Self Organisation in Random Geometric Graph models of Wireless Sensor Networks

Swaprava Nath

under the guidance of
Prof. Anurag Kumar

ME Final Presentation

Department of Electrical Communication Engineering
Indian Institute of Science, Bangalore 560012, India

June 26, 2008
Outline of Talk

1. Theory
   - Review of Geometric Graph (GG)
   - Assumptions in Literature
   - Motivation for Random GG
   - Paradigms of HD-ED Relationship in RGG

2. Simulations illustrating Point-Node Theorem

3. Application in Localisation
   - Theorem Used
   - Algorithm: Hop Count-derived Distance-based Localisation (HCDL)

4. Performance Comparison

5. Conclusion and Future Work
Outline of Talk

1. Theory
   - Review of Geometric Graph (GG)
   - Assumptions in Literature
   - Motivation for Random GG
   - Paradigms of HD-ED Relationship in RGG

2. Simulations illustrating Point-Node Theorem

3. Application in Localisation
   - Theorem Used
   - Algorithm: Hop Count-derived Distance-based Localisation (HCDL)

4. Performance Comparison

5. Conclusion and Future Work
Outline of Talk

1. Theory
   - Review of Geometric Graph (GG)
   - Assumptions in Literature
   - Motivation for Random GG
   - Paradigms of HD-ED Relationship in RGG

2. Simulations illustrating Point-Node Theorem

3. Application in Localisation
   - Theorem Used
   - Algorithm: Hop Count-derived Distance-based Localisation (HCDL)

4. Performance Comparison

5. Conclusion and Future Work
Geometric Graph

Area to monitor, $\mathcal{A}$

Sample deployment $v$

Neighbours in the Geometric Graph $\mathcal{G}(v, r)$

Node locations can be arbitrary or random
Definition of Hop Distance (HD)

Area to monitor, $A$

Hop distance $h$

Euclidean distance $d$
Definition of Hop Distance (HD)

Question: Relation between hop-distance (HD) and Euclidean distance (ED)?
Outline of Talk

1 Theory
   - Review of Geometric Graph (GG)
   - Assumptions in Literature
   - Motivation for Random GG
   - Paradigms of HD-ED Relationship in RGG

2 Simulations illustrating Point-Node Theorem

3 Application in Localisation
   - Theorem Used
   - Algorithm: Hop Count-derived Distance-based Localisation (HCDL)

4 Performance Comparison

5 Conclusion and Future Work
Assumption in HCRL

- Hop Count Ratio-based Localisation (HCRL), proposed by Yang et al. [IEEE SECON 2007]
- Assumption: \( d \propto h \), hence \( \frac{d_1}{d_2} = \frac{h_1}{h_2} \)
- Suppose, node location \((x, y)\), Anchors \((x_{01}, y_{01})\) and \((x_{02}, y_{02})\)

\[
\frac{\sqrt{(x-x_{01})^2 + (y-y_{01})^2}}{\sqrt{(x-x_{02})^2 + (y-y_{02})^2}} \approx \frac{h_1}{h_2} \quad \leftarrow \text{Equation of circle}
\]
Assumption in PDM

- Proximity Distance Map (PDM), proposed by Lim and Hou [IEEE Infocom 2005]
- \( L \) anchors, node \( i \) has HD vector \( h_i \in \mathbb{N}^L \)
- Assumption: ED vector \( d_i = Th_i \)
- ED matrix between anchors, \( D = [d_1, \cdots, d_L] \), is known
- HD matrix between anchors, \( H = [h_1, \cdots, h_L] \), is computed
- \( D = TH \Rightarrow T = DH^T(HH^T)^{-1} \)
- This \( T \) is used for all non-anchor nodes
- Node location estimated from the ED vector \( d_i \)
Outline of Talk

1 Theory
   - Review of Geometric Graph (GG)
   - Assumptions in Literature
   - **Motivation for Random GG**
   - Paradigms of HD-ED Relationship in RGG

