POWER SYSTEMS MARGINAL COST CURVE AND ITS APPLICATIONS

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ABSTRACT

This paper presents a forward recursive procedure to calculate the expected system marginal cost curve (EMC). The EMC formulation allows for multi-state and multi-block dispatch of generating units and is used to determine the optimal energy of pumped-storage units. A new approach is developed to compute the first and second derivatives of the expected generation energy of a thermal unit with respect to the capacity of all thermal units in the system. The salient feature of the proposed approach is that it applies to hydro-thermal systems with multiple limited-energy hydro units.

Keywords–Marginal Cost Curve, Pumped-Storage Units, Probabilistic Power System Dispatch, Derivatives in Probabilistic Production Cost, Emissions Control

1. Introduction

The electric utility industry today is undergoing rapid and irreversible changes resulting from volatile fuel costs, transmission access, less predictable load growth, a more complex regulatory environment, etc. These changes have imposed more stringent economical constraints on power systems. Hence, electric utilities are considering a tighter control on multiple objectives, e.g. economical operation of their facilities, higher reliability and security requirements, minimal impact on environment, etc., which has brought about a sheer necessity for greater sophistication in power systems planning and operation.

In this regard, the simulation of production cost plays a key technology in power systems analyses, which is used widely by power systems planners and operators, energy policy analysts and regulatory agencies. The simulation results are used in utilities for revenue forecasting, generation capacity expansion planning, cost/benefit analysis of conservation and load management programs as well as marginal cost estimation for pricing and evaluating interchange contracts. In addition, production cost simulation can be conducted for a trade-off evaluation of operation costs and emissions resulting from different compliance strategies.

Currently, two basic approaches are used in practice for production cost simulation, which include the Monte Carlo and the load-duration-curve methods. Despite recent progress in Monte Carlo approach to stochastic production cost simulation, analytical models based on equivalent load duration curves remain to be most popular for calculating production cost. In the load-duration-curve model, the effect of random outages of generating units tends to increase the probability that the load will exceed a given value, indicating that the available units will pick up the load not served by units on outage. In [1], we developed a new probabilistic production cost simulation model, which was carried out directly under chronological load curve. In our model, random outages of generating units reflected system generation capability via available capacity probability density function (PDF).

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It is often necessary to know the marginal cost of operating a power system. Marginal cost is the cost bore by utilities for supplying an additional unit of power demand. Marginal cost data are used in electricity rate structures, generation planning and power purchase planning. The marginal cost of power is dependent on the hourly time-of-day profile of the power purchase (or sale), the utility operating system and the term of power purchase (or sale). The expected marginal cost (EMC) at a load level is the sum of the marginal operating cost of individual units weighted by the probability that it is the marginal unit at that load level. The EMC curve is extremely useful for time-of-day pricing [17], evaluation of load management [16] and other applications where the expected cost as a function of load is used. Reference [5] presents an algorithm to calculate these marginal cost curves using a backward recursive procedure. Instead, we obtain the system marginal cost curve by a forward recursion procedure in our proposed simulation model. More important, we give a concise EMC formulation for the dispatch of multi-state and multi-block units.

For each set of decisions, the derivatives of the objective function with respect to decision variables show a way to change the decisions that will reduce the objective function. The first derivative of the expected generation with respect to generating unit capacity has many applications in generation expansion planning [10-12], allocation of resources for reliability improvement [13-14], maintenance scheduling [14] as well as emissions compliance planning [15]. The calculation of the first derivatives of the expected generation energy with respect to generating unit capacity is studied in [10-11] which uses a recursive formula based on the traditional piece-wise linear approximation; the approach is computationally inefficient and numerically unstable. Reference [12] was the first study to use Gram-Charlier series to compute the derivatives; since then other algorithms [2-3, 13-14] have used Gram-Charlier series which have resulted in great accuracy and fast solutions. However, those algorithms are suitable for an entirely thermal units system, and they fail if there are multiple limited-energy hydro units in the system.

In this paper, we present an approach to calculate the production cost curve by using the system available capacity PDF. Usually, the production cost curve has three forms [2]: the marginal cost curve, λ, which shows the incremental cost of generation as a function of load, its integral which represents the total system cost curve, and the derivative of total cost with respect to unit capacity. Section 2 derives a recursive formula for the system marginal cost curve. Section 3 uses the system marginal cost curve to determine the optimal generating energy for pumped-storage units. Section 4 derives the first and second derivatives of total cost and emission with respect to unit capacity, and uses them to evaluate the changes of system cost and emissions; the salient feature is that the derivatives do apply when there are multiple limited-energy hydro units exist in the system. Section 5 gives the test results.

List of Symbols

$\lambda_n(k)$: expected incremental cost at the load interval k after thermal unit n is committed
p_i: availability of unit i
q_i: forced outage rate of unit i
n: thermal unit index

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N_t: total number of thermal units
N_h: total number of hydro units
N_p: total number of pumped-storage units
FOR: forced outage rate of unit
PDF_n: available capacity probability density function for n-unit thermal system
PDF_j^n: available capacity probability density function precluding the random outages of block n from j-unit thermal system
$P_n(k)$: probability value of PDF_n at state k
$P_j^n(k)$: probability value of PDF_j^n at state k
Δc: common factor of all unit capacities
c_n: capacity of unit n
γ_n: segmentation number of unit n's capacity
ω_n: cumulative segmentation number of n-unit system capacity
ω_j^n: cumulative segmentation number of j-unit system capacity precluding block n
k: available capacity state index
Δ: denotation of segmentation number for 1 MW increment, $\Delta = \frac{1}{\Delta c}$
E_k: unserved energy at the available capacity state k
$E_{k+\Delta}$: unserved energy at the available capacity $k \cdot \Delta c$+1 MW
$E_{k+2\cdot\Delta}$: unserved energy at the available capacity $k \cdot \Delta c$+2 MW
UE_j: expected unserved energy for j-unit system
EG_j: expected generating energy of thermal unit j
EG_j^n: change of the expected generating energy of thermal unit j due to a Δc_n change in the capacity of thermal unit n
T: simulation period in hours
UE: expected unserved energy for supply system
l_t: t^{th} hour load
l_{max}: maximum load for the entire study horizon
l_{min}: minimum load for the entire study horizon
INT(x): integer part of x
$k_{l_{max}}$: INT($l_{max}/\Delta c$)
$k_{l_{min}}$: INT($l_{min}/\Delta c$)
E_{hi}: assigned energy of hydro unit i
q_{hi}: forced outage rate of hydro unit i
c_{hi}: capacity of hydro unit i
γ_{hi}: segmentation number of hydro unit i's capacity
θ_{hi}: average available time of hydro unit i, $\theta_{hi} = E_{hi}/((1-q_{hi})c_{hi})$
UT_j: expected unserved energy for the j-unit thermal system
$EH_j(i)$: maximum dispatchable energy of hydro unit i for j-unit thermal system
KH_j: maximum modified available capacity state of a hydro unit for j-unit thermal system
$PDFH_{j,i}$: available capacity probability density function for j-unit thermal and (i-1) unit hydro system
$PDFH_{j,i}^n$: available capacity probability density function for j thermal units and $i-1$ hydro units system precluding thermal block n
$PH_{j,i}(k)$: probability value of $PDFH_{j,i}$ at state k
$PH_{j,i}^n(k)$: probability value of $PDFH_{j,i}^n$ at state k
ET_j: expected generating energy of the j^{th} unit for the thermal system.
η: efficiency of pumped-storage unit

