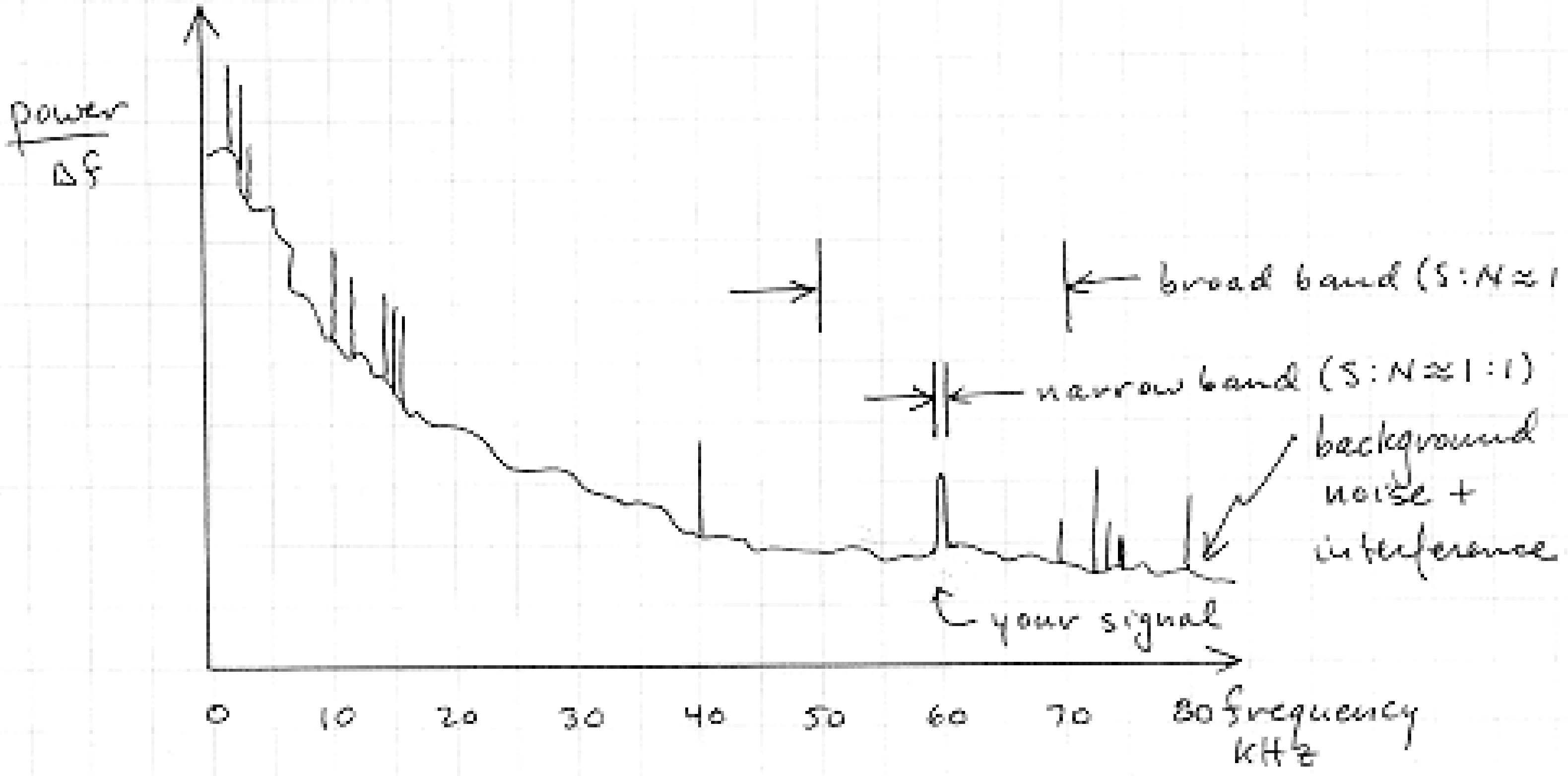
topics we will cover next ...

- how to beat the noise, i.e., ...
- how to make the best possible measurements
  - consistent with the limits imposed by fundamental noise
- *usually* signal accumulates faster than noise
  - e.g., integrated signal power ~ measuring time
    integrated noise power ~ (measuring time)$^{1/2}$
  - *but don't fool yourself into thinking it is always true, e.g., in inertial navigation signal-to-noise ratio generally gets worse with time*

how can we beat the noise?

