SHORT-TERM GENERATION SCHEDULING WITH TRANSMISSION AND ENVIRONMENTAL CONSTRAINTS USING AN AUGMENTED LAGRANGIAN RELAXATION

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ABSTRACT

This paper proposes a new approach based on augmented Lagrangian relaxation for short term generation scheduling problem with transmission and environmental constraints. In this method, the system constraints, e.g. load demand, spinning reserve, transmission capacity and environmental constraints, are relaxed by using Lagrangian multipliers, and quadratic penalty terms associated with system load demand balance are added to the Lagrangian objective function. Then the decomposition and coordination technique is used, and non-separable quadratic penalty terms are replaced by linearization around the solution obtained from the previous iteration. In order to improve the convergence property, the exactly convex quadratic terms of decision variables are added to the objective function as strongly convex, differentiable and separable auxiliary functions. The overall problem is decomposed into N subproblems, multipliers and penalty coefficients are updated in the dual problem and system constraints are satisfied iteratively. The corresponding unit commitment subproblems are solved by dynamic programming, and the economic dispatch with transmission and environmental constraints is solved by an efficient network flow programming algorithm. The augmented Lagrangian relaxation method enhanced by decomposition and coordination technique avoids oscillations associated with piece-wise linear cost functions. Numerical results indicate that the proposed approach is fast and efficient in dealing with numerous system constraints.

Keywords– Power System Operation, Unit Commitment, Economic Dispatch, Augmented Lagrangian Relaxation, Decomposition and Coordination Technique, Transmission Capacity, Environmental Constraint

1. INTRODUCTION

Every modern society mandates an adequate supply of electric energy as economically as possible with a reasonable level of quality and continuity. With the steady growth of demand for power, electric utilities were able to improve the system security by adding generation and transmission facilities to the system; however, as new construction projects are being postponed or canceled due to financial, regulatory and environmental constraints, electric utilities are forced to consider other options for the secure operation of the system. Furthermore, public outcry for clean air [1] has suggested the use of electric mass transit systems in metropolitan areas with congested highways, the conversion of open-hearth furnaces to electric furnaces in steel industry, etc. These projects create additional loads for electric utilities while power generation reserves become more scarce and pollution emission constraints force utilities to curtail older and cheaper power stations near the urban areas, placing more constraints on transmission systems. One possible option for consumers and suppliers is to consider wheeling across third party owned transmission systems [2], which will further overload transmission lines in intermediate utility systems. The constrained transmission capacity may further strain generation dispatch and influence system reliability.

Traditionally, short-term generation scheduling considers two separate entities,

- the generation system, where emphasis lays on generation side of the system, dynamic constraints are taken into account and the cost function is not necessarily differentiable. The network is reduced to only one node, i.e., the coupling constraints, representing the equality between generation and demand as well as the inequality for system spinning reserve consideration. This model is the classical unit commitment problem [3-6]. Since the coupling constraints are not numerous, decomposition methods such as Lagrangian relaxation (LR) have often been used for generating efficient solutions for this kind of problems. Essentially, the LR method involves the decomposition of the problem into a sequence of master problems and easy subproblems, whose solutions converge to a near optimal solution for the original problem. Due to many researchers' efforts, LR has been successfully applied to realistic size systems.
- the transmission system, where emphasis lays on transmission network, constraints are static and the cost function is differentiable. This model is usually called the optimal power flow problem. It provides the on-line generation schedule for the system in each time period. Efficient algorithms [7-9] for OPF have been considered in the past by exploring the particular structure and sparsity of the constraint matrix.

By disregarding the transmission network constraints, the classical unit commitment may not satisfy OPF requirements. However, separating the short-term generation scheduling into two parts and not dealing with static (spatial) and dynamic (temporal) constraints simultaneously has been, to some degree, due to mathematical difficulties in real-time system. These technical difficulties are eminent as the size and nature of the optimization problems in each case are considered for large scale systems. The solution of these problems in real time is a challenge to modern mathematical optimization theory.

One common approach is to consider these two problems in two steps to achieve computational simplicity; the approach ignores transmission network constraints in the unit commitment part of the algorithm, then accounts for network constraints during economic dispatch. However, the approach may not be very efficient since it attempts to determine unit commitment strictly based on its generation cost characteristic, then schedule the generation allocation according to the determined commitment which may require immense computation to adjust the commitment to conform to transmission constraints.

Another approach is to consider these two problems with simultaneous spatial and temporal complexity in one solution process. Reference [10] was the first paper which accounted for transmission constraints in both unit commitment and economic

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dispatch. Its proposed algorithm solved a security-constrained economic dispatch subproblem based on DC power flow to evaluate the lower and upper bound costs for each branch point; however, the unit commitment part was greatly simplified since it did not include start-up/shut-down costs and minimum up-time/down-time constraints. Reference [11] considered transmission constraints by relaxing the transmission constraints and adding them to the objective function; however, computational results were limited to two transmission lines. In addition, the computational time increased considerably when the two lines were added. Practically, the rather slower and unsteady convergence is due to applying LR to generation scheduling. Reference [12-13] presented an augmented Lagrangian technique to deal with generation scheduling; however, inequality (system spinning reserve and transmission) constraints as well as equality (system load demand) constraint were added as quadratic penalty terms. For each inequality constraint, a heuristic method introduces equality constraints by applying slack variables. Since there are a score of transmission lines in large scale power systems, this implementation will introduce a large volume of new variables and increase the dimension of the problem inevitably, thus making the model very difficult to solve. Moreover, transmission constraints are taken into account only at one time period.

