Coalition Formation in a Power Transmission Planning Environment

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Abstract

The study of a decentralized coalition formation scheme in an specific Power Systems transmission expansion scenario is the purpose of this paper. We define first who are the players in the expansion game, and provide a decentralized coalition scheme based on Bilateral Shapley Values. We study the stability properties of our approach to allocate expansion costs to all coalition members. Finally, the resulting coalition scheme from Bilateral Shapley Value negotiation is compared with a centralized approach.

1 Introduction

Transmission planning in power systems addresses the problem of determining the optimal number of lines that should be added to an existing network to supply the forecasted load as economically as possible, subject to operating constraints. “The objective is the minimum cost expansion plan given the base network configuration, the generation facilities, and the forecasted demands for a target year [19].”

Traditionally, the transmission expansion planning problem has been studied using two types of techniques: (1) techniques based on mathematical programming, such as Branch-and-Bound [7, 8, 17], and (2) techniques based on sensitivity analysis [2, 18]. There is a third technique that uses neural networks hybridized with genetic algorithms [27], that has shown promising results. Clearly, the combinatorial nature of the problem makes a formidable task to pursue optimal solutions, making it very hard to find reasonable solutions in short computational time.

Currently, the electric utility industry is facing deregulation to allow transmission open access to suppliers and customers before the year 2002. Addressing that issue, Wu et al. [26] have developed a decentralized algorithm to optimize multilateral trades among generators and customers, although only looking at the operational side. Regarding transmission planning, Bushnell and Stoft [3], and Chao et al. [4] have presented investment incentives and market mechanisms, respectively.

Transmission expansion planning involves decisions taken by some of the actors in the expansion scenario (suppliers, customers and/or transmission line owners) that can and will affect decisions taken by other players. This fact is particularly well-known when a new transmission line is built, and it is shared
by several “players” of the expansion game. The decision whether to build the line or not, and the allocation of costs to the players who will use the line is still an open issue.

The formation of coalitions of players in a transmission expansion investment scenario is a valid approach to solve the expansion problem, as it is shown in [9]. In [9], Gately is concerned with regional cooperation in planning investments and cost allocation using the Shapley value, a well known cooperative game theory concept. Gately’s approach is a centralized one, where there exists a central planner taking the allocation decisions.

Very recently, research on Distributed Artificial Intelligence (DAI) has focused on how the coalitions are formed, and negotiation algorithms amongst players. Again, cooperative game theory concepts are used, but they are suited to decentralized multitask environments [12, 14, 15, 16, 23]. A DAI approach addresses and solves the pending issues in a deregulated power transmission environment:

- Determining which coalitions will be formed
- Implementing a negotiation algorithm
- Allocating total expansion costs to every agent of the transmission game

What we propose in this paper is the creation of a multi-agent system, where the agents cooperate with each other to achieve the optimal common expansion goal. Here, the agents have to fulfill certain number of tasks, i.e. adding new lines, and they want to cooperate forming coalitions to reduce overall costs. Each agent is rational, in the sense of being a utility maximizer, and is “an independently motivated agent, not willing to settle for a plan generated by a centralized planner [12].”

2 Network Expansion Model

We will follow the simple model that Garver used to solve a six buses system, as shown in Figure 1. The generation upper limits and line flow upper limits for existing lines are also shown. The reader is referred to [8, 25] for further details.

When ranking possible additions to the system, we follow the heuristic approach suggested by [18, 25], such that the original linear programming problem is transformed into a quadratic problem subject to linear constraints. The general formulation of the expansion problem can be expressed as:

\[
\min \frac{1}{2} \sum_{j=1}^{M} c_j P_j^2
\]

subject to

\[
B \Theta + K^T P_D = P
\]

\[
|B_L A \Theta| \leq \bar{P}_L
\]

where \(c_j\) is the cost of adding line \(j\) to the network, \(P_j\) is the active power (in p.u.) flowing through the added line \(j\), \(M\) is the number of possible new lines, \(B\) is the matrix whose elements are the imaginary parts of the nodal admittance matrix of the existing network, \(\Theta\) is the phase angle vector, \(K^T\) is the transpose of the node-branch connection matrix, \(P_D\) is the
flow vector for possible lines, \( P \) is the nodal injection power for the overall network, \( B_L \) is a diagonal matrix whose elements are branch admittances, and \( P_L \) is the branch active power vector. The minimization algorithm is run recursively until there are no overloads in the system. Although the optimum value is not always guaranteed, the simplicity of the heuristic algorithm makes it a valid first approach to solve a highly combinatorial problem like this one.

