Hidden Markov Models

Machine Learning 10-601
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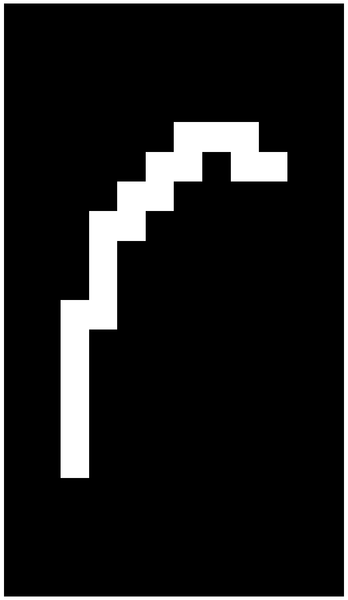
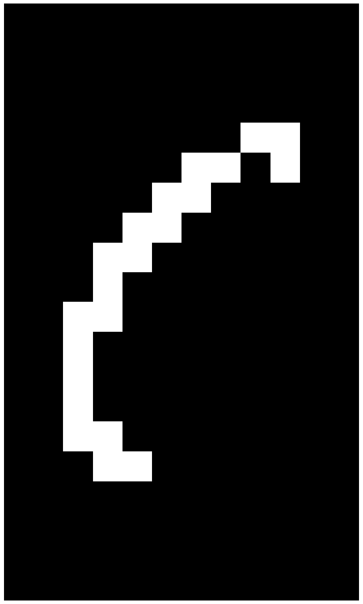
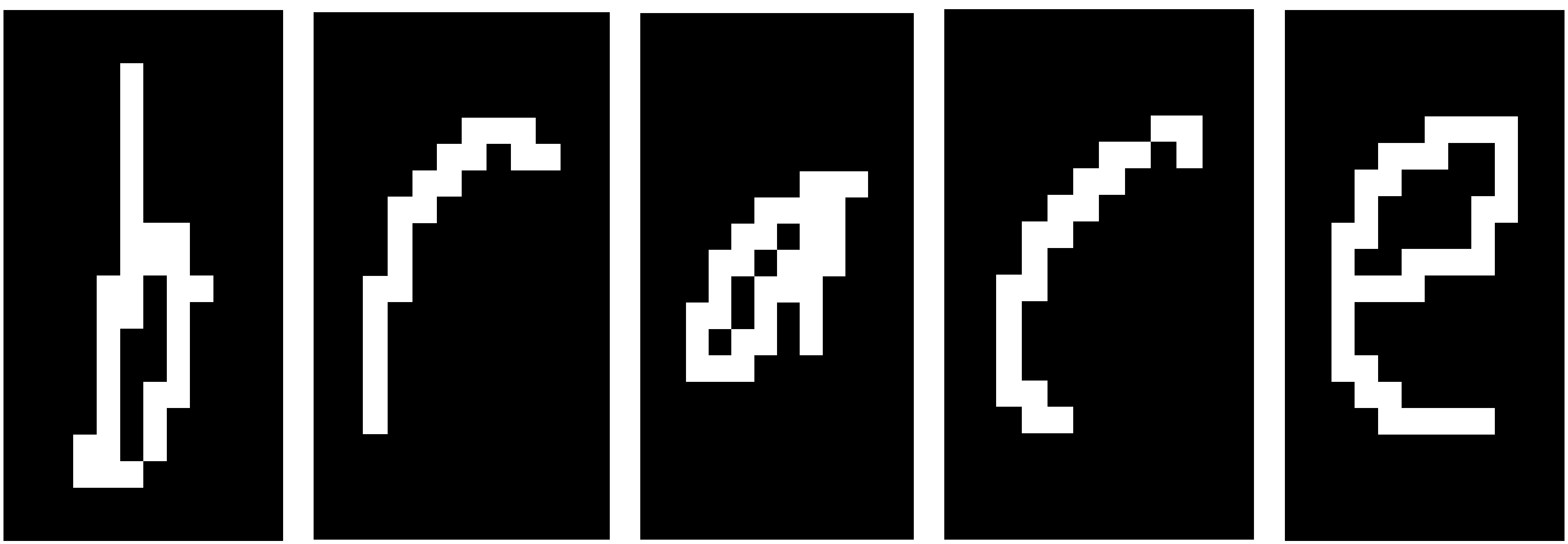
With thanks to Prof. Carlos Guestrin for some of these slides

