Improved Multi-Heuristic A*

for searching with uncalibrated heuristics

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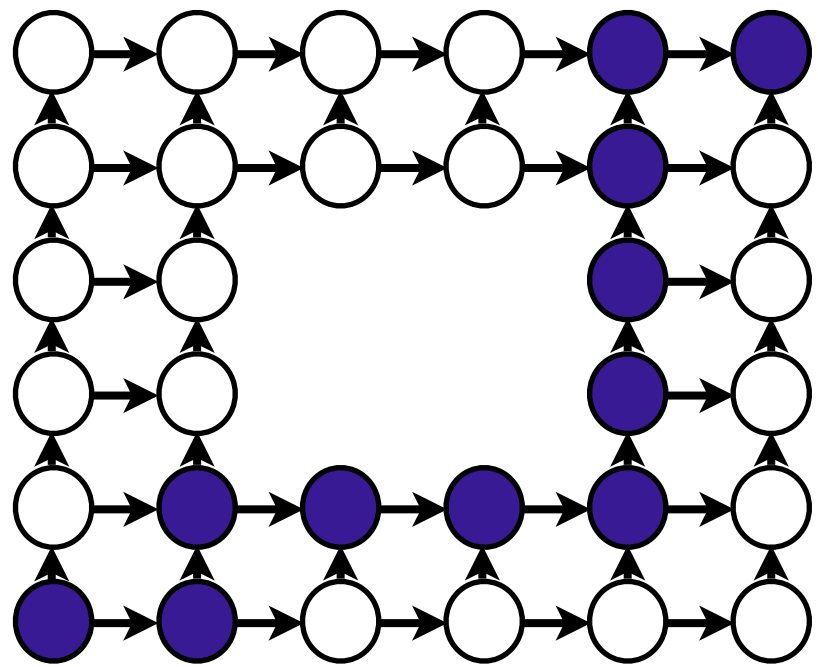
PR2
STANFORD

goal



start

Admissible Heuristics

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Solution quality

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Combining multiple heuristics

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Planning time

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This work: Improved Multi-Heuristic A*

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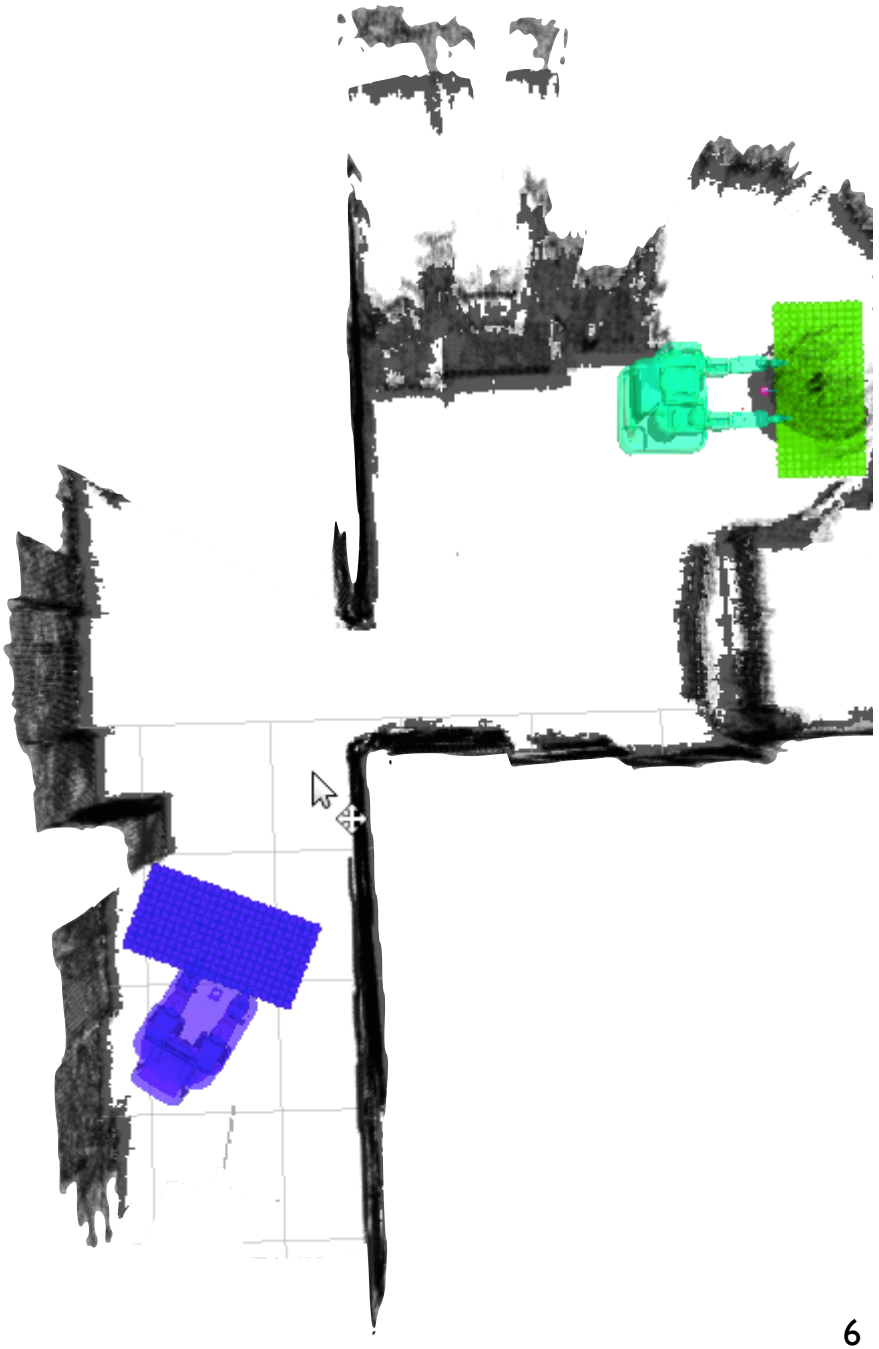
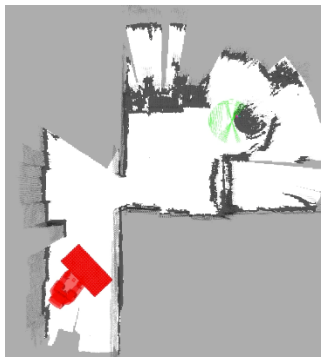
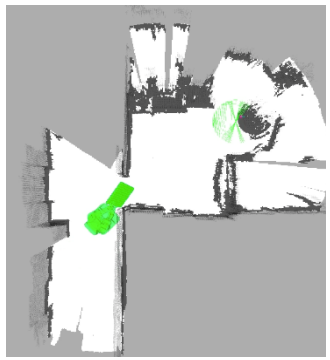
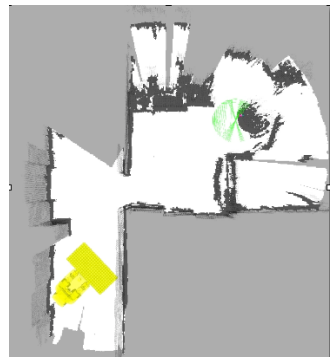
12 DoF Full-body Motion Planning



Calibration Problem







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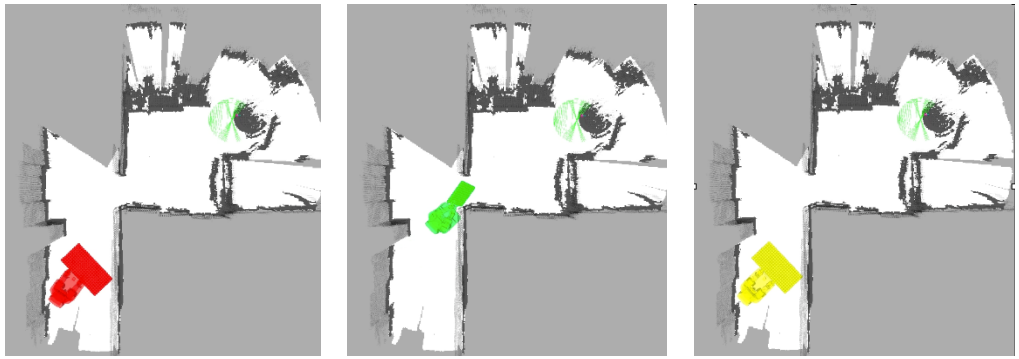

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h_1: base distance (m)
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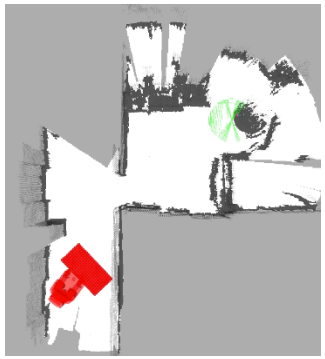

