Computational Design

Stelian Coros

Schedule for presentations

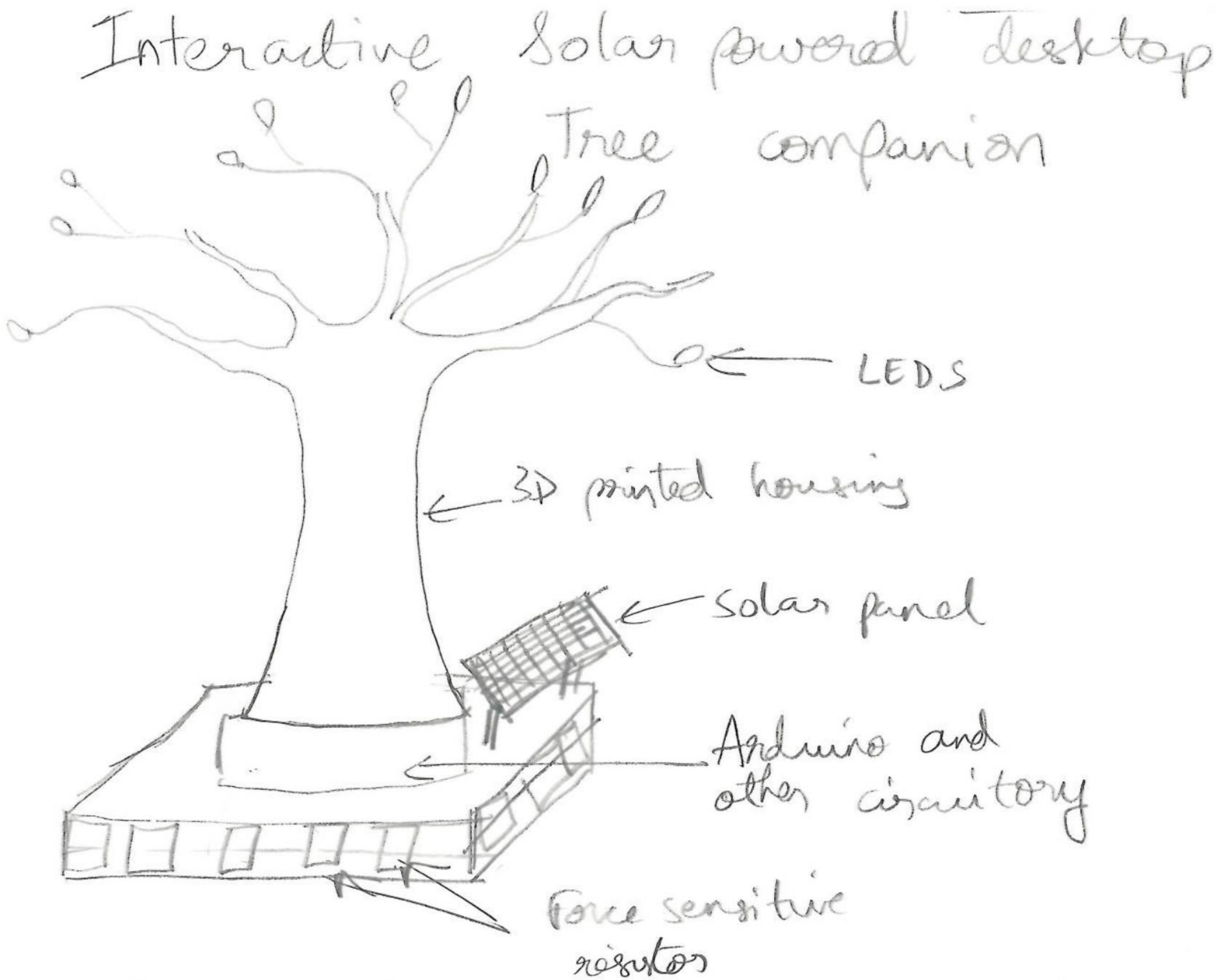
February	3	5	10	12	17	19	24	26	
March	3	5	10	12	17	19	24	26	30
April	2	7	9	14	16	21	23	28	30

Send me:
ASAP: 3 choices for dates + approximate topic (scheduling)
1-2 weeks before your presentation: list of papers you plan to talk about
Day before each presentation: 3 questions for one of the papers that will be discussed

Next class

- Visit the Digital Fabrication Lab

Mini-assignment



Computational Design

- Need a parameterized model
  - Mathematical formulation that predicts behavior of a system
  - Examples?
- CAD systems
  - Expose design parameters
  - Use mathematical model to preview outcome
  - What do you want from a CAD system?

