A Stable and Efficient Buyer Coalition Formation Scheme for E-Marketplaces *

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ABSTRACT

Buyer coalitions are beneficial in e-marketplaces because they allow buyers to take advantage of volume discounts. However, existing buyer coalition formation schemes do not provide buyers with any means to declare and match their preferences or to calculate the division of the surplus in a stable manner. Concepts and algorithms for coalition formation have been investigated in game theory and multi-agent systems research, but because of the computational complexity, they cannot deal with thousands of buyers which could join a coalition in practice. In this paper, we propose a new buyer coalition formation scheme GroupBuyAuction. At GroupBuyAuction, buyers form a group based on a category of items. A buyer can post an OR-asking for multiple items within a category. An OR-asking is a list of items indicating that the buyer would buy any one of the items in the list with some particular reservation price. Sellers bid volume discount prices. The group leader agent splits the group into sub groups (coalitions), selects a winning seller for each coalition, and calculates surplus division among buyers. We prove that this scheme guarantees the stability in surplus division within each coalition in terms of the core in game theory. Simulation results show that, under most conditions, our scheme increases buyers' utility, and allows more buyers to obtain items compared to traditional group buying schemes, such as those used at existing commercial WWW sites.

1. INTRODUCTION

There are several opportunities for buyers to form coalitions on the Internet. By forming a coalition, buyers can advantageously negotiate with sellers and purchase items at volume discount prices. In [12], we showed that both buyers and sellers could benefit from buyer coalitions, analyzed coalition formation models and protocols, and proposed a framework for buyer agents to form coalitions and negotiate with seller agents.

However, existing commercial WWW sites [1] and even our protocols proposed in [12] do not provide buyers with any means to declare and match their preferences or to calculate the division of the surplus in a stable manner. This may prevent buyers from forming a large coalition. Concepts of coalition formation and its stability have been investigated in game theory [4, 5]. Some research on multi-agent systems [7, 8, 10, 9] has applied the concepts from game theory to multi-agent cooperation, and developed algorithms to form stable and beneficial agent coalitions. Some of those algorithms are theoretically applicable to buyer coalition formation, but they cannot be used in practice. Because of their computational complexity, they cannot deal with thousands of buyers which could be expected to join a coalition.

In this paper, we propose a new buyer coalition formation scheme, GroupBuyAuction, which enables a large number of buyers to form coalitions. A buyer sometimes may have several choices of items and wants to purchase any one of them. To answer this kind of buyer's request, a buyer group at GroupBuyAuction is formed for a category of items, not for a particular item. For instance, a 'camera' group invites buyers who want to buy a camera. A buyer within a group can post an asking which contains a specific item name and a reservation price, the maximum price which the buyer is willing to pay for the item. A buyer can also post an OR-asking, a list of single askings, indicating that the buyer would buy any one of the items in the list. For example, a buyer can say "I want to buy either a camera A for $300 or lower, or B for $400 or lower." A seller can make a bid for each item with volume discount prices. A seller's bid is something like "I can sell camera A for $250 each if more than 5 items are sold, for $300 otherwise." A leader agent in a group manages this reverse auction on behalf of buyers. When the auction closes, the leader agent splits the group into sub groups (which we call coalitions) each

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[1] For example, Mercata (http://www.mercata.com/), MobShop (http://www.mobshop.com/), BazaarE (http://www.bazaare.com/) and Volumebuy (http://www.volumebuy.com/).

of which consists of buyers preferring the same item. The leader also selects the winning seller for each coalition, and calculates surplus division among buyers. Buyers in a coalition may pay different prices for the identical item according to their reservation prices.

Desired goals for our coalition formation scheme include: (1) increase the number of buyers who can purchase items, (2) increase group's total utility and individual buyer's utility, and (3) divide the total utility among buyers in a fair and stable way. The first feature meets the primary intention for buyers to join a group, but other two features are also important to invite buyers. Self-interested buyers wish to purchase items at as low price as possible. If a coalition might force a buyer to pay more to support other buyers, a buyer would hesitate to join. We also expect the second feature could motivate a group leader to manage a group because, for example, a leader in practice may get some commission out of the group's total utility.

As these desired features are, in general, computationally too complex as mentioned above, we take the following approach. When forming a coalition configuration, we try to maximize the utility of the most valuable coalition, then maximize the utility of the second valuable one, and continue recursively. Then we divide each coalition's surplus within the coalition. We prove that our coalition formation scheme based on this approach guarantees the stability of surplus division within each coalition in terms of the core in game theory. In addition, our scheme encourages truth telling in buyer reservation price. Simulation results show that, under most conditions, our scheme increases group's total utility and the number of buyers obtaining items compared to a traditional group buying scheme similar to those used at existing commercial WWW sites.

This paper is organized as follows. Section 2 describes prior work. Section 3 outlines GroupBuyAuction and a prototype system developed based on the RETSINA multi-agent framework [11]. In section 4 we describe the coalition formation scheme in detail. Section 5 analyzes the stability of the coalition formation scheme. Section 6 describes the experimental results. Finally, we conclude our discussion in section 7.

2. PRIOR WORK

Works in game theory and microeconomics such as [4, 5] have provided concepts of coalition and its stability. A coalition is a set of agents which cooperate to achive a common goal, and the stability requirement is that the outcome of a coalition be immune to deviations by individual agents or subsets of agents. Those concepts are important as criteria of coalition formation schemes, and we justify our scheme based on the core, one of stability concepts in game theory. However, game theory does not provide efficient algorithms for coalition formation.

