hwang and Likhachev, RSS 2014]

- Consistent Heuristic $h_0$
- Anchor Search: Pick $s$ with minimum $g + w_1 \cdot h_0(s)$

While not done for $i$ in 1 to 3
- If $\min(\text{OPEN}_i) \leq w_2 \cdot \min(\text{OPEN}_0)$, expand from $\text{OPEN}_i$
- Else, expand from $\text{OPEN}_0$

Calibration problem

Bounds on solution quality, re-expansions
Related Work

Focal-A*  [Pearl and Kim, PAMI 1982]

\[ f(s) \leq w \cdot f(s_a) \]

Use distance-to-go estimate to greedily choose from FOCAL list
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Explicit Estimation Search  [Thayer and Rumel, IJCAI 2011]

Use secondary inadmissible heuristic to remove bias in construction of FOCAL list
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Use distance-to-go estimate to greedily choose from FOCAL list

Explicit Estimation Search [Thayer and Ruml, IJCAI 2011]

Use secondary inadmissible heuristic to remove bias in construction of FOCAL list

No calibration problem

Do not combine multiple heuristics

No bounds on re-expansions
Improved MHA*

Multi-heuristic search framework

View shared-search as single queue search: interleave ‘admissible’ and ‘inadmissible’ expansions

Prioritize based on uncalibrated heuristics: no additive combination

Derive bounds from consistent heuristic
procedure MAIN()
OPEN ← ∅
CLOSED_a ← ∅
g(s_{start}) ← 0, g(s_{goal}) ← ∞
Insert \( s_{start} \) in OPEN with PRIORITY(\( s_{start} \))

while not TERM-CRITERION(\( s_{goal} \)) do
    if OPEN.EMPTY() then return null
    \( s_a \) ← OPEN.TOP()
    EXPANDSTATE(\( s_a \))
    CLOSED_a ← CLOSED_a \cup \{ s_a \}

return solution path
Improved MHA*

procedure MAIN()
    OPEN ← ∅
    CLOSED_a ← ∅
    g(s_{start}) ← 0, g(s_{goal}) ← ∞
    Insert s_{start} in OPEN with PRIORITY(s_{start})
    while not TERM-CRITERION(s_{goal}) do
        if OPEN.EMPTY() then return null
        s_a ← OPEN.TOP()
        EXPANDSTATE(s_a)
        CLOSED_a ← CLOSED_a ∪ \{s_a\}
    return solution path
Improved MHA*

procedure MAIN()
    OPEN ← ∅
    CLOSED_a ← ∅
    \text{let } g(s_{start}) ← 0, g(s_{goal}) ← \infty
    \text{Insert } s_{start} \text{ in OPEN with PRIORITY}(s_{start})
    \textbf{while not } \text{TERM-CRITERION}(s_{goal}) \textbf{ do}
        if OPEN.\text{EMPTY}() \textbf{ then return null}
        s_a ← OPEN.\text{TOP}()
        EXPAND\text{STATE}(s_a)
        CLOSED_a ← CLOSED_a \cup \{s_a\}
    \textbf{return solution path}
**Improved MHA**

```plaintext
procedure MAIN()
    OPEN ← \emptyset
    CLOSED_a ← \emptyset
    g(s_{start}) ← 0, g(s_{goal}) ← \infty
    Insert s_{start} in OPEN with PRIORITY(s_{start})
    while not TERM-CRITERION(s_{goal}) do
        if OPEN.EMPTY() then return null
        s_a ← OPEN.TOP()
        EXPAND-STATE(s_a)
        CLOSED_a ← CLOSED_a \cup \{s_a\}
    return solution path
```

**PRIORITY(s)**

\[
g(s) + \frac{w}{2} \cdot h(s)
\]

or

\[
g(s) + w \cdot h(s)
\]