2. System Marginal Cost Curve

Suppose there are N_t thermal units in a generating system and the incremental cost for each block of unit n is assumed to be constant. For a load value equal to x, the incremental cost is characterized as:

$$f_n(x) = f_n \text{ in \$/MWh}, \qquad 0 \le x \le C_n.$$

We form the expected generating marginal cost curve by recursive convolution and/or deconvolution of the system available capacity PDF. In order to decrease the computation burden to obtain PDF, we resort to FFT techniques which have been described in [19]. Assume the generating units are committed one by one in the economic merit order, i.e., the least expensive unit is used first. By definition, the expected marginal cost (EMC) at a load level is the sum of the marginal operating cost of individual units weighted by the probability that it is the marginal unit at that load level. Obviously, EMC is a function of load level and dependent upon the merit order of the committed units.

By committing the first unit, we obtain PDF_1. For each load interval $k = INT(x/\Delta c)$, the EMC is given as:

$$\lambda_1(k) = \begin{cases} 0 & \text{if } k \le 0; \\ p_1 \cdot f_1 & \text{if } 0 < k \le \omega_1; \\ 0 & \text{if } k \ge \omega_1. \end{cases}$$

The significance of this curve is clear, since the expected incremental cost of generating $x \in [0, k \cdot \Delta c]$ MW of load is given as the probability that the unit is available times the cost of operating the unit at that load level, i.e., $p_1 \cdot f_1$.

The second unit is now added into the system, giving,

$$\lambda_2(k) = \lambda_1(k) \quad + \quad p_2 \cdot f_2 \cdot \sum_{i=1}^{\gamma_2} P_1(k-i)$$

here, $k = 1, 2, \cdots, \omega_2$. The first term results from the condition that unit 1 is available at load level $x \in [(k-1)\Delta c, k\Delta c]$ MW. Since unit 1 is less expensive, it will be used to supply the load. The second term results from the condition that at load level $x \in [(k-1)\Delta c, k\Delta c]$ MW, unit 1 is not available or the capacity of unit 1 is not large enough to serve the load with probability $\sum_{i=1}^{\gamma_2} P_1(k-i)$, so unit 2 is used to supply the load at an expected cost of $p_2 \cdot f_2 \cdot \sum_{i=1}^{\gamma_2} P_1(k-i)$.

Thus for a single block unit n, the expected generating marginal cost is,

$$\lambda_n(k) = \lambda_{n-1}(k) \quad + \quad p_n \cdot f_n \cdot \sum_{i=1}^{\gamma_n} P_{n-1}(k-i) \qquad (1)$$

where, $k = 1, 2, \cdots, \omega_n$. Using the same principle, if unit n is multi-state, the expected generating marginal cost is,

$$\lambda_n(k) = \lambda_{n-1}(k) \quad + \quad f_n \cdot \sum_{s=1}^{S_n} [p_{ns} \cdot \sum_{i=1}^{\gamma_{ns}} P_{n-1}(k-i)] \qquad (2)$$

Also, if unit n is multi-block, the expected generating marginal cost is,

$$\lambda_n(k) = \lambda_{n-1}(k) \quad + \quad p_n \cdot f_n \cdot \sum_{i=1}^{\gamma_n} P_{n-1}^j(k-i) \qquad (3)$$

All generating units are convolved in a similar manner, resulting in the final EMC curve. In order to guarantee the monotonic increment of the EMC curve, the last unit N_t+1 should be a unit of infinite capacity, which serves the unserved energy during the outage period, and its cost is higher than that of the most expensive unit in the system.

The total production cost, $Cost$(\$), is equal to the cost of serving the hourly load and is calculated by the following formula,

$$Cost = \sum_{k=1}^{k_{l_{max}+1}} (E_{k-1} - E_k) \cdot \lambda_{N_t+1}(k) \quad (4)$$

We can prove that the operation cost calculated by the above formula is the same as the sum of generation cost of individual units as well as the cost of expected unserved energy (Appendix A),

$$Cost = \sum_{i=1}^{N_t} f_i \cdot EG_i + f_{N_t+1} \cdot UE \quad (5)$$

In the following, we use a simple numerical example which is taken from [1] to illustrate the calculation of the EMC curve. There are three units in the system, two of them are rated at 40 MW with FOR=0.1 and an average generation cost of 0.40$/KWh. The other unit is rated at 20 MW with FOR=0.2 and an average generation cost of 0.45$/KWh. Also, the unserved energy cost is 0.55$/KWh, modeled as a perfectly reliable unit with infinite capacity. We calculate the EMC curve as shown in Table 1.

