15-252

More Great Ideas in Theoretical Computer Science

Spring 2021

Course pages

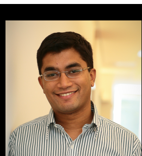
http://www.cs.cmu.edu/~venkatg/teaching/15252-sp21/

piazza.com/cmu/spring2021/15252

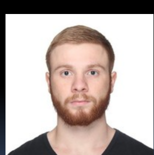
Zoom link (common for all lectures and office hours) communicated via email.

- Add it to your calendar for convenience!

Course Staff



Venkatesan Guruswami



Andrii Riazanov

Office hours: TBA

No office hours this week

- Other times besides OH available upon request

We look forward to getting to know you well, so never hesitate to reach out!

Topics

- Mix of digging into more advanced version of 251 material, and some one-off topics.
- Happy to customize to some extent. The course is for your fun and intellectual enrichment.
- Feedback (on topics, level, speed, etc.) always welcomed.

Grading (Pass/Fail)

Class participation + Homeworks

- Weekly homework, roughly one per lecture, out Friday
- Each with couple of problems
- Submit and graded via gradescope
- HW preferably typeset in LaTeX, but not required
  - If handwritten, please write clearly and legibly. Solutions in rough/poor form will not be graded.
- Collaboration and other rules specified in HW

Feature Presentation

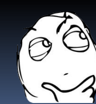
Pancake Sorting

Feel free to ask questions

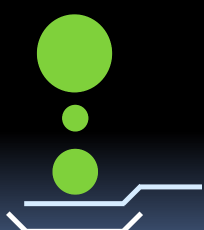
The chef in our place is sloppy; when he prepares pancakes they come out all different sizes.

When the waiter delivers them to a customer, he rearranges them
(so that the smallest is on top, and so on, down to the largest at the bottom).

He does this by grabbing several from the top and flipping them over, repeating this (varying the number he flips) as many times as necessary.



How do we sort this stack?
How many flips do we need?



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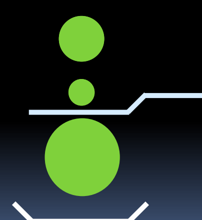
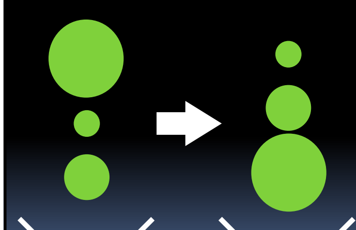
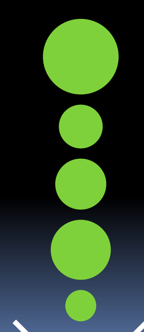


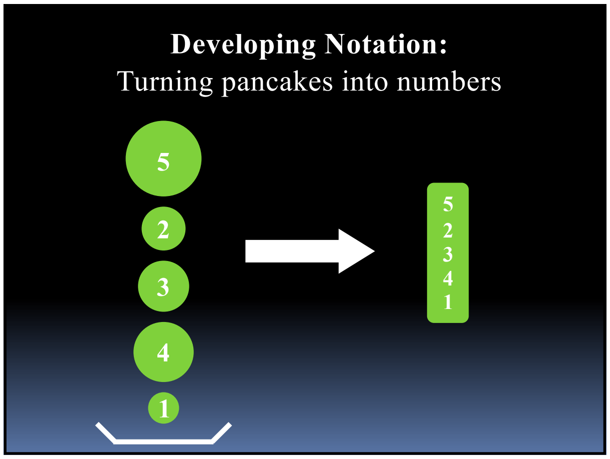
How do we sort this stack?
How many flips do we need?