\(h(s)\): consistent heuristic
Improved MHA*

```
procedure MAIN()
    OPEN ← Ø
    CLOSED_a ← Ø
    g(s_{start}) ← 0, g(s_{goal}) ← ∞
    Insert s_{start} in OPEN with PRIORITY(s_{start})
    while not TERM-CRITERION(s_{goal}) do
        if OPEN.EMPTY() then return null
        s_a ← OPEN.TOP()
        EXPANDSTATE(s_a)
        CLOSED_a ← CLOSED_a ∪ \{s_a\}
    return solution path
```

```
procedure EXPANDESTATE(s)
    Remove s from OPEN
    for all s' ∈ SUCC(s) do
        if s' was not seen before then
            g(s') ← ∞
        if g(s') > g(s) + c(s, s') then
            g(s') ← g(s) + c(s, s')
        if s' ∉ CLOSED_a then
            Insert/Update s' in OPEN
            with PRIORITY(s')
    ```
Improved MHA*

procedure MAIN()
   OPEN ← ∅
   CLOSED_a ← ∅
   \( g(s_{\text{start}}) \leftarrow 0, \ g(s_{\text{goal}}) \leftarrow \infty \)
   Insert \( s_{\text{start}} \) in OPEN with PRIORITY(\( s_{\text{start}} \))
   while not TERM-CRITERION(\( s_{\text{goal}} \)) do
      if OPEN.EMPTY() then return null
      \( s_a \leftarrow \) OPEN.TOP()
      EXPANDSTATE(\( s_a \))
      CLOSED_a ← CLOSED_a ∪ \( \{s_a\} \)
   return solution path
Improved MHA*

```plaintext
procedure MAIN()
    OPEN ← \emptyset
    CLOSED_a ← \emptyset
    g(s_{\text{start}}) ← 0, g(s_{\text{goal}}) ← \infty
    Insert s_{\text{start}} in OPEN with PRIORITY(s_{\text{start}})
    while not \text{TERM-CRITERION}(s_{\text{goal}}) do
        if OPEN.\text{EMPTY}() then \text{return null}
        s_a ← OPEN.\text{TOP}()
        EXPANDSTATE(s_a)
        CLOSED_a ← CLOSED_a \cup \{s_a\}
    \text{return} solution path
```
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    g(s_{start}) ← 0, g(s_{goal}) ← ∞
    Insert s_{start} in OPEN with PRIORITY(s_{start})
    while not TERM-CRITERION(s_{goal}) do
        if OPEN.EMPTY() then return null

        s_a ← OPEN.TOP()
        EXPANDSTATE(s_a)
        CLOSED_a ← CLOSED_a ∪ \{s_a\}
    
    return solution path
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procedure MAIN()
    OPEN ← ∅
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    \( g(s_{\text{start}}) \leftarrow 0 \), \( g(s_{\text{goal}}) \leftarrow \infty \)
    Insert \( s_{\text{start}} \) in OPEN with PRIORITY(\( s_{\text{start}} \))
    while not TERM-CRITERION(\( s_{\text{goal}} \)) do
        if OPEN.EMPTY() then return null
        for \( i = 1, \ldots, n \) do
            \( s_i \leftarrow \arg \min_{s \in \text{OPEN}} \text{RANK}(s, i) \)
            EXPANDSTATE(\( s_i \))
        \( s_a \leftarrow \text{OPEN.TOP()} \)
        EXPANDSTATE(\( s_a \))
        CLOSED_a ← CLOSED_a \cup \{s_a\}
    return solution path

N inadmissible heuristics
Improved MHA*

procedure MAIN()
    OPEN ← ∅
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    \( g(s_{\text{start}}) \leftarrow 0, \ g(s_{\text{goal}}) \leftarrow \infty \)
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            \( s_i \leftarrow \arg\min_{s \in \text{OPEN}} \text{RANK}(s, i) \)
            EXPANDSTATE(\( s_i \))
        \( s_a \leftarrow \text{OPEN.TOP()} \)
        EXPANDSTATE(\( s_a \))
        CLOSED_a ← CLOSED_a ∪ \{\( s_a \)\}
    return solution path

N inadmissible heuristics

if \( h_i \) calibrated
\[
\text{RANK}(s, i) = g(s) + w \cdot h_i(s)
\]
else
\[
\text{RANK}(s, i) = h_i(s)
\]
Improved MHA*

procedure MAIN()
  OPEN ← ∅
  CLOSED_a ← ∅, CLOSED_u ← ∅
  \( g(s_{start}) ← 0 \), \( g(s_{goal}) ← ∞ \)
  Insert \( s_{start} \) in OPEN with PRIORITY(\( s_{start} \))
  while not TERM-CRITERION(\( s_{goal} \)) do
    if OPEN.EMPTY() then return null
    for \( i = 1, \ldots, n \) do
      \( s_i ← \arg\min_{s \in OPEN} RANK(s, i) \)
      EXPANDSTATE(\( s_i \))
      CLOSED_u ← CLOSED_u \cup \{s_i\}
      \( s_a ← \text{OPEN.TOP()} \)
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      CLOSED_a ← CLOSED_a \cup \{s_a\}
  return solution path

N inadmissible heuristics

if \( h_i \) calibrated
  \( RANK(s, i) = g(s) + w \cdot h_i(s) \)
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Improved MHA*

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    Insert s_start in OPEN with PRIORITY(s_start)
    while not TERM-CRITERION(s_goal) do
        if OPEN.EMPTY() then return null
        for i = 1, ..., n do
            s_i ← arg min_{s ∈ OPEN} RANK(s, i)
            EXPANDSTATE(s_i)
            CLOSED_u ← CLOSED_u ∪ {s_i}
        end for
        s_a ← OPEN.TOP()
        EXPANDSTATE(s_a)
        CLOSED_a ← CLOSED_a ∪ {s_a}
    end while
    return solution path
Improved MHA*

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```

\[ f(s) \leq w \cdot f(s_a) \]
Improved MHA*

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procedure MAIN()
    OPEN ← ∅
    CLOSED_a ← ∅, CLOSED_u ← ∅
    g(s_{start}) ← 0, g(s_{goal}) ← ∞
    Insert s_{start} in OPEN with PRIORITY(s_{start})
    while not TERM-CRITERION(s_{goal}) do
        if OPEN.EMPTY() then return null
        for i = 1, ..., n do
            s_i ← arg min_{s ∈ P-SET} RANK(s, i)
            EXPANDSTATE(s_i)
            CLOSED_u ← CLOSED_u ∪ \{s_i\}
        s_a ← OPEN.TOP()
        EXPANDSTATE(s_a)
        CLOSED_a ← CLOSED_a ∪ \{s_a\}
    return solution path
```