Table 1 EMC calculation for 3-unit system ($/MWh)

k	Unit 1		Unit 2		Unit 3		Unit ∞
	$P_1(k)$	$\lambda_1(k)$	$P_2(k)$	$\lambda_2(k)$	$P_3(k)$	$\lambda_3(k)$	$\lambda_\infty(k)$
0	.10		.01		.002		
1	.00	.36	.00	.396	.008	.3996	.4007
2	.90	.36	.18	.396	.036	.3960	.4015
3			.00	.324	.144	.3888	.4141
4			.81	.324	.162	.3240	.4285
5					.648	.2916	.4852

The total system generation cost is,

$$Cost = \sum_{k=1}^{5} (E_{k-1} - E_k) \cdot \lambda_4(k) = \$596.7595$$

which is the same as,

$$Cost = \sum_{i=1}^{4} f_i \cdot EG_i = \$596.7595$$

If we assume that the first unit has two blocks of 20 MW, the average generation costs of the first and second blocks are 0.40$/KWh, 0.50$/KWh, respectively. The merit order is the base block of unit 1 as well as unit 2, unit 3 and the peak block of unit 1. We calculate the EMC curve as shown in Table 2.

Table 2 EMC calculation for 4-block system ($/MWh)

k	Unit 1 (base)		Unit 2		Unit 3		Unit 1 (Peak)			Unit ∞
	$P_1(k)$	$\lambda_1(k)$	$P_2(k)$	$\lambda_2(k)$	$P_3(k)$	$\lambda_3(k)$	$P_3^1(k)$	$\lambda_4(k)$	$P_4(k)$	$\lambda_\infty(k)$
0	.10		.01		.002		.02		.002	
1	.90	.36	.09	.396	.026	.3996	.08	.3996	.008	.4007
2			.09	.360	.090	.3924	.18	.4014	.036	.4069
3			.81	.324	.234	.3564	.72	.3924	.104	.4177
4					.648	.2916		.3726	.162	.4771
5								.3240	.648	.5176

The total system generation cost is,

$$Cost = \sum_{k=1}^{5} (E_{k-1} - E_k) \cdot \lambda_4(k) = \$608.0311$$

which is the same as,

$$Cost = \sum_{i=1}^{5} f_i \cdot EG_i = \$608.0311$$

One application of EMC is to use (4) for a rapid calculation of the change in production cost, resulting from load changes due to demand-side management (DSM) measures such as conservation or load management. As long as such measures do not affect the generating units, the production cost curve remains constant and may be computed beforehand using (1), (2) or (3). However, this procedure does not apply when there are regulating limited-energy hydro units in the system.

3. Pumped-Storage Optimization

Pumped-storage units replace expensive peaking units during peak hours, and are recharged by inexpensive base-load energy during off-peak periods. Obviously, the financial benefits of peak shaving must be weighed against the cost of pumping.

In the deterministic production cost simulation model, it is easy to determine how much energy should be used for peak shaving. The point at which the incremental cost for pumping is equal to the incremental cost for generating power is used as a boundary beyond which the utilization of the pumped storage capacity will not be economical [1]. In our earlier study [1], we used an iterative procedure to find the optimal energy used by pumped-storage unit for peak shaving and pumping, thus many trials and much computation were required. In this paper, we use the marginal cost curve to determine the economic boundary and solve this difficult task in the probabilistic production simulation.

Suppose a pumped-storage unit generates a small amount of energy ΔE_g, at the peak load level x_g. Its avoided cost of generation, ΔF_g, should amount to the operating cost of units which the pumped-storage unit has replaced, $\Delta F_g = \lambda_g \cdot \Delta E_g$. In order to restore the water used in generating ΔE_g, it will pump an energy equal to $\Delta E_p = \Delta E_g/\eta$, with a pumping cost of $\Delta F_p = \lambda_p \cdot \Delta E_p = (\lambda_p/\eta) \cdot \Delta E_g$. Here, λ_g and λ_p are expected marginal costs at load levels x_g and x_p, respectively. So the net benefit of generating and pumping is $\Delta F = \Delta F_g - \Delta F_p = (\lambda_g - \lambda_p/\eta) \cdot \Delta E_g$. Here, λ_g is the pumped-storage generating cost curve which is equal to the system expected marginal cost curve, and λ_p/η is the pumped-storage pumping cost curve which can be obtained by the system expected marginal cost divided by η for the pumped-storage unit. Therefore, in order to determine the optimal energy of the pumped-storage unit for peak-shaving, $\lambda_g(x_g) = \lambda_p(x_p)/\eta$ is the economic boundary beyond which the utilization of the pumped storage capacity will not be economical (Figure 1). In the following, we give the generating and pumping algorithm to determine the optimal energy of pumped-storage units.

Assume l_t^0 denotes the original chronological load curve at hour t, $C_{p_i}, C_{g_i}, E_{p_i}, E_{g_i}$ are the capacity and energy for pumping and generating of the pumped-storage unit i respectively, l_t^i denotes the chronological load curve at hour t after scheduling the pump-storage unit i, $E_{p_t}^t, E_{g_t}^t$ indicate the energy for pumping and generating of the pumped-storage unit i at hour t, respectively.

The generating and pumping algorithm for scheduling pumped-storage units is as follows,

1. Form the pumped-storage generating cost curve λ_g and pumping cost curve λ_p by the system expected marginal

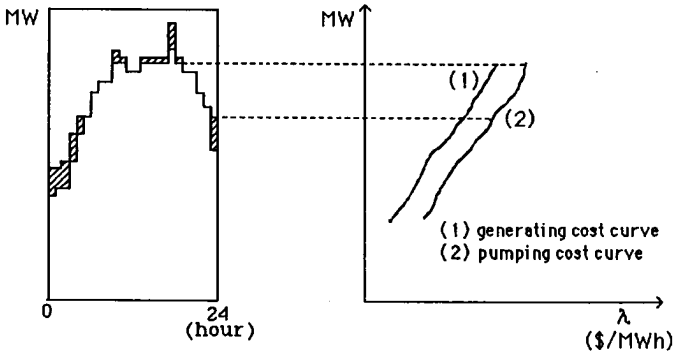

Fig. 1 Economic boundary for pumped-storage units

cost curve, respectively.