2 Simulations illustrating Point-Node Theorem

3 Application in Localisation
   - Theorem Used
   - Algorithm: Hop Count-derived Distance-based Localisation (HCDL)

4 Performance Comparison

5 Conclusion and Future Work
HD-ED Relationship in Arbitrary GG

- **Setting:**
  - $n$ nodes placed on unit area $\mathcal{A}$ arbitrarily
  - $\mathbf{v} = [v_1, v_2, \cdots, v_n] \in \mathcal{A}^n$
  - Geometric graph $\mathcal{G}(\mathbf{v}, r)$ is formed

- **Notation:**
  - $\mathcal{N} = [n] = \{1, 2, \cdots, n\}$, the index set of the nodes
  - $H_{l,i}(\mathbf{v}) =$ HD of node $i$ from $l^{th}$ anchor on $\mathcal{G}(\mathbf{v}, r)$, for the deployment $\mathbf{v}$
  - $D_{l,i}(\mathbf{v}) =$ Euclidean distance of node $i$ from anchor $b_l$ for the deployment $\mathbf{v}$.

$$\overline{D}_l(\mathbf{v}, h_l) = \max_{\{i \in \mathcal{N} : H_{l,i}(\mathbf{v}) = h_l\}} D_{l,i}(\mathbf{v})$$

$$\underline{D}_l(\mathbf{v}, h_l) = \min_{\{i \in \mathcal{N} : H_{l,i}(\mathbf{v}) = h_l\}} D_{l,i}(\mathbf{v})$$
Graphical Illustration

Area to monitor, \( A \)

These paths are on \( G(v, r) \)

\[ D_l(v, h_l) \]

Anchor can be anywhere in \( A \), this is an example

Sample deployment \( v \)

\( l^{th} \) anchor location

Swaprava (ECE, IISc)
Lemma

For arbitrary \( v \) and \( h_l \geq 2 \), \( r < D_l(v, h_l) \leq \overline{D}_l(v, h_l) \leq h_l r \) and both bounds are sharp.

A regular \( h_l + 1 \) sided polygon

Figure: Node placement on the right achieves the lower bound of ED

- HD does not give useful information about ED in Arbitrary GG
Outline of Talk

1. Theory
   - Review of Geometric Graph (GG)
   - Assumptions in Literature
   - Motivation for Random GG
   - Paradigms of HD-ED Relationship in RGG

2. Simulations illustrating Point-Node Theorem

3. Application in Localisation
   - Theorem Used
   - Algorithm: Hop Count-derived Distance-based Localisation (HCDL)

4. Performance Comparison

5. Conclusion and Future Work
Paradigms of HD-ED Relationship in RGG

- HD-ED Relationship between Fixed Points
- HD-ED Relationship between Random Nodes
- HD-ED Relationship between Fixed Point and Random Node
HD-ED Relationship between Fixed Points

- **Setting:**
  - $n$ nodes placed on unit area $\mathcal{A}$ *Uniform i.i.d.*
  - Node location vector $\mathbf{v} = [v_1, v_2, \ldots, v_n] \in \mathcal{A}^n$
  - $\mathbb{P}^n(.)$ is the probability measure
  - Geometric graph $\mathcal{G}(\mathbf{v}, r(n))$ is formed

- **Notation:**
  - $H_{b_1 b_2}(\mathbf{v})$ is the hop distance between any two points $b_1$ and $b_2$ on $\mathcal{A}$, for the sample deployment $\mathbf{v}$
  - We will take $r(n) = c \sqrt{\frac{\log n}{n}}$, $c > \frac{1}{\sqrt{\pi}}$, a constant, to guarantee asymptotic connectivity (Gupta and Kumar, 1998)
Theorem

For all $\epsilon, 1 > \epsilon > 0$, if $c^2(\epsilon) \geq \frac{2}{q \sqrt{1-p^2}}$, where $p$ and $q$ are any two constants satisfying $1 - \epsilon < p < 1$ and $0 < q < p - (1 - \epsilon)$ and $p \geq 2q$, on a unit square $\mathcal{A}$,

$$\lim_{n \to \infty} P^n \left\{ \mathbf{v} : \forall z_1, z_2 \in \mathcal{A}, \frac{z_1 z_2}{r(n, \epsilon)} \leq H_{z_1 z_2}(\mathbf{v}) < \frac{z_1 z_2}{(1 - \epsilon)r(n, \epsilon)} \right\} = 1$$

where $r(n, \epsilon) = c(\epsilon) \sqrt{\log \frac{n}{n}}$.