Finding the maximal group utility can be translated into the weighted set packing problem [1]: Given a set B and collection of its subsets $Col = \{C_0, ..., C_n\}$ such that each $C_i$has its value $v(C_i)$, find a sub collection $SubCol \subset Col$ of pairwise disjoint sets such that $\sum_{C_i \in SubCol} v(C_i)$ is the maximum among all sub collections. We can interpret B as a buyer group, SubCol as a collection of coalitions, and v as coalition's utility gained by group buying. The weighted set packing problem is NP-complete, and several optimization algorithms have been proposed [1, 2]. However, these algorithms rely on the assumption that the maximum size of subsets in $SubCol$ is bounded by a relatively small number k. In the context of group buying, bounding the coalition size by a small number is impractical.

Research on multi-agent systems also has investigated coalition formation of agents. [7] proved that, for a given set B, searching the best coalition configuration among $\{\{B\}\} \cup \{\{B_1, B_2\} \mid B_1 \cup B_2 = B, B_1 \cap B_2 = \emptyset\}$ guarantees that the largest coalition value found is within a bound from optimal one by $|B|$, and that no other search algorithm can establish any bound while searching only $2^{|B|-1}$ coalition configurations or fewer. This result means, without some kind of heuristics or assumptions, bounding the group's total utility is virtually impossible because $|B|$ could be large.

[8, 10] have provided distributed coalition formation schemes for multi-agent systems mainly focusing on increasing the group's total utility. They also limit the highest coalition size by an integer k, which means the algorithms proposed cannot be applied to large coalitions. [9] aims both to increase the total utility and to reach the stable payoff division among agents. Yet, the algorithms restrict the size of each coalition to guarantee the practical computation time.

[3] has proposed a new model of coalition formation, and applied it to coalition formation among buyer agents in an e-marketplace. Their model treats agents as locally interacting entities; an agent may create a coalition when it encounters another agent, join an existing coalition, or leave a coalition. The model describes global behavior of a set of agents from the macroscopic view point by differential equations, and simulates well how buyer coalitions evolve and reach the steady state. However, the model does not assist individual agents to form a coalition nor to negotiate surplus distribution.

3. OVERVIEW OF GROUPBUYAUCTION

GroupBuyAuction is a kind of reverse auction system where buyers (agents and/or humans) pool their demand to maximize their power, and sellers (agents and/or humans) bid discount prices to sell large volumes of products at once. Figure 1 shows the architecture of the GroupBuyAuction system build on the RETSINA multi-agent framework [11]. The GroupBuyAuction agent communicates with buyer agents, seller agents and PriceWatcher agents. The GroupBuyAuction system also has its WWW site (http://charites.cimds.ri.cmu.edu/gbauction) so that human buyers and sellers can directly access the system.

A buyer can ask GroupBuyAuction to create a buyer group. A buyer group is formed based on a product category (see Figure 2). A group does not specify a particular item. Instead, a group has a list of items which some buyers request. A buyer can post an OR-asking, a list of single askings each of which contains a specific item and its reservation price. A buyer would purchase any one of items listed in the OR-asking at her reservation price or lower.

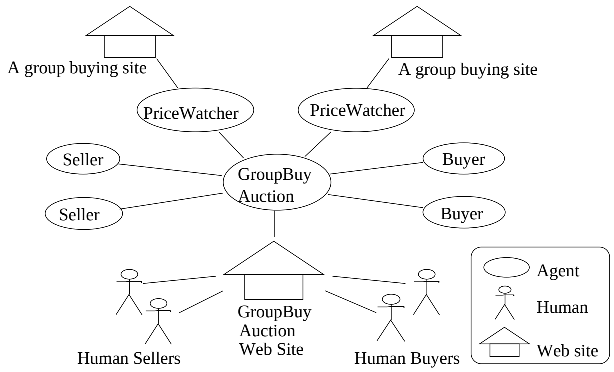

Figure 1: The GroupBuyAuction system architecture

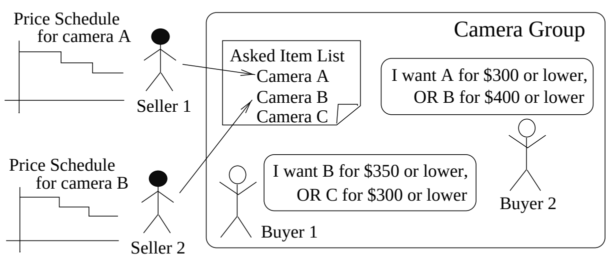

Figure 2: A buyer group

A seller can make bids on items listed at buyer groups. Each bid contains a specific item and the seller's volume discount price schedule. A price schedule is a decreasing function of the number of items sold and its unit price.

Each buyer group has its leader agent which is, under the current implementation, automatically created by the GroupBuyAuction system. The leader opens and closes her group's auction. The leader also asks PriceWatcher agents to retrieve discount prices at other group buying sites on the Internet. When the auction closes, the leader splits the group into sub groups (coalitions) each of which includes buyers preferring the same item, selects the best seller for each coalition, and calculates prices which buyers have to pay. The best seller for a coalition is one of sellers which made bids or one of other group buying sites. In the former case, the best seller can exclusively sell the items to buyers in the coalition. If one of other group buying sites offers the best price, the leader tells buyers to join the site.

4. COALITION FORMATION SCHEME

We begin with a simple example. Assume there are three items in a category which have the same price schedule shown in Figure 3. The horizontal axis shows the number of items, and the vertical axis indicates the unit price when multiple items are sold together. For instance, if three items are sold, the unit price goes down to 90. Table 1 shows five buyers in the group for the category. Each row shows the buyer's OR-asking. For instance, $b4$ agrees to buy any one of $item1$ or $item2$ if the price does not exceed her reservation prices (85 and 95 respectively).