2. Order the pumped-storage units according to their average available time θ_s^i . Let $i = 1$.

3. Determine loading points X_{p_i} and X_{g_i}, which satisfy the following constraints,

$$\sum_t^T E_{p_t}^i(X_{p_i}) = E_{p_i}$$

$$\sum_t^T E_{g_t}^i(X_{g_i}) = E_{g_i}$$

$$E_{g_i} = \eta_i \cdot E_{p_i}$$

here,

$$E_{p_t}^i = \begin{cases} 0 & \text{if } l_t^{i-1} \geq X_{p_i}; \\ X_{p_i} - l_t^{i-1} & \text{if } X_{p_i} \geq l_i^{i-1} \geq X_{p_i} - C_{p_i}; \\ C_{p_i} & \text{if } l_t^{i-1} \leq X_{p_i} - C_{p_i}. \end{cases}$$

$$E_{g_t}^i = \begin{cases} 0 & \text{if } l_t^{i-1} \leq X_{g_i}; \\ l_t^{i-1} - X_{g_i} & \text{if } X_{g_i} \leq l_i^{i-1} \leq X_{g_i} + C_{g_i}; \\ C_{g_i} & \text{if } l_t^{i-1} \geq X_{g_i} + C_{g_i}. \end{cases}$$

If $\lambda_p(X_{p_i}) \leq \lambda_g(X_{g_i})$, goto 4; otherwise, decrease E_{g_i} and E_{p_i} until $\lambda_p(X_{p_i}) = \lambda_g(X_{g_i})$ and goto end.

4. Let $l_t^i = l_t^{i-1} - E_{g_t}^i(X_{g_i}) + E_{p_t}^i(X_{p_i})$

5. If $i < \mathrm{N}_p$, let $i = i+1$ and goto 3; otherwise, let $X_{lp} = X_{p_{N_p}}$ and $X_{lg} = X_{g_{N_p}}$, and goto end.

Once the optimal energy of pumped-storage units is determined by the above algorithm, the pumped-storage unit will act as an additional load on the generation system when it is pumping the water to the upper reservoir; as the unit generates power, it will act like an assigned-energy hydro unit which is dealt with in [1]. We use the peak-shaving and pumping algorithm, which is used in the deterministic simulation model to determine the energy for pumped-storage units. Although the forced outages of generation system in the dispatch of these units are not considered precisely, this energy is near optimal. The above algorithm results in fast computation in modeling pumped-storage units.

Now, we describe the method for modifying the dip in the load curve once the optimal energy of pumped-storage units is determined, taking into account the forced outages of pumped-storage units. First, we illustrate our algorithm for a single pumped-storage unit. Assume the unit forced outage rate is q_s and the optimal energy for pumping is E_p, so the available pumping energy, which must be scheduled so that E_p (on average) is actually pumped given the forced outage rate, is $E_{pa} = E_p/(1 - q_s)$ and the load level for pumping is X_{lp}. Reform the chronological load curve into the load duration curve, but remember the corresponding time of each load (Figure 2). There are t_p hourly loads under X_{lp}. When pumping the lowest load t_1, the load will bifurcate into two branches with probabilities $(1 - q_s)$ and q_s with load increments C_p and 0, respectively. Continue this procedure for loads at $t_2, \cdots, t_{s-1}$; however, due to the limit of X_{lp}, loads at $t_s, \cdots, t_p$ will bifurcate with maximum capacities $(X_{lp} - l_{ts}), \cdots, (X_{lp} - l_{tp})$, respectively, and modified load values at $t_1, \cdots, t_p$ will be the weighted values of the two load values. The total expected change of the modified load value is equal to E_p. The load curve is modified to reflect the fact that the pumped-hydro unit may be unavailable while it should be pumping, but a similar modification is not done for the load of generation while the pumped-hydro unit should be generating; instead the unit will act as an assigned-energy hydro unit at this time, and E_g will be dealt with by the algorithm for any assigned-energy units which is proposed in [1]. This implementation can be easily extended to multiple pumped-storage units with forced outage rates.

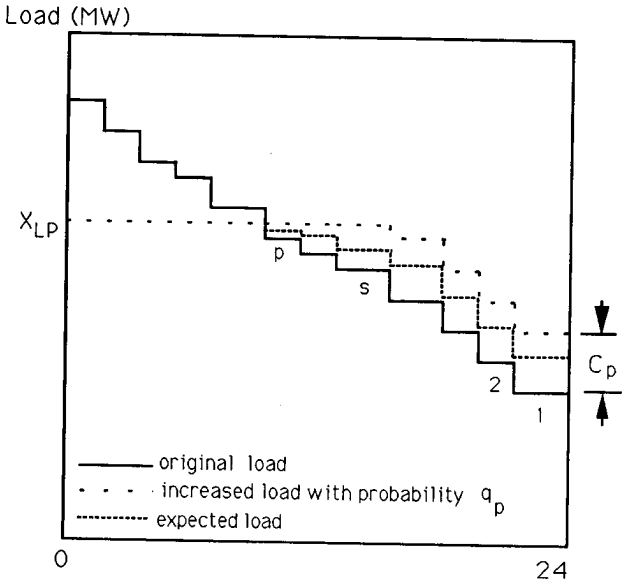

Fig. 2 Modified load curve due to pumping

4. Evaluation of System Cost and Emission Changes

In response to public concern about acid deposition, U.S. Congress recently passed a legislation aimed at reducing SO_2, CO_2 and NO_x emissions from electric generation plants. This legislation points out a new era for the dispatch of electric power, and many utilities will be considering a wide range of options for controlling emissions [6]. Among these strategies are retrofit of emissions control equipment, boiler modifications, fuel switching, coal cleaning, early retirement of dirtier generation units, reboilering and repowering, dispatch of emission (in which the dispatch order is based on a weighted sum of emissions and costs, rather than costs alone), trading of emissions allowances, and energy conservation.

Most of these options would change the cost of power generation and emissions characteristics of generating units. Usually the emission controlled economic dispatch problem can be formulated as a multi-objective stochastic optimization, and solved by the probabilistic production cost simulation method. If we decide to evaluate changes in the system operation cost and emission due to the installation of a retrofit on a generating unit, an ordinary procedure is to run the production siumlation program twice. However, this approach will be time consuming if there are several retrofit options for each unit and many such units exist in the system. In this paper, we use the same approach as in [3], which is based on the second order Taylor's series expansion of the unit output to estimate changes in system costs and emissions; however, we use a new formula based on our chronological load curve probabilistic model to compute the first and second derivatives of the expected generation with respect to the capacity of a unit.