- narrow-banding … if the signal is at (or near) a specific natural frequency … otherwise:
- modulation: give your signal a signature
- move it to higher frequency to overcome 1/f
- locate it far from noisy regions of the environment – especially far from strong “lines” like AC power, radio/TV stations, etc
- filter out the background except in a small window around the modulation frequency
- filter out any signal that has the right frequency but the wrong phase

narrow-banding to squeeze out noise



a. put signal into narrowest possible band
b. fine-tune detector to signal band
c. problem: how do you "lock" them ?

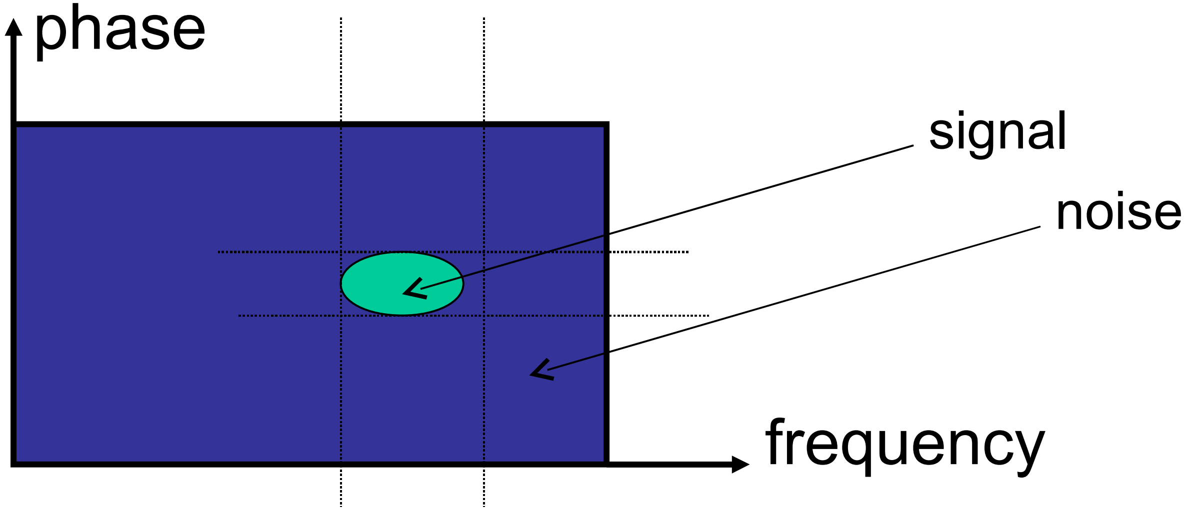
assignment

14) In the slide titled “narrow-banding to squeeze out noise”, estimate (as best you can by measuring appropriate features of the figure) the signal-to-noise power ratios for the indicated “wide band” and “narrow band” measurements.

-- Note I’ve said it is about 20:1 wideband and about 1:1 narrow band; your job is to make (and explain) your own estimates and say whether or not (and why) you agree with mine.

even better: phase-sensitive detection

- narrow-banding squeezes out noise that is not near the expected signal frequency
- "synchronous rectification" *furthermore* squeezes out noise that is not near the expected signal phase



(important) side note …

- at this point in our discussion we aim to measure the signal amplitude
  - and having a handle on the phase is just a trick we can use to improve the signal-to-noise
- but in some later topics the phase will be precisely what we care about
  - e.g., CW (continuous wave) amplitude modulated laser rangefinders, where phase ⇔ time-of-flight ⇔ range

review: Fourier analysis

- given any periodic signal of fundamental frequency f (repetition time T = 1/f) ...
  - $S(t) = \Sigma_n\, a_n \cos(2\pi n\, f\, t) + b_n \sin(2\pi n\, f\, t)$
  - i.e., the signal can be decomposed (*analysed*) into a sum of partial-signals at the harmonics (*multiples*) of the fundamental frequency
  - note that the result is a *line spectrum*
- if the signal is not periodic then the sum is replaced by an integral
  - and the analysis yields a *continuum spectrum*

review: phase

- alternatively, the sin and cos terms at each harmonic can be written as *either* one sin or one cos term with a particular phase φ ...
  - $S(t) = \Sigma_n\, a_n \cos(2\pi n\ f\ t) + b_n \sin(2\pi n\ f\ t)$
  - $s_n(t) = a_n \cos(2\pi n\ f\ t) + b_n \sin(2\pi n\ f\ t)$