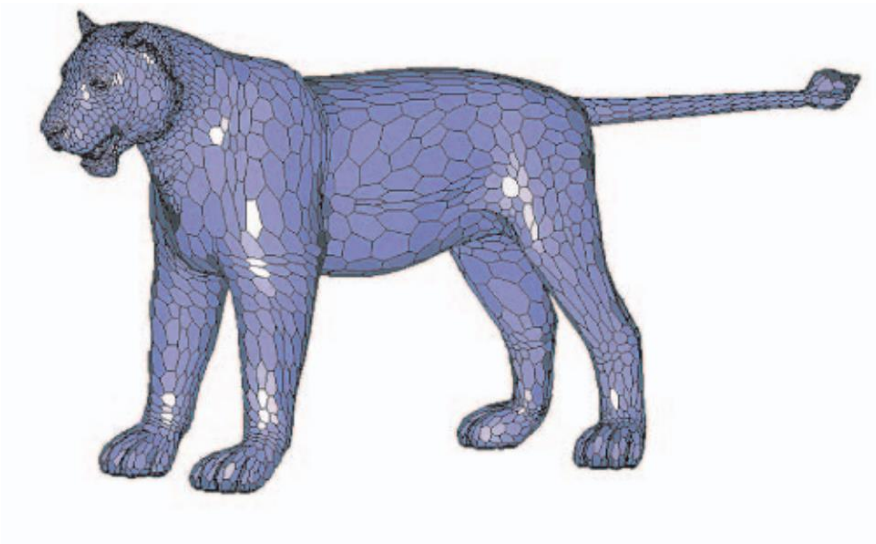
Computational Design

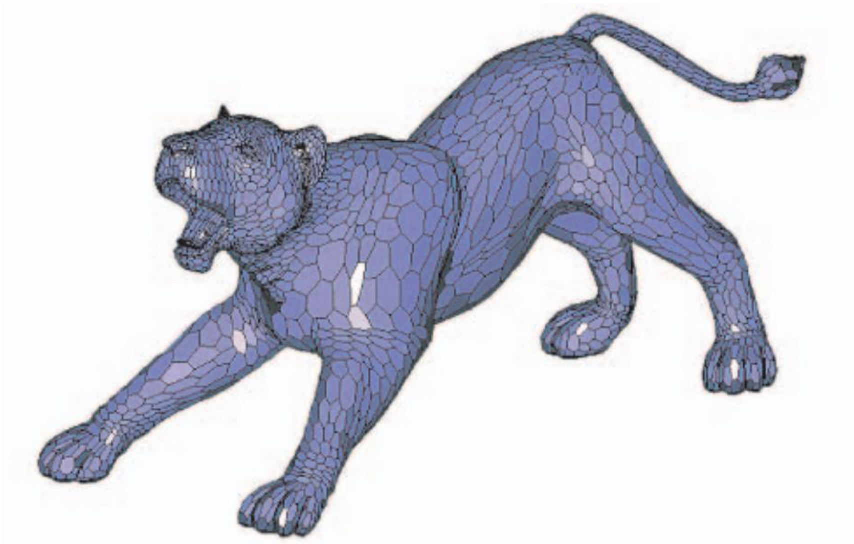
- Forward design: direct manipulation of design parameters
  - Level of abstraction
  - Exploration of design spaces

- Inverse design: automatically infer design parameters from functional specifications
  - Optimization-based