Assume Δc is a common factor of all N unit capacities and changes of all unit capacities due to installation of a retrofit on generating thermal unit. So, the first order derivative of EG_j with respect to C_n, for a totally thermal generating system, is,

$$\frac{\partial EG_j}{\partial C_n} = \begin{cases} 0 & \text{if } n > j \\ p_n \sum_{k=0}^{\omega_{n-1}} [(E_{k+\gamma_n} - E_{k+\gamma_n+\Delta}) \; cdot \\ \qquad P_{n-1}(k)] & \text{if } n = j \\ p_n \; p_j \sum_{k=0}^{\omega_{j-1}^n} [(E_{k+\gamma_n} - E_{k+\gamma_n+\Delta}) \cdot \\ \qquad (P_{j-1}^n(k-\gamma_j) - P_{j-1}^n(k))] & \text{if } n < j \end{cases} \quad (6)$$

The second derivative is:

$$\frac{\partial^2 EG_j}{\partial C_n^2} = \begin{cases} 0 & \text{if } n > j \\ -p_n \sum_{k=0}^{\omega_{n-1}} [(E_{k+\gamma_n} - 2E_{k+\gamma_n+\Delta} + E_{k+\gamma_n+2\Delta}) \cdot P_{n-1}(k)] & \text{if } n = j \\ -p_n\, p_j \sum_{k=0}^{\omega_{j-1}^n} [(E_{k+\gamma_n} - 2E_{k+\gamma_n+\Delta} + E_{k+\gamma_n+2\Delta}) \cdot (P_{j-1}^n(k-\gamma_j) - P_{j-1}^n(k)] & \text{if } n < j \end{cases} \quad (7)$$

Hydro units play important roles in the planning and operation of electric power systems. In [1], we denote hydro units with tighter constraints such as run-of-river units or those with smaller reservoirs as non-regulating hydro units, and prove that the peak-shaving method which is often used in the deterministic production cost simulation model [8] is the most appropriate approach. Hydro units with relatively large reservoirs and few water release constraints are denoted as regulating hydro units, and we present a new approach to scheduling this type of hydro units. If there are multiple regulating limited-energy hydro units in the generation system, we can obtain the first and second derivatives. The corresponding formulas are given in Appendix B.

Assume unit n is committed before unit j, it is obvious that the expected generation of unit j, EG_j, depends on the capacity of unit n. Based on the Taylor's series expansion of EG_j, the change ΔEG_j^n due to a change Δc_n can be calculated as,

$$\Delta EG_j^n \approx \frac{\partial EG_j}{\partial C_n} \cdot \Delta c_n + \frac{1}{2} \cdot \frac{\partial^2 EG_j}{\partial C_n^2} \cdot \Delta c_n^2 \quad (8)$$

Thus, the resulting change in the cost of unit j will be,

$$\Delta Cost_j^n = f_j \cdot \Delta EG_j^n$$

So, the total change in the system operation cost due to a change in the capacity of unit n will be,

$$\Delta Cost_{system} = \sum_{j \geq n} \Delta Cost_j^n \quad (9)$$

The change in the system emission can be obtained similarly by replacing f_j with s_j (lb/MWh) which indicates the emission rate of unit j.

5. Test Results

5.1 Test Case 1

We use a modified version of the IEEE-RTS to calculate its expected system marginal cost curve. The generation model consists of 32 generation units, including 6 hydro units, each with 50 MW capacity (Table 3). The combined capacity of the system is 3400 MW. The peak load is 2850 MW, the minimum load is 1102 MW, and loads are assumed constant over each sampled hour of the day. Week 51 was chosen for load description as defined in [18]. The time duration is 168 hours, and the energy demand is 359.323 GWh. The capacity of the smallest unit is 10 MW, which is a common factor to all other capacities. To avoid problems with hydro scheduling, we assume the available energy of each hydro unit is not limited, so hydro units would be committed before other units. Figure 3 depicts the weekly chronological hourly load curve, Figure 4 shows the system marginal cost curve as a function of load and Figure 5 shows the weekly chronological system marginal cost curve.

Table 3 Modified generation system for IEEE-RTS

Type of unit	Unit capacity (MW)	No. of units	FOR	Incremental Cost ($/MWh)
1 Hydro	50	6	0.01	0.000
2 Nuclear	400	2	0.12	5.450
3 Coal	150	4	0.04	10.704
4 Coal	350	1	0.08	10.883
5 Coal	80	4	0.02	13.494
6 Oil	200	3	0.05	20.730
7 Oil	100	3	0.04	20.853
8 Oil	10	5	0.02	25.875
9 Oil	20	4	0.10	37.500