In this paper, the augmented Lagrangian relaxation method (see Appendix A) using the decomposition and coordination technique is used for generation scheduling with transmission and environmental constraints. The coupling constraints, i.e. load demand, spinning reserve, as well as transmission and environmental limits are relaxed by Lagrangian multipliers and added to the objective function, quadratic penalty terms associated with the system load demand balance are added to the Lagrangian objective function, and the decomposition and coordination technique is used. The corresponding unit commitment subproblems are solved by dynamic programming, and the economic dispatch with transmission and environmental constraints is solved by an efficient network flow programming algorithm. An illustrative example is presented to show the effectiveness of the proposed approach.

2. PROBLEM FORMULATION

The objective of generation scheduling is to minimize the power system operation cost including the cost of fuel for energy generation and starting up processes, while satisfying transmission, environmental as well as other system constraints.

The list of symbols used in this paper is as follows:

$\mathcal{F}$: operating cost of the system

$P_i(t)$: power output of unit i at hour t, (MW)

N_t: total study time in hours

N: total number of units

N_{Bg}: total number of generation buses

N_{Bl}: total number of load buses

N_B: total number of system buses

M: total number of transmission lines

$I_i(t)$: commitment state (1 or 0) of unit i at hour t

$F_i(P_i(t))$: fuel cost of unit i when generating power is $P_i(t)$

$H_i(P_i(t))$: emission function of unit i

$X_i^{on/off}(t)$: time duration for which unit i has been on/off at hour t

$S_i(t)$: start up cost of unit i at hour t

$T_i^{on/off}(t)$: minimum up/down time of unit i

$P_D(t)$: system load demand at hour t, (MW)

$\bar{P}_i$: rated upper generation limit of unit i, (MW)

$\underline{P}_i$: rated lower generation limit of unit i, (MW)

$P_R(t)$: system spinning reserve requirement at hour t, (MW)

$P_m(t)$: power flow of transmission line m, (MW)

$\underline{P}_m(t), \bar{P}_m(t)$: power flow limits of transmission line m, (MW)

k_{mn}: sensitivity coefficient for line m flow with respect to power output of unit n, $P_n(t)$ (see Appendix B for further information)

$\lambda(t)$: Lagrangian multiplier for system power balance constraint

c: positive penalty coefficient for system power balance constraint

$\mu(t)$: Lagrangian multiplier for system spinning reserve constraint

$\gamma_m^+(t), \gamma_m^-(t)$: Lagrangian multiplier for transmission line capacity constraints

ν: Lagrangian multiplier for system emission constraints in unit commitment subproblem

ν_e: Lagrangian multiplier for system emission constraints in economic dispatch subproblem

EM: emission cap for system emission output in the study period

$\hat{C}_i$: selected segment of the incremental cost for unit i

P: bus real power injection vector

P_l: transmission line real power flow vector

P_d: bus load vector

A: power system transmission network incidence matrix

X: power system transmission network basic loop reactance matrix

Mathematically, the optimization problem can be described as follows. The objective of the problem is to minimize,

$$\mathcal{F} = \sum_{i=1}^{N}\sum_{t=1}^{N_t}\Big[I_i(t)F_i(P_i(t)) + S_i(t)\Big] \tag{1}$$

The constraints for the problem are:

1) System power balance

$$\sum_{i=1}^{N} I_i(t)P_i(t) = P_D(t) \qquad t = 1,\ldots,N_t \tag{2}$$

2) System spinning reserve requirements

$$\sum_{i=1}^{N} \bar{P}_i(t)I_i(t) \geq P_D(t) + P_R(t) \qquad t = 1,\ldots,N_t \tag{3}$$

3) Unit generation limits

$$\underline{P}_i \leq P_i(t) \leq \bar{P}_i \qquad \begin{matrix} i = 1,\ldots,N \\ t = 1,\ldots,N_t \end{matrix} \tag{4}$$

4) Thermal unit minimum start up/shut down times

$$\left(X_i^{on}(t-1) - T_i^{on}\right) * \left(I_i(t-1) - I_i(t)\right) \geq 0 \tag{5}$$

$$\left(X_i^{off}(t-1) - T_i^{off}\right) * \left(I_i(t) - I_i(t-1)\right) \geq 0 \tag{6}$$

5) Transmission line capacity limits

$$\underline{P}_m(t) \le P_m(t) \le \bar{P}_m(t)$$
$$P_m(t) = \sum_{i=1}^{N} k_{mi} P_i(t) I_i(t) \qquad m = 1,\dots,M \quad t = 1,\dots,N_t \quad i = 1,\dots,N \tag{7}$$

6) Environmental limits

$$\sum_{t=1}^{N_t}\sum_{i=1}^{N} H_i(P_i(t)) I_i(t) \le EM \qquad t = 1,\dots,N_t \quad i = 1,\dots,N \tag{8}$$