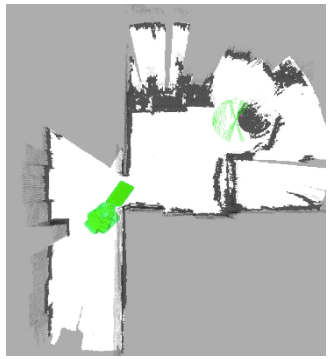

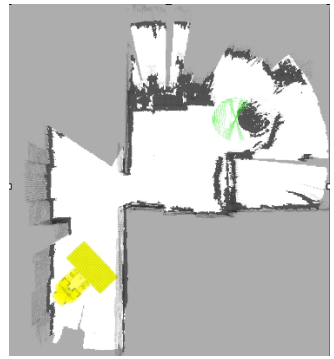

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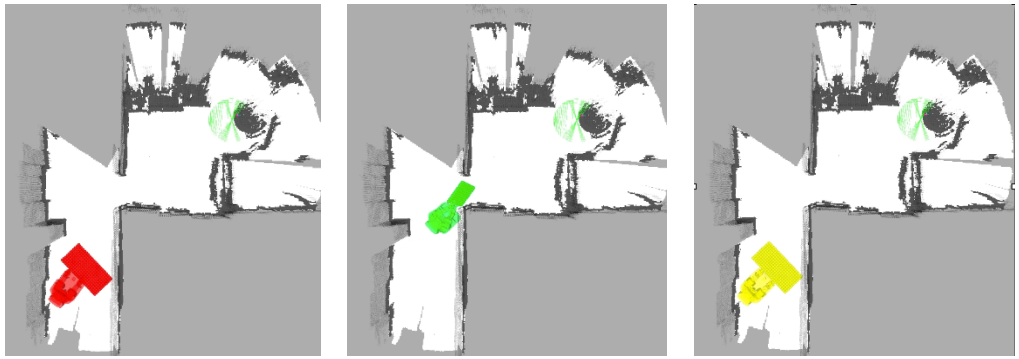

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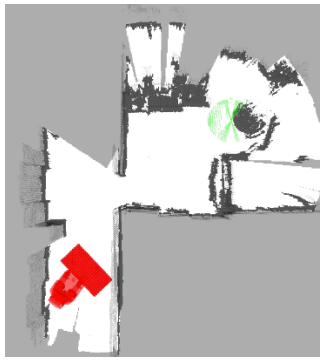
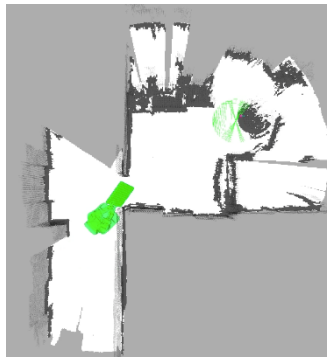
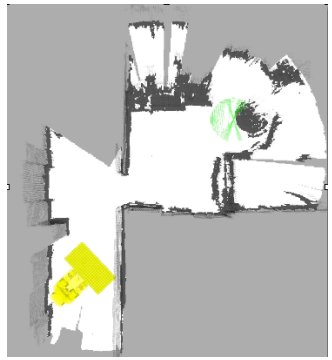
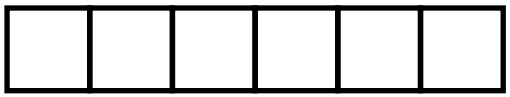
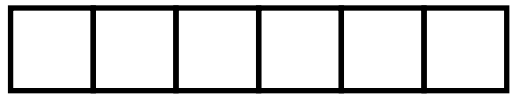
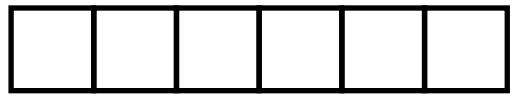
Combining g and h additively not meaningful

Related Work

Related Work

Multi-Heuristic Greedy Best-First Search [Roger and Helmert, ICAPS 2010]

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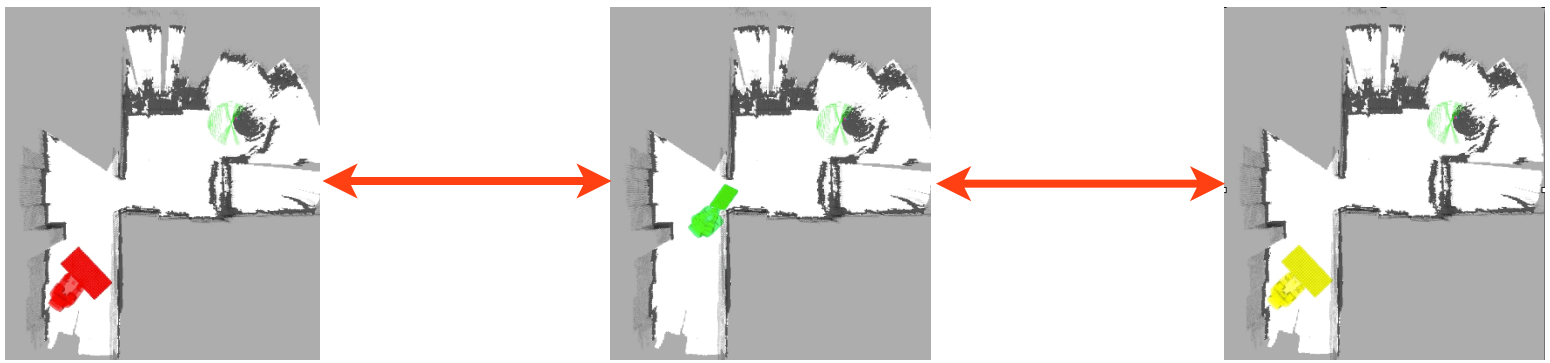

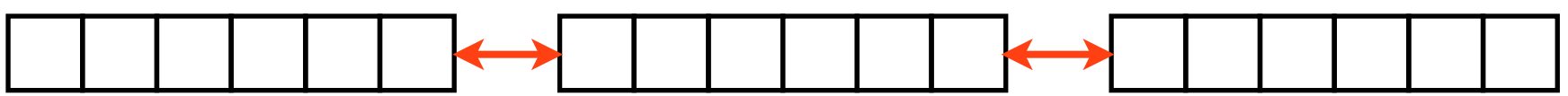

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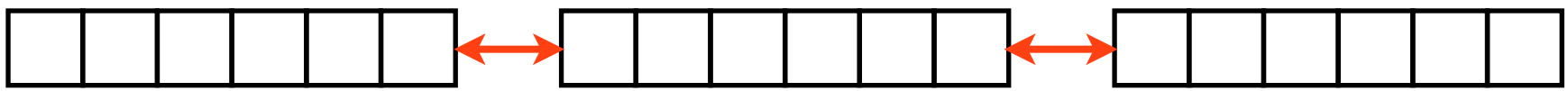

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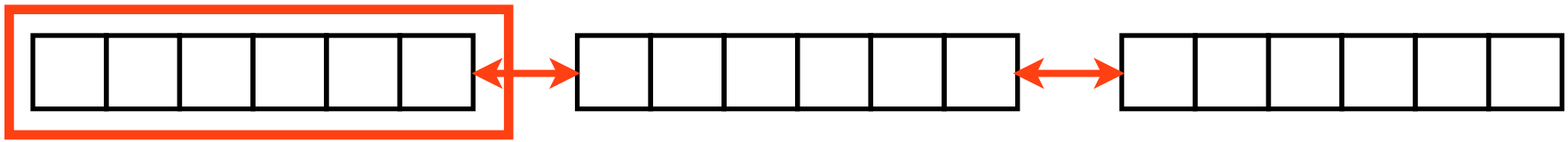

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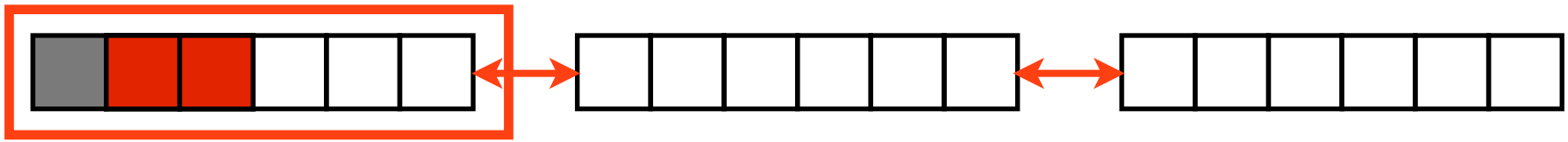