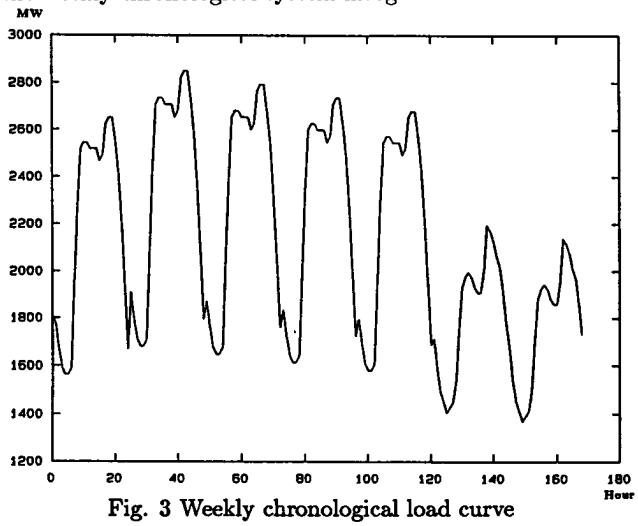

Fig. 3 Weekly chronological load curve

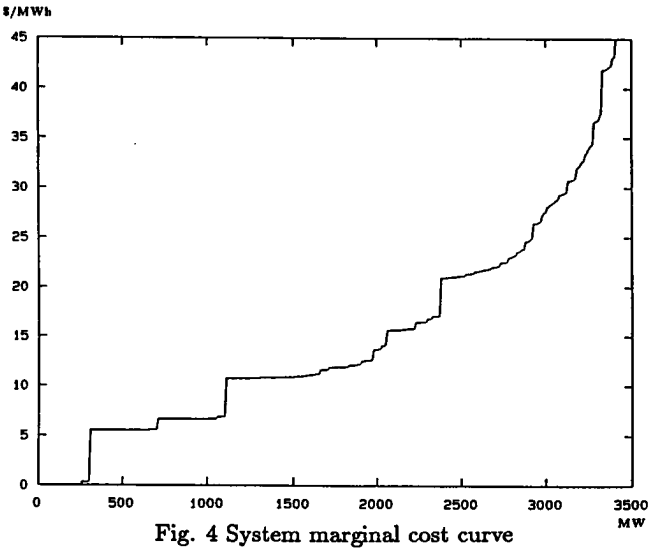

Fig. 4 System marginal cost curve

Now, assume the system is augmented with two pumped-storage units. Each unit has a capacity of 10 MW, efficiency of 66.7%, forced outage rate of 0.01% and a limited energy for generating 10 GWh. Using the proposed pumped-storage optimization method, the generating energy for each unit is 3.39 GWh, and the CPU time is 25 seconds. However, if we use the proposed iterative method in [1] and let the decreased pumped energy be 0.5 GWh, the optimal energy for each unit will be 3.00 GWh after 14 iterations with 180 seconds of CPU time. So, applying the system marginal cost curve to pumped-storage units will result in a much faster and more practical calculation of the optimal energy.

5.2 Test Case 2

In order to compare the derivatives obtained by our proposed approach with those in [3,13], we use the same synthetic utility system used in [3] and [13]; Table 4 describes the re-

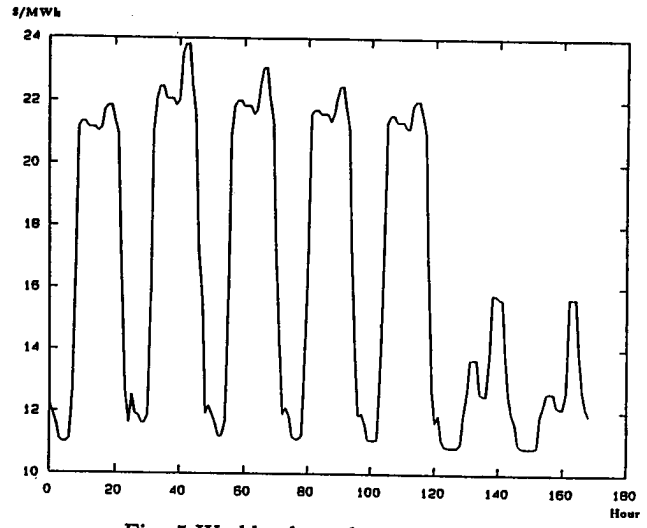

Fig. 5 Weekly chronological marginal cost

sources for the system. There are 52 units in 8 categories. The system load is assumed to be normally distributed with 6267.4 MW mean and 745.4 MW standard deviation. The time period for this study is 728 hours.

Table 5 represents the first derivative of the total expected generation of each category with respect to the capacity of unit 8 using [3,13] and our method. The derivative of the total expected generation of each unit type with respect to the capacity of unit 8 is the sum of the derivatives of the expected generation of individual units in each type with respect to those capacities. In order to demonstrate the procedure for computing the system marginal cost and emissions, we assume the same computational condition, that is, a retrofit, limestone injection multistage burner (LIMB), is being considered for unit 5. The retrofit data is the same as that in [3]. Table 6 shows the first and second derivatives of the expected generation of each type of units with respect to the capacity of unit 5, and the resulting changes in cost and emissions for each type of units. Test results show that, on average, it takes less than five seconds on a 80386 PC to obtain marginal system costs and emissions due to the installation of one retrofit option on a generating unit.

Table 4 Generation system

Type of Resource	Loading Order	Unit Capacity (MW)	Variable Cost ($/MWh)	Emission Rate (lb/MWh)	FOR
1	1-2	1200	10	0	0.150
2	3	800	15	63.33	0.240
3	4-5	600	18	50.00	0.210
4	6-7	400	20	33.33	0.130
5	8	400	50	25.20	0.130
6	9-16	200	55	20.00	0.074
7	17-23	200	60	0	0.074
8	24-52	50	70	0	0.240

Table 5 First derivative w.r.t. resource 5

Resource	(a)	(b)	(c)
1-4	0	0	0
5	592	594	592
6	-267	-264	-269
7	-235	-236	-234
8	-73	-75	-73

(a) method used in [3]
(b) method used in [13]
(c) proposed method

However, if we run production cost simulation twice, one is with the original units, another is with retrofit installed on the specified unit, the CPU time is about 42 seconds. Thus the proposed approach will provide a substantial reduction in computation if there are several retrofit options at each unit and there are many such units in emission compliance planning.

Since there are no hydro units in the synthetic utility system, we return to the modified IEEE-RTS system to test the derivative when regulating limited-energy units exist in the supply system. Assume the available assigned-energy of each hydro unit is limited to 40 GWh for the weeks 1-8 and 48-52 period. Table 7 Gives the first derivative of each thermal unit with respect to the capacity of thermal unit 6.