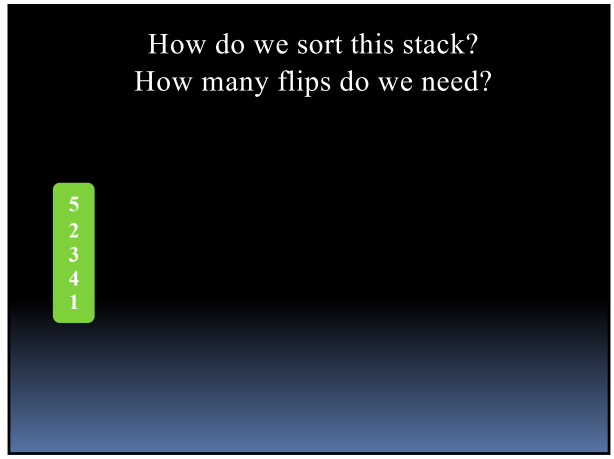


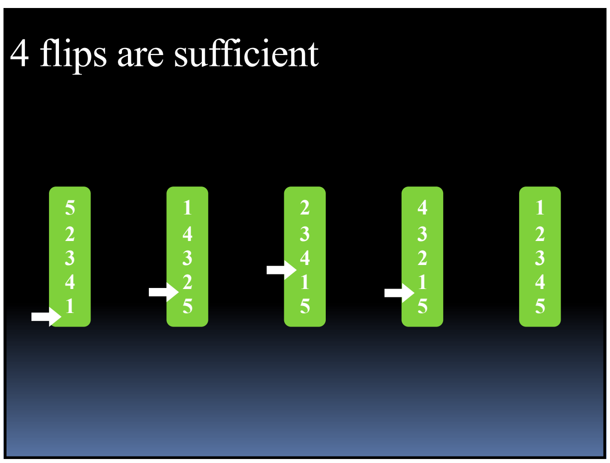
2 flips sufficient
2 flips necessary

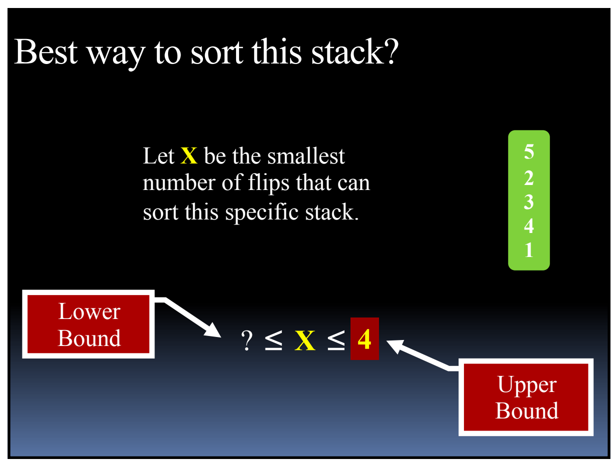
How do we sort this stack?
How many flips do we need?