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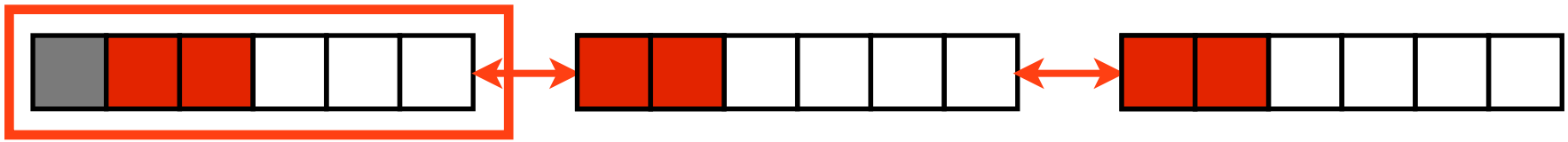

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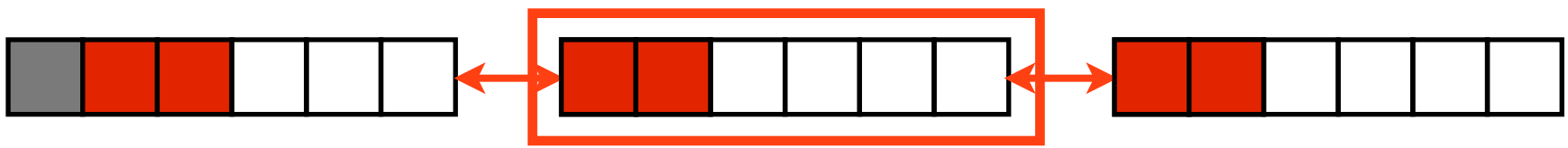

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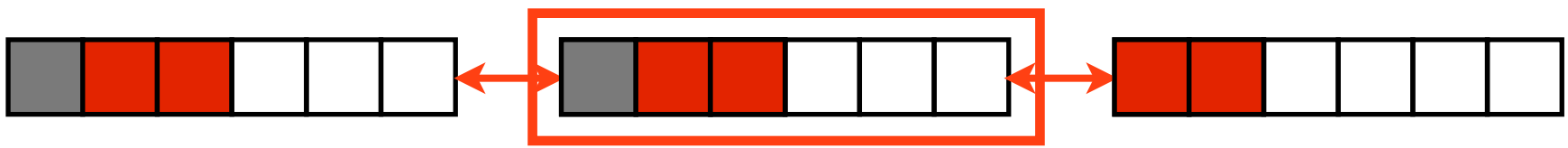

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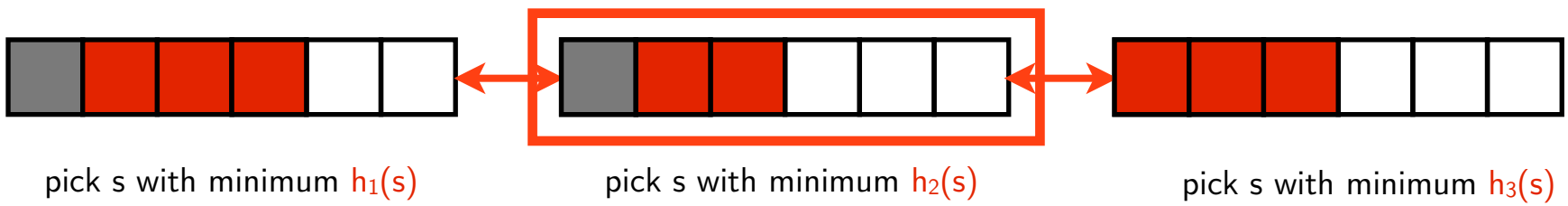

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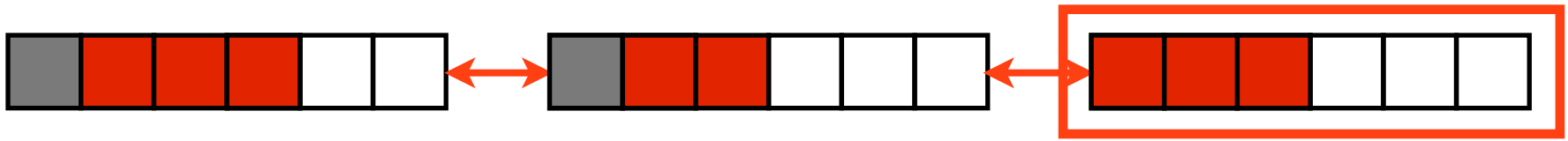

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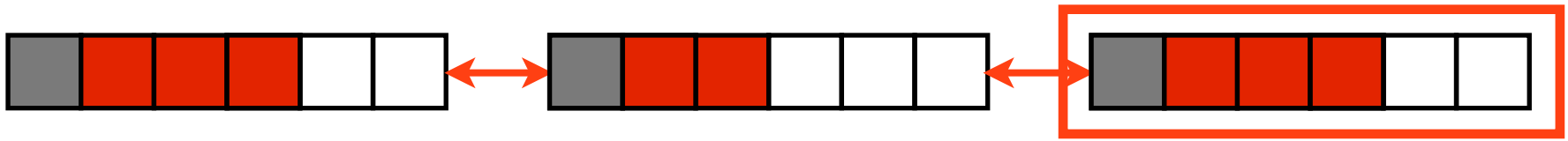

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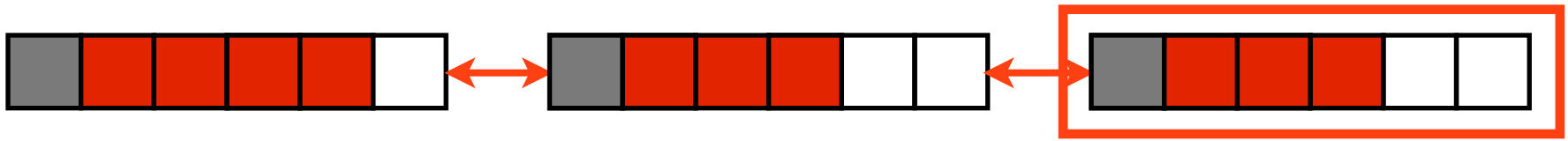

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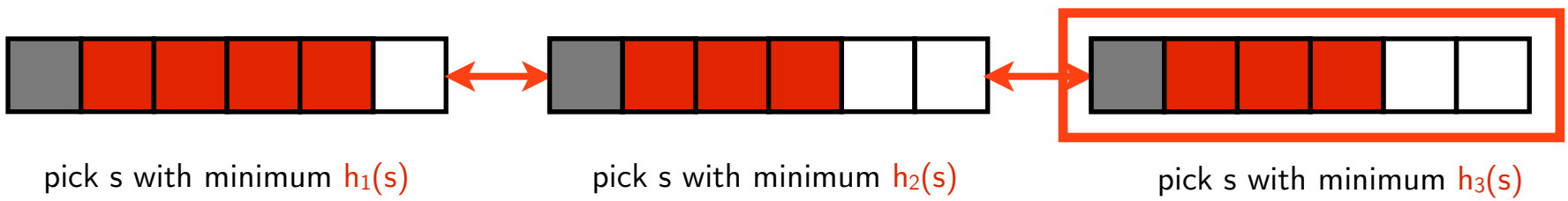

No calibration problem

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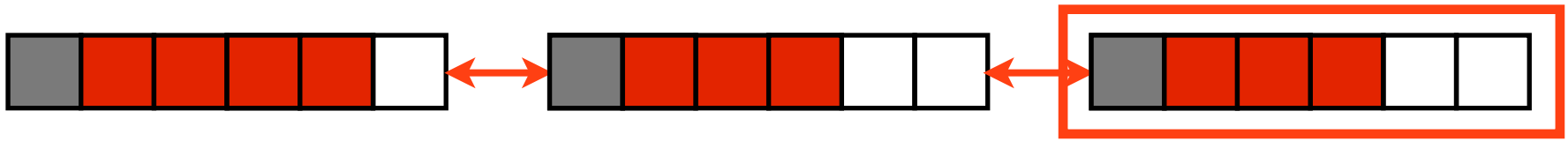

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No calibration problem

No bounds on solution quality

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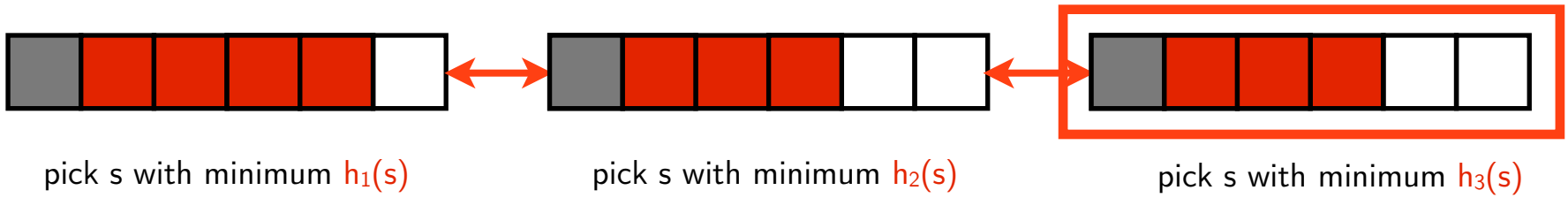