Forward Design

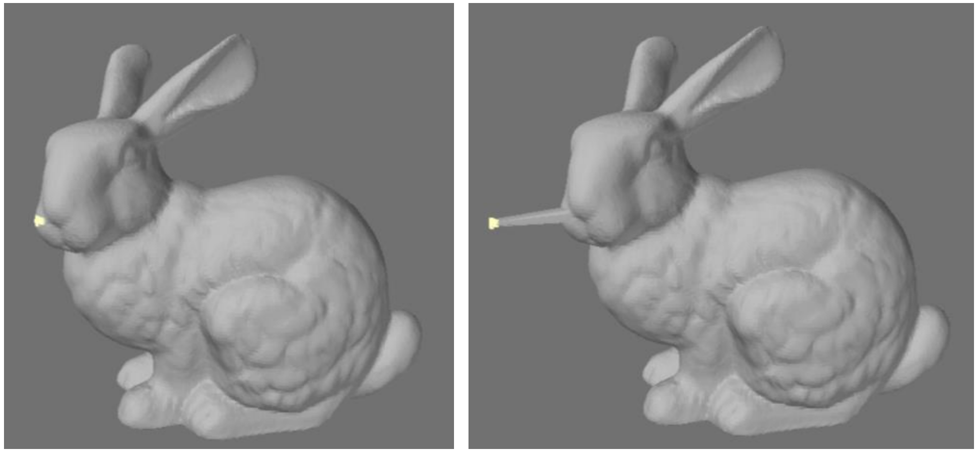
- Editing 3D Models as an example





Editing 3D Models

- Options:
  - Directly edit mesh vertices
    - Finding the right level of abstraction is key!



Editing 3D Models

- Options:
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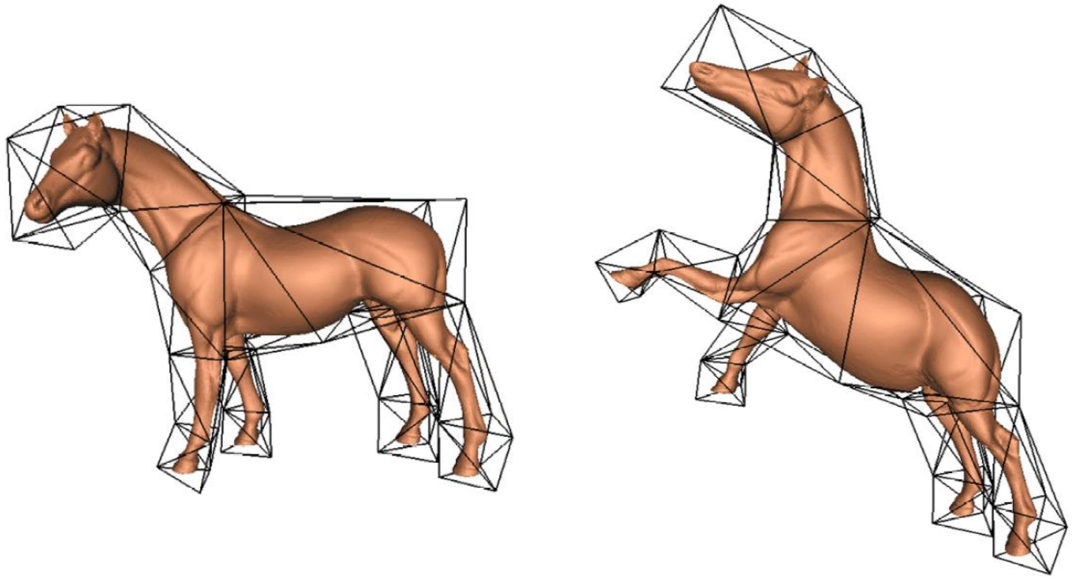
  - Cage-based editing



Cage-based mesh editing

- Embed 3D model in a coarse mesh (cage)
- Edit cage vertices, deform model with it automatically

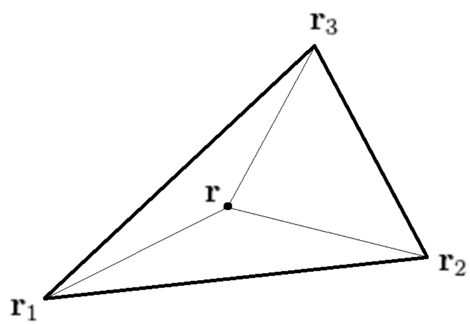
$$v = \sum_i w_i v_i$$

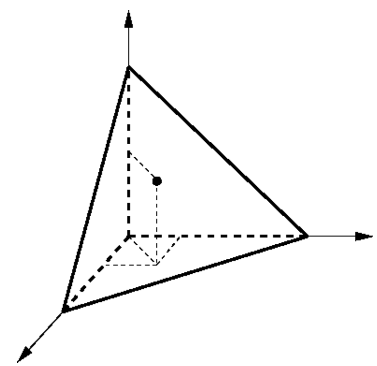
Cage-based mesh editing

- Computing Appropriate Weights
  - Harmonic coordinates, mean value coordinates, bounded biharmonic weights, etc.
  - Want:
    - Smoothness
    - Monotonicity
    - Non-negativity
    - Partition of unity
    - Locality and sparsity
- Simple approach: Barycentric coordinates (volumetric cages)