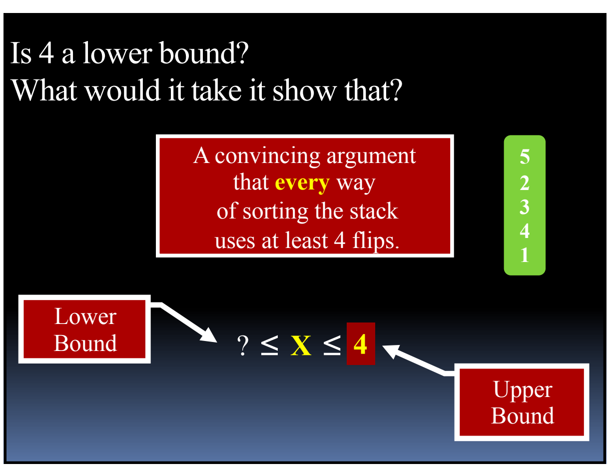


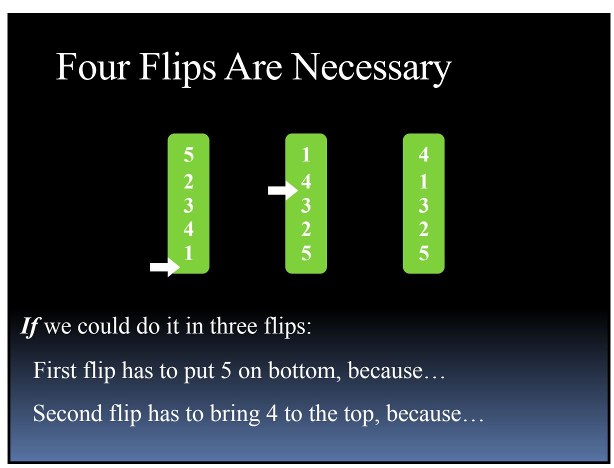

Developing Notation:
Turning pancakes into numbers
5
2
3
4
1
5 2 3 4 1


How do we sort this stack?
How many flips do we need?
5 2 3 4 1


4 flips are sufficient
5 2 3 4 1
1 4 3 2 5
2 3 4 1 5
4 3 2 1 5
1 2 3 4 5


Best way to sort this stack?
Let X be the smallest number of flips that can sort this specific stack.
5 2 3 4 1
Lower Bound
? ≤ X ≤ 4
Upper Bound


Is 4 a lower bound?
What would it take it show that?
A convincing argument that every way of sorting the stack uses at least 4 flips.
5 2 3 4 1
Lower Bound
? ≤ X ≤ 4
Upper Bound


Four Flips Are Necessary
5 2 3 4 1
1 4 3 2 5
4 1 3 2 5
If we could do it in three flips:
First flip has to put 5 on bottom, because…
Second flip has to bring 4 to the top, because…

Best way to sort?

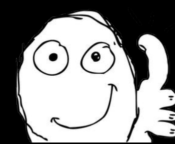
Let **X** be the smallest number of flips that can sort this specific stack.

5
2
3
4
1

Lower Bound → $4 \le X \le 4$ ← Upper Bound

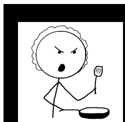
$X = 4$

Matching upper and lower bounds!

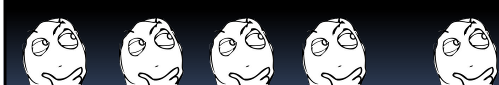


Lower Bound → $4 \le X \le 4$ ← Upper Bound

$X = 4$



5	5	1	5	?
2	4	2	4	?
3	3	3	1	?
4	2	4	2	?
1	1	5	3	?



4	1	0	2	P_5

5th Pancake Number

P_5 = Number of flips required to sort when your worst enemy gives you a stack of 5 pancakes

P_5 = MAX over all 5-stacks **S** of MIN # of flips to sort **S**

5th Pancake Number

Lower Bound → $4 \le P_5 \le ?$ ← Upper Bound

Fact: $P_5 = 5$

To show $P_5 \ge 5$?

1. Show a **specific** 5-stack.
2. Argue that every way of sorting **this** stack uses at least 5 flips.

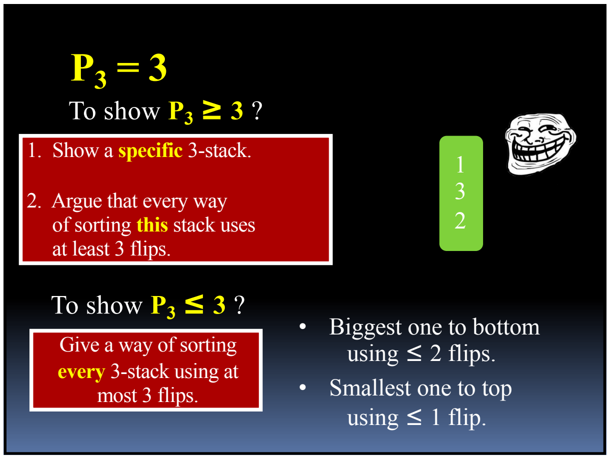
To show $P_5 \le 5$?