Table 1: Sample buyers' Preferences

buyer	item0	item1	item2
b0	100		70
b1	80	95	95
b2		95	
b3		65	
b4		85	95

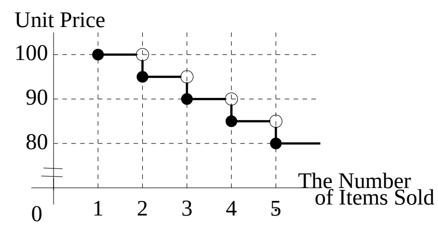

Figure 3: A sample price schedule

The main issues we study are how to split the buyer group into coalitions, and how to distribute the surplus of the group among buyers. In this example, there are one possible coalitions for item0 ($\{b0\}$), three for item1 ($\{b1, b2\}$, $\{b1, b2, b4\}$, $\{b1, b2, b3, b4\}$), and one for item2($\{b1, b4\}$). Our scheme derives the coalition configuration shown in Table 2; $\{b1, b2, b4\}$ as an 'item1' coalition has the largest surplus among all possible coalitions, and $\{b0\}$ as an 'item0' coalition is the only coalition which the rest of buyers can form. Each cell in the table contains the buyer's price to pay and reservation price between parentheses. The prices to pay in a coalition differ depending on buyers' reservation prices. For example, $b1$ pays 92.5 ($b1$' reservation price is 95), while $b4$ pays only 85 ($b4$'s reservation price is 85). If $b4$ did not join the coalition, $b1$ and $b2$ would have to pay 95 for item1. On the other hand, the coalition does not include $b3$ because $b3$ would bring no benefit to others.

Table 2: A sample coalition configuration

buyer	item0	item1	item2
b0	100(100)		
b1		92.5 (95)	
b2		92.5 (95)	
b3			
b4		85.0 (85)	

The rest of this section formally explains this coalition formation scheme.

4.1 Approach

We take the following approach to design the coalition formation scheme.

Principle 1: Maximize the utility of the most valuable coalition, and then maximize the utility of the second valuable one, and continue recursively.

Principle 2: Distribute the surplus of each coalition within the coalition in a stable way.

The first principle does not necessarily maximize the group's total utility. But forming a large coalition is the original intention of group buying, and we expect this principle could implicitly lead buyers to getting together in a large coalition in practice. The second principle assures the stability within each coalition. We do not consider the stability over all the coalitions. If we did so, the coalition with the highest utility might have to give a part of its surplus to smaller coalitions, which would be against the first principle.

4.2 Definition of Terms

Terms and notations are defined as follows.

Items and Price Schedules: $G = \{g_1, g_2, ..., g_m\}$ denotes a set of items in a category. (We deal with only one category throughout this paper, and do not explicitly denote the category to avoid the complexity in notation.) Let N and R be the set of natural number and real number respectively. A price schedule of g_i is represented as a descending function $p_i : N \to R$; $p_i(n)$ is a unit price when n of g_i are sold together. p_i is determined by sellers' bids as explained next.

Sellers: $S = \{s_1, s_2, ..., s_l\}$ denotes a set of sellers. A seller s_h's bid for g_i is in the form of price schedule $p_{ih} : N \to R$. When s_h does not make a bid for g_i, $p_{ih}(n) = \infty$ for $\forall n \in N$. p_i is defined as $p_i(n) \stackrel{\text{def}}{=} min\{p_{ih}(n) \mid s_h \in S\}$

Buyers: $B = \{b_1, b_2, ..., b_n\}$ denotes a group of buyers for a category. (Again, we do not explicitly denote the category to avoid the complexity in notation.) $r_{ki} \geq 0$ represents b_k's reservation price for g_i and a list $(r_{k0}, ..., r_{km})$ does b_k's OR-asking. We consider a reservation price as the buyer's benefit from owning the item, and define b_k's utility gained from buying g_i at the price p as $r_{ki} - p$.

Coalitions: Let $C_i \subset B$ denote a coalition to purchase g_i. A coalition configuration is $Conf = \{C_1, ..., C_m\}$ such that $C_i \cap C_j = \emptyset$ for $i \neq j$. C_i can be empty. $Conf$ does not necessarily satisfy $\cup_{i=1,...,m} C_i = B$; some buyers in B may not belong to any coalitions.

We define $v_i(C)$, utility of $C \subset B$ as a g_i coalition, as surplus derived by serving the coalition;

$$v_i(C) \stackrel{\text{def}}{=} \sum_{b_k \in C} r_{ki} - cost_i(C)$$

where $cost_i(C)$ is the cost to purchase $|C|$ items of g_i; i.e., $cost_i(C) = |C| \cdot p_i(|C|)$. ($|C|$ denotes the cardinality of C.) C can afford to buy $|C|$ items of g_i if and only if $v_i(C) \geq 0$.

4.3 Coalition Configuration Algorithm

A coalition configuration $Conf = \{C_1, ..., C_m\}$ is formed so that the utility of the most valuable coalition is maximized first, and then the utility of the second most one is maximized, etc. This algorithm is formalized as follows.

Algorithm 1: Coalition Configuration

1. Set $Conf = \emptyset$, $RestOfItemIDs = \{1, 2, ..., m\}$ and $RestOfBuyers = B$.
2. For every $i \in RestOfItemIDs$, calculate a candidate coalition $C_i^* \subset RestOfBuyers$, one of the largest sets with the largest utility as a g_i coalition, as follows.

   $$AC_i \stackrel{\text{def}}{=} \{C \subset RestOfBuyers \mid v_i(C) \geq 0\}$$
   $$VC_i \stackrel{\text{def}}{=} \{C \in AC_i \mid v_i(C) \geq v_i(C') \, for \, \forall C' \in AC_i\}$$
   $$LVC_i \stackrel{\text{def}}{=} \{C \in VC_i \mid |C| \geq |C'| \, for \, \forall C' \in VC_i\}$$

   (AC_i is the set of admissible coalitions, VCi the set of the most valuable coalitions, LVC_i the set of the largest coalitions among the most valuable ones.)