Since the objective function (1) has taken into account the effect of the power transmission cost, the line candidate with the largest power flow is most effective in the expanded network\(^1\). Constraint (2) expresses the total nodal injection power as a function of the existing and the possible network parameters, and constraint (3) reflects the thermal limits of the existing network lines. Table 1 presents the basic circuit data.

The solution obtained is the same optimal solution originally given by Garver, without generation rescheduling (generator outputs 1, 3, and 6 are 50, 165 and 545 MW respectively). The optimal solution has a cost of 200 units, and circuit additions are: \( n_{26} = 4 \) circuits, \( n_{35} = 1 \) circuit, and \( n_{46} = 2 \) circuits.

When we allow generation rescheduling, i.e. the real power generation ranging from 0 to the maximum generation available (150, 360, and 600 MW respectively), the optimal solution has a cost of 130 units, and circuit additions are: \( n_{26} = 3 \) circuits, and \( n_{35} = 2 \) circuits\(^2\).

Note that this solutions correspond to what is defined in cooperative game theory language as “grand coalition” schemes. In the next Section, we analyze the rules of the transmission expansion game and the making of feasible coalitions.

### 3 Coalitions and Games in Expansion Planning

To calculate the allocated cost in the Garver 6 Bus example, we define first what the purpose of the game is, who the players are, and what is a coalition in the transmission expansion game. We follow the DAI terminology, and a player will be called an agent from now on.

The purpose of the game, as mentioned earlier, is the expansion of the transmission grid, with the minimum possible cost, subject to the constraints expressed in (2) and (3), and with a “fair” allocation of total cost amongst the agents.

An agent in the game can be either a generator, a load, or an independent third party (for example, an independent company who owns transmission lines). A typical agent in this context is regarded as an independent entity: a customer or group of customer loads, a generator or a set of generators, or a combination of both. For simplicity, we do not consider for now fractional bus loads or generators, although these would be the smallest possible agents. For simplicity, we assume that any set of generation units and loads attached to the same bus belong to a single agent, thus we cannot have two agents sharing the same bus. Therefore, we have a maximum of 6 agents in the expansion game, corresponding to the 6 buses.

Furthermore, a coalition of agents is a set made up of, at least one generator, one load, and one transmission line. There are three axioms that a coalition has to satisfy:

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\(^1\)See [25], pp. 394-400, for a very detailed explanation.

\(^2\)Note that 130 is not the best possible result; the optimum is 110, as shown in [20]. This deficiency is due to the simplicity of the model, that does not explore all possible line combinations.
1. Generator(s) have to meet the demand, i.e. the load has to be always met by the generation.
2. Existing line(s) thermal limits can not be exceeded when running a power flow for the new coalition.
3. There must be one or more transmission lines (either existing or possible candidates) connecting all the buses in the coalition.

![Diagram](image1.png)

Figure 2: Two Examples of Autonomous Coalitions

These three axioms create what we call “autonomous” coalitions, because they can try their own expansion plans without having to negotiate with any other similar entity. It can be anticipated from the given set of axioms that some of the possible coalitions are ruled out. In particular, when we have a single bus in the system, with either a generator, a load, or both, it can not be considered a coalition, because it lacks at least one transmission line. However, we will assume that the single agents will pay for line investments if necessary, so that they will not need to own transmission lines at the beginning of the expansion game.

Also, if we have two buses that meet the first axiom, and there are no line candidates that tie them, it is very likely that axiom 2 will be violated. Finally, if not all the buses in the coalition are tied to at least another one, axiom 3 will be violated. Figure 2 shows some examples of feasible “autonomous” coalitions in the Garver test case.

Note that, for practical purposes, we will use the bus notation when referring to coalition sets. For example, when we say coalition \{1,2\} we are referring to a coalition that combines all generators and loads in buses 1 and 2, and all the lines that interconnect both buses.

