Mechanisms for Coalition Formation and Cost Sharing in an Electronic Marketplace

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ABSTRACT

In this paper we study the mechanism design problem of coalition formation and cost sharing in an electronic marketplace, where buyers can form coalitions to take advantage of discounts based on volume. The desirable mechanism properties include stability (being in the core), and incentive compatibility with good efficiency, concepts from the perspectives of cooperative and non-cooperative game theory. We first analyze the problem from both these perspectives. We show the impossibility to simultaneously satisfy efficiency, budget balance and individual rationality at a Bayesian-Nash equilibrium, and propose a mechanism in the core of the game. We then present a group of reasonable mechanisms that are derived from the two perspectives, and evaluate their performance in incentive compatibility. Empirical results show positive correlation between stability and incentive compatibility(which is in turn related to efficiency). The mechanism which shares the coalition cost in an egalitarian way is the best in terms of both stability and incentive compatibility.

1. INTRODUCTION

Offering quantity based discounts is a simple but effective way of promotion, especially in today's highly competitive markets. For example, Amazon.com, the largest player in online retailing, introduced free shipping on orders of $99 or more and again lowered the free-shipping threshold to $49, and this tide of discounts was followed by Buy.com, Barns&Noble and many other e-tailers [1]. Quantity based discounts help suppliers to attract customers, rev up sales, and coordinate the distribution channel to maximize joint profits [2]. From the perspective of buyers, quantity based discounts provide a huge incentive to form coalitions and take advantage of lower prices without ordering more than their actual demand. By forming a coalition, buyers can also improve their bargaining power and negotiate more advantageously with sellers to purchase at a lower price. Both buyers and sellers can benefit from buyer coalitions[18]. Coalitions of customers exist not only in commerce, but also in other service industries, such as insurance, to pursue better deals for a group.

In this paper we envision a buyer-biased e-marketplace [1] which facilitates coalition formation of buyers, driven by the quantity discounts provided by sellers. Buyers submit their interest to purchase and reservation prices to the market mediator. The mediator matches buyers as buying groups and regulates the cost sharing based on the submitted information. To seek high efficiency of the market, or welfare of the buyer society is an important goal of the mediator, because higher efficiency implies more benefit for the buyers and as a result attracts more customers. As the joining of more buyers brings the unit price down, an efficient coalition may include buyers with reservation prices below the unit price paid by the coalition, and buyers may pay different prices for the identical item according to their reservation prices [20]. As self-interested and autonomous entities, buyers may behave strategically by misrepresenting their willingness to buy in order to maximize their profit. Information manipulation by buyers can complicate the effort to improve the efficiency of the market, and also makes it harder for the buyers to compute the best possible strategy. We want the mechanism to be *incentive compatible*, i.e., it is in the inter-

[1]Some e-marketplaces following the idea of demand aggregation include eWinWin (www.ewinwin.com), UniStar (www.unistarllc.com) and OnlineChoice (www.onlinechoice.com).

est of buyers to be truth telling[2]. Also, a coalition formation and cost sharing rule should provide incentive for buyers in the selected coalition to participate in the coalition without coercion. In other words, it should be fair and result in a *stable* coalition. We call this strategic interaction among the buyers a *Discount Coalitional Game*. A discount coalitional game has the following characteristics: (1) The incentive of coalition formation is the per capita cost discount on purchasing an item; (2) The coalition formation and cost sharing of the coalition depend on the value that each buyer has on the item; (3) This value for the item is private information and buyers may be manipulative on the information revelation; (4) A group of buyers may decline the outcome if it is beneficial for them to deviate and form other coalitions. We focus on the mechanism design issue of coalition formation and cost sharing for the discount coalitional game (the algorithmic issues of coalition formation have been addressed in [10], here we are not considering combinatorial coalition formation though.).

The mechanism properties we are concerned with are related to both cooperative (or coalitional) and non-cooperative game theory. Research on cooperative decision making from a cooperative game theory perspective provides axiomatic solution concepts for surplus (cost) sharing of coalitions. These include the Core, Shapley Value and Nucleolus among other concepts [15, 12]. Based on the agents' (claimed) opportunity costs or benefits from coalitions, these concepts admit a class of surplus (cost) sharing rules based on some widely accepted equity axioms, or properties that need to be satisfied. The stability of a coalition requires that the distribution of surplus among the coalition members is immune to groups of agents refusing to participate and forming their own coalition, and the *core* is the most commonly used concept to characterize this property. However, the concepts only characterize these sharings and in most cases it is hard to find them either analytically or computationally, even if the existence can be proven. Moreover, cooperative game theory focuses on what groups of players can achieve rather than what individuals can do. As a result, it typically assumes complete information and that the grand coalition is formed[3].

On the other hand, mechanism design in non-cooperative game theory aims to design a game that achieves the desired social outcome even when agents are self-interested, strategic and have private information. It deals with information revelation in a hostile environment. The major goal of mechanism design is *efficiency*(Eff): to compute a social choice that maximizes the social welfare. Some other goals include *individual rationality*(IR): no agents lose by participating in the game, and *budget balance*(BB): there are no monetary transfers out of or into the system. A mechanism is called *incentive compatible*(IC) if truth-telling forms a Bayesian-Nash equilibrium. It is called *strategyproof*(SP) if truth-telling is a dominant strategy for all agents. The latter is a stronger concept. The properties of IC or SP are desirable because they mitigate strategic behavior and computation on part of the agents. The *revelation principle* states that if there exists a mechanism that implements an outcome in some equilibrium, then there exists a direct revelation mechanism, that implements the outcome in the same equilibrium concept, with all agents reporting their true types. Therefore it does not lose to restrict attention to truthful direct revelation mechanisms. The family of Groves mechanisms[4] [5] is a group of mechanisms that satisfy Eff and SP. All mechanisms that satisfy Eff and SP are manifestations of the Groves mechanism [4]. The Clarke (or pivotal) mechanism is a special Groves mechanism in which the utility of an agent is equal to the marginal effect she brings to the community[5]. In addition to Eff and SP, the Clarke mechanism also satisfies IR. The biggest disadvantage of Groves mechanisms is their failure to satisfy (weak[6]) BB and IR simultaneously. The stability of the outcome, that could be challenged by some agents approving a change from the current decision rule of coalition formation and cost sharing, is usually not considered in non-cooperative game design, although it is an equally important issue in coalition formation.

