Carnegie Mellon University Instructor: Venkatesan Guruswami
15-252: More GITCS Term: Spring 2020

Problem Set 1
Due date: Friday, January 24

Instructions

- You are allowed to collaborate with one other student taking the class, or do it solo.
- Collaboration is defined as discussion of the lecture material and solution approaches to the problems. Please note that *you are not allowed to share any written material and you must write up solutions on your own.* You must clearly acknowledge your collaborator in the write-up of your solutions.
- Solutions must be typeset in LaTeX and emailed to the TA.
- You should not search for solutions on the web. More generally, you should try and solve the problems without consulting any reference material other than what we cover in class and any provided notes.
- Please start working on the problem set early. Though it is short, the problem(s) might take some time to solve.

1. Suppose we have a much better chef who has much better control over the sizes of the pancakes he makes, which come out in just two sizes, small and large. But the chef has no control over which pancake comes out small or large, so they are jumbled arbitrarily. The natural goal then is to sort the pancakes so all the large ones are stacked below all the small ones. Analogous to P_n from lecture, let B_n be the largest number of flips needed to achieve such a sorting, taken over all possible 2^n configurations of small and large pancakes. Prove that $B_n = n - 1$ for all $n \geq 1$.

2. Now suppose our above chef got little sleep (probably due to 251 homework) and the next morning becomes a bit more sloppy and produces pancakes of three sizes—small, medium, and large. Define the analogous quantity T_n to be the largest number of flips needed to sort an arbitrarily jumbled stack of small-medium-large pancakes. Prove that $T_n \leq 4n/3 + 2$. For extra credit (and maybe some playful research), you may try and see if you can prove $T_n \leq cn + O(1)$ for some smaller constant c.