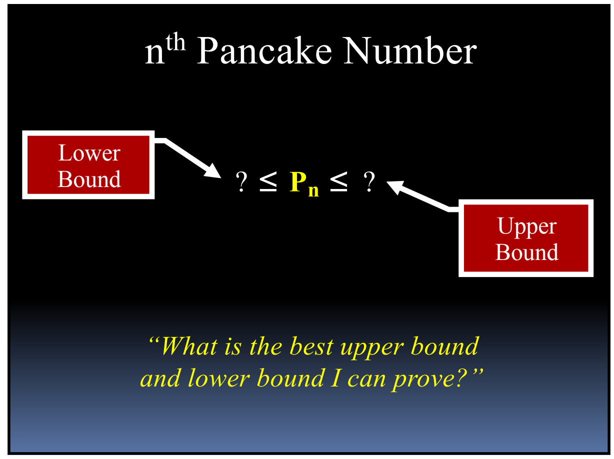
Give a way of sorting **every** 5-stack using at most 5 flips.

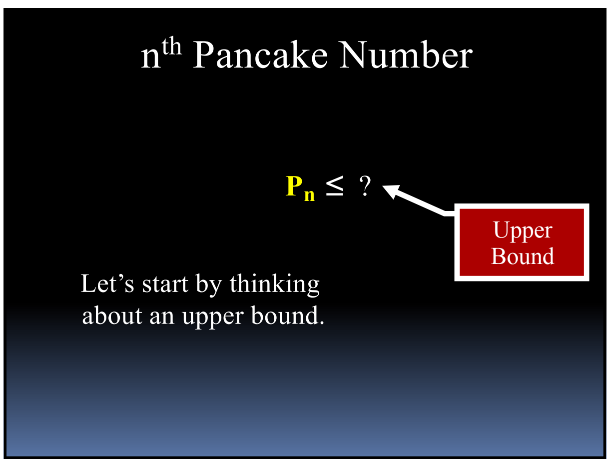
What is P_n for small n?

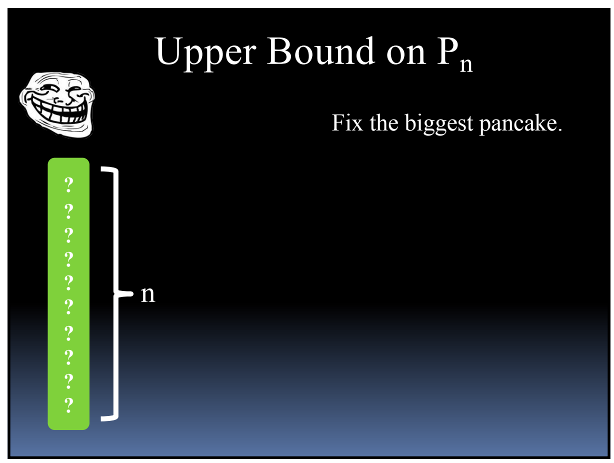
Can you do n = 0, 1, 2, 3 ?