Some concepts in economics, such as the incentive compatible core[3] or durable decision rules [6], have been proposed to examine the stability of an economy in which agents possess private information. These solutions, if they exist, are a subset of the collection of IC mechanisms, and an important but hard question is to find them efficiently. For the discount coalitional game, as we show in this paper, there does not exist a mechanism that satisfies Eff, BB and IR simultaneously at a Bayesian-Nash outcome. Moreover, the effort to find nontrivial IC mechanisms is also impeded by intractability of computation of equilibria. We consider mechanisms that satisfy the hard constraints of BB and IR, but only approximately satisfy IC and Eff. As a weaker stability requirement, we also enforce the core constraint based on the reported information while truth-telling is not enforced. Our goal is to find a mechanism that satisfies IR and BB, and is relatively good in both stability and incentive compatibility (which results in good efficiency).

To our knowledge this paper is the first work in practical mechanism design that studies both stability and incentive compatibility (efficiency) of an economy with incomplete information. This problem has some special features that are different from the traditional mechanism design settings, as we show in Section 3.2. We first analyze the problem from

[2] It would also be interesting to think of the strategic behavior of sellers in pricing. But we do not consider this in the present paper and the goal is to maximize buyers' surplus.

[3] A grand coalition is the coalition of all the agents, assuming the value of a coalition (weakly) increases with the expansion of the coalition.

[4] A Groves mechanism outputs the selection C that maximize the total reported value over all agents. The payment rule in a Groves mechanism is defined as

$$t_n(\hat{r}_n) = -\sum_{m \neq n} v_m(C, \hat{r}_m) + h_n(\hat{r}_{-n})$$

where $t_n(\hat{r}_n)$ is the payment of agent n, who reports the type $\hat{r}_n$, $v_m(C, \hat{r}_m)$ is the value of agent m at the selection C with type $\hat{r}_m$, $h_n(\hat{r}_{-n})$ is an arbitrary function of the reported types of all agents except n.

[5] In the Clarke mechanism $h_n(\hat{r}_{-n}) = v(C_{-n}, \hat{r}_{-n})$, where C_{-n} is the optimal selection that does not contain agent n, $v(C_{-n}, \hat{r}_{-n})$ is the value of the optimal selection according to the reported types, without considering agent n.

[6] There can be monetary transfers out of the system, but not into the system.

both the perspectives of cooperative and non-cooperative game theory. We prove that there does not exist a mechanism which satisfies Eff, BB and (weak ex ante)IR at the outcome of a Bayesian-Nash equilibrium for the discount coalition game. We extend the concept of *core* based on grand coalitions from cooperative game theory to mechanisms with incomplete information, and propose a mechanism which is in the core based on the reported information. Finally we derive a group of reasonable mechanisms, subject to the constraint of BB and IR. Empirical experiments on these mechanisms show that mechanisms that share cost more evenly are better than others in terms of incentive compatibility and efficiency. Moreover, among these, the mechanism that shares cost in an egalitarian way is the best in both stability(in the core) and incentive compatibility(efficiency). We corroborate the relationship between core(a cooperative game theory concept) and incentive compatibility(a non-cooperative game theory concept) by deriving the above mechanisms again with the constraint of being in the core and show that these perform better than their original counterparts.

The rest of the paper is organized as follows: The review of related work is provided in Section 2. In Section 3 the discount coalitional game and the mechanism design problem are formulated, and some special considerations of the problem are presented. In Section 4, we present theoretical analysis about mechanism design of the discount coalitional game from the perspectives of non-cooperative and cooperative game theory. In Section 5 we describe a few mechanisms derived from both the perspectives and compare them empirically. Discussion remarks follow in Section 6.

2. PRIOR WORK

Forges, Minelli & Vohra [3] provide a good survey on the core[7] of an exchange economy with asymmetric information. Incentive compatibility is emphasized in studying the concepts of the core at the ex ante and interim stage. Some analysis about non-emptiness of the core is presented. Some other work related to this topic includes [19] and [6]. While these studies consider incentive compatibility along with stability, the core concepts are restricted to IC mechanisms. Thus they are insufficient for the discount coalitional game, where it is intractable to compute equilibirium for any interesting IC mechanism.

Yamamoto & Sycara [20] propose a cost sharing rule in the core of the optimal coalition for group buying assuming complete information. We extend this rule to the discount coalitional game and show that the cost sharing rule combined with the coalition formation rule is in the core of the game[8].

Green & Laffont [4] show that a strategy proof mechanism can not be both Eff and BB, when preferences are quasi-linear. Along similar lines, Myerson & Satterthwaite [14] show that in an exchange economy with quasi-linear preferences it is impossible to achieve Eff, BB and (interim) IR in a Bayesian-Nash incentive compatible mechanism. Facing these impossibility results, research in non-cooperative mechanism design has focussed on relaxing one property while satisfying others, or balancing between various properties to achieve a reasonable approximation for each of them. Moulin et al[13] analyze the cost sharing of coalitions with non-decreasing and submodular cost functions[9]. Among all mechanisms that satisfy IR, mechanisms that are group strategy proof and budget balanced but not efficient, or strategy proof and efficient but not budget balanced are characterized. The discount coalitional game is not necessarily submodular in cost (it depends on the price schedule), also unlike the cost sharing game, agents in this game are indistinguishable.