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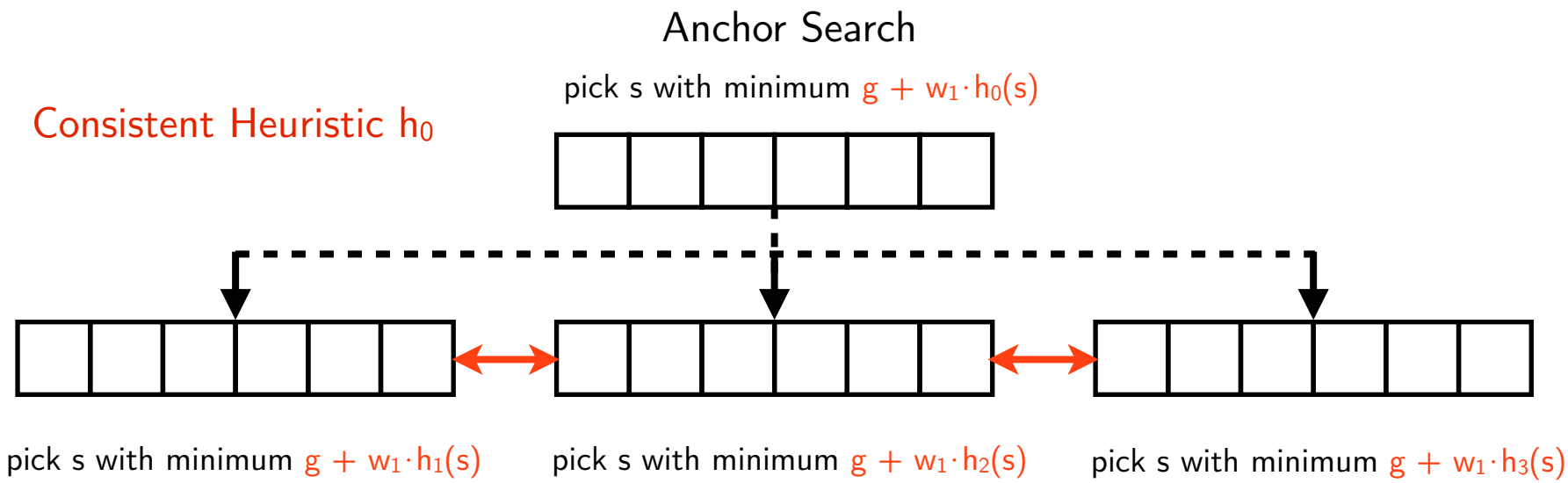

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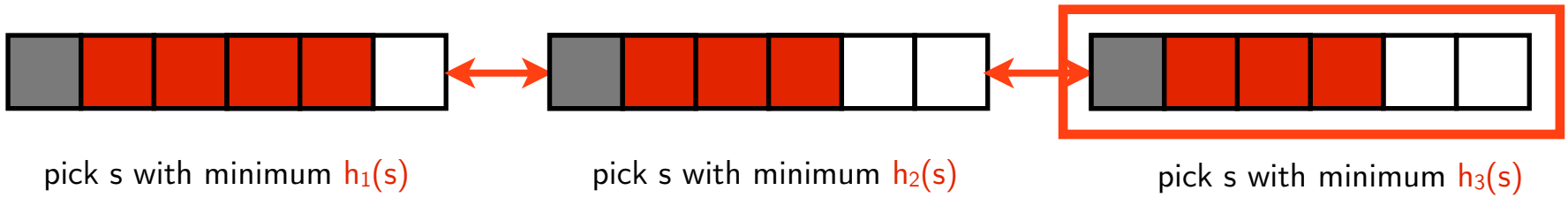

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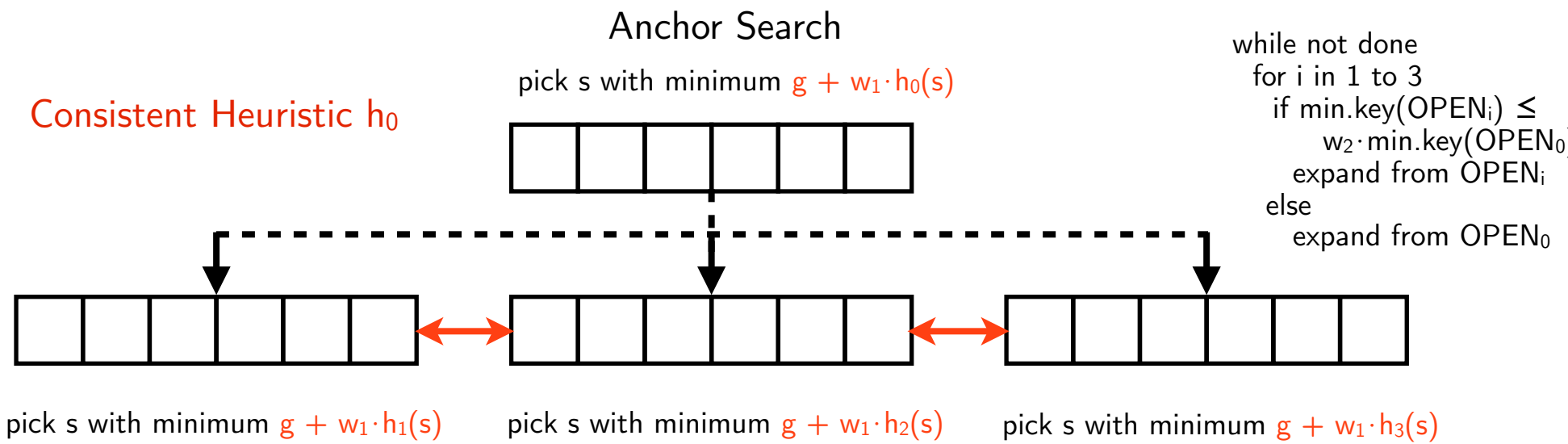

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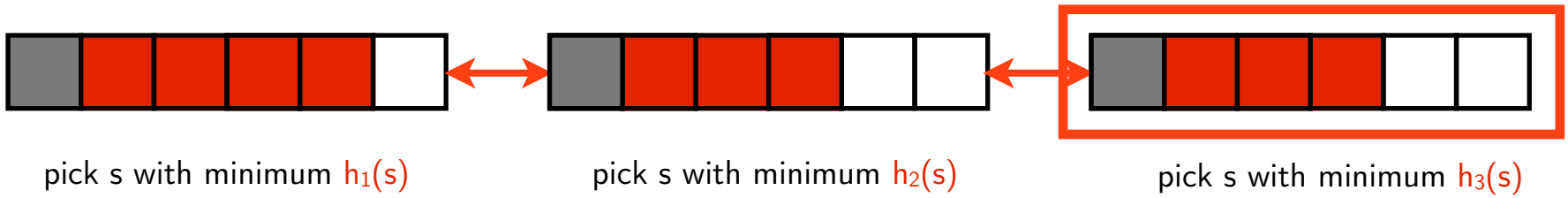

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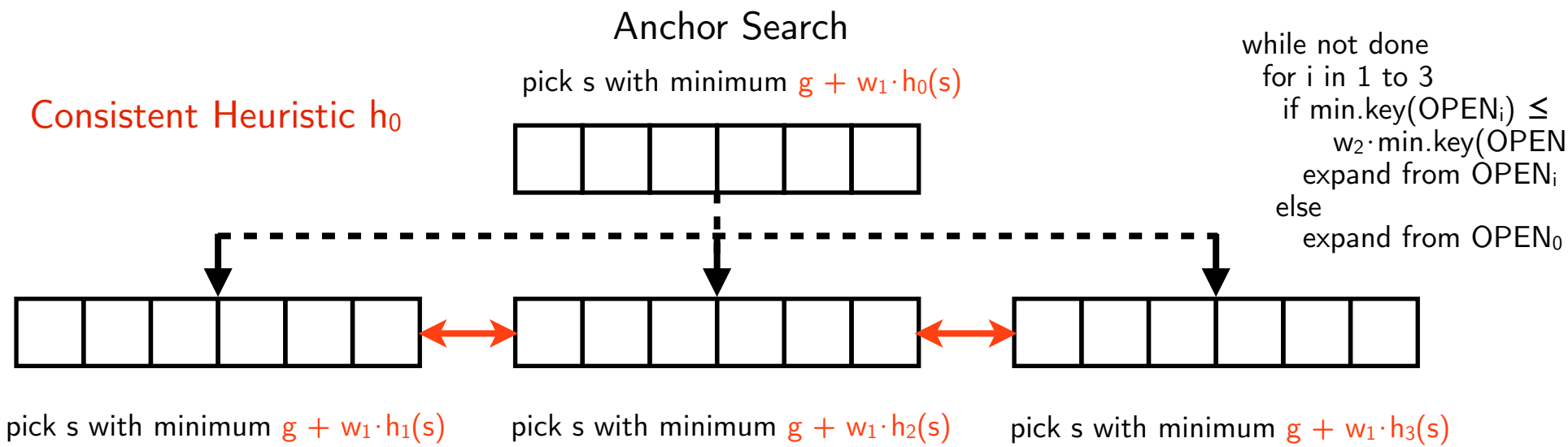