In particular, for the Garver 6 Bus test case allowing rescheduling, the set of feasible coalitions is the following:

1 player: \{2\}, \{4\}, \{5\}, \{6\}
2 players: \{2,6\}, \{3,5\}, \{4,6\}, \{5,6\}
3 players: \{1,2,6\}, \{1,3,5\}, \{1,4,6\}, \{1,5,6\}, \{2,3,6\}, \{2,4,6\}, \{2,5,6\}, \{3,5,6\}, \{4,5,6\}
4 players: \{1,2,3,6\}, \{1,2,4,6\}, \{1,2,5,6\}, \{1,4,5,6\}, \{2,3,4,6\}, \{2,3,5,6\}, \{3,4,5,6\}
5 players: \{1,2,3,4,6\}, \{1,2,3,5,6\}, \{1,2,4,5,6\}, and \{2,3,4,5,6\}
grand coalition: \{1,2,3,4,5,6\}

The other possible coalitions violate at least one of the axioms.

4 Decentralized Coalition Formation between Transmission Expansion Players

The use of decision techniques to analyze DAI problems, like the one posed in Section 3, started in the present decade. However, the Shapley Value (SV) is known since the seminal paper by Lloyd Shapley [21]. Shapley Value is a tool from Cooperative Game Theory to calculate a fair division of the utility among the members of a coalition. It is a solution concept for an n-person cooperative game. Shapley Value is the weighted average of marginal contributions of a member to all possible coalitions in which it may participate. It assumes that the game is superadditive and the grand coalition is formed. The reader is referred to [10, 21] for a detailed explanation of necessary conditions to calculate a meaningful Shapley value.
Value.

The mathematical expression of the Shapley Value, $\phi_i$, is given by:

$$\phi_i = \frac{1}{n} \sum_{q=1}^{n} \frac{1}{c(s)} \sum_{i \in S} [v(s) - v(s - i)] \tag{4}$$

where,

$i = \text{player}$

$s = \text{coalition of players}$

$q = \text{size of a coalition}$

$n = \text{total number of players}$

$v(q) = \text{characteristic function (cost savings) associated with coalition } q$

c(q) = \text{number of coalitions of size } q \text{ containing the designated player } i$, given by,

$$c(q) = \frac{(n - 1)!}{(n - q)! (q - 1)!} \tag{5}$$

In order to avoid the exponential complexity of Shapley Values calculation, Ketchpel introduced the so-called Bilateral Shapley Value (BSV) [12]. Klusch and Shehory [13, 14, 15] adapted this approach for a completely decentralized and bilateral negotiation process among rational information agents using these values. In particular, the algorithm for coalition formation they provided is also useful in the power transmission environment.

Let $CS \subseteq \mathcal{P}(A)$ a coalition structure on a given set of agents $A = \{a_1, \ldots, a_m\}$, $C = C_i \cup C_j \subseteq A$ a (bilateral) coalition of disjoint ($n$-agent) coalitions $C_i$ and $C_j$ ($n \geq 0$). The Bilateral Shapley Value for some coalition $C_i$ in a bilateral coalition $C$ is defined by $b_{SV(C_i,C_j)}(C_i) := \frac{1}{2}v(C_i) + \frac{1}{2} (v(C) - v(C_j))$. Both coalitions $C_i$, $C_j$ are called founders of $C$, and $v(C)$ denotes the self-value of coalition $C$.

It can be seen that the founders will get half of their local contribution, and the other half stemming from cooperative work with the other entity. The second term of the BSV expression reflects the strength of each agent concerning his contribution, therefore avoiding the “free-rider” problem, so common in transmission expansion value allocation schemes. BSVs can be considered extensions of the concept of Shapley Values, creating a fair distribution among the entities, but maintaining the attractive features that SVs offer.

We will present a solution $(C,(x_1, \ldots, x_n))$ for a given cooperative game $(A,v)$ in general environments using these Bilateral Shapley Values. It is based on a special algorithmic calculation of each agent utility $x_i$ ($i \in \{1, \ldots, m\}$) which is not necessarily $b_{SV}$, if $a_i \in C_k$ and $C_k$ is a founder of $C$ in the coalition structure $CS$. In the following we denote coalitions with more than one agent as a (multi-parties) player.

Given all that, the method of coalition formation between transmission expansion agents is based on the approach followed by Klusch and Shehory in [15].