Parkes[17] proposes a heuristic method to achieve budget balance with Clarke based payment schemes in exchanges, while sacrificing both efficiency and incentive compatibility. The idea is to minimize the distance between the payment and the Clarke mechanism, subject to the constraint of BB and IR. The Adjusted Clarke mechanism proposed in this paper is actually inspired by this heuristic.

A problem closely related to the discount coalitional game is that of Bidding clubs. Bidding clubs are a class of mechanisms in auctions that can be exploited by bidders to reduce the intensity of competition and gain benefits for all bidders[8]. Although there are some function similarities between the two, the specific incentives of forming coalitions are different.

3. THE DISCOUNT COALITIONAL GAME AND MECHANISM DESIGN

3.1 Definitions

Let $B = \{1, 2, \ldots, N\}$ denote the collection of buyers. For simplicity we assume each buyer asks for one unit of an item. Each buyer n knows the value of the item r_n to herself, and the price schedule of the item. The item value is a private information and unobservable to other buyers or the mediator, although we can assume it follows an independent and identical distribution for each buyer. The discount coalitional game has two stages. In the first stage each buyer n submits a bid $\hat{r}_n$ to the market mediator which indicates the reservation price, the maximum payment she can afford to pay for the item, without knowing other buyers' bids. If buyer n is truth telling, $\hat{r}_n = r_n$. In the second stage the mediator forms the coalition $C \subseteq B$ of the buyers that would purchase together as a virtual buyer, and determines the payment t_n of each buyer n based on the bids. There is no discrimination among the buyers. The bids become public information to all the buyers in the second stage. Based on the public information, buyers can make valid threats of deviating from the coalition. Throughout the paper, for any set S and quantity x, we use x_S to denote the vector $(x_n)_{n \in S}$, and denote x_B by simply x.

The unit price schedule $p(m) : Z^+ \to R^+$, is a decreasing step function of m, the number of units sold together, subject to the conditions of free disposal, i.e., for $m_1 < m_2$, $p(m_1) \geq p(m_2)$ and $m_1 p(m_1) \leq m_2 p(m_2)$. The cost of a coalition C is $cost(C) = |C| p(|C|)$.

[7] A feasible allocation of an economy belongs to the core if no coalition can improve upon it.

[8] This is different from the concept of core of the coalition, and is described in the following section.

[9] A cost function $c(\cdot)$ is non-decreasing and submodular if (i)$S \subset T \Rightarrow c(S) \leq c(T)$ and (ii) $c(S \cup T) + c(S \cap T) \leq c(S) + c(T)$.

A **mechanism** or outcome rule[10] $o : \hat{r} \to (C, t)$ specifies, based on the reservation prices $\hat{r}$ of the buyers, the coalition $C(\hat{r})$ of buyers to be formed, and the payment $t_n(\hat{r})$ of each buyer n[11].

The **value** of a coalition C is defined as the difference between the sum of item values to the coalition members and the minimum cost needed to satisfy the requests of all the members: $v(C, r) = \sum_{b_n \in C} r_n - |C|p(|C|)$. The **utility** of buyer n at an outcome $o = (C, t)$ takes the quasi-linear form: $u_n(o, r_n) = r_n I_{n \in C} - t_n$, where $I_{n \in C} = 1$ if $n \in C$ and 0 otherwise. If we replace the true values r_n by reservation prices $\hat{r}_n$ in the above formulas, we get the reported value of the coalition, and the reported utility of a buyer.

The desired properties of a good mechanism $o(r) = (C(r), t(r))$ include:

- **(ex-post) Individual Rationality(IR)**: No buyer loses by participating in the game. In other words, their participation is voluntary:

$$u_n(o, r_n) \geq 0 \ \forall n \in B.$$

  It follows that $u_n = 0$ and $t_n = 0$ for $n \notin C$.

- **(ex-post) Budget Balance(BB)**: There are no transfers out of or into the system, i.e., the coalition is charged the cost incurred in the coalition[12]:

$$\sum_{n \in C} t_n(r) = |C|p(|C|).$$

  With BB we have $v(C, r) = \sum_{n \in C} u_n(o, r)$.

- **(ex-post) Efficiency(Eff)**: The sum of buyers' utility, or the value of the coalition formed is maximized[13]:

$$C = \text{argmax}_{S \subset B} v(S, r).$$

  It is easy to see that when each buyer demands a unit quantity, for any $i \in C$ and $j \notin C$, $r_i \geq r_j$. The *Efficiency Loss* of a mechanism (C', t) is defined as $\frac{v(C,r) - v(C',r)}{v(C,r)}$, the percentage decrease in value of the coalition from using the mechanism (C', t) against the optimal value.

- **Stability**: No set of players pays more than the amount they would pay if they form a coalition among themselves. This leads to the formation of a stable coalition. Note that this sounds similar to the concept of *core* from cooperative game theory. There is some difference that we will point out in the following subsection.

[10] Based on the revelation principle, we can restrict to direct revelation mechanisms.
[11] With quasi-linear utility functions, cost sharing and surplus sharing are equally transferrable.
[12] Gains of buyers are allowed with BB since some buyers may pay lower than their reservation prices.
[13] The exchange interaction between sellers and buyers is not a concern in the paper. The discount price schedules are given externally by the sellers.

3.2 Special considerations

We analyze some of the special features of the discount coalitional game to differentiate it from the traditional mechanism design settings. We also describe the gap between some standard concepts in economics and the feasibility of computationally tractable solutions, and thus justify the approximate approaches we take in this paper.

