Carnegie Mellon University Instructor: Venkatesan Guruswami
15-252: More GITCS Term: Spring 2021

Problem Set 1

Due date: Friday, February 12 (by midnight)

Instructions

- You are allowed to collaborate with one other student taking the class, or do it solo.
- Collaboration is defined as discussion of the lecture material and solution approaches to the problems. Please note that *you are not allowed to share any written material and you must write up solutions on your own.* You must clearly acknowledge your collaborator in the write-up of your solutions.
- Solutions must be submitted on gradescope (with each problem on a separate page). Typesetting in LaTeX is recommended but not required. If submitting handwritten work, please make sure it is a legible and polished final draft.
- You should not search for solutions on the web. More generally, you should try and solve the problems without consulting any reference material other than what we cover in class and any provided notes.
- Please start working on the problem set early. Though it is short, the problem(s) might take some time to solve.

1. (This is exercise 1.8 from the lecture notes): Suppose we are allowed to take any contiguous set of pancakes and flip them in place (they need not be on the top of the stack). Let Q_n be the maximum over stacks of size n of the minimum number of flips required to sort the stack, using this new flipping operation. Show that $\frac{n}{2} \leq Q_n \leq n-1$ for all $n \geq 2$.
2. Suppose we have a much better chef who has much better control over the sizes of the pancakes he makes, which come out in just two sizes, small and large. But the chef has no control over which pancake comes out small or large, so they are jumbled arbitrarily. The natural goal then is to sort the pancakes so all the large ones are stacked below all the small ones. Analogous to P_n from lecture, let B_n be the largest number of flips needed to achieve such a sorting, taken over all possible 2^n configurations of small and large pancakes. Prove that $B_n = n-1$ for all $n \geq 1$.