A summary of the steps in the coalition process is given as follows:

1. **Self-Calculation Phase**: Each individual agent gathers information to determine its self-value. Calculation of the self-value determines the monetary cost of line expansion for individual agents, following the three axioms from Section 3. It is possible that some of the agents are unwilling to use new lines on their own, and this fact is reflected by a cost of zero. For the 6 Bus Garver example, where each single agent is attached to a bus, we can observe that agents 1 and 3 are self-sufficient, because the load is met by the existing node generation. However, agents 2, 4, and 5 need to use extra lines to meet their own demand. Finally, bus 6 needs to be hooked to the network, not to become isolated. These facts are reflected in their self-values. In the case of different initial settings, agents 3 and 5 being a single initial

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3Similar work has been done by Kraus and Shehory [22]. In contrast to our work, it does not provide a solution for cooperative games and a given coalition structure with other than 2-agent coalitions.

4Note that $b_{SV(C_i,C_j)}(C_i) = v(C)$, and $v(\emptyset) := 0$.

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6General environments allow for both, superadditive as well as subadditive games. A game $(A,v)$ is superadditive iff $\forall C = C_1 \cup C_2 \subseteq \mathcal{P}(A) : v(C) \geq v(C_1) + v(C_2)$; it is subadditive iff it is not superadditive.
agent for instance, costs would change, reflecting different initial conditions.

2. Communication Phase: Once each agent has calculated its own self-value, it is time to determine the joint value that he will have when cooperating with another agent. Unfortunately, an agent is not necessarily aware of the environment surrounding him, thus a coordinator is needed to gather this information. This is certainly the case of transmission expansion planning, where the entire network is not known completely by any agent. In order to calculate the BSVs, agents need to send all their proposed line addition(s) to this central figure, and then receive the adequate number of new lines for proposed coalitions, via messages sent by the coordinator to all of them. It is perfectly possible that two agents reach an agreement that is satisfactory for both of them but detrimental to the security of the system. That is why the figure of an independent coordinator is needed in order to check that reliability of the system and quality of service are assured. Usually, a power flow of the system subject to security constraints should be enough. It is also possible to bypass Step 2 had all the agents access to everyone else’s information, but this is generally not the case, due to the complexity of the network. On the other hand, agents freely exchange self-values with each other.

3. BSV Calculation Phase: Now the agents know their own self-values, and every possible value with other coalition partners via Step 2. After getting these messages from the coordinator, they proceed to calculate BSVs if teaming with the agent that sends the offer to team. Then, the agents determine individually rational lists of preferred agents: ordered list of local agents’s BSVs for two-entity coalitions. These lists will change whenever Step 3 is called again, with new multi-parties players acting as agents.

4. Bilateral Negotiation Phase: Each agent looks at the head of his ordered list, and extends an offer to the preferred partner. The offer consists of sending the partner’s BSV: the value that he would attain for collaborating with the sender. If it happens that the sender receives also a message from the preferred partner, and they both find it is beneficial to join, they do. They create a multi-parties player that will behave like one agent from then on. Every other agent is also informed, for him to erase the members of the multi-parties players from his own preference list.

The coalition formation process is repeated, starting from Step 2 are known from the beginning), by all agents and multi-parties players until no more coalitions are possible. If no coalition is possible at one particular step, the agents look at the second best partner if still not possible, to the third, etc., until reaching the end of the list.

There are several features of this negotiation algorithm that are only relevant to power expansion planning. First, this is a general environment, neither superadditive nor subadditive. The grand coalition will not necessarily be formed. Second, previous negotiation algorithms used a utilitarian coalition building scheme, where the agents have to satisfy their tasks via cooperation, thus increasing their benefits. The utilitarian coalition formation in the transmission expansion domain is done in a cost-oriented view. Finally, even if common resources are shared (the lines), it does not mean that all agents can have access to them. In the case of multi-parties players, for example load at 2 and generation at 6 forming a single 2-6 agent, we disregard the lines connecting this agent to the other ones (for example possible line 4-6). However, when considering a single agent expansion plan, the lines that connect him to other agents are ignored, except if the single agent cannot meet his own demand; in that case, he pays for all the building expenses to the other party. This is an artifact to cope with the fact that agents’ actions will affect everyone else in the game. It has to be revised in future research.