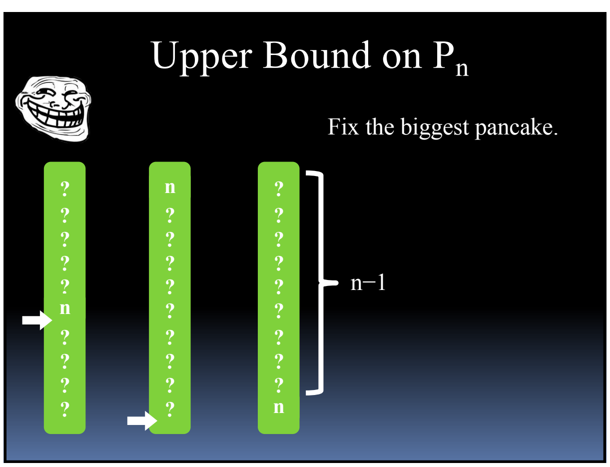
n	0	1	2	3
P_n	0	0	1	3

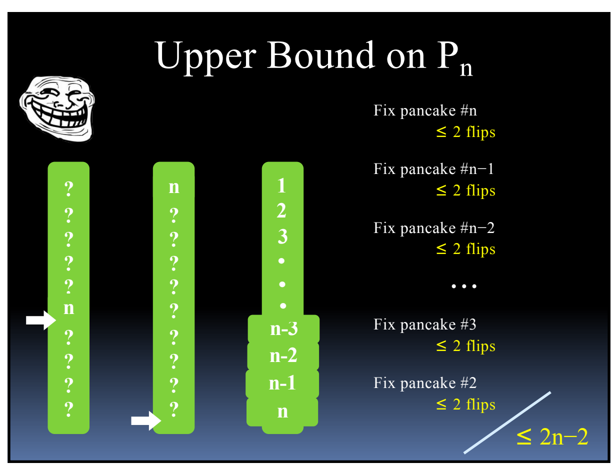

$P_3 = 3$
To show $P_3 \geq 3$?
1. Show a specific 3-stack.
2. Argue that every way of sorting this stack uses at least 3 flips.
1
3
2
To show $P_3 \leq 3$?
Give a way of sorting every 3-stack using at most 3 flips.
• Biggest one to bottom using ≤ 2 flips.
• Smallest one to top using ≤ 1 flip.


n^{th} Pancake Number
Lower Bound
? ≤ P_n ≤ ?
Upper Bound
"What is the best upper bound and lower bound I can prove?"


n^{th} Pancake Number
P_n ≤ ?
Upper Bound
Let's start by thinking about an upper bound.


Upper Bound on P_n
Fix the biggest pancake.
n


Upper Bound on P_n
Fix the biggest pancake.
n−1


Upper Bound on P_n
Fix pancake #n
≤ 2 flips
Fix pancake #n−1
≤ 2 flips
Fix pancake #n−2
≤ 2 flips
…
Fix pancake #3
≤ 2 flips
Fix pancake #2
≤ 2 flips
≤ 2n−2

Upper Bound on P_n

? ? ? ? ? ? n ? ? ? ?

n ? ? ? ? ? ? ? ? ? ?

? ? 3 • • • n-3 n-2 n-1 n

Fix pancake #n
$\leq$ 2 flips

Fix pancake #n−1
$\leq$ 2 flips

Fix pancake #n−2
$\leq$ 2 flips

…

Fix pancake #3
$\leq$ 2 flips

Fix pancake #2
$\leq$ **1 flip**

$\leq$ 2n−3

Upper Bound on P_n

<u>Bring-To-Top Method</u>

shows $P_n \leq 2n-3$

assuming $n \geq 2$.

Fix pancake #n
$\leq$ 2 flips

Fix pancake #n−1
$\leq$ 2 flips

Fix pancake #n−2
$\leq$ 2 flips

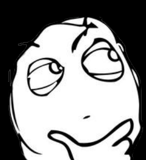
…

Fix pancake #3
$\leq$ 2 flips

Fix pancake #2
$\leq$ **1 flip**

$\leq$ 2n−3

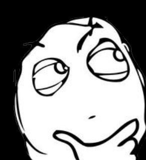
$? \leq P_n \leq 2n-3$



Let's think about a lower bound for P_n

1. Show a **specific** n-stack.
2. Argue that every way of sorting **this** stack uses a lot of flips.

Lower Bound on P_n



This stack seems pretty painful!

• • • 17 15 13 11 9 7 5 • • •

Breaking-Apart Argument

Suppose the stack has an adjacent pair which should not be adjacent in the end.

Spatula must go between them at least once.

("Adjacent pair" includes bottom pancake and the plate.)

Each flip can achieve at most 1 "break-apart".

• • • 17 15 13 11 9 7 5 • • •

Proof of $P_n \geq n$

S: n • • • 6 4 2 n-1 • • • 5 3 1

Case 1: n is even.

S contains n adjacent pairs which need to be broken apart, each necessitating at least one flip.

Detail: Assuming n > 4.

4 2 3 1

Proof of $P_n \geq n$

S

n-1
•
•
•
6
4
2
n
•
•
•
5
3
1

Case 2: n is odd.

S contains n adjacent pairs which need to be broken apart, each necessitating at least one flip.

Detail: Assuming n > 3.