\[
f(s) ≤ w \cdot f(s_a)
\]

P-SET ← \{s : s ∈ OPEN ∧ s ∉ CLOSED_u ∧ P-CRITERION(s)\}
**Improved MHA* Variants**

```plaintext
procedure MAIN()
    OPEN ← ∅
    CLOSED_a ← ∅, CLOSED_u ← ∅
    g(s_{start}) ← 0, g(s_{goal}) ← ∞
    Insert s_{start} in OPEN with PRIORITY(s_{start})
    while not TERM-CRITERION(s_{goal}) do
        if OPEN.EMPTY() then return null
        for i = 1, ..., n do
            s_i ← \arg\min_{s \in P-SET} RANK(s, i)
            EXPANDSTATE(s_i)
            CLOSED_u ← CLOSED_u ∪ \{s_i\}
        s_a ← OPEN.TOP()
        EXPANDSTATE(s_a)
        CLOSED_a ← CLOSED_a ∪ \{s_a\}
    return solution path
```

\[ f(s) \leq w \cdot f(s_a) \]

\[ P-SET ← \{ s : s \in OPEN \land s \notin CLOSED_u \land P-CRITERION(s) \} \]
Improved MHA* Variants

**procedure** MAIN()

```
OPEN ← ∅
CLOSED_a ← ∅, CLOSED_u ← ∅
g(s_{start}) ← 0, g(s_{goal}) ← ∞
Insert s_{start} in OPEN with PRIORITY(s_{start})
while not TERM-CRITERION(s_{goal}) do
    if OPEN.EMPTY() then return null
    for i = 1, ..., n do
        s_i ← arg min_{s ∈ P-SET} RANK(s, i)
        EXPANDSTATE(s_i)
        CLOSED_u ← CLOSED_u ∪ {s_i}
    s_a ← OPEN.TOP()
    EXPANDSTATE(s_a)
    CLOSED_a ← CLOSED_a ∪ {s_a}
return solution path
```

**Decision points**

- **PRIORITY(s)**
- **TERM-CRITERION(s)**
- **P-CRITERION(s)**

\[ f(s) \leq w \cdot f(s_a) \]

\[ P-SET ← \{ s : s ∈ OPEN \land s \notin CLOSED_u \land P-CRITERION(s) \} \]
# Improved MHA* Variants