Given these premises, we have run a simulation for the Garver 6 Bus test case implementing the Bilat-

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7 Individual rationality means that the agent wants to have a new value that is, at least, as good as the one that he could attain alone.

8 This means that the agents in the coalition will vote for a representative agent which in the future takes decisions for this coalition.

9 Related work can be found in the DAI area for task-oriented domains with cost functions by Rosenmarch and Zlotkin.
eral Negotiation algorithm. Cost functions are given in Table 2. Note that when using SVs and BSVs, values are negative to reflect positive expansion costs in monetary units. Omitted costs equal zero. Note that the respective coalition values of this (subadditive) game are negative reflecting the utility of a coalition in a cost-oriented view.\(^{10}\)

The first simulation starts with the individual agents 1 to 6 creating lists of preferences. We assume for the first two simulations that whenever a tie occurs between going solo or teaming, teaming is preferred. The final resulting coalition is \(\{1,2,3,4,5,6\}\), and the order of coalition creation is the following: \([1,2,3,6,5,4]\) in the first round, \([1,\{2,3,5,6\},4]\) in the second, \([1,\{2,3,5,6\},4]\) in the third, and \([1,\{2,3,4,5,6\}\] in the fourth and last round. Once the grand coalition is created, we follow a backward-induction method to calculate value allocations via BSVs. Starting from the grand coalition, we divide the team into the two founding members: \([1,2,3,6,4,5,4]\), and split the total value of -130. Let \(b_{sv}(1,2,3,6,4,5,4)\) be the value allocated to agent 1 using the Bilateral Shapley value rationale:

\[
\begin{align*}
b_{sv}(1,2,3,6,4,5,4)(12356) &= \frac{1}{2} v(\{12356\}) \\
&\quad + \frac{1}{2} (v(\{123456\}) - v(\{4\})) \\
&\quad - v(\{12356\})
\end{align*}
\]

where \(v(\{i\})\) is the value of coalition \(i\), as per Table 2 when we reverse signs. Going backwards one more step, we find the following values for the next sub-division of \([1,2,3,5,6]\) into their founders, \([\{1\}\], and \([2,3,5,6]\):\n
\[
\begin{align*}
b_{sv}(1,2,3,5,6)(1) &= \frac{1}{2} v(\{1\}) + \frac{1}{2} (b_{sv}(1,2,3,6,5,4)(12356) \\
&\quad - v(\{2356\}))
\end{align*}
\]

The process is followed until the values for individual agents are found: \((12.5, -49.375, 10.625, -55, -29.375, -19.375)\), adding up to -130 total value. Note that in the second and subsequent steps of the backward algorithm, the total value to be split is given by the previous Bilateral Shapley Value, as shown in (8) and (9), by using \(b_{sv}(1,2,3,5,6)(12356)\) instead of \(v(\{12356\})\). This backward process can be used as a general calculation scheme for every agent utility in its coalition.

This recursive division of Bilateral Shapley Values within each coalition can be written as follows:

1. \(BCLC := \{(C, (C_1, b_{sv}(C_1)), (C_2, b_{sv}(C_2))) : C \in C\}\)
2. \(F_S := \text{occ}(S) = \{S_1, S_2\}\)
3. \(ARUL = \{(a_1, u) : u \in R, a_i \in A\}\)

\(^{10}\)E.g., the value (cost) for the grand coalition is -130 (130).

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Cost</th>
<th>Coalition</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>([1,2,3,6,4])</td>
<td>90</td>
<td>([2,3,5,6])</td>
<td>101</td>
</tr>
<tr>
<td>([5])</td>
<td>40</td>
<td>([4,5])</td>
<td>304</td>
</tr>
<tr>
<td>([6])</td>
<td>60</td>
<td>([1,2,3,6])</td>
<td>120</td>
</tr>
<tr>
<td>([2,6])</td>
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<td>([1,2,4,5,6])</td>
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<td>161</td>
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<tr>
<td>([1,2,3,6,4])</td>
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<td>([1,2,3,4,6])</td>
<td>90</td>
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<tr>
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<td>([2,3,6,4])</td>
<td>150</td>
<td>([1,2,3,4,5,6])</td>
<td>160</td>
</tr>
</tbody>
</table>

Table 2: Coalitions Expansion Costs
where $BCL$ is a set containing the history of coalition structures and monetary profits, $F$ is the set of founders, and $ARUL$ is the set of agent’s utility distribution.