Impossibility results: The impossibility of having a mechanism for agents with quasi-linear preference that satisfy Eff, BB and IR in Bayesian-Nash equilibria is proved, as far as we know, in two types of economies: exchange [14] and public project [7](see [16] for a summary of the impossibility results in mechanism design). The discount coalitional game is different from both economies. In the exchange economy, the sum of the probabilities of the agents owning a good is one, which is a critical condition to derive the impossibility result. But in the discount coalitional game the sum of the probabilities is not a constant and depends on the social choice function. The public project economy can only be admitted as a very special case of a discount coalitional game with a particular price schedule such that $p(1) = 2p(2) = \cdots = Np(N)$. In that case the cost of a coalition is a constant and the social choice function is trivial: either to include all agents or exclude all. The impossibility result for the public project economy does not apply to the general conditions of the discount coalitional game. Although the impossibility result for the discount coalitional game may seem not surprising, the proof of such a result is not a trivial derivation from the existing theorems.

Equilibrium computation and IC mechanisms: Because of the complexity of the social choice decision, it is hard to derive analytically the expected probability of being included in the coalition and the expected payment as (discontinuous) functions of the reported value in nontrivial mechanisms. Therefore the nontrivial Bayesian-Nash equilibria can not be analyzed by generally solving differential equations, as is usually done in auctions or exchange economies. Because the equilibrium computation is not tractable, it is not feasible to design a nontrivial IC mechanism. But for the same reason we do not expect agents with bounded rationality to play at the equilibrium strategies either. This leads us to empirically examine and compare the efficiency loss of mechanisms with heuristic strategy responses of players, which do not construct an equilibrium. Instead of designing a complex social choice rule based on the computationally intractable equilibrium outcome, we use a simple social choice rule which maximizes the reported value of the coalition. By appropriately designing the cost sharing rule we want the mechanisms to be fairly good in containing the deviation of agents and consequently achieving high efficiency. The experimental setting and results are described in Section 5.

Core: The *core* is the most commonly used concept in characterizing the stability of an economy, but it is originally defined on cost(surplus) sharing in a grand coalition with complete information. A budget balanced cost sharing rule t is in the core of a coalition C if it satisfies that any subset of the coalition can get at least as much by joining the coalition C as the value of the coalition formed by the members of the subset, i.e., $v(S, r) \leq \sum_{n \in S} u_n(t, r)$ for $\forall S \subset C$ and

$v(C, r) = \sum_{n \in C} u_n(t, r)$. While considering the core in the discount coalitional game, we have to be careful about two issues: non-grand coalition and incomplete information:

- *Non-grand coalition*: The outcome of the discount coalitional game is not necessarily the grand coalition because adding more buyers to a coalition does not necessarily increase its value. Hence it also provides opportunities of deviation by forming coalitions consisting of not only some members, but also buyers outside the outcome coalition. For differentiation, if a core is based on a grand coalition, we call it the *core of a coalition*, otherwise we call it the *core of a game*.
- *Incomplete information*: Since it is not feasible to compute the Bayesian-Nash equilibria or design a nontrivial IC mechanism, the true values are not deducible and truth telling is not enforced. This raises the question of what information the agents would use in speculating the benefit of deviation from the coalition. Even if communication is possible between the buyers to reveal their types and facilitate the deviation decision, buyers can still lie in the communication. The answer to the question include two options: to use the expected value information or the reported value information. With the first approach buyers make the deviation decision based on the *expected* additional profit they could make by deviation. The core calculated is similar to the coarse core in [19]. The difference is that in the discount coalitional game the expectation of the true value can be adjusted based on the reported value, which becomes a lower bound of the true value[14]. With the second approach buyers are more conservative(or risk averse). If a deviation is beneficial based on the reported values, so it is necessarily beneficial based on the true values or expected values. If a mechanism is good in IC, the core based on reported values is also a good approximation of the core based on expected or true values. Actually the approximation only relates to the values of the coalition non-members involved in the deviation, as is shown in 3.2. For simplicity we adopt the second option of using the reported information in this paper.

The core for the discount coalitional game based on the reported values is defined as follows:

Definition [Core of the Discount Coalitional Game] A budget balanced mechanism (C, X) is in the *core* if it satisfies the *stand-alone principle*: no members of the coalition C can form a coalition by themselves or with other buyers, such that the reported value of the new coalition exceeds the sum of the members' reported utility:

$$\sum_{n \in C_1} u_n(C, \hat{r}_n) \geq v(C_1 \cup C_2, \hat{r})$$

$\forall\ C_1 \in C$ and $C_2 \in B \setminus C$. It is equivalent to

$$\sum_{n \in C_1} t_n \leq cost(C_1 \cup C_2) - \sum_{n \in C_2} \hat{r}_n$$

[14]The truth telling strategy dominates the strategy of over bidding.

4. ANALYSIS

In this section we analyze the discount coalitional game theoretically. Specifically we answer the following two questions: (1) Does there exist an IC mechanism that satisfies all Eff, BB and IR; (2) What is a mechanism in the core of the discount coalitional game, if it exits.

We start with the following impossibility theorem (proven in appendix):

PROPOSITION 4.1. *There does not exist an IC mechanism for the discount coalitional game that satisfies efficiency (Eff), (interim) individual rationality (IR) and ex-ante weak budget balance (BB) at the same time.*

Facing this impossibility result, we propose some reasonable mechanisms that satisfy IR and BB, while sacrificing both Eff and IC. The following describes one such mechanism that is in the core of the discount coalitional game(proof in appendix).

