Cost Semantics for Space Usage in a Parallel Language

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(Joint work with Guy Blelloch & Robert Harper)

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Understanding *How* Programs Compute

Interested in intensional behavior of programs

- more than just final result
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State-of-the-art = compile, run, & profile

- ✖ architecture specific (*e.g.* # cores)
- ✖ dependent on configuration (*e.g.* scheduler)
- ✖ compilers for functional languages are complex (*e.g.* closure, CPS conversion)

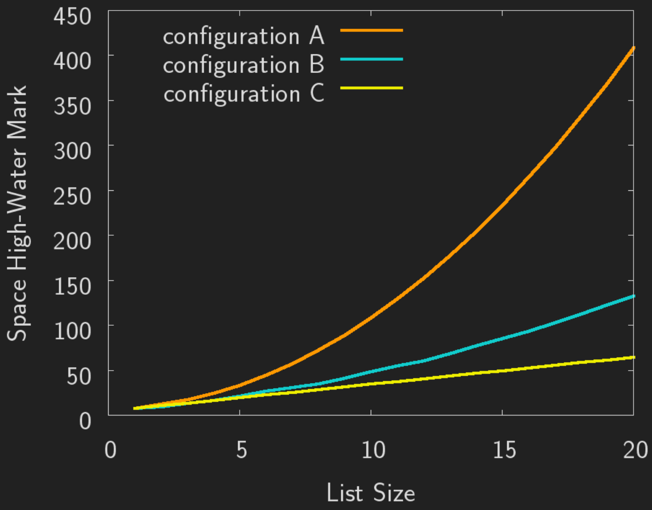
Motivating Example: Quicksort

Assume fine-grained parallelism

- pairs $< e_1 \mid\mid e_2 >$ *may* evaluate in parallel
- schedule determined by compiler & run-time

```
fun qsort xs =
  case xs of nil => nil
           | x::xs =>
    append <qsort (filter (le x) xs) ||
            x::(qsort (filter (gt x) xs))>
```

Quicksort: High-Water Mark for Heap



Approach

Cost Semantics

- define execution costs for high-level language
- account for *parallelism* & *space*

Provable Implementation

- make parallelism explicit
- translate to lower-level language
- prove costs are preserved at each step
- consider scheduler, GC implementation

Approach Talk Outline

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Background: Cost Semantics

A *cost semantics* is a dynamic semantics

- *i.e.* execution model for high-level language

Yields a cost metric, some abstract measure of cost

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We will a consider a cost model that accounts for parallelism and space.

Source Language

Consider a pure, functional language.

- includes functions, pairs, and booleans

Pair components evaluated in parallel.

- denoted $< e_1 \mid\mid e_2 >$

Values are disjoint from source language.

- values are labeled to make sharing explicit
  e.g. $(v_1, v_2)^\ell$

Parallel Cost Semantics

Cost semantics is a big-step (evaluation) semantics

- yields two graphs: computation and heap
- sequential, unique result per program

$$e \Downarrow v; g; h$$

Expression e evaluates to value v with computation graph g and heap graph h.

Computation Graphs

Track control dependencies using a DAG with distinguished start and end nodes.

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$1 \qquad [n] \qquad g_1 \oplus g_2 \qquad g_1 \otimes g_2$

Heap Graphs

Track heap dependencies using a directed graph

$$h = E$$

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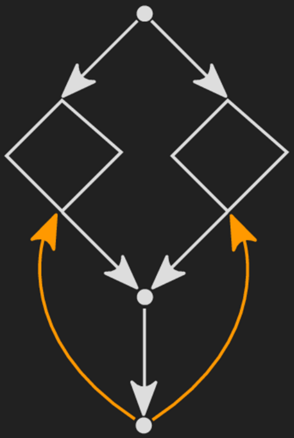

Using Cost Graphs

Cost graphs are tools for programmers.

- relate execution costs to source code
- later: simulate runtime behavior

Many concrete metrics possible

- considered maximum heap size in example
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However, this reasoning is only valid if the implementation respects these costs.

Provable Implementation

Guaranteed to faithfully mirror high-level costs

- "implementation" $=$ lower-level semantics

Costs $\Rightarrow$ contract for lower-level implementations

- *e.g.* environment trimming, tail calls
- can guide concrete implementation on hardware

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This work: transition semantics defines parallelism

- several (non-)deterministic versions
- can incorporate specific scheduling algorithms

Transition Semantics

Non-deterministic, parallel, small step semantics

- parallel construct for in-progress computations

$$
\begin{array}{lrcl}
\text{(expressions)} & e & ::= & \ldots \mid \text{let par } d \text{ in } e \\
\text{(declarations)} & d & ::= & x = e \mid d_1 \text{ and } d_2
\end{array}
$$

Transition Semantics

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- declarations simulate a call "stack"
- allows unbounded parallelism, *e.g.*

$$\frac{d_1 \longmapsto d_1' \quad d_2 \longmapsto d_2'}{(d_1\ \mathsf{and}\ d_2) \longmapsto (d_1'\ \mathsf{and}\ d_2')}$$

Schedules

Define a schedule of g as any covering traversal from n_{start} to n_{end}.