Agent $a_x$ of coalition entity $C_i$ in a bilateral coalition $C_k \in C, \text{ce}(C_k) = \{C_i, C_j\}$ receives a monetary reward $ru_x \in \mathbb{R}$ of the joint utility $bsv_{C_k}(C_i)$ for $C_i$:

$$ru_x := \text{recutil}(C, BCL, a_x)$$  (10)

based on the functions

\[
\text{recutil}(C: \text{CoalitionStructure}, BCL: \text{SetCoalTuple}, a: \text{Agent}): \text{real};
\]

\[
\begin{align*}
\text{begin} & \quad \text{ARUL} := \emptyset; \\
\text{for each} & \quad C \in \text{coaltuple}[1] \in BCL \text{ do} \\
& \quad \text{for each} \quad (S, bsv_{C}(S)) \in \text{coaltuple}[2] \\
& \quad \quad \text{bcu}(S, bsv_{C}(S)); \\
& \quad \text{for each} \quad \{(a_i), u\} \in \text{ARUL} \text{ do} \\
& \quad \quad \text{if} \ a_i = a_x \text{ then begin recutil} := u; \text{break}; \text{end}; \\
\text{end}; \\
\text{bcu}(S: \text{Coalition}, bsv: \text{real}); \\
\text{begin} & \quad \text{if} \ |S| > 1 \text{ then begin} \\
& \quad \quad v(S) := bsv; \\
& \quad \quad \text{for each} \quad K \in F_S \text{ do} \\
& \quad \quad \quad \text{begin} ru := bsv_{C}(K); \\
& \quad \quad \quad \quad \text{if} \ |K| > 1 \text{ then bcu}(K, ru) \\
& \quad \quad \quad \quad \text{else \ ARUL} := \text{ARUL} \cup \{(K, ru)\}; \\
& \quad \quad \text{end} \quad \text{end}; \\
& \quad \text{else} \quad \text{ARUL} := \text{ARUL} \cup \{(S, bsv)\}; \text{end};
\end{align*}
\]

Cost allocation results for this example are shown in Figure 3.

A second simulation is run, since player 6 is indifferent to team with 2 or 4 at the beginning. The process is now: $\{\{1\},\{2\},\{3-5\},\{4-6\}\} \rightarrow \{\{1-3-5\},\{2-4-6\}\} \rightarrow \{\{1-2-3-4-5-6\}\}$. The value allocation is given by: $(21.25, -49.375, 6.25, -55, -33.75, -19.375)$.

Note that not all simulations drive the coalition to a grand coalition scheme. Here is one example:

\[
\begin{align*}
\{\{1\},\{2-6\},\{3\},\{4\},\{5\}\} \rightarrow \{\{1-2-6\},\{3\},\{4\},\{5\}\} \rightarrow \{\{1-2-3-6\},\{4\},\{5\}\}.
\end{align*}
\]

But value allocation is still up to -130: $(22.5, -48.75, 15, -60, -40, -18.75)$.

In this example, we have considered agents belonging to the transmission network. However, in a deregulated environment, we should consider agents that invest money in building lines, and want to recover that investment. This is the case of third party transmission line owners. For the sake of the example, suppose that line 2-6 belongs to a party that does not own anything else in the network. These are the steps that a decentralized algorithm would follow for the line owner to recover her investment:

1) Determine if the line is beneficial for the entire network. The coordinator runs a transmission line expansion, as shown in Section 2.

2) If the line is beneficial, run the coalition formation algorithm, as shown in Section 3.

3) Recover part, or all investment costs, from the parties that are directly using the line: agents 2 and 6. If they do not provide enough, distribute the remaining costs among the other agents in a fair manner (for example, splitting them in equal parts).

For all simulations, buses 1 and 3 receive monetary units for their contribution to the welfare of the
system, whilst the remaining buses must contribute, although less that what they would pay on their own. This is a clear incentive for all players to play this game.

5 Coalition Stability

Following the decentralized coalition formation algorithm shown in Section 4, it remains to be proved that the resulting coalitions are stable. To say that a coalition is stable means that none of its members desires to leave the group to obtain a better payoff. In our particular 6 bus example, there are only three sets of coalitions that can divide the total cost in an optimal way:

1) Grand Coalition: \{1-2-3-4-5-6\};
2) \{1-2-3-4-6\},\{5\}; and
3) \{1-2-3-6\},\{4\},\{5\}

All of them divide a total value (cost) of (-)130, but the negotiation process in this game for forming the "grand coalition" is not unique.

