Maximizing Revenue Under Market Shrinkage and Market Uncertainty

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Examples of shrinking markets

- Cord cutters
- Retail stores
- Labor markets among a shrinking population

**Examples:**

- Cord cutters: Streaming services like Netflix, Hulu, Amazon Prime, and HBO Now are becoming more popular, leading to traditional cable and satellite providers losing subscribers.

- Retail stores: With the rise of e-commerce and online shopping, traditional brick-and-mortar stores are facing challenges.

- Labor markets: As the population ages and the birth rate decreases, the labor force participation rate has been declining, affecting job growth and economic vitality.

**Sources:**

- Labor Market: Where Is Everybody? (The Shrinking Labor ...)
  - Sep 24, 2020 — Simply put, the labor force participation rate has been falling. The rate for men has been trending downward for nearly 60 years, from 86.7% in ...

- Shrinking labor force explains drop in unemployment
  - In her analysis of the report, labor economist Heidi Shierholz explained that most of that decline can be explained by the drop in the labor force participation rate ...

- Covid Shrinks the Labor Market, Pushing Out Women and ...
  - Dec 3, 2020 — Nearly four million Americans have stopped working or looking for jobs, a 2.2% contraction of the U.S. work force. A smaller labor market leaves ...
Modeling a shrinking market

- Fixed set $S = \{v_1, \ldots, v_n\}$ of bidder valuations
- Seller knows $S$
- Each bidder in $S$ shows up independently with probability $p$

What fraction of revenue can the seller guarantee?

$$\sup_M \mathbb{E}[\text{Rev}_M(S_0)] \geq (?) \cdot W(S)$$
Revenue loss can be drastic

- At first glance answer might appear to be $p$ (or even higher, if revenue thought to have diminishing returns in number of buyers)

- Example 1: $E[\text{Rev}_{\text{VCG}}(S_0)] = p^2 \text{Rev}_{\text{VCG}}(S) = p^2 (W(S) - \varepsilon)$
  - Due to reduced competition among buyers

VCG gets payment of $c - \varepsilon/m$ for each item so $\text{Rev}_{\text{VCG}}(S) = mc - \varepsilon = W(S) - \varepsilon$

But

$$E[\text{Rev}_{\text{VCG}}(S_0)] = \sum_{\text{item } i} E[\text{Rev from item } i] = p^2 (mc - \varepsilon)$$
Revenue loss can be drastic

If valuations can depend on what other bidders receive, things are even worse

**Theorem** (Balcan, Prasad, Sandholm NeurIPS’22). For any $\epsilon > 0$ there exists a set $S$ of bidders with allocational valuations such that

\[
\sup E[Rev_M(S_0)] \leq p^{m/2} \cdot (Rev_{VCG}(S) + 2\epsilon) + \epsilon
\]

where the supremum is over all possible auctions $M$. 

Escaping large revenue loss

Enabled by two main assumptions:

• *Winner monotonicity*
  – if bidder $i$ wins in VCG, and $j$ leaves, $i$ still wins in VCG

• *Welfare submodularity*
  – efficient welfare a submodular function

  e.g. bidders with gross-substitutes valuations
How much revenue can be preserved?

General possibility result: rich enough set of mechanisms always contains one robust to shrinkage

**Theorem** (Balcan, Prasad, Sandholm NeurIPS’22). Exists auction $M$ s.t.

$$
E[\text{Rev}_M(S_0)] \geq \Omega \left( \frac{p^2}{k^{1+\log_{1/\gamma}(4/p)}} \right) \cdot W(S)
$$

$\gamma$ a constant depending on $S$, $k \approx \max$ number of winners in VCG

A shrinkage-robust auction can be computed by sampling simulated shrunken markets and maximizing empirical revenue
Techniques

- **Winner diagram**: concise way of capturing all meaningful executions of an auction
- Randomize over a high-welfare subgraph of the winner diagram
Practically-motivated applications

• Our result yields refined guarantees when the mechanism designer:
  – Limits the number of winners
  – Places bundling constraints on the items
Conclusions

• First formal model of market shrinkage in combinatorial auctions

• Can serve as a testbed for many other mechanism design questions with market uncertainty