- i.e., with high probability, ED between any two points is roughly equal to HD $\times$ radius of the RGG, where the radius is larger than the critical radius by a constant factor
Setting:
- \( n \) nodes placed on unit area \( \mathcal{A} \) Uniform i.i.d.
- Node location vector \( \mathbf{v} = [v_1, v_2, \ldots, v_n] \in \mathcal{A}^n \)
- \( \mathbb{P}^n(.) \) is the probability measure
- Geometric graph \( \mathcal{G}(\mathbf{v}, r(n)) \) is formed

Notation:
- \( \mathcal{N} = [n] = \{1, 2, \ldots, n\} \), the index set of the nodes, i.e., node \( i \in \mathcal{N} \) has a location \( v_i \) on \( \mathcal{A} \).
- \( D_{a,b}(\mathbf{v}) \): The Euclidean distance on \( \mathcal{A} \) between two nodes \( a \) and \( b \), \( a, b \in \mathcal{N} \), for the sample deployment \( \mathbf{v} \).
- \( H_{a,b}(\mathbf{v}) \): The hop distance on \( \mathcal{G}(\mathbf{v}, r(n)) \) between two nodes \( a \) and \( b \), \( a, b \in \mathcal{N} \), for the sample deployment \( \mathbf{v} \).

\[
\overline{D}(\mathbf{v}, h) = \max_{\{(a,b)\in\mathcal{N}^2:H_{a,b}(\mathbf{v})=h\}} D_{a,b}(\mathbf{v})
\]

\[
\underline{D}(\mathbf{v}, h) = \min_{\{(a,b)\in\mathcal{N}^2:H_{a,b}(\mathbf{v})=h\}} D_{a,b}(\mathbf{v})
\]
We want bounds on the ED between any pair of nodes which are at a hop-distance \( h \) from each other.

**Area to monitor, \( A \)**

RGG is formed with radius \( r(n) \)

Two different pairs of nodes both \( h \) hops away from each other

We want the EDs of all such pairs to be bounded between \([kr(n), hr(n)]\) simultaneously, \( k < h \) is a constant.
Define,

\[ E_h(n, \epsilon) = \left\{ v : \left( (1 - \epsilon)(h - 1) - \frac{1}{2} \right) r(n, \epsilon) \leq D(v, h) \leq \overline{D}(v, h) \leq h r(n, \epsilon) \right\} \]

**Theorem**

For \( 1 > \epsilon > 0 \), if \( c^2(\epsilon) \geq \frac{1}{g(\epsilon)} \), where

\[
\begin{align*}
g(\epsilon) &= q(\epsilon)\sqrt{1 - p^2(\epsilon)}, \\
p(\epsilon) &= \frac{1 - \epsilon + \sqrt{(1 - \epsilon)^2 + 8}}{4}, \\
q(\epsilon) &= \frac{-3(1 - \epsilon) + \sqrt{(1 - \epsilon)^2 + 8}}{4},
\end{align*}
\]

\[
\mathbb{P}^n(E_h(n, \epsilon)) = 1 - O\left( \frac{n^{1 - c^2(\epsilon)g(\epsilon)}}{\ln n} \right)
\]

Thus, \( \lim \mathbb{P}^n(E_h(n, \epsilon)) = 1 \)
HD-ED Relationship between Fixed Point and Random Node

Setting:
- $n$ nodes placed on unit area $\mathcal{A}$ Uniform i.i.d.
- Node location vector $\mathbf{v} = [v_1, v_2, \cdots, v_n] \in \mathcal{A}^n$
- $\mathbb{P}^n(.)$ is the probability measure
- Geometric graph $G(\mathbf{v}, r(n))$ is formed

Notation:
- $\mathcal{N} = [n] = \{1, 2, \cdots, n\}$, the index set of the nodes
- $H_{l,i}(\mathbf{v}) =$ HD of node $i$ from $l^{th}$ anchor on $G(\mathbf{v}, r(n))$, for the deployment $\mathbf{v}$
- $D_{l,i}(\mathbf{v}) =$ Euclidean distance of node $i$ from anchor $b_l$ for the deployment $\mathbf{v}$.

\[
\overline{D}_l(\mathbf{v}, h_l) = \max_{\{i \in \mathcal{N} : H_{l,i}(\mathbf{v}) = h_l\}} D_{l,i}(\mathbf{v})
\]

\[
\underline{D}_l(\mathbf{v}, h_l) = \min_{\{i \in \mathcal{N} : H_{l,i}(\mathbf{v}) = h_l\}} D_{l,i}(\mathbf{v})
\]
We want bounds on the ED between a fixed point and all nodes at a hop-distance $h$ from the point.