The stability method proposed by Aumann and Myerson in [1] is applicable for some cases, however, it does not provide a general rule for stability. It is particularly valuable in cases where the games present specific properties, as shown in Section 5 of [1].

For a more general criterion, we use a cooperative standard of fairness (SOF), as presented in Chapter 5 of [11]. Quoting Kahan and Rapoport, "a SOF for a game is a vector-valued function with t elements \( \Psi = \psi_1(P),\psi_2(P),\ldots,\psi_t(P) \), defined for each partition \( P = (P_1,P_2,\ldots,P_t) \) of the \( n \) players into t negotiation groups (\( 1 \leq t \leq n \)). \( P_j \) is a nonempty negotiation group in the partition \( P \) and \( \psi_j(P) \) designates the fair share of that group \( P_j \) according to the rules generating the SOF. The SOF function \( \Psi(P) \) is assumed to satisfy two conditions:

\[
\psi_j(P) \geq v(P_j) \text{ for any } P \text{ and all } P_j \in P \tag{11}
\]

\[
\psi_1(P) + \psi_2(P) + \cdots + \psi_t(P) = \hat{v}(N) \text{ for all } P \tag{12}
\]

where \( \hat{v}(N) \) is the superadditive cover of a coalition:

\[
\hat{v}(N) = \max \left\{ \sum_{j=1}^{P} v(P_j) \right\}
\]

(13)

That is, coalition \( P \) divides itself up into subcoalitions in such a way that the sum of the values of the subcoalitions is a maximum. Clearly, for superadditive games, the superadditive cover of a coalition is the value of the coalition.

Condition (11) states that any negotiation group can command as a fair share at least the value it would obtain were it to bind itself to a coalition. Condition (12) defines the entity being divided into fair shares as the maximum joint payoff available in the game, i.e., the superadditive cover of the grand coalition.

In our case, the backward induction algorithm to allocate values (costs) is more specific, because a coalition structure \( \mathcal{C} = \{C_1,\ldots,C_n\} \), and its sequence bilateral assignments (of coalition founders) \( ce = (ceCS(C_1),\ldots,ceCS(C_n)) \), \( |ceCS(C_i)| = 2 \) is given for each coalition formation round, i.e. the history of the bilateral assignments for each coalition structure is known. In particular, the coalition algorithm (see Sect. 4) allows only for bilateral coalitions with individually rational founders in each round. Thus, given two players \( a \) and \( b \) as founders of \( C \) in some coalition structure \( \mathcal{C} \), \( ceCS \) (with \( ceCS(C) = \{a,b\} \) it holds that:

\[
b_{sv\{a,b\}}(a) - v(a) = b_{sv\{a,b\}}(b) - v(b) = \frac{1}{2}(v(a,b) - v(a) - v(b)) \geq 0\tag{14}
\]

It is guaranteed that at any step, agents \( a \) and \( b \), as founders of the created coalition, are individually rational and the subgame is superadditive, as shown in (14). Thus, given a partition \( P = \{P_1,P_2\} \) with a bilateral assignment \( ce \), we have that: for each \( P_j \in P, C_k \in ce(P_j) \):

\[
b_{sv\{P_j\}}(C_k) \geq v(C_k)\tag{15}
\]
However, there is no proof that the individual rationality condition expressed in (15) implies the individual rationality of the single agents in C.

Note that (12) is always met when the grand coalition is formed, since the superadditive cover is also equal to the coalition value (cost) in that case. In case the grand coalition is not formed, (12) is also met, because the final coalitions are indifferent whether to team or stay solo; thus the maximum (minimum) possible value (cost) is obtained, and it is also equal to v(\{12345\}). Note that in this particular case, (14) is equal to zero. The reader can check this assertion for the cases: \([\{1-2-3-4-6\},\{3\}]\) and \([\{1-2-3-6\},\{4\},\{5\}]\).

Therefore, we can guarantee for a general environment that the proposed algorithm meets the SOF that any cooperative fair cost allocation game has to meet. To compare with other definitions of SOFs, the reader is referred to Section 5.1.2. of [11].