3. APPROACH TO UNIT COMMITMENT

3.1 Augmented Lagrangian Function For Unit Commitment

At present, one of the potential approaches for solving the proposed problem is the LR method. The basic idea of LR is to relax the system constraints, i.e. power demand, reserve requirements, as well as transmission and environmental limits by using Lagrangian multipliers. The relaxed problem is then decomposed into N subproblems. The search process is an iterative algorithm that solves relaxed subproblems and updates Lagrangian multipliers according to the extent of violation of system constraints. This implementation is systematic and efficient, but the quality of the final solution depends on the sensitivity of the commitment to Lagrangian multipliers [14]. Unless a proper modification of multipliers is ensured in every iteration, unnecessary commitment of generating units may occur, which may result in higher operation costs. These difficulties are often explained by the nonconvexity of this type of the optimization problem. Recent optimization theory shows that this explanation is true but not complete. Even some convex problems cannot be solved by this approach. In earlier studies, the cost functions were considered as piece-wise linear, which resulted in a rather slow and unsteady convergence. These difficulties can be overcome by the augmented Lagrangian method and decomposition and coordination technique [15]. In this method, quadratic penalty terms associated with power demand are added to the objective function to improve the algorithm convergence.

So, the augmented Lagrangian function for unit commitment is,

$$\begin{aligned}
&\mathcal{L}(P_i(t), I_i(t), \lambda(t), \mu(t), \gamma_m(t)^+, \gamma_m(t)^-, \nu) \\
&= \sum_{i=1}^{N}\sum_{t=1}^{N_t}\Big[F_i(P_i(t))I_i(t) + S_i(t)))\Big] \\
&- \sum_{t=1}^{N_t}\lambda(t)\Big(\sum_{i=1}^{N} I_i(t)P_i(t) - P_D(t)\Big) \\
&- \frac{c}{2}\sum_{t=1}^{N_t}\Big(\sum_{i=1}^{N} I_i(t)P_i(t) - P_D(t)\Big)^2 \\
&- \sum_{t=1}^{N_t}\mu(t)\Big(\sum_{i=1}^{N}\bar{P}_i(t)I_i(t) - P_D(t) - P_R(t)\Big) \\
&- \sum_{t=1}^{N_t}\sum_{m=1}^{M}\gamma_m^+(t)(\bar{P}_m(t) - P_m(t)) \\
&- \sum_{t=1}^{N_t}\sum_{m=1}^{M}\gamma_m^-(t)(P_m(t) - \underline{P}_m(t)) \\
&+ \nu\Big(\sum_{t=1}^{N_t}\sum_{i=1}^{N} H_i(P_i(t))I_i(t) - EM\Big)
\end{aligned} \tag{9}$$

with constraints expressed by (4), (5) and (6).

Obviously, the generalized penalty term $\frac{c}{2}\sum_{t=1}^{N_t}\Big(\sum_{i=1}^{N} I_i(t)P_i(t) - P_D(t)\Big)^2$ disrupts the decomposition ability of the problem. By applying the decomposition and coordination technique, the nonseparable quadratic penalty terms are linearized around the solution obtained from the previous iteration in the solution algorithm. Moreover, exactly convex quadratic terms of decision variables are added to the objective function as auxiliary functions which is strongly convex, differentiable and separable to improve the convergence property. Thus the objective of the $(k+1)$th iteration is given as:

$$\begin{aligned}
&\mathcal{L}(P_i(t), I_i(t), \lambda(t), \mu(t), \gamma_m(t)^+, \gamma_m(t)^-, \nu) \\
&= \sum_{i=1}^{N}\sum_{t=1}^{N_t}\Big[F_i(P_i(t))I_i(t) + S_i(t)\Big] \\
&- \sum_{t=1}^{N_t}\lambda(t)\Big(\sum_{i=1}^{N} I_i(t)P_i(t) - P_D(t)\Big) \\
&- \sum_{t=1}^{N_t}\mu(t)\Big(\sum_{i=1}^{N}\bar{P}_i(t)I_i(t) - P_D(t) - P_R(t)\Big) \\
&- \sum_{t=1}^{N_t}\sum_{m=1}^{M}\gamma_m^+(t)(\bar{P}_m(t) - P_m(t)) \\
&- \sum_{t=1}^{N_t}\sum_{m=1}^{M}\gamma_m^-(t)(P_m(t) - \underline{P}_m(t)) \\
&+ \nu\Big(\sum_{t=1}^{N_t}\sum_{i=1}^{N} H_i(P_i(t))I_i(t) - EM\Big) \\
&+ \frac{1}{2\varepsilon}\sum_{t=1}^{N_t}\sum_{i=1}^{N}\Big([I_i(t)P_i(t) - I_i^k(t)P_i^k(t)]^2\Big) \\
&- c\cdot\sum_{t=1}^{N_t}\Big([\sum_{i=1}^{N} I_i(t)P_i(t) - P_D(t)] \\
&\quad\cdot[\sum_{i=1}^{N} I_i^k(t)P_i^k(t) - P_D(t)]\Big)
\end{aligned} \tag{10}$$

where $I_i^k(t)$, $P_i^k(t)$ are the results obtained from the previous iteration, ε is a positive number.