Consider the cost sharing in which coalition members share cost as evenly as possible, subject to the constraints of IR and BB. The cost sharing rule is given by the following:

$$t_n = \begin{cases} h_C & (n \in \overline{C}) \\ \hat{r}_n & (n \notin \overline{C}) \end{cases} \tag{1}$$

where h_C and $\overline{C}$ satisfy:

$$|C| p(|C|) = |\overline{C}| \cdot h_C + \sum_{n \in C \setminus \overline{C}} \hat{r}_n$$

$$\overline{C} = \{n \in C | h_C \leq \hat{r}_n\}.$$

The cost sharing rule is demonstrated in Figure 1.

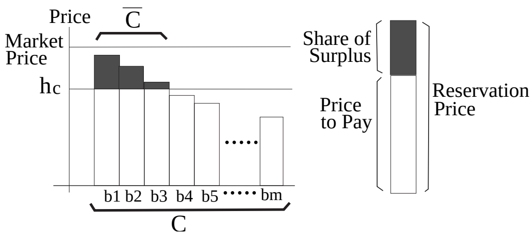

Figure 1: A cost sharing rule in the core (Egalitarian Sharing)

PROPOSITION 4.2. *The cost sharing rule specified by Equation 1 and the coalition formation rule specified by $C(\hat{r}) = argmax_{S \subseteq N} v(S, \hat{r})$ construct a mechanism in the core of the discount coalitional game.*

With the mechanism in the core, then a question is: how does it perform with respect to IC and Eff, given that it does not satisfy IC or Eff(following Proposition 4.1). The next section answers this question by simulation.

5. EXPERIMENTS

5.1 Methodology

In this section, we describe and motivate a few reasonable mechanisms for the discount coalitional game. All these mechanisms have the property that they compute the optimal coalition on reported values and then divide the cost among selected buyers in such a way that no buyer pays more than his reported bid(i.e., the mechanisms satisfy IR and BB). The goal of the experiment is to evaluate the different mechanisms in terms of incentive compatibility and efficiency, and determine if there is a correlation between the concept of core or equitable cost sharing and incentive compatibility. Based on the social choice rule, incentive compatibility and efficiency are consistent. The outcome would be efficient if buyers all tell the truth, or bad in efficiency if buyers reveal information far away from the true values. We use the average efficiency loss over the profiles of true values of buyers as a negative index of the incentive compatibility of a mechanism.

To investigate the correlation, we design mechanisms with different properties and compare them to the egalitarian cost sharing mechanism described in Section 4, which is in the core. From each of the basic mechanisms that are not in the core, we also derive mechanisms in the core and compare the efficiency in the two cases.

Theoretically we should examine the efficiency of a mechanism by calculating the value of the coalition in the Bayesian-Nash equilibrium. However in the discount coalitional game, the computation of the Bayesian-Nash equilibrium is not tractable, even the iterative computation of the best response does not converge because of the discontinuity of the best response strategy. Instead, we adopt the following heuristic approach – we compute the best strategy of an agent assuming that all other agents are telling the truth. Then using this best response as a strategy for *every* player, we compute the average efficiency loss of the mechanisms. We do not claim that numbers obtained in this way are representative of the results expected in equilibrium. In fact the efficiency loss is *over-estimated* using the best response calculated in this way. All the same, we believe that the results demonstrate the relative effectiveness of the different mechanisms at preventing the agents from manipulation.

We first describe the mechanisms in the following subsection and then present simulation results in the next subsection.

5.2 The Mechanisms

Let the buyers in the outcome coalition C be ordered in order of decreasing reservation price, $\hat{r}_1 \geq \cdots \geq \hat{r}_n$. Let C^+ denote the set of buyers in C that have $\hat{r}_i > p(|C|)$, where $p(|C|)$ is the unit price of the item when $|C|$ units are sold together, and C^- denote the rest of the buyers in C. All our mechanisms, except for Proportional Cost, will charge a buyer $i \in C^-$ her reported value $\hat{r}_i$. The rest of the price will be divided among the rest of the buyers.

1. **Egalitarian Sharing**

   Each buyer in C^* pays the amount specified by Equation 1, as illustrated in Figure 1. This mechanism is in the core. Note that as long as a buyer bids more than $h(C)$, the price paid by the buyer is independent of his bid. So this mechanism to some extent motivates the agents not to lie.

2. **Backward Cost Sharing**

   Instead of having buyers in C^+ share the cost as equally as possible, as in Egalitarian Sharing, this mechanism tries to break the evenness and provide substantial incentive for buyers to bid higher. Buyers with low reservation prices are required to pay as much as possible, until the sum of the prices paid equals the cost of the coalition. For example, if the cost to be shared by C is 14, while the reported values of 3 buyers in C are 10, 9 and 6, then we charge the third buyer 6, the second buyer 8 and the first buyer 0. Note that b_1 gets remarkable profit while b_2 and b_3 pay relatively high amounts. This mechanism gives incentive to buyers with high values to bid high, but also encourages buyers with medium reservation prices to bid low to avoid sharing much remaining cost of C^+.

3. **Proportional Subsidy**

   Let deficit $= |C^-|p(|C|) - \sum_{i \in C^-} \hat{r}_i$, which is equal to the amount that buyers in C^+ have to subsidize the buyers in C^- in order to cover their expense. In this mechanism we divide the deficit among buyers in C^+ in proportion to their reservation price. The motivation behind this mechanism is to obtain a fair allocation in the sense that buyers with high reservation prices obtain a higher surplus, but also pay more than buyers with low reservation prices.

4. **Proportional Cost**

   The cost of the coalition is divided among the coalition members in proportion to their reported values. In this mechanism the cost division is more uneven compared to Proportion Subsidy.