It is also possible to determine coalition stability using the “propensity to disrupt” concept given as in [9]. Gately defined this concept as the ratio between how much a set of players in a coalition would lose if player \(i\) refused to cooperate to how much \(i\) would lose if it refused to cooperate. For a bilateral coalition, the ratio is equal to 1 for both players, as seen in (14). For any step of the algorithm, the players are “fairly” rewarded, since their respective propensities to disrupt are equal.

6 Centralized vs. Decentralized Cost allocation: SVs vs. BSVs

The main advantage that a centralized approach offers to the expansion game is the simplicity. Using a central planner, we do not need a synchronized algorithm to exchange information amongst coordinator and agents, neither among the agents themselves. The role of the coordinator becomes more important in a centralized approach, thus. He not only calculates joint values, but also Shapley Values for all the agents, given all possible combinations, as shown in Table 2. Were we implementing that scheme, these would be the allocation values: \((16.85, -45.86, 54.73, -19.41, -42.48, -93.81)\).

Comparing this result with the three decentralized simulations of the previous Section, we can observe that there is a clear mismatch between buses 3 and 6 allocations. The reason is that, when we use pure SVs to allocate values, we consider all combinations of agents: 3 becomes a valuable agent for some coalitions: \(\{3, 5\}, \{1, 3, 5\}\), and \(\{1, 2, 3, 6\}\); but 6 is a worthless agent: \(\{2, 5, 6\}, \{4, 5, 6\}\). On the other hand, SVs are suited for superadditive environments, which is not the case. By contrast, using BSVs, only the best combinations will form, and the concepts of “valuable” and “worthless” will depend on the strategy that a player follows. For instance, agent 6, knowing his value in the game, will team either with 2 or 4 to reduce his own final cost.

We can recall that in our decentralized environment the offers are only bilateral, building up to larger coalitions. By building only 2 member coalitions we are missing some combinations that may be more beneficial to the agents. Therefore, not all combinations are considered, like in a SV centralized environment, but the coalition formation algorithm terminates in polynomial time.

It can be inferred that the skill of the agents to choose the right coalition is crucial to get the largest slice of the allocation pie. By relinquishing his right to choose the right coalition, an agent may end up with a value that does not reflect his true power in the game.

Therefore, a decentralized approach will reflect agents’ strengths, and will also set a scheme for negotiation. Finally, were we stopping the negotiation process, we could do it at any time, and calculate the BSVs for the current scheme. This in turn provides a new approach in power transmission planning for self-organizing regional cooperations within reasonable time.

\(^{11}\) See Table 2; a valuable player for a coalition refers to his positive marginal contribution.
7 Software Implementation

The network expansion program has been coded in Scilab, a MATLAB\textsuperscript{12} clone developed at INRIA [29]. A MATLAB version is currently in progress. Mathematica\textsuperscript{13} is the cost allocation software that has been used for this project. The interested reader is referred to Chapter 8 of [24] for implementation details. Towards an implementation of Transmission Planning Agents, an Interactive Development Environment for the specification and simulation of Agent Systems IDEAS [14] has been recently implemented on a network of Sun workstations.

8 Conclusions

This paper has presented a new application of decentralized coalition formation and cooperative cost allocation in the power systems transmission planning area. A simple heuristic transmission planning problem has been solved to illustrate the application. Furthermore, an “autonomous” coalition set of players has been defined. A simple 6 bus case has been studied using centralized cost allocation techniques, based on the Shapley Value, and a Distributed Artificial Intelligence concept: the Bilateral Shapley Value. Results using both techniques have been presented and analyzed. Future work will refine coalition formation schemes, analyze coalition stability, and will explore classical Game Theory concepts, like the kernel [16, 23], applied to decentralized expansion games. We are currently integrating a transmission expansion software in a common platform, the Ptolemy [28] environment. The interested reader can see some of the capabilities of this graphical software framework in Power Systems applications [5, 6].

References


\textsuperscript{12}MATLAB is a registered trademark of The MathWorks, Inc.
\textsuperscript{13}Mathematica is a registered trademark of Wolfram Research, Inc.


[28] *Ptolemy Reference Manual*, V. 0.6, EECS Department, University of California at Berkeley, 1996.