The search for the optimal commitment schedule under certain system conditions, that is for a given $\lambda(t), \mu(t), \gamma_m^+(t), \gamma_m^-(t)$ and ν as $t = 1,\dots,N_t$ which are denoted by $\hat{\lambda}(t), \hat{\mu}(t), \hat{\gamma}_m^+(t), \hat{\gamma}_m^-(t)$ and $\hat{\nu}$ becomes a minimization process for variables $P_i(t)$ and $I_i(t)$. Hence,

$$\begin{aligned}
&\min\mathcal{L}(P_i(t), I_i(t), \hat{\lambda}(t), \hat{\mu}(t), \hat{\gamma}_m^+(t), \hat{\gamma}_m^-(t), \hat{\nu}) \\
&= \sum_{i=1}^{N}\Big\{\sum_{t=1}^{N_t}[F_i(P_i(t))I_i(t) + S_i(t)))] \\
&\qquad - \sum_{t=1}^{N_t}\hat{\lambda}(t)I_i(t)P_i(t) - \sum_{t=1}^{N_t}\hat{\mu}(t)\bar{P}_i(t)I_i(t) \\
&+ \frac{1}{2\varepsilon}\sum_{t=1}^{N_t}\Big([I_i(t)P_i(t) - I_i^k(t)P_i^k(t)]^2\Big) \\
&- c\cdot\sum_{t=1}^{N_t}\Big(I_i(t)P_i(t)\cdot[\sum_{i=1}^{N} I_i^k(t)P_i^k(t) - P_D(t)]\Big) \\
&+ \hat{\nu}\sum_{t=1}^{N_t} H_i(P_i(t))I_i(t)\Big\} + c\cdot\sum_{t=1}^{N_t}[P_D(t)\cdot\sum_{i=1}^{N} I_i^k(t)P_i^k(t)]
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{t=1}^{N_t}\sum_{m=1}^{M}(\hat{\gamma}_m^+(t)-\hat{\gamma}_m^-(t))P_m(t))-c\cdot\sum_{t=1}^{N_t}P_D(t)^2\\
&+\sum_{t=1}^{N_t}\Big[\hat{\lambda}(t)P_D(t)+\hat{\mu}(t)\big(P_D(t)+P_R(t)\big)\Big]\\
&-\sum_{t=1}^{N_t}\sum_{m=1}^{M}(\hat{\gamma}_m^+(t)\bar{P}_m(t)+\hat{\gamma}_m^-(t)\underline{P}_m(t))-\hat{\nu}EM\\
&=\sum_{i=1}^{N}\Big\{\sum_{t=1}^{N_t}[F_i(P_i(t))I_i(t)+S_i(t)))]\\
&\qquad-\sum_{t=1}^{N_t}\hat{\lambda}(t)I_i(t)P_i(t)-\sum_{t=1}^{N_t}\hat{\mu}(t)\bar{P}_i(t)I_i(t)\\
&+\frac{1}{2\varepsilon}\sum_{t=1}^{N_t}\Big([I_i(t)P_i(t)-I_i^k(t)P_i^k(t)]^2\Big)\\
&-c\cdot\sum_{t=1}^{N_t}\Big(I_i(t)P_i(t)\cdot[\sum_{i=1}^{N}I_i^k(t)P_i^k(t)-P_D(t)]\Big)\\
&+\hat{\nu}(\sum_{t=1}^{N_t}H_i(P_i(t))I_i(t)\Big\}+c\cdot\sum_{t=1}^{N_t}[P_D(t)\cdot\sum_{i=1}^{N}I_i^k(t)P_i^k(t)]\\
&+\sum_{t=1}^{N_t}\sum_{m=1}^{M}(\hat{\gamma}_m^+(t)-\hat{\gamma}_m^-(t))(\sum_{i=1}^{N}k_{mi}P_i(t)I_i(t))-c\cdot\sum_{t=1}^{N_t}P_D(t)^2\\
&+\sum_{t=1}^{N_t}\Big[\hat{\lambda}(t)P_D(t)+\hat{\mu}(t)\big(P_D(t)+P_R(t)\big)\Big]\\
&-\sum_{t=1}^{N_t}\sum_{m=1}^{M}(\hat{\gamma}_m^+(t)\bar{P}_m(t)+\hat{\gamma}_m^-(t)\underline{P}_m(t))-\hat{\nu}EM \qquad (11)
\end{aligned}
$$

In this regard, the optimization problem can be decoupled into N subproblems, each corresponding to the optimal unit commitment of individual units over the entire study period, which is expressed as,

$$
\begin{aligned}
&\min\mathcal{L}\big(P_i(t),I_i(t),\hat{\lambda}(t),\hat{\mu}(t),\hat{\gamma}_m^+(t),\hat{\gamma}_m^-(t),\hat{\nu}\big)\\
&=\sum_{t=1}^{N_t}\Big(F_i(P_i(t)-\hat{\lambda}(t)P_i(t)-\hat{\mu}(t)\bar{P}_i(t)\\
&-c\cdot P_i(t)\cdot[\sum_{i=1}^{N}I_i^k(t)P_i^k(t)-P_D(t)]\\
&+\sum_{m=1}^{M}(\hat{\gamma}_m^+(t)-\hat{\gamma}_m^-(t))k_{mi}P_i(t)+\hat{\nu}H_i(P_i(t))\Big)I_i(t)\\
&+\sum_{t=1}^{N_t}\Big(S_i(t)+c\cdot P_D(t)\cdot I_i^k(t)P_i^k(t)\\
&+\frac{1}{2\varepsilon}[I_i(t)P_i(t)-I_i^k(t)P_i^k(t)]^2\Big) \qquad (12)
\end{aligned}
$$

subject to (4), (5) and (6).