Handwriting recognition

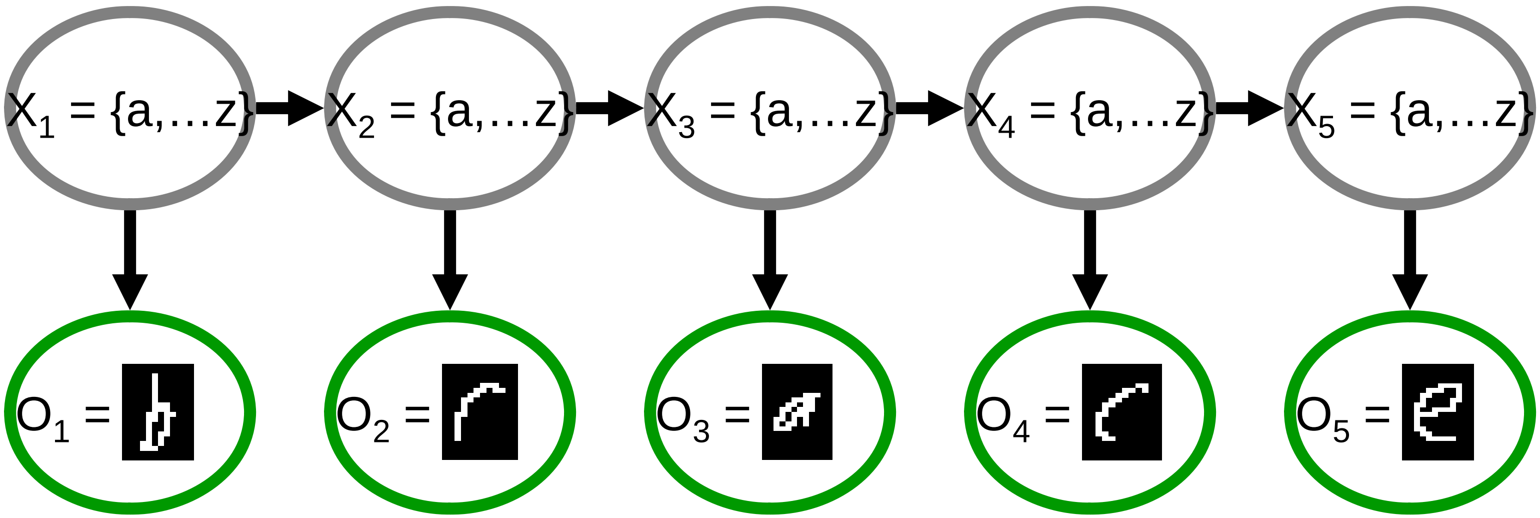
 

Character recognition, e.g., logistic regression

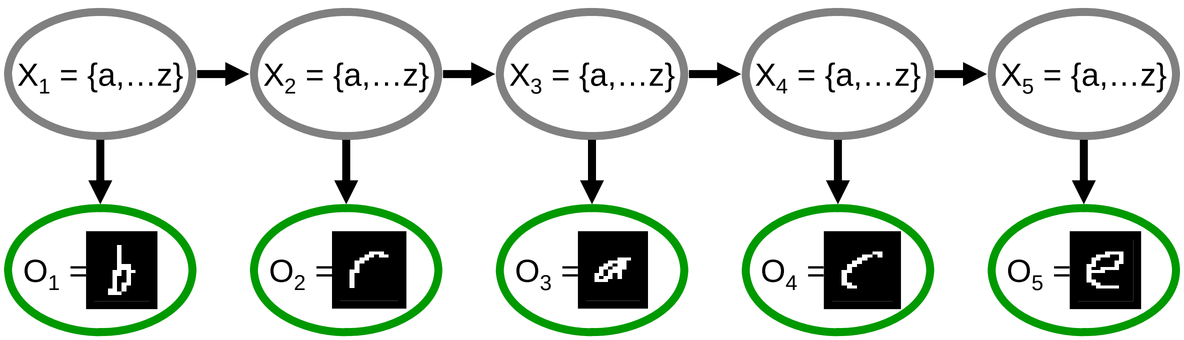
Example of a hidden Markov model (HMM)