Bounds on solution quality, re-expansions

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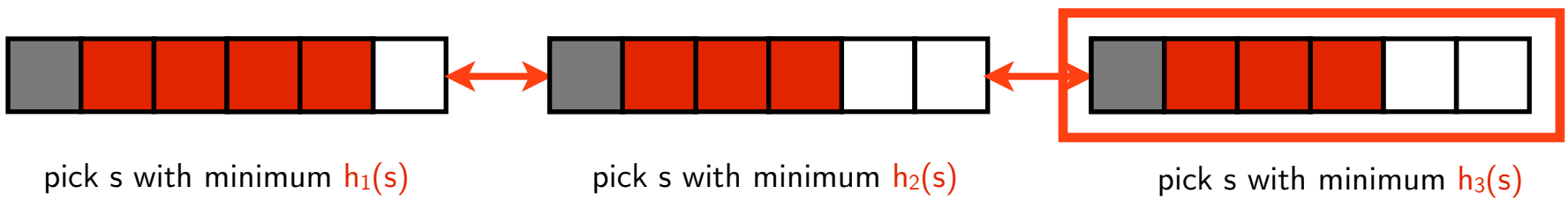

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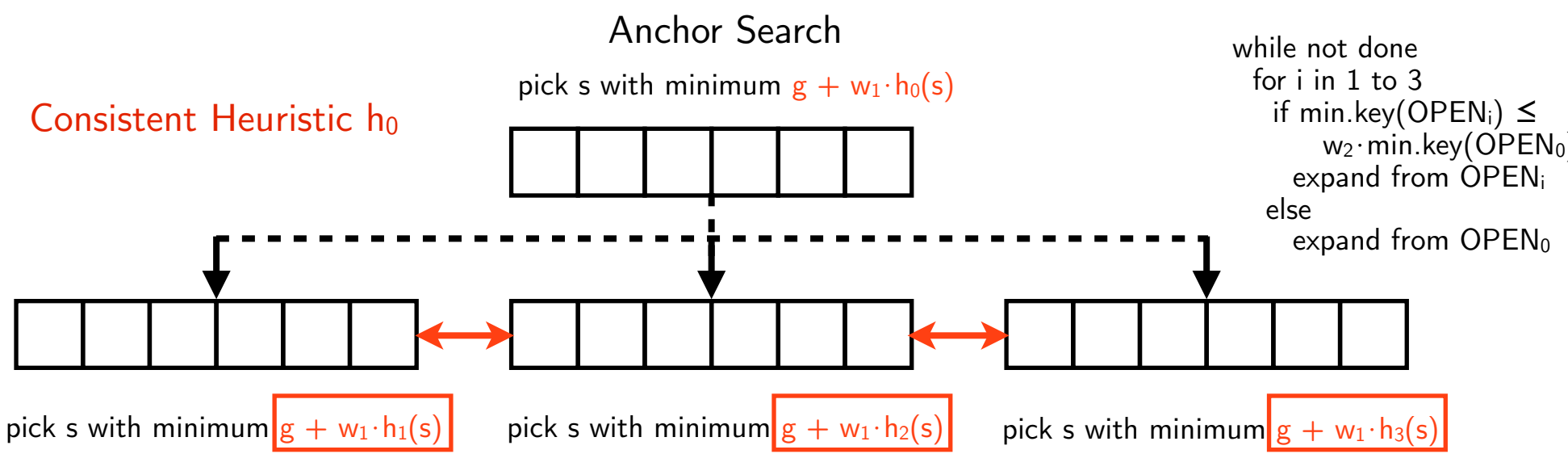
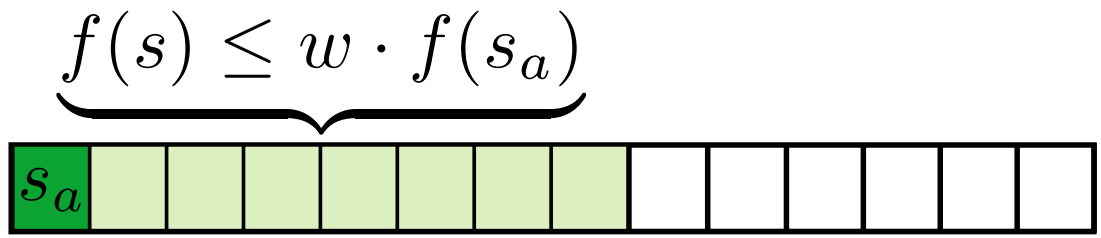


Calibration problem

Bounds on solution quality, re-expansions

Related Work

Focal-A* [Pearl and Kim, PAMI 1982]



Use distance-to-go estimate to greedily choose from FOCAL list

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$$\underbrace{f(s) \leq w \cdot f(s_a)}$$

Use distance-to-go estimate to greedily choose from FOCAL list

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Use secondary inadmissible heuristic to remove bias in construction of FOCAL list

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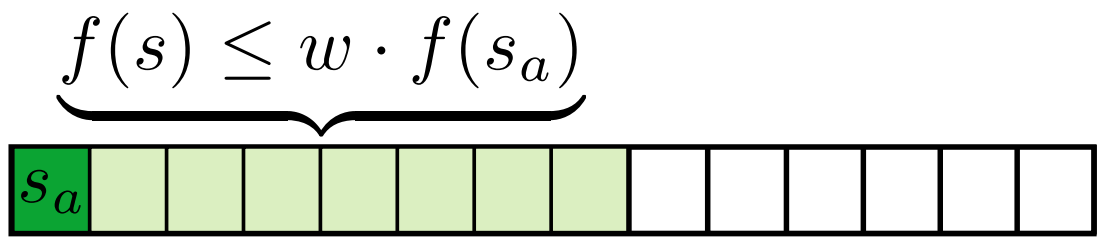

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Use secondary inadmissible heuristic to remove bias in construction of FOCAL list

No calibration problem

Do not combine multiple heuristics

No bounds on re-expansions

Improved MHA*

Multi-heuristic search framework

View shared-search as single queue search: interleave ‘admissible’ and ‘inadmissible’ expansions

Prioritize based on uncalibrated heuristics: no additive combination

Derive bounds from consistent heuristic

Improved MHA*

procedure MAIN()
OPEN $\leftarrow \emptyset$
CLOSED$_a$ $\leftarrow \emptyset$
$g(s_{start}) \leftarrow 0,\ g(s_{goal}) \leftarrow \infty$
Insert s_{start} in OPEN with PRIORITY(s_{start})
while not TERM-CRITERION(s_{goal}) **do**
if OPEN.EMPTY() **then return null**
$s_a \leftarrow$ OPEN.TOP()
EXPANDSTATE(s_a)
CLOSED$_a \leftarrow$ CLOSED$_a \cup \{s_a\}$
return solution path

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PRIORITY(s)

$g(s) + h(s)$

or

$g(s) + w \cdot h(s)$

$h(s)$: consistent heuristic

Improved MHA*

procedure MAIN()
OPEN $\leftarrow \emptyset$
CLOSED$_a$ $\leftarrow \emptyset$
$g(s_{start}) \leftarrow 0,\ g(s_{goal}) \leftarrow \infty$
Insert s_{start} in OPEN with PRIORITY(s_{start})
while not TERM-CRITERION(s_{goal}) **do**
if OPEN.EMPTY() **then return null**
$s_a \leftarrow$ OPEN.TOP()
EXPANDSTATE(s_a)
CLOSED$_a \leftarrow$ CLOSED$_a \cup \{s_a\}$
return solution path

procedure EXPANDSTATE(s)
Remove s from OPEN
for all $s' \in$ SUCC(s) **do**
if s' was not seen before **then**
$g(s') \leftarrow \infty$
if $g(s') > g(s) + c(s, s')$ **then**
$g(s') \leftarrow g(s) + c(s, s')$
if $s \notin$ CLOSED$_a$ **then**
Insert/Update s' in OPEN
with PRIORITY(s')