We want an upper and a lower bound on the ED.

Area to monitor, $\mathcal{A}$
Define, $E_{h_l}(n) = \{v : (1 - \epsilon)(h_l - 1)r(n) \leq D_l(v, h_l) \leq \overline{D}_l(v, h_l) \leq h_l r(n)\}$

**Theorem**

For a given $1 > \epsilon > 0$, and $r(n) = c \sqrt{\frac{\ln n}{n}}$, $c > \frac{1}{\sqrt{\pi}}$, 

$$\mathbb{P}^n(E_{h_l}(n)) = 1 - O\left(\frac{1}{ng(\epsilon)c^2}\right)$$

where $g(\epsilon) = q(\epsilon)\sqrt{1 - p^2(\epsilon)}$, $p(\epsilon) = \frac{1-\epsilon+\sqrt{(1-\epsilon)^2+8}}{4}$, 

$q(\epsilon) = \frac{-3(1-\epsilon)+\sqrt{(1-\epsilon)^2+8}}{4}$.

Hence, $\lim_{n \to \infty} \mathbb{P}^n(E_{h_l}(n)) = 1$

Since $g(\epsilon) \downarrow$ as $\epsilon \downarrow$, the rate of convergence slows down
Proof Techniques (1/4)

Figure: Construction using the blades cutting the circumference of the circle of radius $h_l r(n)$. 
Proof Techniques (2/4)

\[ u(n) = \sqrt{1 - p^2 r(n)} \]

\[ r(n) \]

\[ t(n) = qr(n) \]

\[ t(n) = qr(n) \]

\[ (p - q)(h_l - 1)r(n) \]

all nodes that fall here will have hop distance \( \leq h_l - 1 \) from \( b_l \)

**Figure:** The construction with \( h_l \) hops.

\[ A^l_{i,j} = \{ v : \exists \text{ at least one node in the } i^{th} \text{ strip of } B^l_j \} \]

\[ \{ \cap_{j=1}^{J(n)} \cap_{i=1}^{h_l-1} A^l_{i,j} \} \]

\[ \subseteq \{ v : (p - q)(h_l - 1)r(n) \leq D_l(v, h_l) \leq \overline{D}_l(v, h_l) \leq h_l r(n) \} \]
\[ \Pr^n \left( \bigcap_{j=1}^{J(n)} \bigcap_{i=1}^{h_l-1} A_{i,j}^l \right) = 1 - \Pr^n \left( \bigcup_{j=1}^{J(n)} \bigcup_{i=1}^{h_l-1} A_{i,j}^l \right) \]

\[ \geq 1 - \sum_{j=1}^{J(n)} \sum_{i=1}^{h_l-1} \Pr^n \left( A_{i,j}^l \right) \]

\[ \geq 1 - (h_l - 1) \left[ \frac{\pi h_l}{2 \sqrt{1 - p^2}} \right] (1 - u(n)t(n))^n \]

\[ = 1 - (h_l - 1) \left[ \frac{\pi h_l}{2 \sqrt{1 - p^2}} \right] e^{-nu(n)t(n)} \]

\[ = 1 - (h_l - 1) \left[ \frac{\pi h_l}{2 \sqrt{1 - p^2}} \right] e^{-nq \sqrt{1 - p^2} r^2(n)} \]

\[ \rightarrow 1 \]
We take $p - q = 1 - \epsilon$, and maximise $q \sqrt{1 - p^2}$.

Gives $p(\epsilon) = \frac{1 - \epsilon + \sqrt{(1 - \epsilon)^2 + 8}}{4}$, $q(\epsilon) = \frac{-3(1 - \epsilon) + \sqrt{(1 - \epsilon)^2 + 8}}{4}$.