Table 6 Results of retrofit to unit 5

Type of Unit	First Derivative (MWh/MW)	Second Derivative (MWh/MW^2)	Variable Cost Change ($)	Emissions Change (Ton)
3	573.30	-0.0060	1616080	-5900
4	-16.30	-0.0041	48716	41
5	-24.10	-0.0403	28360	7
6	-252.97	-0.1223	332821	61
7	-207.89	0.1128	301569	0
8	-59.26	0.0747	101063	0

Table 7 First derivative of the expected generation of thermal unit w.r.t. C_6

Unit No.	Thermal Part	Hydro Part	Total
1-5	0	0	0
6	1923.400	0	1923.400
7	-481.000	0	-481.000
8	-113.600	0	-113.000
9	-106.900	0	-106.900
10	-90.950	0	-90.950
11	-92.920	0	-92.920
12	-302.600	410.590	-713.190
13	-281.000	-48.170	-232.830
14	-261.110	-77.130	-138.930
15	-76.300	-113.628	+37.328
16	-83.330	-70.439	-12.891
17	-42.652	-22.407	-20.245
18	-4.811	-3.819	-0.992
19	-2.326	-1.618	-0.708
20	-6.561	-5.120	-1.441
21	-4.668	-4.009	-0.659
22	-2.064	-1.590	-0.474
23	-5.403	-3.788	-1.615
24	-8.627	-6.520	-2.109
25	-4.780	-3.694	-1.086
26	-4.221	-2.661	-1.560

6. Conclusions

In this paper, we present a new method to calculate the system marginal cost curve, and the derivative of the system generation cost with respect to the capacity of various units by using the available capacity probability density function. We use the marginal cost curve to determine the optimal energy of pumped-storage units; even though the result is suboptimal, the solution can be obtained very fast. The proposed approach for calculating the total production cost and its derivatives can give results which agree closely with those of other methods, with a substantial reduction in computational effort when these calculations are to be repeated several times for a lot of planning problems such as generation expansion planning, maintenance scheduling, emission compliance planning, etc. The salient feature of our proposed method is that it can calculate the derivatives of generation energy for a thermal unit with respect to the capacity of all thermal units when there are multiple limited-energy hydro units in the system.

7. Acknowledgements

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APPENDIX A

Let n=N_t+1 and substitute Eq. 1 into Eq. 4; so

$$\begin{aligned} Cost &= \sum_{k=1}^{k_{l_{max}+1}} (E_{k-1} - E_k)\,[\lambda_{N_t}(k) + f_{N_t+1} \sum_{i=1}^{\gamma_{N_t+1}} P_{N_t}(k-i)] \\ &= \sum_{k=1}^{k_{l_{max}+1}} [(E_{k-1} - E_k)\,\lambda_{N_t}(k) \\ &\quad + f_{N_t+1} \sum_{i=1}^{\gamma_{N_t+1}} P_{N_t}(k-i)\,(E_{k-1} - E_k)] \end{aligned} \tag{10}$$

The last term of this equation can be rewritten as follows,

$$\begin{aligned} & f_{N_t+1} \sum_{k=1}^{k_{l_{max}+1}} (\sum_{i=1}^{\gamma_{N_t+1}} P_{N_t}(k-i)\,(E_{k-1} - E_k)) \\ &= f_{N_t+1}\,[(E_0 - E_1)\,P_{N_t}(0) \\ &\quad + (E_1 - E_2)\,(P_{N_t}(0) + P_{N_t}(1)) + \cdots \\ &\quad + (E_k - E_{k+1})\,(P_{N_t}(0) + P_{N_t}(1) + \cdots + P_{N_t}(k)) + \cdots \\ &\quad + (E_{k_{l_{max}}} - E_{k_{l_{max}+1}})\,(P_{N_t}(0) + P_{N_t}(1) + \cdots + P_{N_t}(k_{l_{max}}))] \\ &= f_{N_t+1}\,[E_0\,P_{N_t}(0) + E_1\,P_{N_t}(1) + \cdots + E_{k_{l_{max}}}\,P_{N_t}(k_{l_{max}})] \\ &= f_{N_t+1} \sum_{k=0}^{k_{l_{max}}} E_k\,P_{N_t}(k) \\ &= f_{N_t+1}\,UE \end{aligned} \tag{11}$$

which denotes the cost of expected unserved energy. Let n=N_t, N_t-1,$\cdots$, 1, then we can prove that the last term of each substitution is the generation cost, $f_n \cdot EG_n$, of each unit n.

APPENDIX B

Assume there are only thermal units in the supply system. If thermal unit j is dispatched before thermal unit n, then the expected generation energy of unit j is independent of the change of capacity of unit n, thus,

$$\frac{\partial EG_j}{\partial C_n} = 0 \tag{12}$$

If thermal unit j is dispatched after thermal unit n, then the first order derivative of the expected generation energy of unit j with respect to the capacity of unit n is the difference between the expected generating energy of unit j with 1 MW increment in its generating capacity and unchanged capacity of unit n. Hence,

$$\begin{aligned} \frac{\partial EG_j}{\partial C_n} &= (UE_{j-1}^{new} - UE_j^{new}) - (UE_{j-1} - UE_j) \\ &= [(UE_j - UE_j^{new}) - (UE_{j-1} - UE_{j-1}^{new})] \end{aligned} \tag{13}$$

$$\Delta UE_j = UE_j - UE_j^{new}$$

$$= \left(\sum_{k=0}^{\omega_j}[E_k \cdot P_j(k)] - \sum_{k=0}^{\omega_j+1}[E_k\ P_j^{new}(k)]\right)$$

$$= \left(\sum_{k=0}^{\omega_j^n}[q_n\ E_k\ P_j^n(k)] + \sum_{k=-\gamma_n}^{\omega_j-\gamma_n}[p_n\ E_{k+\gamma_n}\ P_j^n(k)]\right.$$

$$\left. - \sum_{k=0}^{\omega_j^n}[q_n\ E_k\ P_j^n(k)] - \sum_{k=-\gamma_n-1}^{\omega_j-\gamma_n}[p_n\ E_{k+\gamma_n+\Delta}\ P_j^n(k)]\right)$$

$$= p_n \sum_{k=0}^{\omega_j^n}[(E_{k+\gamma_n} - E_{k+\gamma_n+\Delta})\ P_j^n(k)] \quad (14)$$

In the above derivation, we chose 1 MW as the state incremental step; however, for the calculation of $P_j^n(k)$, we use Δc as the incremental step.