2
3
1

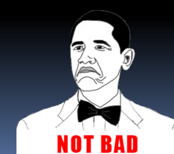
$n \leq P_n \leq 2n-3$

(for n > 2)

Lower Bound

Upper Bound

Upper and lower bounds are within a factor of 2.



Slight Digression

From **any** n-stack to **sorted** n-stack in $\leq P_n$

From **sorted** n-stack to **any** n-stack in $\leq P_n$?

Reverse the sequence of flips used to sort!

Hence: **any** n-stack to **any** n-stack in $\leq 2P_n$

Is there a better way?

Any Stack S to **Any** Stack T in $\leq P_n$

S: 4,3,5,1,2

3,4,1,2,5

"new S"

T: 5,1,4,3,2

1,2,3,4,5

Rename the pancakes in T to be 1,2,3,…,n

Rewrite S using the new naming scheme

In $\leq P_n$ flips can sort "new S".

The same sequence of flips also brings S to T.

The Known Pancake Numbers

n	P_n
1	0
2	1
3	3
4	4
5	5
6	7
7	8
8	9
9	10
10	11
11	13
12	14
13	15
14	16
15	17
16	18
17	19
18	20
19	22

P_{20} is unknown

It is either 23 or 24, we don't know which.

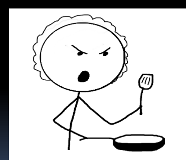
$20 \cdot 19 \cdot 18 \cdots 2 \cdot 1 = 20!$ possible 20-stacks

$$20! = 2.43 \times 10^{18}$$

(2.43 exa-pancakes)

Brute-force analysis would take forever!

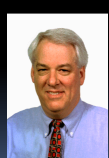
Is This Really Computer Science?

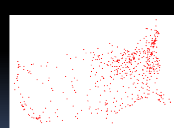


Posed in *Amer. Math. Monthly* 82(1), 1975,
by "Harry Dweighter" (haha).

AKA **Jacob Goodman**,
a computational geometer.

In 1977,
the observations we have made so far
were published by
Mike Garey, David Johnson, & Shen Lin



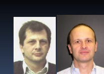


Bounds For Sorting By Prefix Reversal

Discrete Mathematics 27(1), 1979

$(17/16)n \leq P_n \leq (5/3)n+5/3$

by:
William H. Gates (Microsoft, Albuquerque NM)
Christos Papadimitriou (UC Berkeley)



upper bound also
by Győri & Turán

"On the Diameter of the Pancake Network"
Journal of Algorithms 25(1), 1997

$(15/14)n \leq P_n \leq (5/3)n+5/3$

by Hossain Heydari and Hal Sudborough

"An (18/11)n Upper Bound For Sorting By Prefix Reversals"
Theoretical Computer Science 410(36), 2009

$(15/14)n \leq P_n \leq (18/11)n$

by B. Chitturi, W. Fahle, Z. Meng, L. Morales,
C.O. Shields, I.H. Sudborough, W. Voit
@ UT Dallas

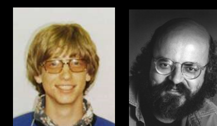
Worst Case: There is an algorithm using ≤ (18/11)n flips, even when your worst enemy gives you stack of n pancakes

Average Case: There is an algorithm using ≤ (17/12)n flips on average when given a random stack of n pancakes

(Josef Cibulka, 2009)

Burnt Pancakes

$(3/2)n-1 \leq BP_n \leq 2n+3$



Burnt Pancakes

$(3/2)n \leq BP_n \leq 2n-2$

"On The Problem Of Sorting Burnt Pancakes"
Discrete Applied Math. 61(2), 1995

by David X. Cohen and Manuel Blum

Applications

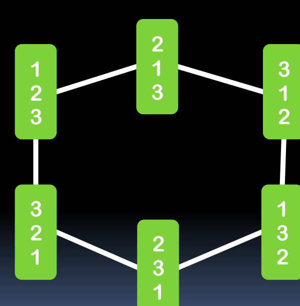
"The Pancake Network"

on n! nodes

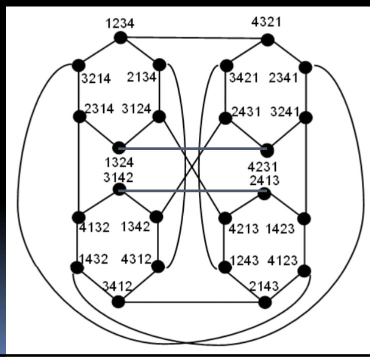
Nodes are named after the n! different stacks of n pancakes