   Select any one of $C_i^* \in LVC_i$ if $LVC_i \neq \emptyset$, $C_i^* = \emptyset$ otherwise. $Cand \stackrel{\text{def}}{=} \{C_i^* \mid i \in RestOfItemIDs\}$ denotes the set of all candidates.
3. If every $C_i^* \in Cand$ is empty, stop this procedure.
4. If there exist non empty candidates in $Cand$, select one of them with the largest utility within $Cand$; that is, select C_i^* such that $v_k(C_k^*) \geq v_i(C_i^*)$ for $\forall C_i^* \in Cand$. Let $Conf = Conf \cup \{C_k^*\}$, $RestOfItemIDs = RestOfItemIDs \backslash \{k\}$, and $RestOfBuyers = RestOfBuyers \backslash C_k^*$.
5. Go back to Step 2 if $RestOfItemIDs \neq \emptyset$ and $RestOfBuyers \neq \emptyset$. Otherwise, stop this procedure.

This algorithm can be considered as a variation of the greedy algorithm for the weighted set packing problem [2]. In general, finding a subset of B which has the largest utility among all subsets could require $O(2^n)$ computations at worst.

However, we have an efficient algorithm to calculate our coalition configuration with order $O(n \cdot \log n)$, where n is the number of buyers in B, and we assume the number of items in a category can be bounded from above by a positive number K independently from n. This assumption makes sense even for very large coalitions. The complexity of searching C_i^* at each recursion is $O(n \cdot \log n)$ computations as explained below, each recursion includes at most K times of the search, and all coalitions are configured within K recursions. Thus, the entire complexity of the coalition configuration is $O(n \cdot \log n)$ computations.

To search C_i^* at each recursion, first arrange all buyers in $RestOfBuyers$ in the descending order in terms of reservation price for $g_i (O(n \cdot \log n)$ computations). Then calculate the utility of subsets $C_{ij} \subset RestOfBuyers$ for $j = 1, ..., t$ (t is at most n) which includes the top j buyers in terms of reservation price for g_i, and select C_i^* out of $\{C_{i1}, ..., C_{it}\}$. This requires $O(n)$ computations. (This algorithm is supported by Proposition 2 in the next section.)

4.4 Surplus Sharing in a Coalition

Buyers in a coalition share their surplus within the coalition. When a coalition C_i has surplus $v_i(C_i) > 0$ and the share of $b_k \in C_i$ is $x_k \geq 0$, b_k actually pays $r_{ki} - x_k$, where r_{ki} is b_k's reservation price. The surplus sharing rule is defined as follows.

Definition 1: Surplus Sharing Rule When a coalition C_i has surplus $v_i(C_i) > 0$, the share x_k of $b_k \in C_i$ is

$$x_k \stackrel{\text{def}}{=} \begin{cases} r_{ki} - h_{C_i} & (b_k \in \overline{C_i}) \\ 0 & (b_k \notin \overline{C_i}) \end{cases}$$

where h_{C_i} and $\overline{C_i}$ satisfies the following conditions:

$$cost_i(C_i) = |\overline{C_i}| \cdot h_{C_i} + \sum_{b_k \in C_i \setminus \overline{C_i}} r_{ki},$$
$$\overline{C_i} \stackrel{\text{def}}{=} \{b_k \in C_i \mid h_{C_i} \leq r_{ki}\}.$$

Figure 4 illustrates this definition. The graph shows each buyer's reservation price, her share of surplus, and her actual price to pay. Buyers in $\overline{C_i}$ pay h_{C_i} which is equal to or lower than their reservation prices. Others in $C_i \setminus \overline{C_i}$ pay just their reservation prices.

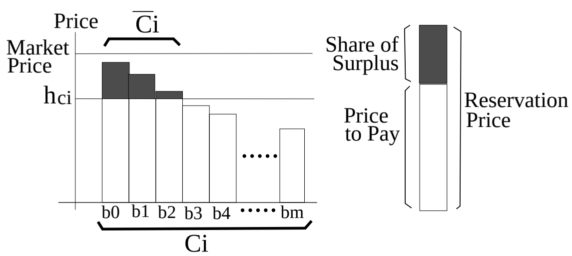

Figure 4: The surplus sharing rule

5. STABILITY OF COALITION CONFIGURATION

As buyers in a coalition pay different prices under our scheme, a fair share of surplus is essential to invite buyers to the coalition. If buyers do not trust the fairness, they may not join a buyer group, nor provide their reservation prices truthfully, which could prevent successful coalition formation.

In this section, we discuss our scheme's stability in terms of the core in game theory[6, 4]. (Note that, following Principle 2 in 4.1, we only consider the stability of each coalition, not the stability over all the coalitions.) The core is defined as follows.

Definition 2: The Core [6]
A coalitional game with transferable payoff consists of (1) a finite set C of players, and (2) a utility function v which associates with every nonempty subset $S \subset C$ a real number $v(S)$. The core of the coalitional game with transferable payoff $< C, v >$ is

$$Core = \{(x_b)_{b \in C} \mid v(C) = \sum_{b \in C} x_b, v(S) \leq \sum_{b \in S} x_b \; for \; \forall S \subset C\}$$

In general, the core can contain multiple elements, and also can be empty. In our case each coalition C_i has the nonempty core; the surplus distribution calculated by our surplus sharing rule is within the core as the next proposition states.