Improved MHA*

procedure MAIN()
  OPEN $\leftarrow \emptyset$
  CLOSED$_a$ $\leftarrow \emptyset$
  $g(s_{start}) \leftarrow 0,\ g(s_{goal}) \leftarrow \infty$
  Insert s_{start} in OPEN with PRIORITY(s_{start})
  while not TERM-CRITERION(s_{goal}) **do**
    if OPEN.EMPTY() **then return null**
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  Insert s_{start} in OPEN with PRIORITY(s_{start})
  while not TERM-CRITERION(s_{goal}) **do**
    if OPEN.EMPTY() **then return null**
    for $i = 1, \ldots, n$ **do**
      $s_i \leftarrow \arg\min_{s \in \text{OPEN}}$ RANK(s, i)
      EXPANDSTATE(s_i)
    $s_a \leftarrow$ OPEN.TOP()
    EXPANDSTATE(s_a)
    CLOSED$_a \leftarrow$ CLOSED$_a \cup \{s_a\}$
  return solution path

N inadmissible heuristics

Improved MHA*

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      EXPANDSTATE(s_i)
    $s_a \leftarrow$ OPEN.TOP()
    EXPANDSTATE(s_a)
    CLOSED$_a$ $\leftarrow$ CLOSED$_a \cup \{s_a\}$
  return solution path

N inadmissible heuristics

if h_i calibrated

$$\text{RANK}(s, i) = g(s) + w \cdot h_i(s)$$

else

$$\text{RANK}(s, i) = h_i(s)$$

Improved MHA*

procedure MAIN()
  OPEN $\leftarrow \emptyset$
  CLOSED$_a \leftarrow \emptyset$, CLOSED$_u \leftarrow \emptyset$
  $g(s_{start}) \leftarrow 0$, $g(s_{goal}) \leftarrow \infty$
  Insert s_{start} in OPEN with PRIORITY(s_{start})
  while not TERM-CRITERION(s_{goal}) **do**
    if OPEN.EMPTY() **then return null**
    for $i = 1, \ldots, n$ **do**
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      EXPANDSTATE(s_i)
      CLOSED$_u \leftarrow$ CLOSED$_u \cup \{s_i\}$
    $s_a \leftarrow$ OPEN.TOP()
    EXPANDSTATE(s_a)
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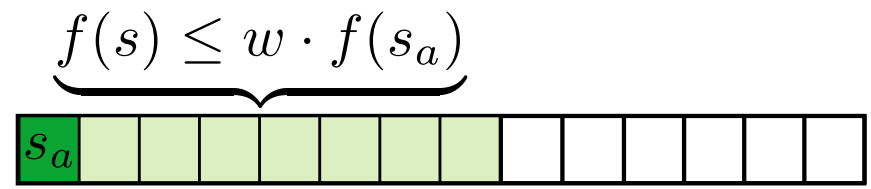

Improved MHA*

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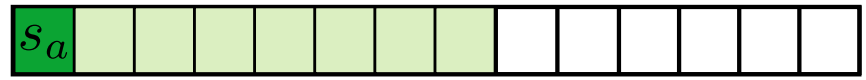
  $g(s_{start}) \leftarrow 0$, $g(s_{goal}) \leftarrow \infty$
  Insert s_{start} in OPEN with PRIORITY(s_{start})
  while not TERM-CRITERION(s_{goal}) **do**
    if OPEN.EMPTY() **then return null**
    for $i = 1, \ldots, n$ **do**
      $s_i \leftarrow \arg\min_{s \in \text{P-SET}}$ RANK(s, i)
      EXPANDSTATE(s_i)
      CLOSED$_u \leftarrow$ CLOSED$_u \cup \{s_i\}$
    $s_a \leftarrow$ OPEN.TOP()
    EXPANDSTATE(s_a)
    CLOSED$_a \leftarrow$ CLOSED$_a \cup \{s_a\}$
  return solution path

$f(s) \leq w \cdot f(s_a)$



P-SET $\leftarrow \{s : s \in$ OPEN
$\wedge\ s \notin$ CLOSED$_u$
$\wedge$ P-CRITERION$(s)\}$

Improved MHA* Variants

procedure MAIN()
  OPEN $\leftarrow \emptyset$
  CLOSED$_a \leftarrow \emptyset$, CLOSED$_u \leftarrow \emptyset$
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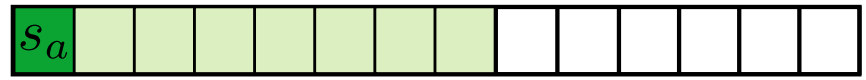
  while not TERM-CRITERION(s_{goal}) **do**
    if OPEN.EMPTY() **then return null**
    for $i = 1, \ldots, n$ **do**
      $s_i \leftarrow \arg\min_{s \in \text{P-SET}}$ RANK(s, i)
      EXPANDSTATE(s_i)
      CLOSED$_u \leftarrow$ CLOSED$_u \cup \{s_i\}$
    $s_a \leftarrow$ OPEN.TOP()
    EXPANDSTATE(s_a)
    CLOSED$_a \leftarrow$ CLOSED$_a \cup \{s_a\}$
  return solution path

$f(s) \leq w \cdot f(s_a)$



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Improved MHA* Variants

procedure MAIN()
  OPEN $\leftarrow \emptyset$
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  Insert s_{start} in OPEN with PRIORITY(s_{start})
  while not TERM-CRITERION(s_{goal}) **do**
    if OPEN.EMPTY() **then return null**
    for $i = 1, \ldots, n$ **do**
      $s_i \leftarrow \arg\min_{s \in \text{P-SET}}$ RANK(s, i)
      EXPANDSTATE(s_i)
      CLOSED$_u \leftarrow$ CLOSED$_u \cup \{s_i\}$
    $s_a \leftarrow$ OPEN.TOP()
    EXPANDSTATE(s_a)
    CLOSED$_a \leftarrow$ CLOSED$_a \cup \{s_a\}$
  return solution path

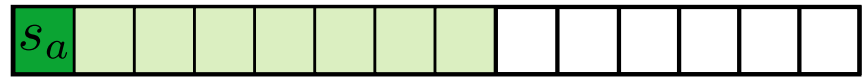
Decision points

PRIORITY(s)

TERM-CRITERION(s)

P-CRITERION(s)

$f(s) \leq w \cdot f(s_a)$



P-SET $\leftarrow \{s : s \in$ OPEN
$\wedge\ s \notin$ CLOSED$_u$
$\wedge$ P-CRITERION$(s)\}$

Improved MHA* Variants

	$\textsc{Priority}(s)$	$\textsc{P-Criterion}(s)$	$\textsc{Term-Criterion}(s)$
MHA*++	$g(s) + w \cdot h(s)$	$g(s) + h(s) \leq \max_{s \in \text{CLOSED}_a} \textsc{Priority}(s)$	$g(s) \leq \max_{s \in \text{CLOSED}_a} \textsc{Priority}(s)$
Focal-MHA*	$g(s) + h(s)$	$g(s) + h(s) \leq w \cdot \min_{s \in \text{OPEN}} \textsc{Priority}(s)$	$g(s) \leq w \cdot \min_{s \in \text{OPEN}} \textsc{Priority}(s)$
Unconstrained MHA*	$g(s) + w \cdot h(s)$	none	$g(s) \leq \max_{s \in \text{CLOSED}_a} \textsc{Priority}(s)$

