Carnegie Mellon University Instructor: Venkatesan Guruswami
15-252: More GITCS Term: Spring 2021

Problem Set 2

Due date: Friday, February 19 (by midnight EST)

Instructions

- You are allowed to collaborate with one other student taking the class, or do it solo.
- Collaboration is defined as discussion of the lecture material and solution approaches to the problems. Please note that *you are not allowed to share any written material and you must write up solutions on your own.* You must clearly acknowledge your collaborator in the write-up of your solutions.
- Solutions must be submitted on gradescope (with each problem on a separate page). Typesetting in LaTeX is recommended but not required. If submitting handwritten work, please make sure it is a legible and polished final draft.
- You should not search for solutions on the web. More generally, you should try and solve the problems without consulting any reference material other than what we cover in class and any provided notes.
- Please start working on the problem set early. Though it is short, the problem(s) might take some time to solve.

1. ($4 \times 5 = 20$ points) Let $\Sigma = \{1, 2, \ldots, k\}$ for some $k \geq 3$. Consider the language

$$A_1 = \{x \in \Sigma^* \mid x \text{ omits at least one symbol of } \Sigma\} .$$

   (a) Construct an NFA with at most $k+1$ states that accepts the language A_1.

   (b) Describe a DFA with 2^k states that accepts the language A_1.

   (c) Prove that there is no DFA with fewer than 2^k states that accepts A_1.

   (d) Now consider the language

$$A_2 = \{x \in \Sigma^* \mid x \text{ omits at least two symbols of } \Sigma\} .$$

   Prove that any NFA that accepts A_2 must have at least $\binom{k}{2}$ states.

   Hint: Think of a set S of $\binom{k}{2}$ strings (a natural choice works) such that each $w \in S$ must lead the NFA to a state not reachable on the remaining strings in $S \setminus \{w\}$.