Understanding the HMM Semantics



HMMs semantics: Details



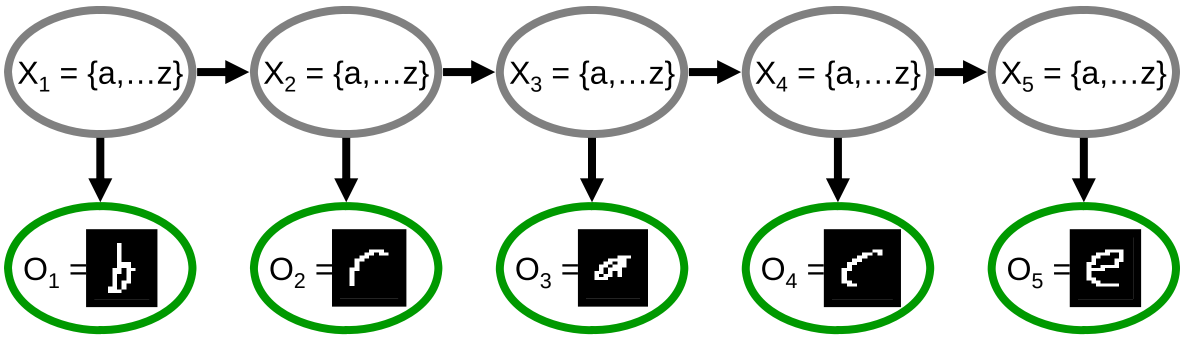
Just 3 distributions:

$$P(X_1)$$

$$P(X_i \mid X_{i-1})$$

$$P(O_i \mid X_i)$$

Using and Learning HMM's



$$P(X_i \mid o_{1..n})$$

Core HMM questions:

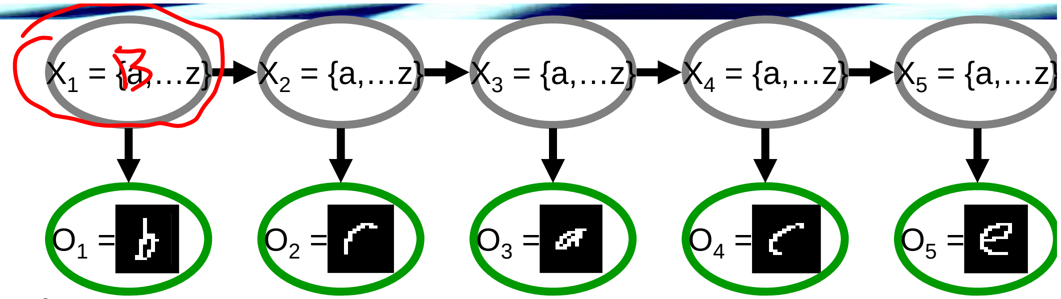
1. How do we calculate $P(o_1, o_2, \ldots o_n)$?

2. How do we calculate argmax over $x_1, x_2, \ldots x_n$ of $P(x_1, x_2, \ldots x_n \mid o_1, o_2, \ldots o_n)$?

3. How do we train the HMM, given its structure and
   3a. Fully observed training examples: $< x_1, \ldots x_n, o_1, \ldots o_n >$

   3b. Partially observed training examples: $< o_1, \ldots o_n >$

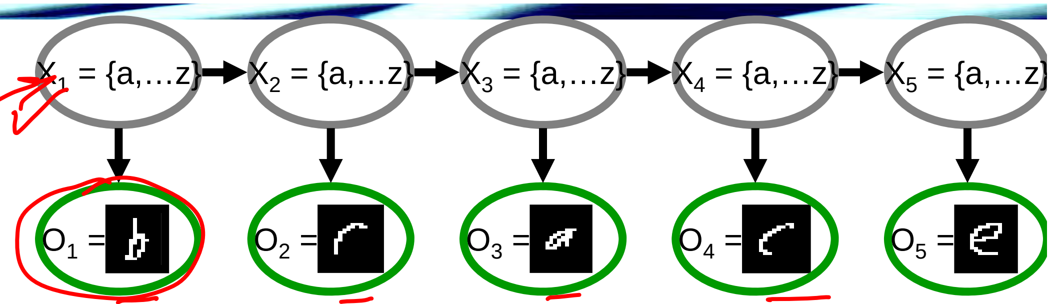
How do we generate a random output sequence following the HMM $P(o_1, o_2, \ldots o_T)$