Proposition 1 (Stability of a coalition)
For $\forall C_i \in Conf$, the surplus distribution $(x_k)_{b_k \in C_i}$ calculated using the surplus sharing rule (Definition 1) is in the core of the coalitional game with transferable payoff $< C_i, v_i >$. That is, $v_i(S) \leq \sum_{b_k \in S} x_k$ holds for $\forall S \subset C_i$.

The stability condition defined by the core is that no subset of buyers in a coalition can obtain utility that exceeds the sum of the current utility of the members in the subset. Thus, even self-interested buyers in a coalition would not be motivated to deviate from the coalition.

There can be multiple surplus distributions within the core. Proposition 2 and 3 below characterize our surplus distribution, and we expect these propositions will encourage a buyer to tell her reservation price truthfully. (Note that Proposition 1 above is proved via Proposition 2 and 3. The proof is provided in Appendix.)

Proposition 2 (Members in a coalition)
At each recursion of coalition configuration in Algorithm 1, for $\forall b_k \in RestOfBuyers$ and $\forall i \in RestOfItemIDs$, if $\exists b_h \in C_i^*$ such that $r_{ki} > r_{hi}$, then $b_k \in C_i^*$.

Proposition 2 means that C_i^* consists of the top $|C_i^*|$ buyers in terms of reservation price. The higher a buyer's reservation price is, the more likely it is she will be able to join a coalition.

Proposition 3 (Price sharing)
At each recursion of coalition configuration in Algorithm 1, for $\forall i \in RestOfItemIDs$ and $\forall C \in AC_i$, $h_{C_i^*} \leq h_C$.

The last proposition assures that, at each recursion, the highest price anybody in C_i^* pays, $h_{C_i^*}$, is the lowest among all the prices afforded by any sets of buyers.

6. EVALUATION

We have conducted a series of simulations to evaluate the effectiveness of our coalition formation scheme in increasing buyers' benefits. We simulated buyers' behaviors under three group buying schemes (our scheme, a traditional scheme and an optimal scheme) under particular conditions, and compared them using following evaluation criteria: (1) group's total utility, and (2)the number of buyers who can obtain items.

6.1 Assumptions

We make the following assumptions.

Items and Price Schedules: Items are common commodities (e.g., consumer electronic devices, stationaries, etc.). There exists a market price for each item. There is no limit to how many items one seller can provide. Each item is accompanied by a price schedule whose value ranges from its market price to the best discount price.

Buyers: A buyer has several choices of items. We model the distribution of preferences for multiple items by RBMI (the Ratio of Buyers who prefer Multiple Items). RBMI is an array $(rb_1, ..., rb_m)$, where m is the number of items and $rb_1 + ... + rb_m = 1$ holds. rb_i denotes the ratio of buyers who prefer i items out of m items. For instance, in the example shown by Table 1 in Section 4, RBMI is (0.4, 0.4, 0.2); out of five buyers, two buyers prefer only one item, two buyers prefer two items, and one buyer prefers three items. RBMI does not specify which particular items each buyer prefers. A buyer randomly selects preferred items.

Every buyer knows every item's market price. Some of reservation prices for a given item are equal to its market price. We call this ratio as RRMP (the Ratio of Reservation prices which are the Market Price). Other reservation prices for the item are randomly distributed between its market price and a certain lower price. We denote the lowest reservation price as LRP. The environment (other buyers' behaviors, price schedules, etc.) does not affect buyers' preferences or reservation prices.

An Optimal Scheme: At every simulation, we calculate an optimal coalition configuration for comparison. The optimal group buying scheme searches all possible coalition configurations and selects one of the configurations which has the largest utility. When there are multiple configurations with the largest utility, the optimal scheme selects one in which the largest number of buyers can purchase items. Buyers in a coalition share their surplus within the coalition, but the optimal scheme does not care about how to share.

A Traditional Scheme Under a traditional group buying scheme, there is no notion of a group or an OR-asking. Instead, buyer selects one item out of her preferred items by herself, and posts a single asking to a coalition formed to purchase the item. All buyers in a coalition who can purchase the item pay the same discount price which is as low as possible for the coalition.

A buyer can know the price schedule, current discount price and the number of buyers at every coalition at any time. A buyer b_k selects one item out of her preferred items by following one of the selection rules listed below.

Random Rule: Randomly Select an item.
Lowest Price Rule: Select an item whose current price is the lowest in proportion to the market price.
Highest Reservation Price Rule: Select an item with the highest reservation price in proportion to the market price.
Highest Value Rule: Select an item which currently brings her the highest utility (reservation price - current price).

6.2 Simulation and Parameters

For every set of parameters, we simulate buyers' behavior under our scheme, the optimal scheme and the traditional scheme 1000 times, and calculate the average data for the evaluation criteria. For the traditional scheme, we simulate four experimental conditions. At every condition, all buyers follow the same selection rule out of four rules listed above.

Table 3 summarizes the simulation parameters in the evaluation. The range of the number of items is 1,3 and 5. We assign the identical price schedule to all items such that the market price is 100, the lowest discount price is 80, and the price decreases by 5 in proportion to the number of buyers. We only vary a price decreasing ratio (PDR), the ratio of 'the least number of buyers which assures the lowest discount price' to 'the number of buyers in a group.' PDR characterizes how steeply the price decreases. Figure 5 shows sample price schedules with PDR of 0.4 and 1.0, and 100 buyers in a group. In the simulation, PDR varies among 0.2, 0.4, 0.6, 0.8 and 1.0.

Note that too small PDR is not realistic from the seller's point of view. A small PDR means that the seller sells items cheap even if a large volume of items are not sold. This is against the basic idea of volume discount; sellers sell things cheap in exchange for buyers purchasing many. The market with too small PDR is also trivial for group buying schemes because buyers need little assistance; buyers could easily get the lowest discount price even if they are randomly distributed over the items.