    $= c_{nc} \cos(2\pi n\ f\ t + \varphi_c)$

    $= c_{ns} \sin(2\pi n\ f\ t + \varphi_s)$
- using simple trigonometric identities, find
  - $c_{nc} = (a_n{}^2 + b_n{}^2)^{½}$ and $\varphi_c = \tan^{-1}(-a_n/b_n)$, etc

application: impedance

- $V = \{L\, dI/dt,\ R\, I,\ \int I dt/C\} = \{L\, d/dt,\ R,\ \int dt/C\}\, I$
  - if only R, then V/I is *constant* called resistance
  - otherwise V/I depends (dramatically!) on time
- but for any real component of the signal $I_n(t) = I_{n0} \cos(2\pi n\, f\, t + \Phi_n)$ at frequency f you are free to add an imaginary part

  $$jI_{n0} \sin(2\pi n\, f\, t + \Phi_n) \quad \text{where } j = (-1)^{1/2}$$

  i.e., pretend $I_n(t)=I_{n0}\, e^{j(\omega_n t+\Phi n)}$ where $\omega_n=2\pi f$
- then $dI/dt = j\omega_n I_n(t)$ and $\int I dt = I_n(t)/j\omega_n$

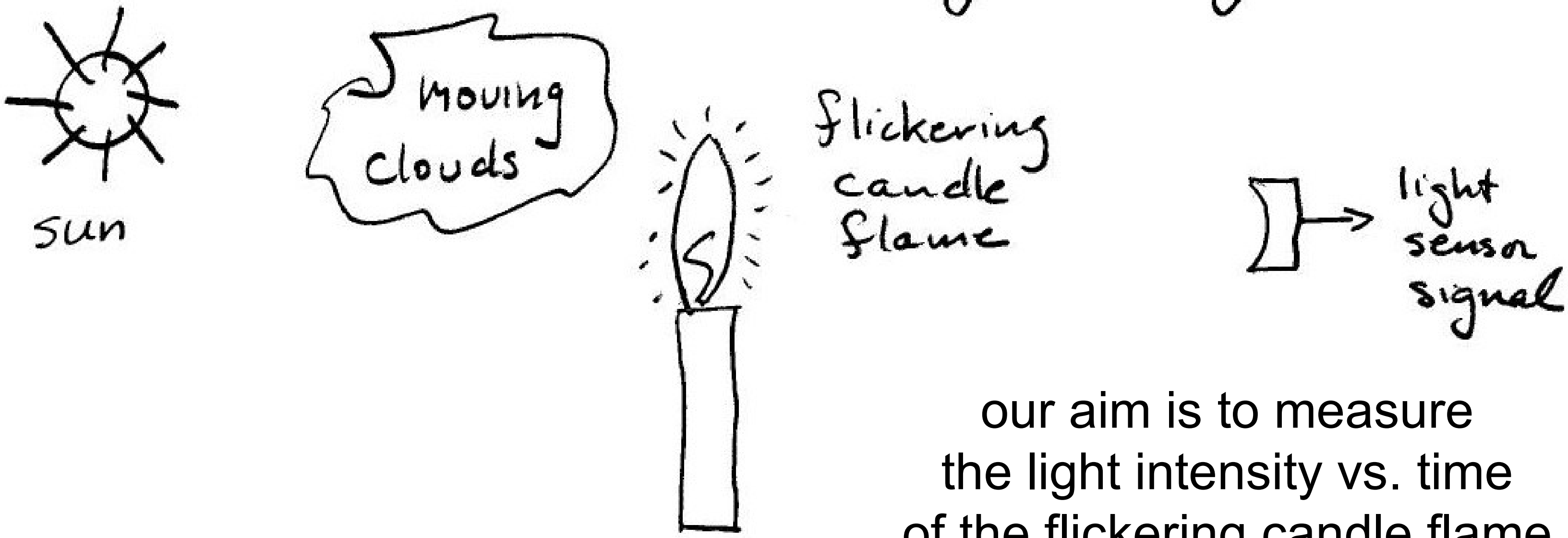
  $V_n = \{j\omega_n L,\ R,\ 1/j\omega_n C\}I_n$

  $V_n/I_n \rightarrow$ ***complex*** *constant* called impedance

phase sensitive detection
a.k.a.
synchronous detection
a.k.a
lock-in detection
or, in each case,
rectification or *amplification*
or *detection*