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Definition (Schedule)

A *schedule* of a graph $g = (n_{start}, n_{end}, E)$ is a sequence of sets of nodes $N_0, \ldots, N_k$ such that $n_{start} \notin N_0$, $n_{end} \in N_k$, and for all $i \in [0, k)$,

- $N_i \subseteq N_{i+1}$, and
- for all $n \in N_{i+1}$, $\mathrm{pred}(n) \subseteq N_i$.

Theorem

Every schedule corresponds to a sequence of derivations in the transition semantics.

Theorem

If $e \Downarrow v; g; h$ *then,*

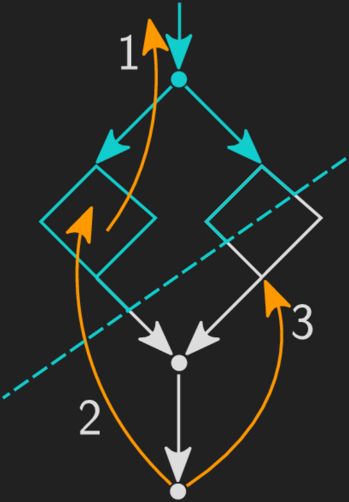
$$N_0, \ldots, N_k \textit{ is a schedule of } g$$
$$\Leftrightarrow$$
$$\textit{there exists a sequence of } k \textit{ transitions}$$
$$e \longmapsto \ldots \longmapsto v \textit{ such that } i \in [0, k],$$
$$\mathit{roots}(N_i; h) = \mathit{labels}(e_i).$$

Measuring Space Usage

GC roots determined by heap graph h and schedule

- roots = edges that cross schedule frontier

Reachable values determined by reachability in h.



Measuring Space Usage (con't)

Note that edges in h correspond to direct dependencies as well as "possible last uses."

$$\frac{e_1 \Downarrow \mathsf{false}^{\ell_1}; g_1; h_1 \quad e_3 \Downarrow v_3; g_3; h_3 \quad (n \text{ fresh})}{\begin{array}{r}\mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 \Downarrow v_3; 1 \oplus g_1 \oplus [n] \oplus 1 \oplus g_3 \\ h_1 \cup h_3 \cup \{(n, \ell_1)\} \cup \{(n, \ell)\}_{\ell \in \mathrm{labels}(e_2)}\end{array}}$$

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Heap graphs have a "static" character

- necessary to simulate GC decisions

Scheduling Algorithms

Transition semantics (above) allowed *all* possible parallel executions.

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Given finite processors, which sub-expressions should be evaluated?

E.g. depth- and breadth-first & work stealing

- DF and BF traversals of cost graph g

Formalized as *deterministic* transition semantics

- abstract presentation: no queues, &c.

Quicksort: Revisited

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append <qsort (filter (le x) xs) ||
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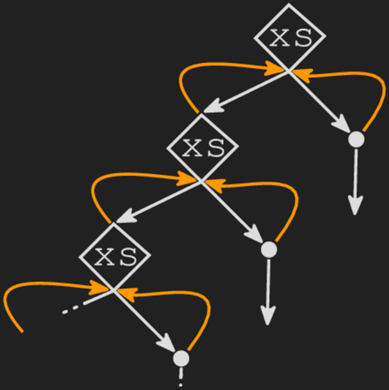

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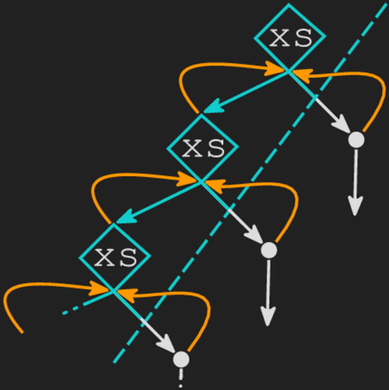

Quicksort: Revisited

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let val (ls, gs) = <filter (le x) xs ||
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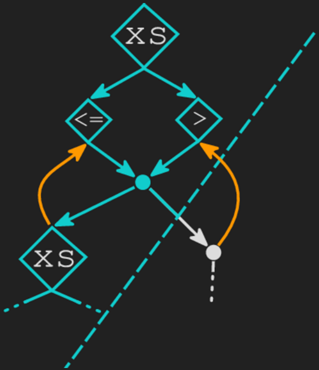

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$\leadsto$ (via inlining)

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Related Work

Greiner & Blelloch measure time and space together [ICFP '96, TOPLAS '99]

- upper bounds based on size and depth of DAG

Minamide shows CPS conversion preserves space usage [HOOTS '99]

- constant overhead *independent* of program

Gustavsson & Sands give laws for reasoning about program transformations in Haskell [HOOTS '99]

- formalize "safe for space" as cost semantics

Future Work

Empirical evaluation

- full-scale implementation, predict & measure performance (different GCs, schedulers)
- killer app?

Language extensions

- static discipline to help control (or at least make explicit) performance costs
- *e.g.* distinguish implementations of quicksort

Summary

Functional programming:

- traditionally, easy to reason about result
- . . . but hard to reason about performance

In this work, we have

- related parallelism & space usage to source
- proved costs preserved by implementation
- considered effects of scheduler, collector

Ongoing: reason about performance in parallel ML