Table 3: Simulation Parameters

	Parameter	Range
Items	The number of items	1, 3, 5
Price Schedule	PDR (price decreasing ratio)	0.2, 0.4, 0.6, 0.8, 1.0
Buyers	The number of buyers	100, 200, 400, 800
	RBMI (the ratio of buyers preferring multiple items)	(1), (1, 0, 0), (.7, .2, .1), (.5, .3, .2), (1/3, 1/3, 1/3), (1, 0, 0, 0, 0), (.7, .2, .05, .03, .02), (.5, .3, .1, .05, .05), (.2, .2, .2, .2, .2)
	RRMP (the ratio of reservation prices which are the market price)	0, 0.25
	LRP (the lowest reservation price)	70, 80 (the best discount price)

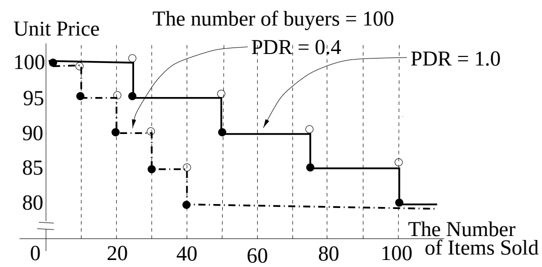

Figure 5: Sample price schedules

The range of the number of buyers is 50, 100, 200 and 400. We also vary RBMI, RRMP and LRP as shown in Table 3 so that the effect of the buyer's preference distribution can be observed. Note that the optimal scheme can handle only the cases with 50 buyers and RBMI of (1), (1,0,0) or (0.7, 0.2, 0.1) because of its computational complexity.

6.3 Results

For a given number of buyers and items, the three schemes showed common relations between buyers' benefits and the simulation parameters. The factors which affected buyers favorably included smaller PDR, larger RRMP and LRP, and more distributed RBMI (for instance, (1/3, 1/3, 1/3) brought larger utility to buyers than (1, 0, 0) did). Among them, PDR brought a clear contrast between the three schemes. Here, we analyze the simulation results focusing on PDR. (Lack of space prohibits showing results with other parameters.)

Out of the four experimental conditions for the traditional scheme, the one where all buyers followed the highest value rule produced the highest utility in almost all simulations. Thus, in this section we refer only to this condition as the traditional scheme's output.

Optimality: First, we compare our scheme to the optimal one by examining the case that the number of items is 3, the number of buyers is 50 and RBMI=(0.7, 0.2, 0.1). In summary, (1) our scheme came out more than 80 percent of the optimal utility under all conditions on average, and (2) as PDR became larger, the difference between our scheme and the optimal one became smaller; when PDR = 1.0, our scheme's outputs were nearly the same as the optimal ones.

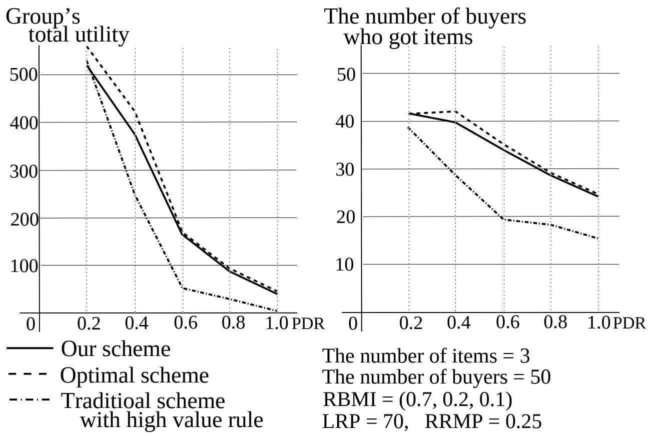

Figure 6: Comparison between our scheme, the optimal one and the traditional one

Figure 6 shows the simulation output under the conditions where LRP = 70 and RRMP = 0.25. [2] The graph on the left shows the average group's total utility, and the right graph shows the average number of buyers who got items. The horizontal axis of each graph is PDR, and the vertical axis is the value for each criterion. When PDR is 0.2, the group's utility gained by our scheme was slightly worse than the one by the optimal scheme and even the one by the traditional scheme. But, the average utility under our scheme was still above 91 percent of the optimal one. As PDR became larger (the market condition became worse for buyers), our scheme performed better in the sense that the buyers' benefits became close to the optimal ones. When PDR $\geq$ 0.6, both the group's utility and the number of buyers that got items they wanted are within 96 percent from the optimal ones. On the other hand, the traditional scheme became much worse when PDR was 0.4 or larger. When PDR = 1.0, the traditional scheme scarcely brought utility to buyers.

Cases with a large number of buyers: Next, we examine the cases that 400 buyers are involved in a group. (We compare only ours and the traditional scheme. Our implementation of optimal scheme could not handle such large number of buyers.) Regardless of the number of buyers in a group, the comparison results showed the same tendency as the previous case of 50 buyers: (1) when PDR=0.2, ours and the traditional one brought the best benefits to buyers, and the traditional scheme slightly overcame ours under some conditions, and (2) as PDR became larger, our scheme performed better than the traditional one.

Figure 7 supports the above statements. The two graphs

[2]We got similar results for other combinations of LRP (70 or 80) and RRMP (0 or 0.25).