Define $g(\epsilon) = q(\epsilon) \sqrt{1 - p^2(\epsilon)}$, Hence,

$$\mathbb{P}^n \{ v : (1 - \epsilon)(h_l - 1)r(n) \leq D_l(v, h_l) \leq \bar{D}_l(v, h_l) \leq h_l r(n) \}$$

$$= 1 - \mathcal{O} \left( \frac{1}{ng(\epsilon)\epsilon^2} \right)$$
Outline of Talk

1 Theory
   - Review of Geometric Graph (GG)
   - Assumptions in Literature
   - Motivation for Random GG
   - Paradigms of HD-ED Relationship in RGG

2 Simulations illustrating Point-Node Theorem

3 Application in Localisation
   - Theorem Used
   - Algorithm: Hop Count-derived Distance-based Localisation (HCDL)

4 Performance Comparison

5 Conclusion and Future Work
Figure: The dashed line shows the ED bounds given by the point-node theorem, the solid line shows $\text{ED} \left( h_1 - 1 \right) r(n)$ from $b_1$ for 1000 nodes, 5 hops, $\epsilon = 0.4$, $\mathbb{P}^n(E_1(n)) \geq 0.37$. $r(n) = \frac{4}{\sqrt{\pi}} \sqrt{\frac{\ln n}{n}}$. 

1000 nodes : 5 hops
Figure: The dashed line shows the ED bounds given by the point-node theorem, the solid line shows ED $(h_1 - 1)r(n)$ from $b_1$ for 5000 nodes, 5 hops, $\epsilon = 0.4$, $\mathbb{P}^n(E_1(n)) \geq 0.79$. $r(n) = \frac{4}{\sqrt{\pi}} \sqrt{\frac{\ln n}{n}}$. 
5000 nodes : 10 hops

Figure: The dashed line shows the ED bounds given by the point-node theorem, the solid line shows ED \((h_1 - 1)r(n)\) from \(b_1\) for 5000 nodes, 10 hops, \(\mathbb{P}^n(E_1(n)) \geq 0.80\). \(r(n) = \frac{4}{\sqrt{\pi}} \sqrt{\frac{\ln n}{n}}.\)
Outline of Talk

1. Theory
   - Review of Geometric Graph (GG)
   - Assumptions in Literature
   - Motivation for Random GG
   - Paradigms of HD-ED Relationship in RGG

2. Simulations illustrating Point-Node Theorem

3. Application in Localisation
   - Theorem Used
     - Algorithm: Hop Count-derived Distance-based Localisation (HCDL)

4. Performance Comparison

5. Conclusion and Future Work
Outline of Talk

1. Theory
   - Review of Geometric Graph (GG)
   - Assumptions in Literature
   - Motivation for Random GG
   - Paradigms of HD-ED Relationship in RGG

2. Simulations illustrating Point-Node Theorem

3. Application in Localisation
   - Theorem Used
   - Algorithm: Hop Count-derived Distance-based Localisation (HCDL)

4. Performance Comparison

5. Conclusion and Future Work
Theorem Used

- For localisation we need multiple (say $L$) anchors
- hop-distance vector, $\mathbf{h} = [h_1, \cdots, h_l, \cdots, h_L] \in \mathbb{N}^L$
- For $L$ anchors, all possible $\mathbf{h}$ vectors are not feasible
- $\mathcal{H}(n)$: set of all feasible $\mathbf{h}$ vectors (it depends on $n$)

Recall, $E_{h_l}(n) = \{v : (1 - \epsilon)(h_l - 1)r(n) \leq D_l(v, h_l) \leq \overline{D}_l(v, h_l) \leq h_l r(n)\}$

**Theorem**

For a given $1 > \epsilon > 0$, and $r(n) = c \sqrt{\frac{\ln n}{n}}$, $c > \frac{1}{\sqrt{\pi}}$, $\forall \mathbf{h} = [h_1, \cdots, h_l, \cdots, h_L] \in \mathcal{H}(n)$,

$$\Pr^n \left( \bigcap_{l=1}^L E_{h_l}(n) \right) = 1 - O \left( \frac{1}{n g(\epsilon) \epsilon^2} \right)$$

where $g(\epsilon) = q(\epsilon) \sqrt{1 - p^2(\epsilon)}$, $p(\epsilon) = \frac{1 - \epsilon + \sqrt{(1 - \epsilon)^2 + 8}}{4}$, $q(\epsilon) = \frac{-3(1 - \epsilon) + \sqrt{(1 - \epsilon)^2 + 8}}{4}$
Illustration