<table>
<thead>
<tr>
<th></th>
<th>PRIORITY($s$)</th>
<th>P-CRITERION($s$)</th>
<th>TERM-CRITERION($s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MHA</strong>*++</td>
<td>$g(s) + w \cdot h(s)$</td>
<td>$g(s) + h(s) \leq \max_{s \in \text{CLOSED}_a} \text{PRIORITY}(s)$</td>
<td>$g(s) \leq \max_{s \in \text{CLOSED}_a} \text{PRIORITY}(s)$</td>
</tr>
<tr>
<td><strong>Focal-MHA</strong>*</td>
<td>$g(s) + h(s)$</td>
<td>$g(s) + h(s) \leq \min_{s \in \text{OPEN}} \text{PRIORITY}(s)$</td>
<td>$g(s) \leq \min_{s \in \text{OPEN}} \text{PRIORITY}(s)$</td>
</tr>
<tr>
<td><strong>Unconstrained MHA</strong>*</td>
<td>$g(s) + w \cdot h(s)$</td>
<td>none</td>
<td>$g(s) \leq \max_{s \in \text{CLOSED}_a} \text{PRIORITY}(s)$</td>
</tr>
</tbody>
</table>
## Improved MHA* Properties

<table>
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<tr>
<th>MHA++</th>
<th>Focal-MHA*</th>
<th>Unconstrained MHA*</th>
</tr>
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</table>


 Improved MHA* Properties

MHA*+++  Focal-MHA*  Unconstrained MHA*

Theorem 1

For all three variants, solution is bounded sub-optimal

\[ g(s_{goal}) \leq w \cdot g^*(s_{goal}) \]
Improved MHA* Properties

MHA*++  Focal-MHA*  Unconstrained MHA*

Theorem 1
For all three variants, solution is bounded sub-optimal
\[ g(s_{goal}) \leq w \cdot g^*(s_{goal}) \]

Theorem 2
For all three variants, no state is expanded more than twice
Improved MHA* Properties

MHA*++  Focal-MHA*  Unconstrained MHA*

Theorem 1
For all three variants, solution is bounded sub-optimal
\[ g(s_{goal}) \leq w \cdot g^*(s_{goal}) \]

Theorem 2
For all three variants, no state is expanded more than twice

Theorem 3
For MHA*++ and Focal-MHA*, no state with
\[ g(s) + h(s) > w \cdot g^*(s_{goal}) \] will be expanded
Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot
Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot

1 consistent heuristic + 19 inadmissible heuristics

100 trials with random start-goal pairs

Trial ‘success’ if solution found in 1 minute

Comparisons with MH-GBFS\textsuperscript{[1]}, RRT-Connect\textsuperscript{[2]}

\textsuperscript{[1]} Roger and Helmert, ICAPS 2010  \hspace{1cm} \textsuperscript{[2]} Kuffner and LaValle, ICRA 2000
Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot

<table>
<thead>
<tr>
<th></th>
<th>++</th>
<th>Focal</th>
<th>Uncons</th>
<th>Original</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success (%)</td>
<td>84</td>
<td>75</td>
<td>84</td>
<td>61</td>
</tr>
<tr>
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Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot

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Improved MHA*: Experiments

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MHA*: success rate goes down with w
### Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot

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### Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot

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**RRT-C**: success rate affected by narrow passages
**Improved MHA*: Experiments**

12 DoF full-body motion planning for the PR2 robot

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Run time: RRT-C fast when it succeeds
## Improved MHA*: Experiments

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MH-GBFS, RRT-C: No bounds on quality
Improved MHA*: Experiments

Sliding Tile Puzzles: 8x8, 9x9 and 10x10

Consistent heuristic: Manhattan distance + linear conflicts

8 Inadmissible heuristics: ‘waypoint’ pattern database heuristics

100 random trials, 5 minute timeout
Improved MHA*: Experiments

Solution cost (#moves) vs. Sub-optimality bound w

- 8x8
- 9x9
- 10x10

- 2600
- 2275
- 1950
- 1625
- 1300

- 50
- 10
- 5
Improved MHA*: Experiments

9x9 Sliding Tile Puzzles

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Summary

Improved MHA*: allows use of multiple inadmissible, uncalibrated heuristics

Completeness, bounded sub-optimality, bounded re-expansions

Experimental validation on full-body motion planning and large sliding tile puzzles

Extensions to round-robin: Meta-A*, Dynamic Thompson Sampling

Future work: parallelization, anytime