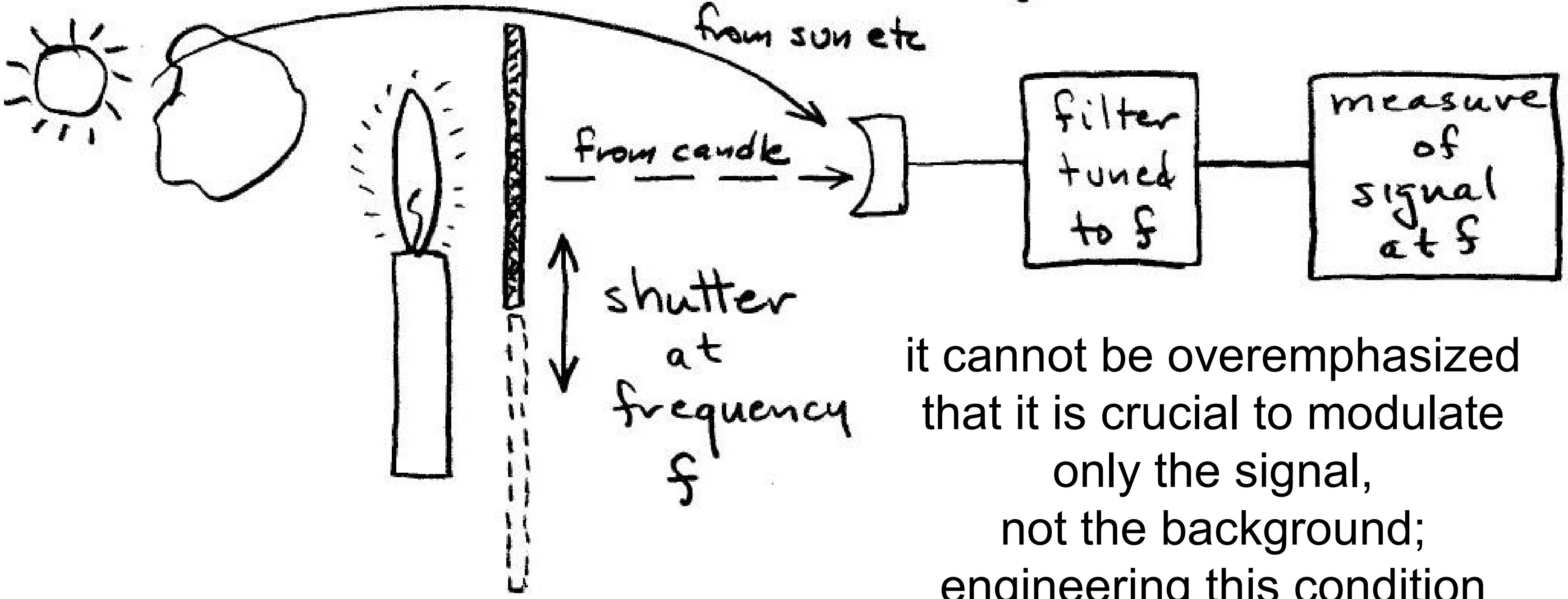
how to hear a hummingbird in a hurricane

how to see a candle in bright sunlight



our aim is to measure
the light intensity vs. time
of the flickering candle flame
in the presence of an overpowering
background signal (= noise)
from the sunlight
which is itself varying with time