As we do not consider economic dispatch at this stage, $P_i(t)$ is set equal to the unit power generation corresponding to the system power incremental cost $\lambda(t)$. Consequently, the integer variables $I_i(t)$, $t=1,\ldots,N_t$ are the remaining unknown factors. We adopt a dynamic programming model to find the suitable values of $I_i(t)$, which minimize (12) and satisfy (4), (5) and (6).

An iterative procedure is used to search for a suitable solution of $\mathcal{L}(P_i(t), I_i(t), \lambda(t), \mu(t), \gamma_m^+(t), \gamma_m^-(t), \nu)$ for the relaxed energy balance constraints, etc. The procedure for finding a suitable unit commitment schedule is as follows:

Step 1 Assign an initial value to $\lambda(t)$, $\mu(t)$, $\gamma_m^+(t)$, $\gamma_m^-(t)$, and ν $t=1,\ldots,N_t$ denoted by $\hat{\lambda}(t)$, $\hat{\mu}(t)$, $\hat{\gamma}_m^+(t)$, $\hat{\gamma}_m^-(t)$ and $\hat{\nu}$.

Step 2 Find the suitable unit commitment schedule under current system operating status. That is,

$$\min\mathcal{L}\big(P_i(t),I_i(t),\hat{\lambda}(t),\hat{\mu}(t),\hat{\gamma}_m^+(t),\hat{\gamma}_m^-(t),\hat{\nu}\big)$$

where the solution is denoted by $\hat{P}_i(t)$ and $\hat{I}_i(t)$.

Step 3 If (2), (3), (7) and (8) are satisfied, we have obtained a satisfactory unit commitment schedule; proceed with the economic dispatch algorithm; otherwise, goto Step 4.

Step 4 Corresponding to the unit states $\hat{P}_i(t)$ and $\hat{I}_i(t)$, use the subgradient method to modify $\lambda(t)$, $\mu(t)$, $\gamma_m^+(t)$, $\gamma_m^-(t)$, and ν. Once the updated $\hat{\lambda}(t)$, $\hat{\mu}(t)$, $\gamma_m^+(t)$, $\gamma_m^-(t)$, and ν are obtained, go back to Step 2.

4. APPROACH TO ECONOMIC DISPATCH

4.1 Problem formulation for economic dispatch

Once the generation in a power system is scheduled, it is necessary to determine the optimal allocation of the system demand among generating units. Mathematically, economic dispatch is much simpler than unit commitment since it is a continuous variable optimization process with several sets of coupling constraints, e.g., system energy balance constraints [16]. Since we adopt a piecewise linearized fuel cost function, the system generation cost will be expressed as a linear function. The problem is stated as follows:

Objective function:

$$\mathcal{F}=\sum_{i=1}^{N}\sum_{t=1}^{N_t}F_i(P_i(t)) \qquad (13)$$

Transmission line capacity constraints:

In economic dispatch, we model transmission line capacity constraints through a DC power flow with bounds on power injections at nodes and flows on lines. So, for each time period,

$$A\cdot P_l=P-P_d \qquad (14)$$

$$X\cdot P_l=0 \qquad (15)$$

$$\underline{P}\le P\le\bar{P} \qquad (16)$$

$$\underline{P}_l\le P_l\le\bar{P}_l \qquad (17)$$

Here, (14) and (15) represent the electric network transmission with a DC power flow model. (14) corresponds to Kirchhoff Current Law (KCL) at each node and (15) corresponds to Kirchhoff Voltage Law (KVL) in each basic loop; the number of basic loops in the transmission network is the same as the number of branches minus the number of buses plus one. (16) and (17) represent limits on power injections and flows.

Environmental limits:

$$\sum_{t=1}^{N_t}\sum_{i=1}^{N}H_i(P_i(t))\le EM \qquad (18)$$

4.2 Solution for economic dispatch

In order to solve (13)-(18), a decomposition approach is used. The Lagrangian function which relaxes the environmental constraints is,

$$\min\mathcal{L}\big(P_i(t),\hat{\nu}_e\big)=\sum_{i=1}^{N}\sum_{t=1}^{N_t}F_i\big(P_i(t)\big)$$

$$+\hat{\nu}_e(\sum_{t=1}^{N_t}\sum_{i=1}^{N} H_i(P_i(t)) - EM) \quad (19)$$

Therefore, the objective function for each time period is to minimize,

$$\sum_{i=1}^{N}[\hat{C}_i P_i(t) + \hat{\nu}_e H_i(P_i(t))] \quad (20)$$

subject to (14)-(17).

Figure 1 shows the flow chart of the proposed approach. The external loop carries out the dual solution of the problem with respect to the environmental constraint. The internal loop performs the optimization process which considers the electrical network transmission constraints in a time sequence.

