Carnegie Mellon University Instructor: Venkatesan Guruswami
15-252: More GITCS Term: Spring 2020

Problem Set 4
Due date: Friday, February 14

Instructions

- You are allowed to collaborate with one other student taking the class, or do it solo.
- Collaboration is defined as discussion of the lecture material and solution approaches to the problems. Please note that *you are not allowed to share any written material and you must write up solutions on your own.* You must clearly acknowledge your collaborator in the write-up of your solutions.
- Solutions must be typeset in LaTeX and emailed to the TA.
- You should not search for solutions on the web. More generally, you should try and solve the problems without consulting any reference material other than what we cover in class and any provided notes.
- Please start working on the problem set early. Though it is short, the problem(s) might take some time to solve.

Note: All logarithms are to base 2.

1. In this problem, our goal is to show that there exist long sequences of zeroes in incompressible strings. (For this problem, you can assume n is large enough.)

   (a) Prove that the number of n bit strings without any consecutive sequence of $\frac{\log n}{2}$ zeroes is at most $2^n e^{-\frac{1}{\sqrt{n}}\lfloor\frac{2n}{\log n}\rfloor}$.
   (Hint- the following fact is useful: $e^x \geq 1 + x$ for all real numbers x.)

   (b) Let n be a positive integer. Suppose that an n bit string p satisfies $K(p) \geq n$. Prove that there exist a consecutive sequence of $\frac{\log n}{2}$ zeroes in p.

2. One might hope that Kolmogorov complexity is subadditive i.e. $K(xy) \leq K(x)+K(y)$, where xy denotes the concatenation of the strings x and y. However, it turns out that this may not be true.

   Using the previous problem, prove that for every constant c, there exist strings x and y such that $K(xy) \geq K(x) + K(y) + c$.