5. **Adjusted Clarke Mechanism**

   Because the Clarke mechanism is not budget-balanced, we define a variant – the Adjusted Clarke mechanism, which is derived by minimizing the norm-2 distance of the cost sharing vector to the one induced by the Clarke mechanism, subject to the constraints of BB and IR [17]. The idea behind the Adjusted Clarke mechanism is to inherit benefits of efficiency and strategy-proofness from the Clarke mechanism while still remaining BB and IR. Let τ denote the cost sharing rule of the Clarke mechanism, $C_{-n}(\hat{r}_{-n})$ be the optimal coalition without considering buyer n. Then $\tau_n(\hat{r}) = \hat{r}_n - v(C, \hat{r}) + v(C_{-n}, \hat{r}_{-n})$. The cost sharing rule t of the Adjusted Clarke mechanism is as follows, where

   $$t_i = \begin{cases} \hat{r}_i & \text{if } \tau_i + \mu > \hat{r}_i \\ \tau_i + \mu & \text{else} \end{cases} \qquad (2)$$

   where μ is a positive constant number such that the above solution gives budget balance.

Figure 2 gives an example of the cost distribution by the above mechanisms for a coalition of three buyers.

Call the mechanisms described above *Original* mechanisms. Each of the mechanisms has a computational complexity $O(N \log N)$, except the Adjusted Clarke mechanism, which costs $O(N^2)$. Each of the above mechanisms except the Egalitarian Sharing mechanism is not guaranteed to be in the core. For each of the above mechanisms M, we derive a new mechanism M' that is in the core and close to M.[15](The derivation process is described in Appendix) We call these *Derived* mechanisms - Backward Cost Core, Proportional Subsidy Core, Proportional Cost Core, Adjusted Clarke Core mechanism respectively.

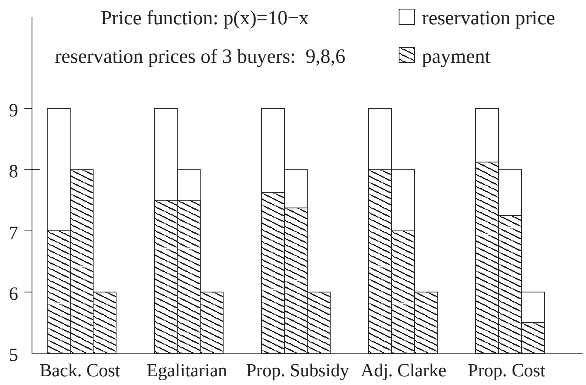

Figure 2: An example of cost division by the original mechanisms.

5.3 Simulation Setup

In the experiments we set the maximum unit price $p(1)$ to be 10, the bids of buyers vary from 0 to 10 with M=100 discrete values. We performed simulations for the mechanisms with 3, 4 and 5 buyers. Computation for more buyers takes too much time with the computation complexity $M^N\zeta$, where ζ is the complexity to compute the coalition and cost sharing for one bidding profile. However, we observe the same pattern for 3, 4 and 5 players and therefore, expect these results to be fairly representative. In order to make sure that the results are not specific to the price functions used, we used two price function families. The first is a linear decreasing price function, that is, $p(n+1)-p(n) = p(m+1)-p(m) = 1$, $\forall n \neq m$. The second function we use is generated randomly subject to the constraint that unit price is always decreasing and total price $mp(m)$ is increasing. We also tried two different probability distributions of the buyer types: the uniform distribution, and the normal distribution with varying means. We did not observe any difference *in the trends* for the different price functions or type distributions. For brevity, we only report results for the linear price function with uniform distribution.

5.4 Simulation Results

The best-response strategy curves[16] of different mechanisms based on the linear price function and uniform distribution are shown in Figure 3. We judge incentive compatibility of the mechanisms by noting the distance between the corresponding strategy curve and the "$x = y$" line. From Figure 3 we notice that no single mechanism dominates another in terms of incentive compatibility: if it gives better incentive for truth-telling to high valuation buyers, then it performs worse for the buyers with medium values, and vice versa. For buyers with high values, the mechanisms can be listed as follows in the decreasing order of IC: Backward Cost, Egalitarian Sharing, Proportional Subsidy, Adjusted Clarke and Proportional Cost.

Table I gives the average efficiency losses for the original mechanisms with 3, 4 and 5 agents. We note that the Egalitarian Cost Sharing mechanism which is also in the core has the least efficiency loss, and so the best incentive compatibility. Also observe the cost allocation given by the different mechanisms for the example shown in Figure 2. We notice that as the cost sharing becomes more and more uneven, the efficiency of the mechanism drops.

Mechanism	3 buyers	4 buyers	5 buyers
Egalitarian Sharing	0.455	0.629	0.832
Proportional Subsidy	0.457	0.685	0.916
Backward Cost Sharing	0.506	0.688	0.832
Adjusted Clarke	0.650	0.812	0.932
Proportional Cost	0.690	0.850	0.934

Table I: Average efficiency loss

Next we examine the correlation between efficiency and core more closely by comparing the efficiency loss of the original and derived mechanisms, as summarized in Table II. From Table II we can see that enforcing the core condition never worsens the efficiency of the mechanism, and in many cases improves the efficiency. In the cases where there is no improvement, we observe from the experiments that the core constraint is never violated, thus the cost division in the two cases is exactly the same. Whenever enforcing the core constraint changes the cost division, the efficiency of the resulting mechanism improves.