How do we compute

$$P(o_1, o_2, \ldots o_T)$$

N poss values



1 2 . . . T

1. brute force:

2. Forward algorithm (dynamic progr., variable elimination):

$N^2 T$

define $\alpha_t(k) = \boxed{P(o_1, o_2, \ldots o_t, X_t = k)}$

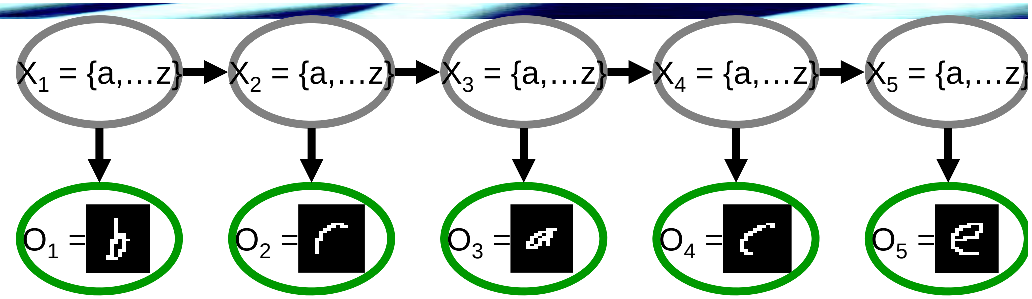
$$\alpha_1(k) = P(X_1=k)\, P(O_1=o_1 \mid X_1=k) = P(o_1, X_1=k)$$

$$\alpha_{t+1}(k) = \left[\sum_{j=1}^{N} \alpha_t(j)\, P(X_{t+1}=k \mid X_t=j)\right] P(O_{t+1}=o_{t+1} \mid X_{t+1}=k)$$

$$P(o_1, o_2, \ldots o_T) = \sum_k \alpha_T(k)$$

How do we compute

$$P(X_t = k | o_1, o_2, \ldots o_T)$$



2. Backward algorithm (dynamic progr., variable elimination):

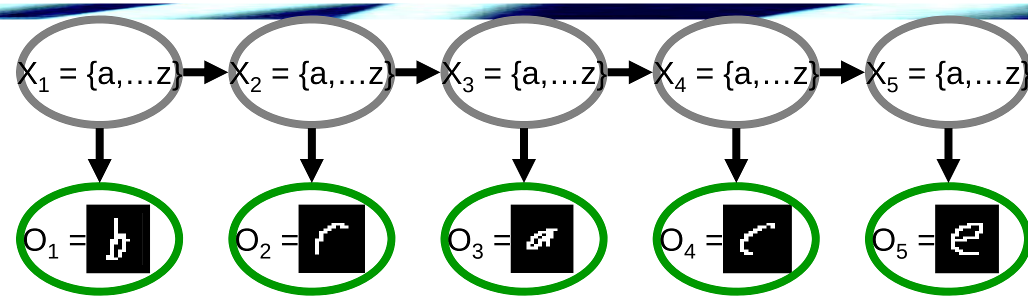
$$\alpha_t(k) = P(o_1, o_2, \ldots o_t, X_t = k)$$

define $\beta_t(k) = P(o_{t+1}, o_{t+2}, \ldots o_T | X_t = k)$

$\alpha_t(k)\,\beta_t(k)$

$$P(X_t = k | o_1, o_2, \ldots o_T) = \frac{P(X_t = k, o_1, o_2, \ldots o_T)}{P(o_1, o_2, \ldots o_T)}$$