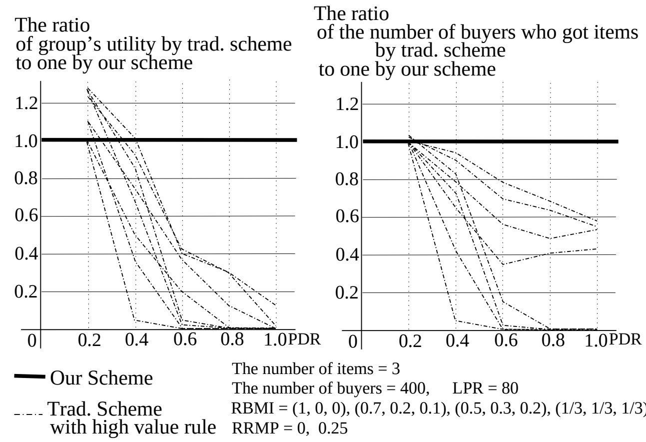

Figure 7: Comparison between our scheme and the traditional scheme

show the performance ratio of the traditional scheme to ours. The left graph shows the ratio of the group's utility by the traditional scheme to the one by our scheme. The right graph shows the ratio of the number of buyers who obtained items by the traditional scheme to the one by our scheme. The horizontal axis of each graph is PDR. The vertical axis is the ratio for each criterion; the value 1.0 means two schemes have the same performance, the value under 1.0 indicates our scheme is better, and the value above 1.0 does the opposite. Each graph includes the data under eight conditions; RBMI = (1,0,0), (0.7, 0.2, 0.1), (0.5, 0.3, 0.2) or (1/3, 1/3, 1/3), and RRMP = 0 or 0.25. Other parameters are fixed (three items in a category, 400 buyers in a group, and LRP = 80). In terms of group's utility, the traditional scheme overcame ours only when PDR = 0.2. When PDR $\geq$ 0.4, our scheme was better under all conditions. Similarly, when PDR = 0.2, ours and the traditional scheme showed almost the same performance regarding the number of buyers who could purchase items. As PDR became larger, our scheme supported more buyers than the traditional one.

7. CONCLUSIONS AND FUTURE WORK

In this paper, a buyer coalition formation scheme GroupBuyAuction was proposed. At GroupBuyAuction, buyers with different preferences and values form a group to purchase possibly different items. The group leader agent splits the group into coalitions each of which consists of buyers preferring the same item, and calculates surplus division among buyers. We showed that our scheme has enough scalability to handle a large number of buyers, guarantees the stability in surplus division within each coalition, and performs better in increasing buyers' utility and allowing more buyers to obtain items compared to a traditional group buying scheme similar to those used at existing commercial WWW sites.

Future work includes to investigate strategies of buyers/sellers and group buying auction designs. In the evaluation reported in this paper, we simply assumed buyers' preferences and reservation prices were not affected by others. Buyers and/or sellers, however, can affect each other if an auctioneer (a group leader in our context) publishes some information about buyers' askings and/or sellers' bids. We need to examine the relations between auction designs and buyer/seller strategies to effectively run a group buying auction.

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APPENDIX

A. PROOF OF PROPOSITIONS

Proof of Prop.2. Suppose $\exists b_k \notin C_i^*, \exists b_h \in C_i^*$ such that $r_{ki} > r_{hi}$. From the definition of v_i, $v_i(C_i^* \cup \{b_k\} \setminus \{b_h\}) > v_i(C_i^*)$ holds, which contradicts the definition of C_i^* ($v_i(C_i^*)$ be the largest). □

Lemma 1. For $\forall C \subset B$ and $\forall b_k \notin C$, if $h_C \leq r_{ki}$ then (1) $h_{C \cup \{b_k\}} \leq h_C$, and (2) $b_k \in \overline{C \cup \{b_k\}}$, where h_X and $\overline{X}$ for any X are calculated as a $g_i$coalition.

Proof of Lemma 1 (1). Suppose $h_{C \cup \{b_k\}} > h_C$, and we will show it leads to a contradiction, $cost_i(h_{C \cup \{b_k\}}) < cost_i(h_{C \cup \{b_k\}})$. Let $D \stackrel{\text{def}}{=} C \cup \{b_k\}$. Then we have

$cost_i(D) \stackrel{\text{def}}{=} sum_{b_h \in C_i \setminus \overline{C_i}} r_{hi} + |\overline{D}| \cdot h_D$
$= \sum\{r_{hi} \mid b_h \in D, r_{hi} < h_D\} + |\overline{D}| \cdot h_D$
$> \sum\{r_{hi} \mid b_h \in D, r_{hi} < h_D\} + |\overline{D}| \cdot h_C$ (since $h_C < h_D$)
$= \sum\{r_{hi} \mid b_h \in D, r_{hi} < h_D\}$
$+ \sum\{r_{hi} \mid b_h \in D, h_C \leq r_{hi} < h_D\} + |\overline{D}| \cdot h_C$
$= \sum\{r_{hi} \mid b_h \in C, r_{hi} < h_C\} + \sum\{r_{hi} \mid b_h \in D, h_C \leq r_{hi} < h_D\}$
$+ |\{b_h \in D \mid \{h_D \leq r_{hi}\}| \cdot h_C$ (by $h_C \leq r_{ki}$ and Def. of $\overline{D}$)
$\geq \sum\{r_{hi} \mid b_h \in C, r_{hi} < h_C\} + |\{b_h \in D \mid h_C \leq r_{hi}\}| \cdot h_C$
$\geq \sum\{r_{hi} < h_C \mid b_h \in C\} + (|\overline{C}| + 1)h_C$
$= cost_i(C) + h_C$ (from Def. of $\overline{C}$)
$\geq |C| \cdot p_i(|C|) + p_i(|C|)$ (from Def. of $cost_i$ and $p_i(|C|) \leq h_C$)
$\geq |C| \cdot p_i(|D|) + p_i(|D|) = |D| \cdot p_i(|D|)$ (since $|D| = |C| + 1$)
$= cost_i(D)$.□

Proof of Lemma 1 (2). From $h_C \leq r_{ki}$ and (1)$h_D \leq h_C$, we have $h_D \leq r_{ki}$, which means $b_k \in \overline{D} = \overline{C \cup \{b_k\}}$. □

Lemma 2 (A general form of Lemma 1). For $\forall C \subset B$ and $\forall D \subset \{b_k \in B \mid h_C \leq r_{ki}\}$, (1)$h_{C \cup D} \leq h_C$, and (2)$D \subset \overline{C \cup D}$, where h_X and $\overline{X}$ for any X are calculated as a $g_i$coalition.

