Carnegie Mellon University Instructor: Venkatesan Guruswami
15-252: More GITCS Term: Spring 2021

Problem Set 3

Due date: Friday, February 26 (by midnight EST)

Instructions

- You are allowed to collaborate with one other student taking the class, or do it solo.
- Collaboration is defined as discussion of the lecture material and solution approaches to the problems. Please note that *you are not allowed to share any written material and you must write up solutions on your own.* You must clearly acknowledge your collaborator in the write-up of your solutions.
- Solutions must be submitted on gradescope. Typesetting in LaTeX is recommended but not required. If submitting handwritten work, please make sure it is a legible and polished final draft.
- You should not search for solutions on the web. More generally, you should try and solve the problems without consulting any reference material other than what we cover in class and any provided notes.
- Please start working on the problem set early. Though it is short, the problem(s) might take some time to solve.

1. We defined the Church numerals in lecture as

$$0 := \lambda f.\lambda x.\ x$$
$$1 := \lambda f.\lambda x.\ f\ x$$
$$2 := \lambda f.\lambda x.\ f\ (f\ x)$$

and so on, with n corresponding to applying the function f iteratively n times on x. CMU Professor Emertius and 1976 Turing Award winner Dana Scott defined, in the 1960's, numerals in the following alternate way:

$$\underline{0} := \lambda f.\lambda x.\ x \text{ (the same as Church numeral)}$$
$$\underline{1} := \lambda f.\lambda x.\ f\ \underline{0}$$
$$\underline{2} := \lambda f.\lambda x.\ f\ \underline{1}$$

and so on.

   (a) Write down a lambda expression that serves the role of the successor function Succ for the Scott numerals.

   (b) The Scott numerals have the property that $\underline{n}\ E\ F = F$ if $n = 0$, and $\underline{n}\ E\ F = E\underline{m}$ if $n = m + 1$. This has the advantage that the predecessor of a numeral can be defined readily. Can you specify a lambda expression for the predecessor function Pred?

   (c) Verify that your lambda expressions above satisfy $\mathsf{Pred}\ (\mathsf{Succ}\ \underline{n}) = \underline{n}$ for all integers $n \geq 0$.

   (d) Give a lambda expression implementing the isZero function for Scott numerals, and argue why your expression satisfies $\mathsf{isZero}\ \underline{n}$ is TRUE when $n = 0$ and FALSE when $n > 0$ (here TRUE and FALSE are the Boolean values defined in lecture).