Improved MHA* Properties

MHA*++

Focal-MHA*

Unconstrained MHA*

Improved MHA* Properties

MHA*++ **Focal-MHA*** **Unconstrained MHA***

Theorem 1

For all three variants, solution is bounded sub-optimal

$$g(s_{goal}) \leq w \cdot g^*(s_{goal})$$

Improved MHA* Properties

MHA*++ **Focal-MHA*** **Unconstrained MHA***

Theorem 1

For all three variants, solution is bounded sub-optimal

$$g(s_{goal}) \leq w \cdot g^*(s_{goal})$$

Theorem 2

For all three variants, no state is expanded more than twice

Improved MHA* Properties

MHA*++ **Focal-MHA*** **Unconstrained MHA***

Theorem 1

For all three variants, solution is bounded sub-optimal

$$g(s_{goal}) \leq w \cdot g^*(s_{goal})$$

Theorem 2

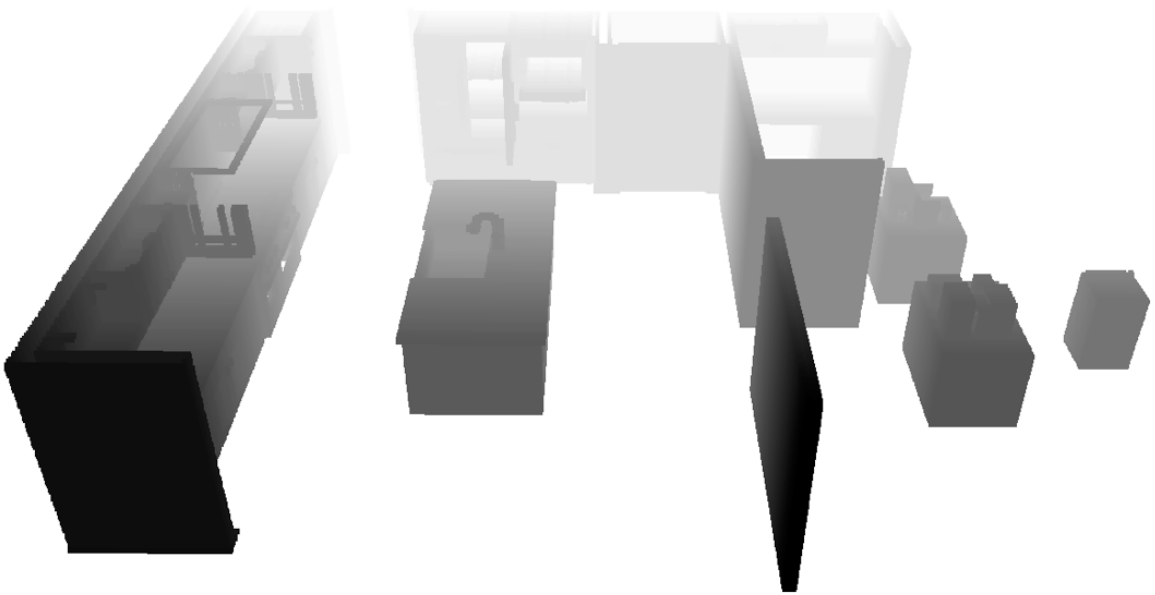
For all three variants, no state is expanded more than twice

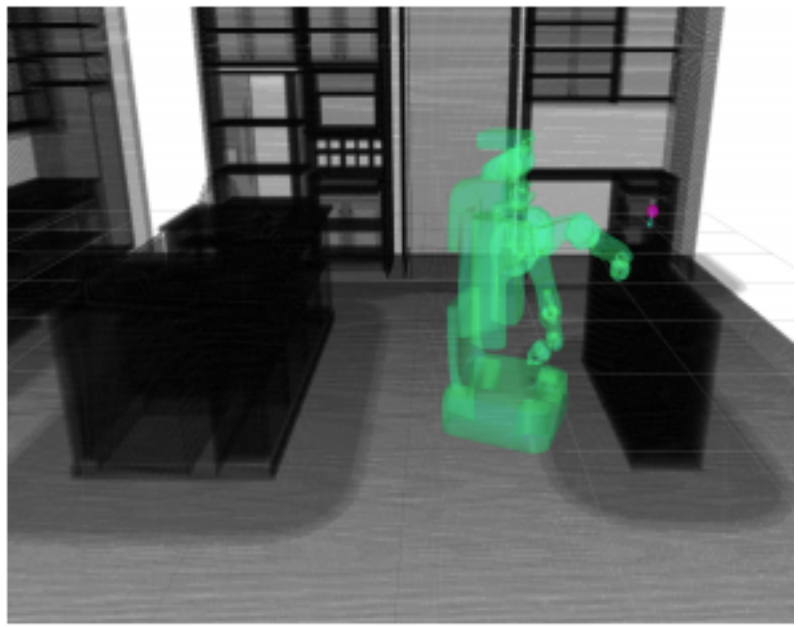
Theorem 3

For MHA*++ and Focal-MHA*, no state with $g(s) + h(s) > w \cdot g^*(s_{goal})$ will be expanded

Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot





Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot

1 consistent heuristic + 19 inadmissible heuristics

100 trials with random start-goal pairs

Trial 'success' if solution found in 1 minute

Comparisons with MH-GBFS[1], RRT-Connect[2]

[1] Roger and Helmert, ICAPS 2010

[2] Kuffner and LaValle, ICRA 2000

Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot

	$w = 100$			
	++	Focal	Uncons	Original
Success (%)	84	75	84	61
States Expanded	2415	3058	2415	5179
Run Time (s)	36.89	47.44	36.98	75.63
Base Cost (m)	5.33	5.47	5.33	5.47
Arm Cost (rad)	6.32	6.45	6.32	6.02

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Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot

	$w = 100$				$w = 10$				$w = 5$			
	++	Focal	Uncons	Original	++	Focal	Uncons	Original	++	Focal	Uncons	Original
Success (%)	84	75	84	61	83	74	83	0	66	74	60	0
States Expanded	2415	3058	2415	5179	3293	3086	3227	-	2378	3086	2472	-
Run Time (s)	36.89	47.44	36.98	75.63	47.60	47.88	46.96	-	37.92	48.08	38.31	-
Base Cost (m)	5.33	5.47	5.33	5.47	5.32	5.52	5.33	-	4.77	5.52	4.49	-
Arm Cost (rad)	6.32	6.45	6.32	6.02	6.17	6.49	6.17	-	5.70	6.49	5.41	-

Improved MHA*: Experiments

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	$w = 100$				$w = 10$				$w = 5$			
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Base Cost (m)	5.33	5.47	5.33	5.47	5.32	5.52	5.33	-	4.77	5.52	4.49	-
Arm Cost (rad)	6.32	6.45	6.32	6.02	6.17	6.49	6.17	-	5.70	6.49	5.41	-

MHA*: success rate goes down with w

Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot

	$w = 100$				$w = 10$				$w = 5$			
	++	Focal	Uncons	Original	++	Focal	Uncons	Original	++	Focal	Uncons	Original
Success (%)	84	75	84	61	83	74	83	0	66	74	60	0
States Expanded	2415	3058	2415	5179	3293	3086	3227	-	2378	3086	2472	-
Run Time (s)	36.89	47.44	36.98	75.63	47.60	47.88	46.96	-	37.92	48.08	38.31	-
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Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot

	$w = 100$				$w = 10$				$w = 5$					
	++	Focal	Uncons	Original	++	Focal	Uncons	Original	++	Focal	Uncons	Original	MH-GBFS	RRT-C
Success (%)	84	75	84	61	83	74	83	0	66	74	60	0	85	45
States Expanded	2415	3058	2415	5179	3293	3086	3227	-	2378	3086	2472	-	2752	n/a
Run Time (s)	36.89	47.44	36.98	75.63	47.60	47.88	46.96	-	37.92	48.08	38.31	-	41.98	21.95
Base Cost (m)	5.33	5.47	5.33	5.47	5.32	5.52	5.33	-	4.77	5.52	4.49	-	5.45	5.22
Arm Cost (rad)	6.32	6.45	6.32	6.02	6.17	6.49	6.17	-	5.70	6.49	5.41	-	6.60	8.26

Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot

	$w = 100$				$w = 10$				$w = 5$					
	++	Focal	Uncons	Original	++	Focal	Uncons	Original	++	Focal	Uncons	Original	MH-GBFS	RRT-C
Success (%)	84	75	84	61	83	74	83	0	66	74	60	0	85	45
States Expanded	2415	3058	2415	5179	3293	3086	3227	-	2378	3086	2472	-	2752	n/a
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Arm Cost (rad)	6.32	6.45	6.32	6.02	6.17	6.49	6.17	-	5.70	6.49	5.41	-	6.60	8.26

RRT-C: success rate affected by narrow passages

Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot

	$w = 100$				$w = 10$				$w = 5$					
	++	Focal	Uncons	Original	++	Focal	Uncons	Original	++	Focal	Uncons	Original	MH-GBFS	RRT-C
Success (%)	84	75	84	61	83	74	83	0	66	74	60	0	85	45
States Expanded	2415	3058	2415	5179	3293	3086	3227	-	2378	3086	2472	-	2752	n/a
Run Time (s)	36.89	47.44	36.98	75.63	47.60	47.88	46.96	-	37.92	48.08	38.31	-	41.98	21.95
Base Cost (m)	5.33	5.47	5.33	5.47	5.32	5.52	5.33	-	4.77	5.52	4.49	-	5.45	5.22
Arm Cost (rad)	6.32	6.45	6.32	6.02	6.17	6.49	6.17	-	5.70	6.49	5.41	-	6.60	8.26