Mechanism	3 buyers		4 buyers		5 buyers	
	O.	D.	O.	D.	O.	D.
Prop. Subs.	**0.457**	**0.456**	**0.685**	**0.681**	0.916	0.916
Back. Cost	**0.506**	**0.440**	**0.688**	**0.648**	**0.832**	**0.811**
Adj. Clarke	0.650	0.650	**0.812**	**0.811**	0.932	0.932
Prop. Cost	**0.690**	**0.689**	**0.850**	**0.847**	0.934	0.934

Table II: A comparison of average efficiency loss of original (O.) and derived (D.) mechanisms

In conclusion, the experimental results suggest that there is positive correlation between the core and incentive compatibility of a mechanism. The Egalitarian Sharing mechanism, which is in the core of the discount coalitional game, is also the best in incentive compatibility and efficiency.

6. DISCUSSION

The correlation between core and incentive compatibility can be explained intuitively: With quasi-linear preferences

[15]It is intractable to derive a mechanism exactly in the core for the Backward Cost mechanism. In the Backward Cost Core mechanism we only ensure the core condition not violated for individual buyers, i.e., $t_n \leq p(1)$.

[16]The actual equilibrium strategies are expected to be higher than the strategies represented by these curves.

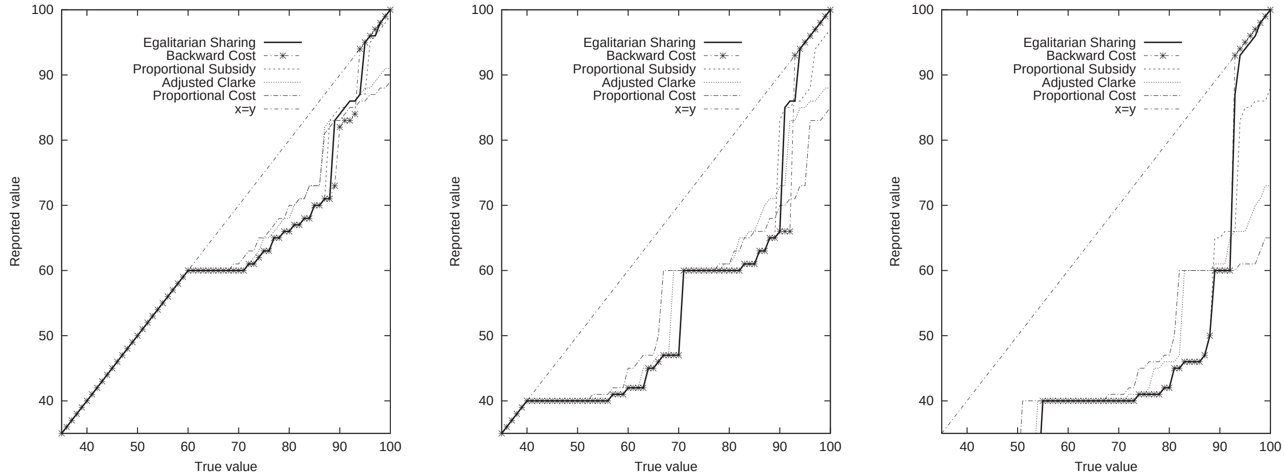

Figure 3: Strategy Curves for 3, 4 and 5 players with uniformly distributed bids and linear price function

and unitarian social welfare, the core constraint is not influenced by the agent values or bids. So a non-discriminatory mechanism in the core tends to distribute the cost evenly. But in a mechanism which distributes the cost (almost) evenly, the bidding strategy has little impact on the utility of a buyer if the buyer is selected in the coalition, and so it results in good incentive compatibility. This connection suggests the equitable cost sharing principle, i.e., the Egalitarian Sharing mechanism, which shares the cost evenly subject to the constraints of IR and BB. The extreme of equitable sharing is to have all coalition members pay equally. Then only the buyers with reservation prices higher than the unit price are included in the coalition, and they all pay the discounted unit price. As we show in [9], this extreme equitable mechanism is what we must have if IR, BB and SP are enforced. Interestingly it is also identical to the mechanism derived from the Shapley value. We do not consider this mechanism because firstly it is not in the core, and secondly the efficiency loss is unbounded.

Acknowledgments

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Appendix

Proof of Proposition 4.1:

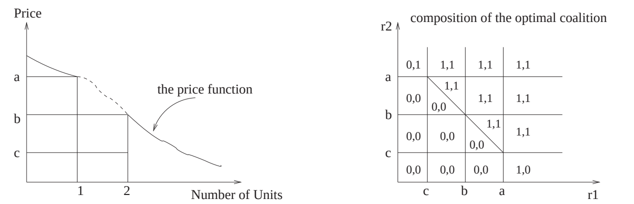

Figure 4: The price function curve and optimal coalition composition

Consider a discount coalitional game with two buyers b_1, b_2 and the discount price function shown in the left part of Figure 4. a,b are the unit prices when one and two units are sold; $c = 2b - a$ is the minimum reservation price a buyer needs to report to be included in the coalition. The item value to each buyer, r_i, $i = 1, 2$, follows the iid distribution with c.d.f. $\Phi_i(r_i)$ and p.d.f. $\phi_i(r_i)$, and $r_i \in [\underline{r}_i, \overline{r}_i]$. Let the configuration of a coalition denoted by $y(\hat{r}) = (y_1(\hat{r}), y_2(\hat{r}))$, where $y_i = 1$ if the buyer b_i is included in the coalition, and $y_1 = 0$ otherwise. The right part of Figure 4 lists the configurations of the optimal coalitions with all possible combination of the value reports. Each buyer's utility function takes the form: $u_i(r_i) = r_i \cdot y_i + t_i$. Define $\bar{t}_i(\hat{r}_i) = E_{r_{-i}}[t_i(\hat{r}_i, r_{-i})]$and $\bar{y}_i(\hat{r}_i) = E_{r_{-i}}[y_i(\hat{r}_i, r_{-i})]$. Then the expected utility of b_i when she is in type r_i and announces $\hat{r}_i$, and the other buyer tells the truth, is:

$$E_{r_{-i}}[u_i(C^*(\hat{r}_i, r_{-i}), r_i)|r_i] = r_i\bar{y}_i(\hat{r}_i) + \bar{t}_i(\hat{r}_i).$$

Define $U_i(r_i) = r_i\bar{y}_i(r_i) + \bar{t}_i(r_i)$.