Proof of Lemma 2. The proof of Lemma 2 (1) is by induction on the cardinality of D. Begin with the first step. When $|D| = \{b_k\}$ and $b_k \in C$, (1) is trivial. If $b_k \notin C$, (1) is supported directly by Lemma 1. For the inductive step, suppose (1) holds for all D such that $|D| \leq n$, and we will show that (1) holds for $D \cup \{b_k\}$ where$b_k \notin D$ and $h_C \leq r_{ki}$. By the induction hypothesis, we have $h_{C \cup D} \leq h_C \leq r_{ki}$. In the case $b_k \notin C$, the above inequation and $b_k \notin C \cup D$ lead $h_{C \cup D \cup \{b_k\}} \leq h_{C \cup D}$ by using Lemma 1 (1). In the case $b_k \in C$, $h_{C \cup D \cup \{b_k\}} \leq h_{C \cup D}$ also holds since $C \cup D \cup \{b_k\} = C \cup D$. Using the induction hypothesis again, we have $h_{C \cup D \cup \{b_k\}} \leq h_C$. (2) follows trivial by (1). □

Proof of Prop.3. Suppose $\exists C \in AC_i$ s.t. $h_C < h_{C_i^*}$ ... (1). By applying $\overline{C_i^*}$ to D in Lemma 2, we have $h_{C \cup \overline{C_i^*}} \leq h_C$ ... (2), and $\overline{C_i^*} \subset \overline{C \cup \overline{C_i^*}}$ ... (3). Using (1), (2) and (3), we see $v_i(C \cup \overline{C_i^*}) > C_i^*$ as follows, which contradicts that C_i^*be the largest by its definition.

$v_i(C \cup \overline{C_i^*}) = \sum_{b_k \in \overline{C \cup \overline{C_i^*}}} (r_{ki} - h_{C \cup \overline{C_i^*}})$
$> \sum_{b_k \in \overline{C \cup \overline{C_i^*}}} (r_{ki} - h_{C_i^*})$ (by combining (1) and (2))
$\geq \sum_{b_k \in \overline{C_i^*}} (r_{ki} - h_{C_i^*}) = v_i(C_i^*)$ (from (3)).□

Lemma 3. For any coalition C_i and any subset $S \subset C_i$, $cost_i(S) \geq |S \cap \overline{C_i}| \cdot h_{C_i} + \sum_{b_k \in S \setminus \overline{C_i}} r_{ki}$.

Proof of Lemma 3. By Prop. 3, $h_{C_i} \leq h_S$...(1) holds. Then, the following two equations are straightforwardly proved using (1): $\overline{S} = \overline{S} \cap \overline{C_i}$, and $(S \setminus \overline{S}) \setminus \overline{C_i} = S \setminus \overline{C_i}$. Therefore,

$cost_i(S) = |\overline{S}| \cdot h_S + \sum_{b_k \in S \setminus \overline{S}} r_{ki}$
$= |\overline{S}| \cdot h_S + \sum_{b_k \in (S \setminus \overline{S}) \cap \overline{C_i}} r_{ki} + \sum_{b_k \in (S \setminus \overline{S}) \setminus \overline{C_i}} r_{ki}$
$\geq |\overline{S}| \cdot h_{C_i} + \sum_{b_k \in (S \setminus \overline{S}) \cap \overline{C_i}} h_{C_i} + \sum_{b_k \in (S \setminus \overline{S}) \setminus \overline{C_i}} r_{ki}$
$= |\overline{S} \cap \overline{C_i}| \cdot h_{C_i} + |(S \setminus \overline{S}) \cap \overline{C_i}| \cdot h_{C_i} + \sum_{b_k \in (S \setminus \overline{S}) \setminus \overline{C_i}} r_{ki}$
$= |S \cap \overline{C_i}| \cdot h_{C_i} + \sum_{b_k \in (S \setminus \overline{S}) \setminus \overline{C_i}} r_{ki}$
$= |S \cap \overline{C_i}| \cdot h_{C_i} + \sum_{b_k \in S \cap (C_i \setminus \overline{C_i})} r_{ki}$.□

Proof of Prop.1. By Lemma 3 and the definition of group utility $v_i(S) \stackrel{\text{def}}{=} \sum_{b_k \in S} r_{ki} - cost_i(S)$, we have

$\sum_{b_k \in S} r_{ki} - v_i(S) \geq |S \cap \overline{C_i}| \cdot h_{C_i} + \sum_{b_k \in S \setminus \overline{C_i}} r_{ki}$.

Using Definition 2, this inequation yields

$v_i(S) \leq \sum_{b_k \in S} r_{ki} - \sum_{b_k \in S \setminus \overline{C_i}} r_{ki} - |S \cap \overline{C_i}| \cdot h_{C_i}$
$= \sum_{b_k \in S \cap \overline{C_i}} r_{ki} - |S \cap \overline{C_i}| \cdot h_{C_i}$
$= \sum_{b_k \in S \cap \overline{C_i}} (r_{ki} - h_{C_i}) = \sum_{b_k \in S} x_k$. □