Run time: RRT-C fast when it succeeds

Improved MHA*: Experiments

12 DoF full-body motion planning for the PR2 robot

	$w = 100$				$w = 10$				$w = 5$					
	++	Focal	Uncons	Original	++	Focal	Uncons	Original	++	Focal	Uncons	Original	MH-GBFS	RRT-C
Success (%)	84	75	84	61	83	74	83	0	66	74	60	0	85	45
States Expanded	2415	3058	2415	5179	3293	3086	3227	-	2378	3086	2472	-	2752	n/a
Run Time (s)	36.89	47.44	36.98	75.63	47.60	47.88	46.96	-	37.92	48.08	38.31	-	41.98	21.95
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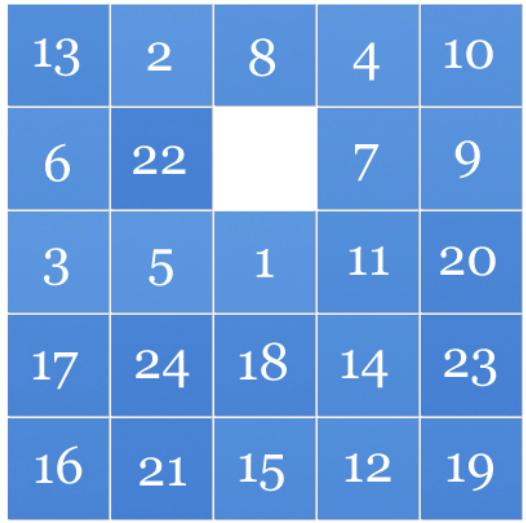
Arm Cost (rad)	6.32	6.45	6.32	6.02	6.17	6.49	6.17	-	5.70	6.49	5.41	-	6.60	8.26

MH-GBFS, RRT-C: No bounds on quality

Improved MHA*: Experiments

Sliding Tile Puzzles: 8x8, 9x9 and 10x10

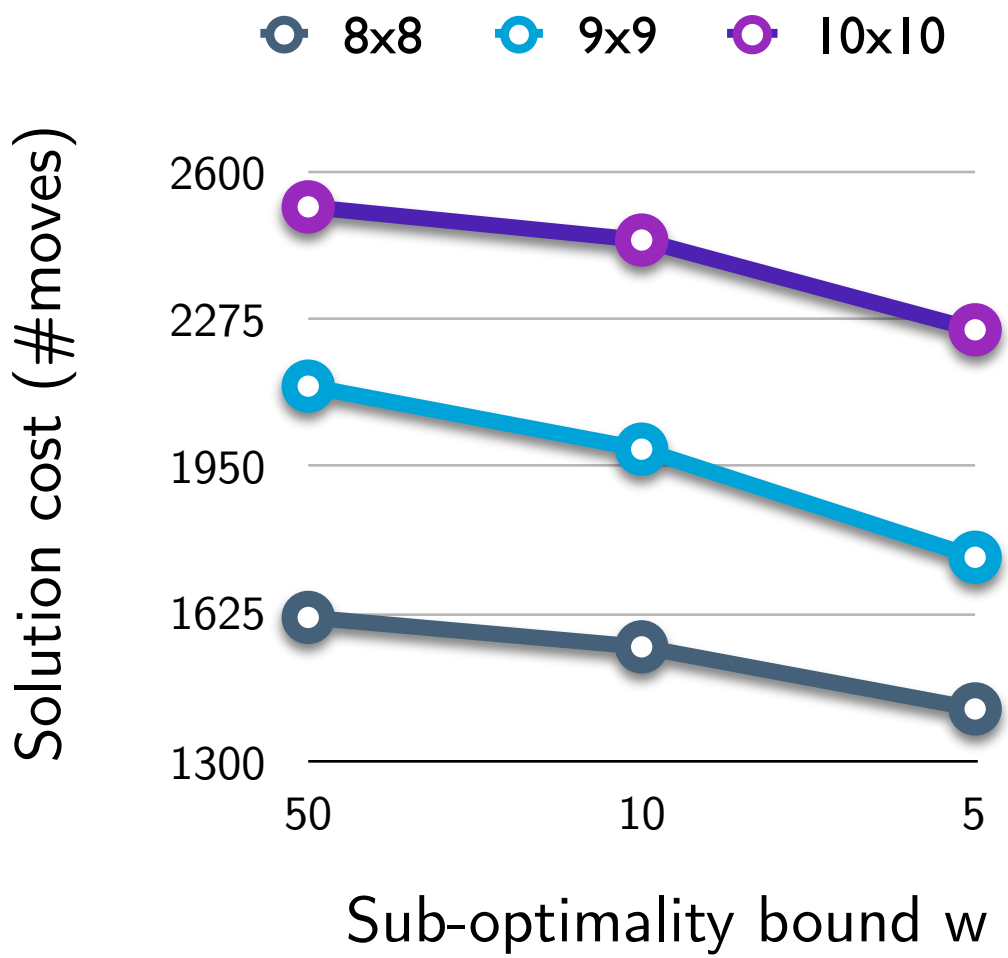


Consistent heuristic: Manhattan distance + linear conflicts

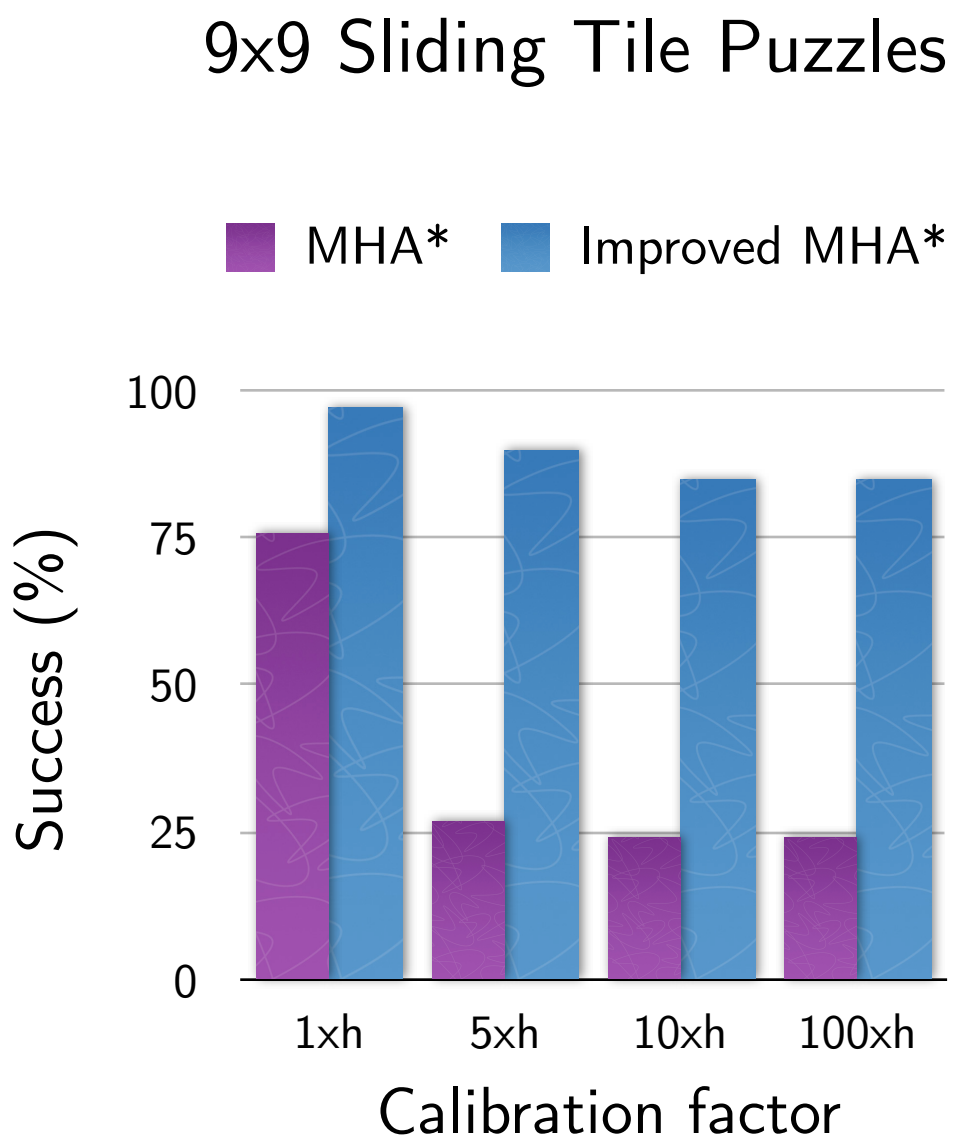
8 Inadmissible heuristics: 'waypoint' pattern database heuristics

100 random trials, 5 minute timeout

Improved MHA*: Experiments



Improved MHA*: Experiments



Summary

Improved MHA*: allows use of multiple inadmissible, uncalibrated heuristics

Completeness, bounded sub-optimality, bounded re-expansions

Experimental validation on full-body motion planning and large sliding tile puzzles

Extensions to round-robin: Meta-A*, Dynamic Thompson Sampling
[Phillips M, Narayanan V, Aine S, and Likhachev M, IJCAI 2015]

Future work: parallelization, anytime