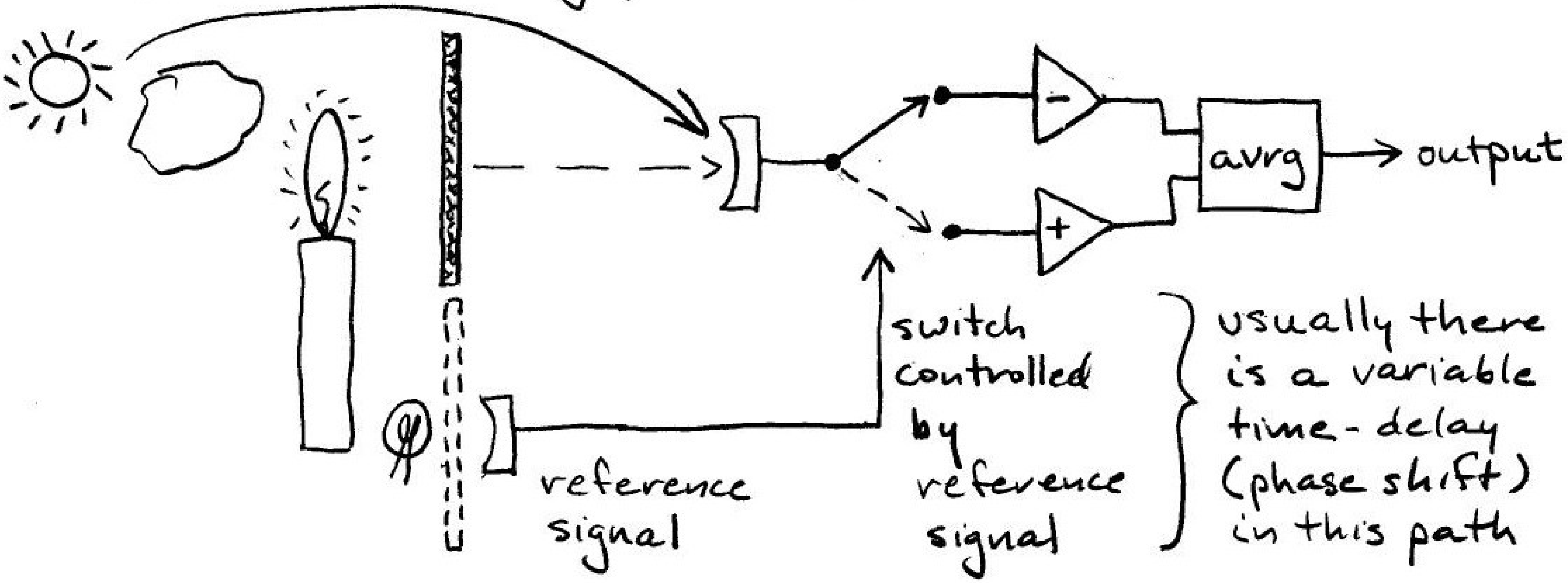
① modulate the candle light rapidly w/r its fluctuations
(but don't modulate the background noise!)



it cannot be overemphasized
that it is crucial to modulate
only the signal,
not the background;
engineering this condition
may be very difficult!

② modulation + frequency-sensitive detection squeezes out noise at wrong frequency

synchronous detection also squeezes out noise with wrong phase w/r modulation



the reference signal may drive the modulator, or
the modulator may be “free running” and the reference signal obtained by monitoring it (as shown here), or
it can (with difficulty) be extracted from the signal itself

③ when shutter is open averaging circuit receives $(S+N)$

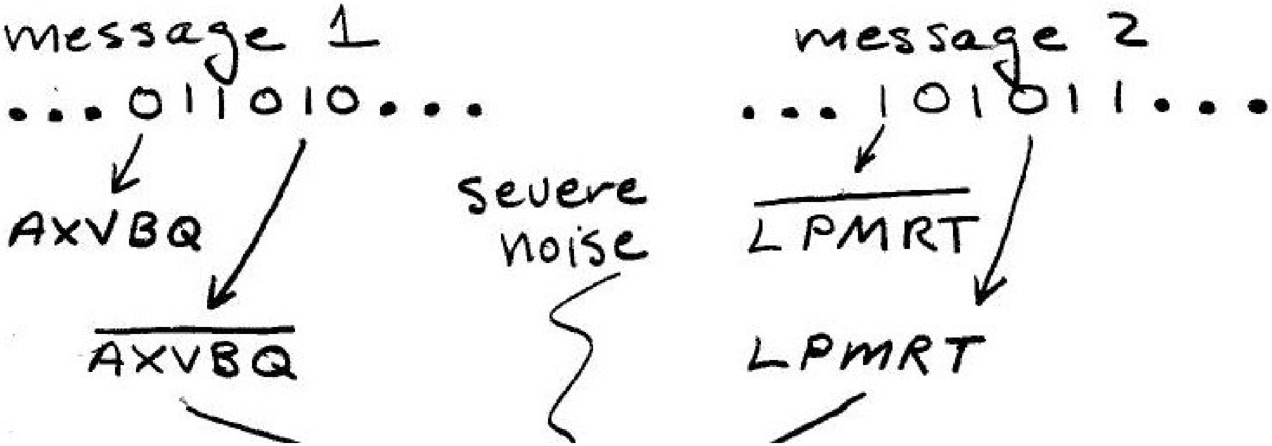
when shutter is closed averaging circuit receives $-N$