From Proposition 23.D.2 on [11], the social choice function $f(\cdot) = (C^*(\cdot), t_1(\cdot), t_2(\cdot))$ is Bayesian incentive compatible only if, for $i = 1, 2$,

$$U_i(r_i) = U_i(\underline{r}_i) + \int_{\underline{r}_i}^{\overline{r}_i} \bar{y}_i(s)ds$$

for all r_i.

Following the same derivation as in Proposition 23.D.3 of [11], we have

$$E[-\bar{t}_1(r_1)] \quad = \quad [\int_{\underline{r}_1}^{\overline{r}_1} \bar{y}_1(r_1)(r_1 - \frac{1-\Phi(r_1)}{\phi(r_1)})\phi(r_1)dr_1] - U_1(\underline{r}_1)$$

and the similar form for $E[-\bar{t}_1(r_2)]$.

Interim IR implies that, for $i = 1, 2$,

$$U_i(\underline{r}_i) \geq 0,$$

and ex-ante weak BB implies that

$$E[-\bar{t}_1(r_1)] + E[-\bar{t}_2(r_2)] \geq E[cost(C^*(r_1, r_2))].$$

Let $E[-\bar{t}_1^1(r_1)]$, $E[-\bar{t}_1^2(r_1)]$, $E[-\bar{t}_1^3(r_1)]$ denote the expected payment of buyer 1 when $r_1 \leq c$, $r_1 \in (c, a]$ and $r_1 > a$ respectively. They sum up with $E[-\bar{t}_1(r_1)]$. Under these three situations, the expected probabilities to be involved in the coalition are: $\bar{y}_1^1(r_1) = 0$, $\bar{y}_1^2(r_1) = 1 - \Phi(2b - r_1)$, $\bar{y}_1^3(r_1) = 1$. Assume the buyers are symmetric, then $E[-\bar{t}_1(r_1)] = E[-\bar{t}_2(r_2)]$. It follows that

$$E[-\bar{t}_1^1(r_1)] = 0.$$

$$E[-\bar{t}_1^2(r_1)] = [1 - \Phi(c)][1 - \Phi(a)](c - a) - \int_c^a r_1(\Phi(r_1) - 1)\phi(2b - r_1)dr_1$$

$$E[-\bar{t}_1^3(r_1)] = a(1 - \Phi(a))$$

But

$$E[cost(C^*(r_1, r_2))] = 2a(1 - \Phi(a)) + (c - a)[1 - \Phi(a)] [1 - \Phi(c)] + b[(1 - \Phi(c))^2 - (1 - \Phi(a))^2]$$

From the above equalities,

$$E[-\bar{t}_1(r_1)] + E[-\bar{t}_2(r_2)] - E[cost(C^*(r_1, r_2))] = [1 - \Phi(c)] [1 - \Phi(a)](c - a) + 2\int_c^a [1 - \Phi(r_1)]\phi(2b - r_1)(r_1 - b)dr_1 - 2U(\underline{r})$$

The above term could be nonegative under some probability distribution Φ. Consider an extereme case: Let $U(\underline{r}) = 0$, $\Phi(a) = 1$ and $\Phi(r) = 0$ for $r < b$. Then $E[-\bar{t}_1(r_1)] + E[-\bar{t}_2(r_2)] - E[cost(C^*(r_1, r_2))] = 2\int_b^a [1 - \Phi(r_1)]\phi(2b - r_1)(r_1 - b)d\ r_1 - 2U(\underline{r}) \geq 0$. $\square$

Proof of Proposition 4.2: Since the cost sharing rule is in the core of the coalition, we only need to inspect the deviation of coalitions formed by members of $S \subset C$ with bidders not in C ($S' \subset B \setminus C$). Note that by the definition of C, $\forall i \in S, j \in S'$ we have $\hat{r}_i \geq \hat{r}_j$. Now assume to the contrary that $\hat{v}(S \cup S') > \hat{u}(S)$. This implies $h(S \cup S') < h(C)$. Now let us replace members of S' by members in $C \setminus S$, as many as possible, to get the set S''. Then, it is easy to see that $h(S \cup S'') \leq h(S \cup S') < h(C)$. Note that every member of $S \cup S''$ pays no more than every member of C. If $S \cup S'' \subseteq C$, this would contradict the fact that C is in the core of the coalition. On the other hand, if $C \subset S \cup S''$, then this implies $v(C, \hat{r}) < v(S \cup S'', \hat{r})$, which contradicts the optimality of C. $\square$

The process to derive a mechanism M' in the core from a mechanism M not in the core

For all but the Backward Cost mechanism, in order to find M', we first find the cost sharing given by M. Then we check the deviation of subsets $S \cup (B \setminus C)$ with $S \subseteq C$, where S includes the k highest paying buyers (these also correspond to the k highest reservation prices) for k varying from 1 to $|C| - 1$(It is easy to see that checking this condition is necessary and sufficient.). Once a core violation is detected, the excess payment e of S is deducted by reducing the payment of each buyer in S evenly (without negative payment). e is then distributed among buyers in $C \setminus S$ according to the corresponding mechanism M, and the core constraint is checked recursively for the resulting cost sharing on $C \setminus S$. Note that, for the Backward Cost mechanism, it is intractable to derive this mechanism, since in this case, the payment of buyers does not increase with their reservation price as in the other mechanisms. Thus in this case, to obtain the derived mechanism we only check the core condition for individual buyers.