$\sum_k \alpha_T(k)$



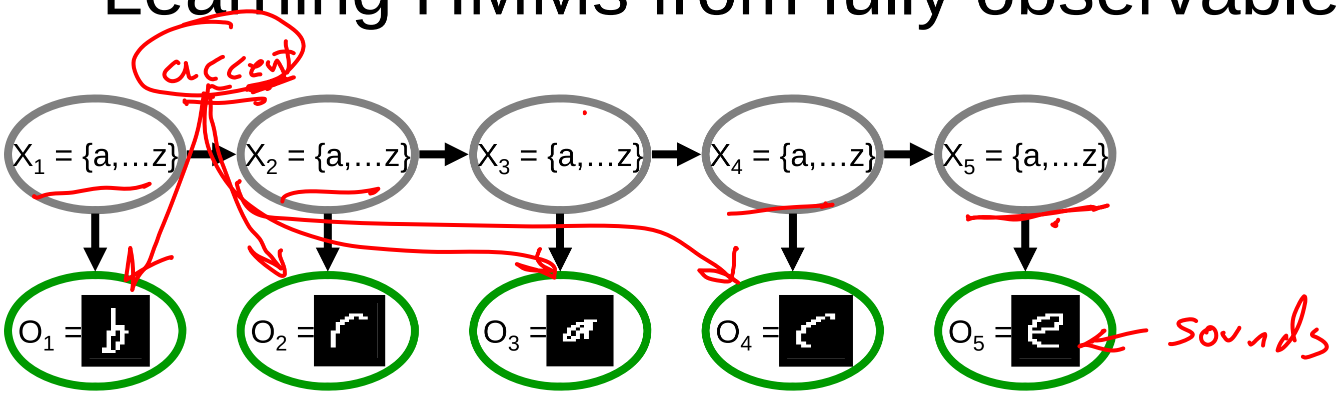
How do we compute

$$\arg\max_{x_1,\ldots x_T} P(x_1,\ldots x_T \mid o_1, o_2, \ldots o_T)$$

Viterbi algorithm, based on recursive computation of

$$\delta_t(k) = \max_{x_1,\ldots x_{t-1}} P(x_1,\ldots x_{t-1} X_t = k, o_1, o_2, \ldots o_t)$$

Learning HMMs from fully observable data: easy



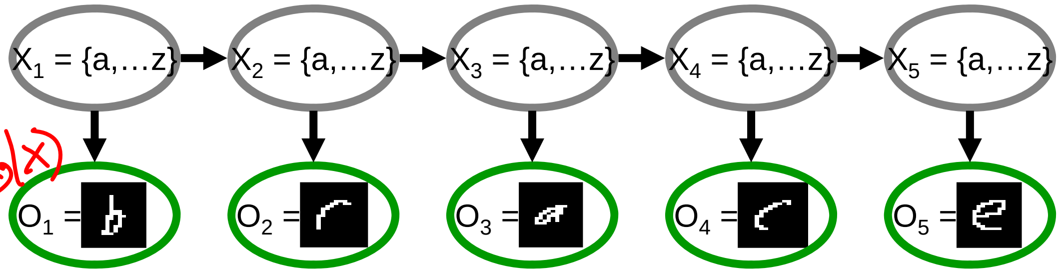
Learn 3 distributions:

$$P(X_1)$$

$$P(O_i \mid X_i)$$

$$P(X_i \mid X_{i-1})$$

Learning HMMs when only observe o1...oT



$P(o|x)$

} unobs Z

} obs Y

EM

Initial model params θ

Loop to convergence.

Estep: use current θ to calc. expected values of Z

(eg. Viterbi alg)

$E[\#\text{ times trans from } x_i \text{ to } x_j]$

$E[\#\text{ times in } x_i]$

$E[\#\text{ times in } x_i \text{ and obs } o_j]$

M step re-maximize likelihood by choosing θ'

$$\theta' \leftarrow \underset{\theta'}{\arg\max}\ E\left[P(o_1, o_2 \dots o_T, \hat{x}_1, \dots \hat{x}_T \mid \theta')\right]$$

$P(z \mid Y, \theta)$

What you need to know

- Hidden Markov models (HMMs)
  - Very useful, very powerful!
  - Speech, OCR, time series, …
  - Parameter sharing, only learn 3 distributions
  - Trick reduces inference from $O(n^2)$ to $O(n)$
  - Special case of Bayes net
  - Dynamic Bayesian Networks

Thanks to Carlos Guestrin for many slides