$$\frac{\partial EG_j}{\partial C_n} = \Delta UE_j - \Delta UE_{j-1}$$

$$= (UE_j - UE_j^{new}) - (UE_{j-1} - UE_{j-1}^{new})$$

$$= \left(p_n \sum_{k=0}^{\omega_j^n}[(E_{k+\gamma_n} - E_{k+\gamma_n+\Delta})\ P_j^n(k)]\right)$$

$$- \left(p_n \sum_{k=0}^{\omega_{j-1}^n}[(E_{k+\gamma_n} - E_{k+\gamma_n+\Delta})\ P_{j-1}^n(k)]\right)$$

$$= p_n \sum_{k=0}^{\omega_j^n}[(E_{k+\gamma_n} - E_{k+\gamma_n+\Delta})\ (P_j^n(k) - P_{j-1}^n(k))]$$

$$= p_n\ p_j \sum_{k=0}^{\omega_{j-1}^n}[(E_{k+\gamma_n} - E_{k+\gamma_n+\Delta})$$

$$(P_{j-1}^n(k-\gamma_j) - P_{j-1}^n(k))] \quad (15)$$

The first order derivative of expected energy of thermal unit n with respect to its own capacity n is,

$$\frac{\partial EG_n}{\partial C_n} = (UE_{n-1}^{new} - UE_n^{new}) - (UE_{n-1} - UE_n)$$

$$= \sum_{k=0}^{\omega_n}[E_k\ P_n(k)] - \sum_{k=0}^{\omega_n+1}[E_k\ P_n^{new}(k)]$$

$$= \sum_{k=0}^{\omega_{n-1}}[q_n\ E_k\ P_{n-1}(k)] + \sum_{k=-\gamma_n}^{\omega_n-\gamma_n}[p_n\ E_{k+\gamma_n}\ P_{n-1}(k)]$$

$$- \sum_{k=0}^{\omega_{n-1}}[q_n\ E_k\ P_{n-1}(k)] - \sum_{k=-\gamma_n-1}^{\omega_n-\gamma_n}[p_n\ E_{k+\gamma_n+\Delta}\ P_{n-1}(k)]$$

$$= \sum_{k=0}^{\omega_{n-1}}[p_n\ E_{k+\gamma_n}\ P_{n-1}(k)] - \sum_{k=0}^{\omega_{n-1}}[p_n\ E_{k+\gamma_n+\Delta}\ P_{n-1}(k)]$$

$$= p_n \sum_{k=0}^{\omega_{n-1}}[(E_{k+\gamma_n} - E_{k+\gamma_n+\Delta})\ P_{n-1}(k)] \quad (16)$$

Assume there are N_h regulating energy-limited hydro units in the supply system.

If thermal unit j is dispatched earlier than thermal unit n, then the first derivative of the expected generation energy of unit j with respect to the capacity of unit n is 0.

If thermal unit j is dispatched later than thermal unit n, then the first order derivative of the expected generation energy of unit j with respect to the capacity of unit n is the difference between the expected generation energy of unit j with 1 MW increment in its generating capacity and unchanged capacity of unit n. Hence,

$$\frac{\partial EG_j}{\partial C_n} = [(UE_{j-1}^{new} - UE_j^{new}) - \sum_{i=1}^{N_h}(EH_{j-1}^{new}(i) - EH_j^{new}(i))]$$

$$- [(UE_{j-1} - UE_j) - \sum_{i=1}^{N_h}(EH_{j-1}(i) - EH_j(i))]$$

$$= [(UE_j - UE_j^{new}) - (UE_{j-1} - UE_{j-1}^{new})]$$

$$- \sum_{i=1}^{N_h}[(EH_j(i) - EH_j^{new}(i)) - (EH_{j-1}(i) - EH_{j-1}^{new}(i))] \quad (17)$$

Here, the calculation of the first term (thermal part) of eq. (17) is the same as that of eq. (15); for the second term (hydro part), we assume that the maximum dispatchable energy of the hydro unit i for the m-unit thermal system is the same as its assigned energy, and that for a thermal system with $(m+1)$ units is smaller than its assigned energy, then,

If $j \le m$, we have,

$$\Delta EH_j(i) = EH_j(i) - EH_j^{new}(i) = 0 \quad (18)$$

If $j = m+1$, we have,

$$\Delta EH_j(i) = EH_j(i) - EH_j^{new}(i)$$

$$= p_{hi}\ \left(\sum_{k=0}^{k_{l_{max}}}[(E_k - E_{k+\gamma_{hi}})\ PH_{j,i}(k)]\right.$$

$$\left. - \sum_{k=0}^{k_{l_{max}}}[(E_k - E_{k+\gamma_{hi}})\ PH_{j,i}^{new}(k)]\right)$$

$$= p_{hi}\ p_n\ \left(\sum_{k=0}^{k_{l_{max}}-\gamma_n}[(E_{k+\gamma_n} - E_{k+\gamma_n+\gamma_{hi}})\right.$$

$$\left. - (E_{k+\gamma_n+\Delta} - E_{k+\gamma_n+\gamma_{hi}+\Delta})]\ PH_{j,i}^n(k)\right) \quad (19)$$

$$\Delta EH_{j-1}(i) = 0 \quad (20)$$

If $j > m+1$, we have,

$$\Delta EH_j(i) - \Delta EH_{j-1}(i)$$

$$= (EH_j(i) - EH_j^{new}(i)) - (EH_{j-1}(i) - EH_{j-1}^{new}(i))$$

$$= p_{hi}\ p_n\ \left(\sum_{k=0}^{k_{l_{max}}-\gamma_n}[(E_{k+\gamma_n} - E_{k+\gamma_n+\gamma_{hi}})\right.$$

$$\left. - (E_{k+\gamma_n+\Delta} - E_{k+\gamma_n+\gamma_{hi}+\Delta})]\ PH_{j,i}^n(k)\right)$$

$$- p_{hi}\ p_n\ \left(\sum_{k=0}^{k_{l_{max}}-\gamma_n}[(E_{k+\gamma_n} - E_{k+\gamma_n+\gamma_{hi}})\right.$$

$$\left. - (E_{k+\gamma_n+\Delta} - E_{k+\gamma_n+\gamma_{hi}+\Delta})]\ PH_{j-1,i}^n(k)\right)$$

$$= p_{hi}\ p_n\ \left(\sum_{k=0}^{k_{l_{max}}-\gamma_n}[(E_{k+\gamma_n} - E_{k+\gamma_n+\gamma_{hi}})\right.$$

$$\left. - (E_{k+\gamma_n+\Delta} - E_{k+\gamma_n+\gamma_{hi}+\Delta})]\ [PH_{j,i}^n(k) - PH_{j-1,i}^n(k)]\right) \quad (21)$$

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