For each ν_e, the internal loop provides a feasible schedule that consumes certain amounts of fuel for generation. If the environmental constraint is violated in a given period, the multiplier ν_e is updated as:

$$\nu_e^{k+1} = \nu_e^k + \epsilon^k \cdot [\sum_{t=1}^{N_t}\sum_{i=1}^{N} H_i(P_i(t)) - EM] \quad (21)$$

The economic dispatch, which minimizes (20) subject to (14)-(17) in each time period, is a network flow problem with additional linear constraints. The efficiency of the approach is highly dependent upon the efficiency of the above problem, so we use an efficient algorithm [17-23] which exploits the particular structure of such a problem to obtain the optimal solution. In our opinion, network flow programming is much faster than linear programming since network flow can be viewed as a special linear programming problem. In this application, another advantage of the network based algorithm is the feasibility of the initial solution. The lower bounds of load and generation buses are zero and the upper bounds are the required load demands and power generations. So, the trivial zero solution is a feasible initial solution and the generation and load arcs are considered as the initial spanning tree. Since several economic dispatches are to be solved in each period, the efficiency of the proposed approach can be improved if proper initial solutions are obtained. In the first iteration of relaxation for the first ν_e, the final solution of the economic dispatch at time period t can be used as the initial solution for time period t+1.

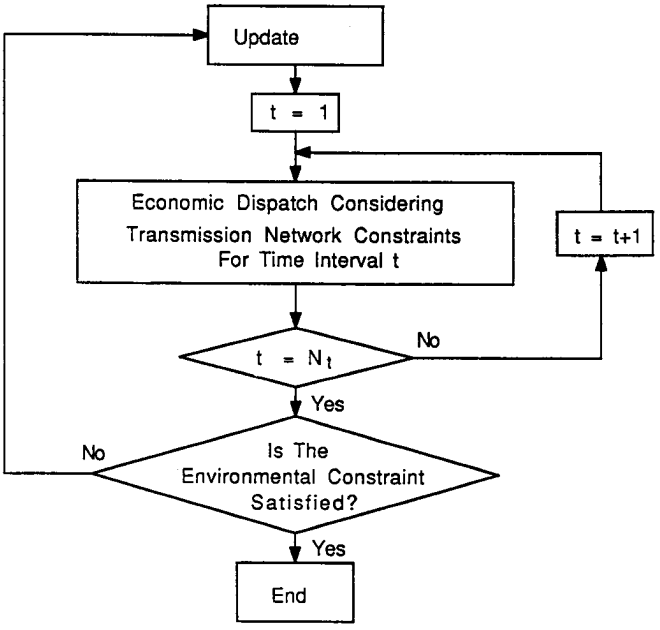

Figure 1 Economic dispatch flow chart

5. TEST RESULTS

The proposed approach is applied to the short term generation scheduling problem of the modified IEEE 24-bus system [24-25] shown in Figure 2. The data for the 26 units are given in Tables 1 & 2. The hourly peak loads are listed in Table 3. The percentage of system load at each bus as well as the line data are given in [25]. The emission coefficients are the same as those for corresponding unit fuel cost curves and the emission cap is the same as the peak load, all multiplied by a conversion factor of 0.8. In this composite system, load in area 1 is 46.70% of the total load, however generation capacity in this area is only 20.08% of the total generation capacity. Thus the power flow is from area 2 to area 1. In order to show the effect of line flow limits on generation scheduling, flow limits on lines 11-14, 11-13, 12-13, 12-23 and 15-24 are reduced to 350 MW. If we disregard the transmission lines limits and reduce the transmission network to one node, the commitment schedule will not use the most expensive combustion units and the calculated hourly generation cost will be $843629.18. However, the commitment schedule will overload the corresponding lines during the peak load period, especially those lines connected to buses 3 and 9. As we consider line flow limits in our algorithm, the calculated hourly generation cost will be $877126.39, which will make use of cycling units at bus 1 and bus 7. In addition, if we disregard line flow limits in unit commitment, i.e. flow constraints are only considered in economic dispatch, the calculated generation schedule will start up the most expensive combustion units at bus 1 to meet the peak load. The hourly generation cost in this case will be $884254.67, which is higher than that of our proposed approach. The cost savings will be more significant as we apply this method to more practical systems.