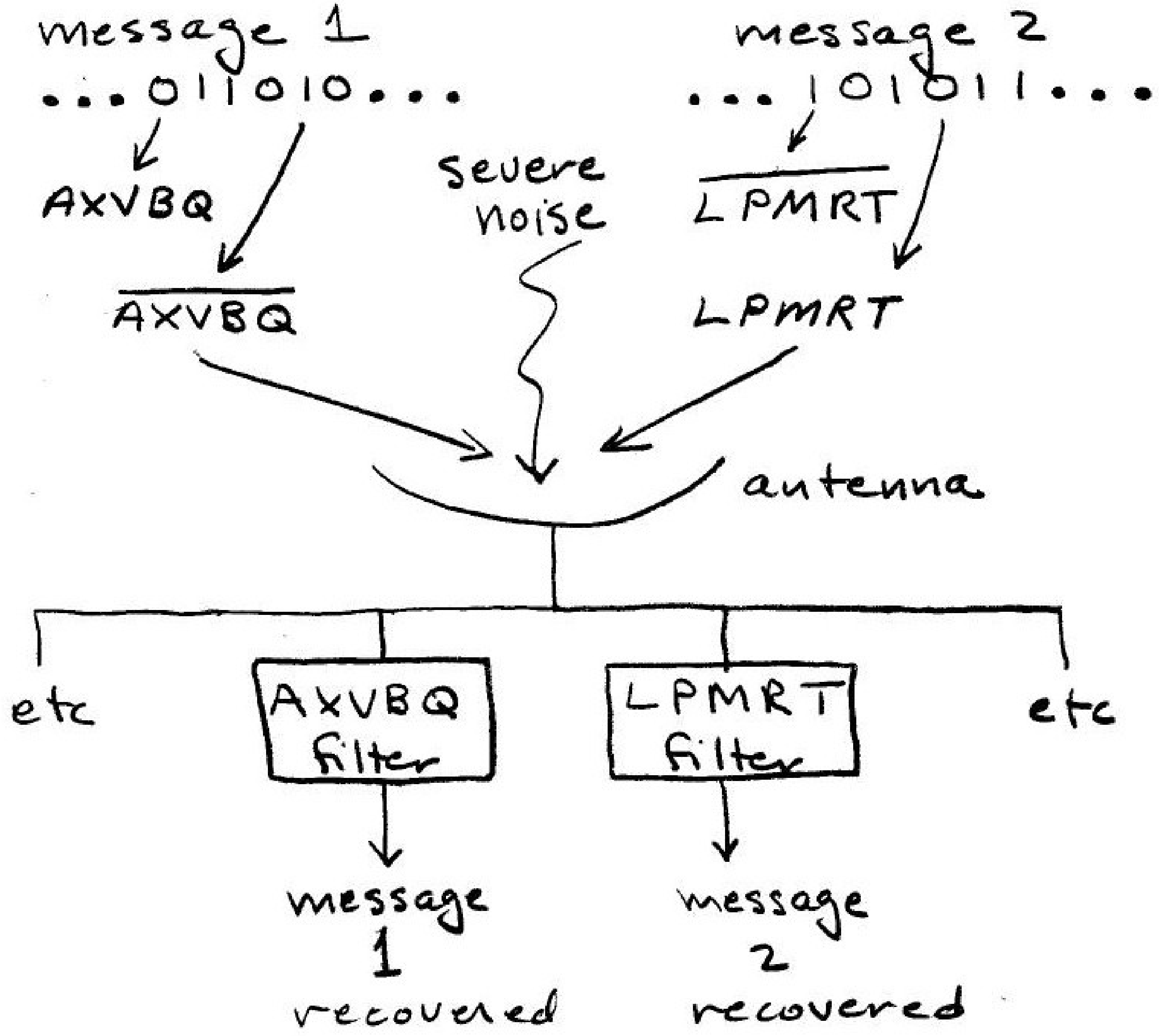
averaging circuit output $= \frac{(S+N)-N}{2} = \frac{S}{2}$

(again: as long as you can arrange to modulate the signal but not the background noise!)

④ complex modulation patterns allow even greater improvement + superposition and separation of multiple signals on one channel

(principle of CDMA cell phones, GPS, etc)




message 1
...011010...
AXVBQ
$\overline{AXVBQ}$
severe noise
message 2
...101011...
$\overline{LPMRT}$
LPMRT
antenna
etc
AXVBQ filter
LPMRT filter
etc
message 1 recovered
message 2 recovered