The node lies in this region w. p. $1 - O(n^{-g(e)c^2})$.
Outline of Talk

1 Theory
   - Review of Geometric Graph (GG)
   - Assumptions in Literature
   - Motivation for Random GG
   - Paradigms of HD-ED Relationship in RGG

2 Simulations illustrating Point-Node Theorem

3 Application in Localisation
   - Theorem Used
     - Algorithm: Hop Count-derived Distance-based Localisation (HCDL)

4 Performance Comparison

5 Conclusion and Future Work
Algorithm: Hop Count-derived Distance-based Localisation (HCDL)

- **STEP 1: (Initialisation)** Each node finds the hop-distance vector \( h = [h_1, \cdots, h_L] \)

- **STEP 2: (Region of Intersection)** For a certain node, set an \( \epsilon \), small enough, and find the region of intersection formed by the annuli of radii \([(1 - \epsilon)(h_l - 1)r(n), h_l r(n)]\) centred at the \( l^{th} \) anchor location, \( l = 1, \cdots, L \)

- **STEP 3: (Terminating Condition)**
  - IF there is an intersection, declare the centroid of the region of intersection as the estimate of the node. GO TO **STEP 4**.
  - ELSE increase \( \epsilon \) by an amount \( k \), \( 0 < k < 1 \). GO TO **STEP 2**.

- **STEP 4: (Repetition)** Repeat **STEP 2** to **STEP 3** for all \( n \) nodes.

- **STEP 5: STOP**

---

\(^1\)This is a joint work with Venkatesan N.E. and Prof. P. Vijay Kumar
Outline of Talk

1. Theory
   - Review of Geometric Graph (GG)
   - Assumptions in Literature
   - Motivation for Random GG
   - Paradigms of HD-ED Relationship in RGG

2. Simulations illustrating Point-Node Theorem

3. Application in Localisation
   - Theorem Used
   - Algorithm: Hop Count-derived Distance-based Localisation (HCDL)

4. Performance Comparison

5. Conclusion and Future Work
HCRL (Yang et al. 2007): Localisation Error Pattern

Localization error pattern using $G_{\text{crit}}$, HCRL

1000 nodes, avg error = 0.084
PDM (Lim and Hou, 2005): Localisation Error Pattern

Localization error pattern using $G_{\text{crit}}$, PDM

1000 nodes, avg error = 0.055
HCDL: Localisation Error Pattern

Localization error pattern using $G_{\text{crit}}$, HCDL

1000 nodes, 50 \( \times \) 50 bins, avg error = 0.045
Cumulative Distribution of Error

Location error CDF for the 3 approaches

- HCDL
- PDM
- HCRL

Empirical CDF

location error (1000 nodes, 20 runs)
Outline of Talk

1. Theory
   - Review of Geometric Graph (GG)
   - Assumptions in Literature
   - Motivation for Random GG
   - Paradigms of HD-ED Relationship in RGG

2. Simulations illustrating Point-Node Theorem

3. Application in Localisation
   - Theorem Used
   - Algorithm: Hop Count-derived Distance-based Localisation (HCDL)

4. Performance Comparison

5. Conclusion and Future Work
Conclusion and Future Work

- Assumed a Geometric Graph model of the Wireless Sensor Network
- HD is not a good measure for ED for arbitrary node placements
- Three paradigms of HD-ED proportionality for random node placements
  - Sufficiency theorems for ED-HD relationships in point-point, node-node and point-node paradigms
  - For point-point and node-node cases, the radius of the GG is larger than the critical radius
  - For point-node case, theorem is valid for critical radius too
- For point-node theory, given $HD = h$, $(1 - \epsilon)(h - 1)r < ED \leq hr$ with high probability
- Proposed algorithm HCDL based on this theory
- Performs better than HCRL and PDM
Thank You