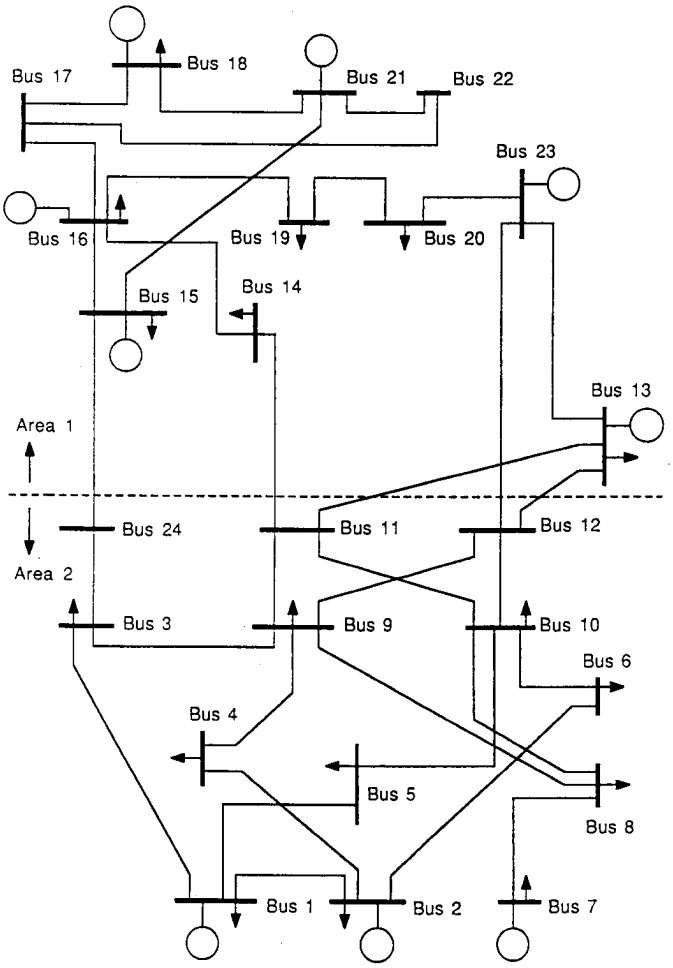

Figure 2 IEEE 24-Bus System

Table 1 Generating unit Data *

Unit	$\underline{P_i}$(MW)	$\overline{P_i}$(MW)	a_i(k$/MW^2)	b_i(k$/MW)	c_i(k$)	Bus No.
1	2.4	12.0	0.02533	25.5472	24.3891	15
2	2.4	12.0	0.02649	25.6753	24.4110	15
3	2.4	12.0	0.02801	25.8027	24.6382	15
4	2.4	12.0	0.02842	25.9318	24.7605	15
5	2.4	12.0	0.02855	26.0611	24.8882	15
6	4.0	20.0	0.01199	37.5510	117.7551	1
7	4.0	20.0	0.01261	37.6637	118.1083	1
8	4.0	20.0	0.01359	37.7770	118.4576	2
9	4.0	20.0	0.01433	37.8896	118.8206	2
10	15.2	76.0	0.00876	13.3272	81.1364	1
11	15.2	76.0	0.00895	13.3538	81.2980	1
12	15.2	76.0	0.00910	13.3805	81.4641	2
13	15.2	76.0	0.00932	13.4073	81.6259	2
14	25.0	100.0	0.00623	18.0000	217.8952	7
15	25.0	100.0	0.00612	18.1000	218.3350	7
16	25.0	100.0	0.00598	18.2000	218.7752	7
17	54.25	155.0	0.00463	10.6940	142.7348	15
18	54.25	155.0	0.00473	10.7154	143.0288	16
19	54.25	155.0	0.00481	10.7367	143.3179	23
20	54.25	155.0	0.00487	10.7583	143.5972	23
21	68.95	197.0	0.00259	23.0000	259.1310	13
22	68.95	197.0	0.00260	23.1000	259.6490	13
23	68.95	197.0	0.00263	23.2000	260.1760	13
24	140.0	350.0	0.00153	10.8616	177.0575	23
25	100.0	400.0	0.00194	7.4921	310.0021	18
26	100.0	400.0	0.00195	7.5031	311.9102	21

$* F_i(\ P_i(t)\ ) = a_i\ P_i^2(t) + b_i\ P_i(t) + c_i$

Table 2 Generating units operating parameters*

unit	min up (h)	min down (h)	init. cond. (h)	α_i ($)	β_i ($)	τ_i (h)
1--5	0	0	-1	0	0	1
6--9	0	0	-1	20	20	2
10--13	3	-2	3	50	50	3
14--16	4	-2	-3	70	70	4
17--20	5	-3	5	150	150	6
21--23	5	-4	-4	200	200	8
24	8	-5	10	300	200	8
25--26	8	-5	10	500	500	10

$* S_i(t) = I_i(t)\ (1 - I_i(t-1))\left[\alpha_i + \beta_i\left[1 - \exp\left(X_i^{off}(t)/\tau_i\right)\right]\right]$

Table 3 Hourly Load Demand

HOUR	LOAD MW	HOUR	LOAD MW	HOUR	LOAD MW
1	2223.0	9	2280.0	17	2593.5
2	2052.0	10	2508.0	18	2850.0
3	1938.0	11	2565.0	19	2821.5
4	1881.0	12	2593.5	20	2764.5
5	1824.0	13	2565.0	21	2679.0
6	1825.5	14	2508.0	22	2622.0
7	1881.0	15	2479.5	23	2479.5
8	1995.0	16	2479.5	24	2308.5

6. CONCLUSIONS

An efficient algorithm based on the augmented Lagrangian relaxation method and the decomposition and coordination technique has been developed to solve the integrated transmission and environmental constrained generation scheduling problem. The algorithm overcomes the oscillations problem caused by linear cost functions when the Lagrangian relaxation approach is used. Test results show that this algorithm is fast, efficient and robust, and provides reasonable results in practical size systems.

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APPENDICES

A. Augmented Lagrangian Relaxation Method

The augmented LR method is a mix of penalty and LR methods. In the penalty method, in order to obtain the optimal solution, we must increase the penalty parameter c and solve the sequential optimization problem; on the other hand, the algorithm based on the Lagrangian duality theory, such as the Uzawa algorithm, will be effective only if $\mathcal{L}(x,\lambda)$ has only one optimization value at the neighboring domain of the duality optimal value λ^*. The augmented LR maintains the advantages of LR and penalty methods and overcomes the disadvantages of both methods.

For nonlinear programming problem,

Minimize $f(x)$

subject to $h(x) = 0$

The augmented Lagrangian function is defined as,

$$\mathcal{L}(x,\lambda,c) = f(x) + \lambda^T \cdot h(x) + \frac{1}{23} \cdot c \cdot h^2(x) \qquad (A.1)$$

Compared with LR, the augmented LR method has the following advantages:

1) $\mathcal{L}(x,\lambda,c)$ has the same saddle point as that of $\mathcal{L}(x,\lambda)$.