Put a link between two nodes if you can go between them with one flip

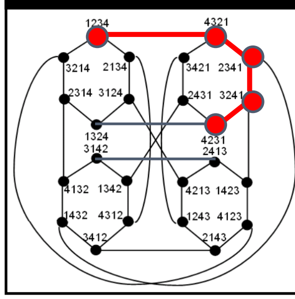
Pancake Network, n = 3



Pancake Network, n = 4



Pancake Network: Message Routing Delay



What is the maximum distance between two nodes in the pancake network?

P_n

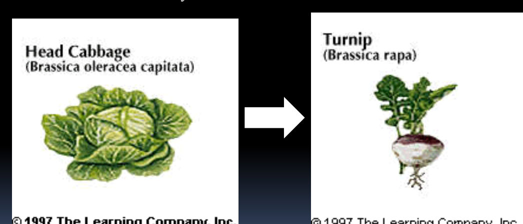
Pancake Network: Reliability

If up to n−2 nodes get hit by lightning, the network remains connected, even though each node is connected to only n−1 others

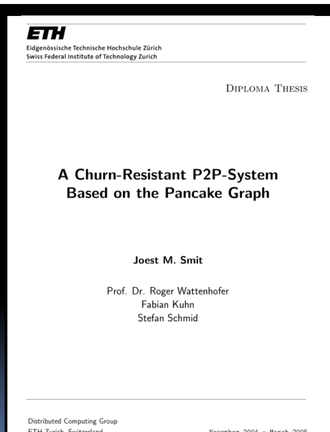
The Pancake Network is **optimally** reliable for its number of nodes and links

Computational Biology

Transforming Cabbage Into Turnip:
polynomial algorithm for sorting signed permutations by reversal
by S. Hannenhalli & P. Pevzner



Of Mice And Men:
algorithms for evolutionary distances between genomes with translocation
by J. Kececioglu and R. Ravi



ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Diploma Thesis

A Churn-Resistant P2P-System Based on the Pancake Graph

Joest M. Smit

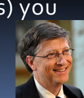
Prof. Dr. Roger Wattenhofer
Fabian Kuhn
Stefan Schmid

Distributed Computing Group
ETH Zurich, Switzerland

November 2004 - March 2005

Lessons

- Simple puzzles might be hard to solve and hold exciting mysteries
- Simple puzzles can have unforeseen applications
- By studying pancakes (theory puzzles) you may become a billionaire



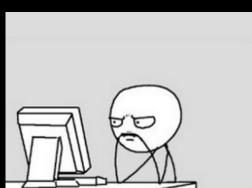
Analogy with computation

- *Input*: initial stack
- *Output*: sorted stack
- *Computational problem*: (input, output) pairs
  pancake sorting problem
- *Computational model*: specified by allowed operations on input *(flip top segment of stack)*
- *Algorithm*: a precise description of how to obtain output from input *(precise sequence of flips)*
- Computability: is it always possible to sort the stack?
- Complexity: how many operations (flips) are needed?

High Level Point

Computer Science is not merely about computers or programming — it is about *mathematically modeling* computational scenarios in our world,
about finding *better and better ways* to solve problems, and
understanding *fundamental limits* of how well we can solve problems

Today's lecture is a microcosm of this.



Review

Definitions of:
- n^{th} pancake number
- upper bound
- lower bound

Proof of:
- Bring-To-Top
- Breaking-Apart
- ANY to ANY in $\leq P_n$

HW 1 will be posted on course webpage by tomorrow
(with accompanying alert on Piazza)
Due midnight next Friday (Feb 12).