2) The dual function of $\mathcal{L}(x,\lambda,c)$, $w(\lambda)$ is a concave function of λ, and differentiable. For $\bar{\lambda} \in R^{m+}$, $h(\bar{x})$ is the gradient of $w(\lambda)$ to $\bar{\lambda}$, and $\bar{x}$ is the optimal value of $\mathcal{L}(x,\lambda,c)$.

3) If $\bar{\lambda} \geq 0$ is the optimal value of the dual problem, $\bar{x}$ is the optimal value of $\mathcal{L}(x,\lambda,c)$, then $(\bar{x},\bar{\lambda})$ must be the saddle point of the primal problem and $\bar{x}$ is the optimal value of the primal problem.

The quadratic penalty term $\frac{1}{2} \cdot c \cdot h^2(x)$ disrupts the separability of the problem; however, the decomposition and coordination technique in [16-17] is an efficient tool to maintain the good feature of the augmented LR, in the meantime, obtain the separable form of the nonlinear programming problem.

B. Calculation of $k_{mn}, \underline{P}_m(t), \bar{P}_m(t)$

The DC load flow relates the net real power injection P to bus voltage angles δ by neglecting line resistances and assuming voltage magnitudes to be 1 per unit. The matrix equation is given by,

$$P = B' \cdot \delta \qquad (B.1)$$

or

$$\delta = A' \cdot P \qquad (B.2)$$

where B' is the bus susceptance matrix, $b'_{ij} = -\frac{1}{x_{ij}}$, for $i \neq$ ref and $j \neq$ ref; $b'_{ij} = 0.0$ for $i =$ ref or $j =$ ref; $b'_{ii} = -\sum_{\substack{j=1 \\ j \neq i}}^{n} b'_{ij}$ for $i \neq$ ref; $b'_{ii} = 0.0$ for $i =$ ref. B' is a singular matrix, and if there are N_B buses, we only have N_B-1 linearly independent equations. Thus, we have a $(N_B\text{-}1)\times(N_B\text{-}1)$ submatrix of B', which can be inverted, and A' represents the inverse of the submatrix of B' plus a zero row and column corresponding to the reference bus. The power flows P_{ij} are given by

$$P_{ij} = \frac{\delta_i - \delta_j}{x_{ij}} \qquad (B.3)$$

If we use a_i and a_j representing the ith and the jth rows of A', then,

$$\underline{P}_{ij} \leq P_{ij}(t) = [\frac{a_i - a_j}{x_{ij}}]P(t) = \sum_{n=1}^{N_B} k_{mn} P_n(t) \leq \bar{P}_{ij} \qquad (B.4)$$

so

$$k_{mn} = k_{ij} = \frac{a_{in} - a_{jn}}{x_{ij}}$$

We separate system buses as generation and load buses, then the transmission line constraints can be expressed as follows:

$$\underline{P}_{ij} \leq P_{ij}(t) = \sum_{n=1}^{N_{Bg}} k_{mn} P_n(t) - \sum_{l=1}^{N_{Bl}} k_{ml} P_l(t) \leq \bar{P}_{ij} \qquad (B.5)$$

We adjust the left and the right hand limits of (B.5) to obtain (7) which represents power flow limits in a closed form. The values of the distribution factors k_{mn} and of transmission line capacity limits are calculated in advance.

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DISCUSSION

D. M. VINOD KUMAR (Indian Institute of Technology, Kanpur, India):

The authors are to be complimented for their effort to use a novel approach to minimize the power system operation cost considering various constraints. The discusser appreciates the authors response on the following:

(a) While considering various constraints authors pointed out algorithm is fast, efficient and robust. To substantiate these, provide total CPU time and give test results of a simple practical system.

(b) Is authors observed CPU time, by considering each constraint one by one. If so provide CPU time with individual constraints.

(c) Is this algorithm is faster even on a moderate size practical power system consisting of a large number of buses and generators, for real-time operation.

Mohammad Shahidehpour and Sasan Mokhtari– We would like to express our appreciation to the discusser for his interest in this paper. In the following, we will address the points raised in the discussion.

(a) The augmented Lagrangian method described in this paper is fast and efficient as it reshapes the objective function to reduce the duality gap. We found that the coefficient of the augmented term, c, can also be used for updating the Lagrange multipliers which devises a practical methodology for optimizing the objective function. For $c = 0.009$ the approach converges in 5 iterations and 19 CPU seconds (on a SPARC10 workstation). However, if c is decreased to 0.006, the solution requires 17 iterations to converge. In our case study, when the objective function is not augmented (the conventional Lagrangian relaxation approach), the solution does not converge after 100 iterations.

(b) When transmission constraints are added, the approach requires 18 iterations to converge in 30 CPU seconds. When the system emission constraint is included in the original formulation, the approach requires 29 iterations in 41 CPU seconds. One type of emission is considered in this study, however, the proposed approach can accommodate several types of emission constraints (e.g. SO2, NOx, CO2, etc).

(c) We have tested our approach on certain utility systems with 37 and 57 units for study periods up to one week. In most cases, our approach converges within 3 to 5 CPU minutes for a typical size power system. This is an important feature of our approach as compared with more conventional methods which suffer from a long CPU time. More recently, we have enhanced our approach by incorporating additional constraints such as ramp rate limits, maintenance schedule, limits on different types of system and regional emission, as well as fuel constraints.

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