Barycentric coordinates

- Ratio of areas (volumes in 3D)





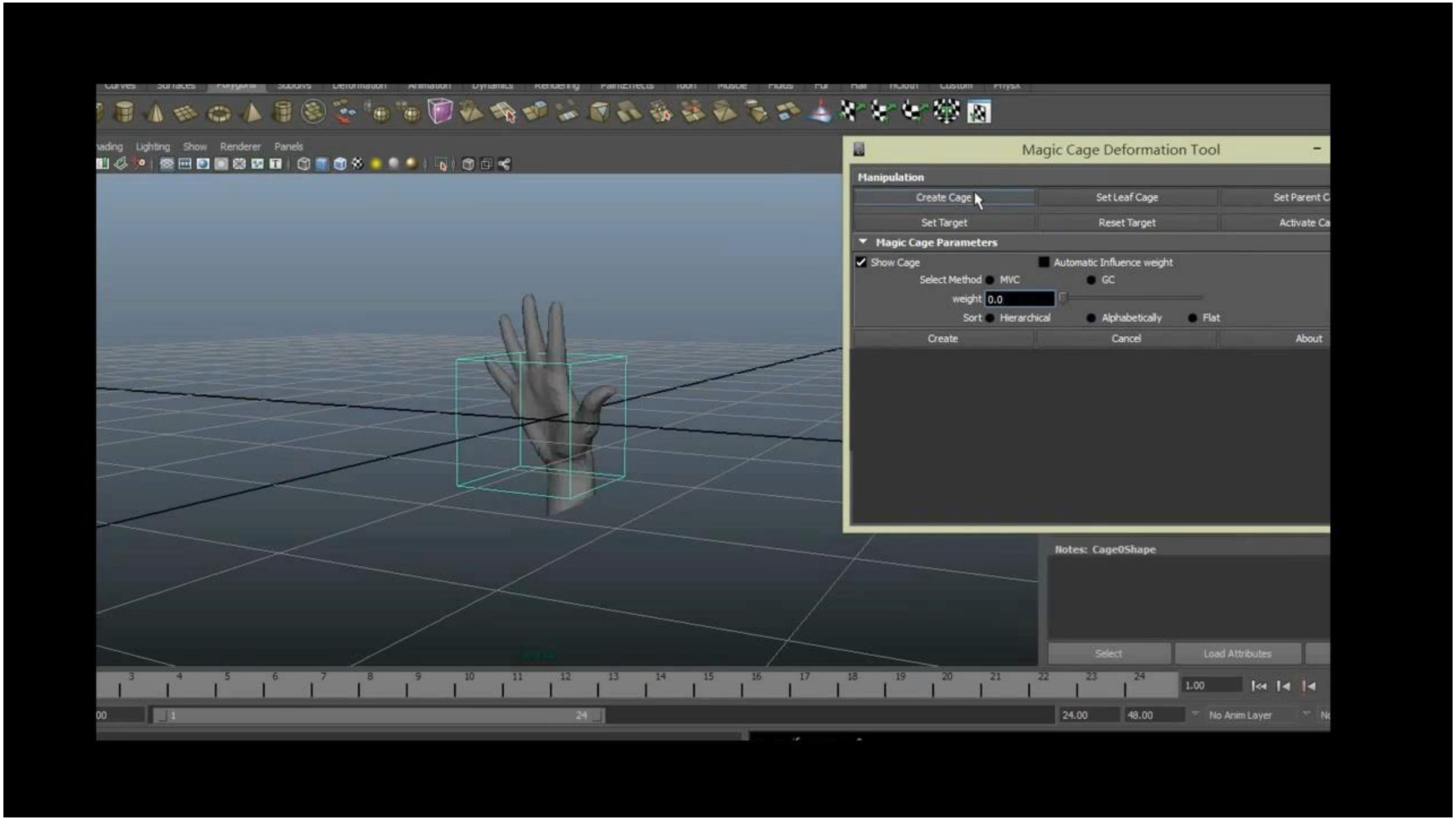
$$\mathbf{r} = \lambda_1 \mathbf{r}_1 + \lambda_2 \mathbf{r}_2 + \lambda_3 \mathbf{r}_3,$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \mathbf{T}^{-1}(\mathbf{r} - \mathbf{r}_3)$$

Conceptually the same
in higher dimensions

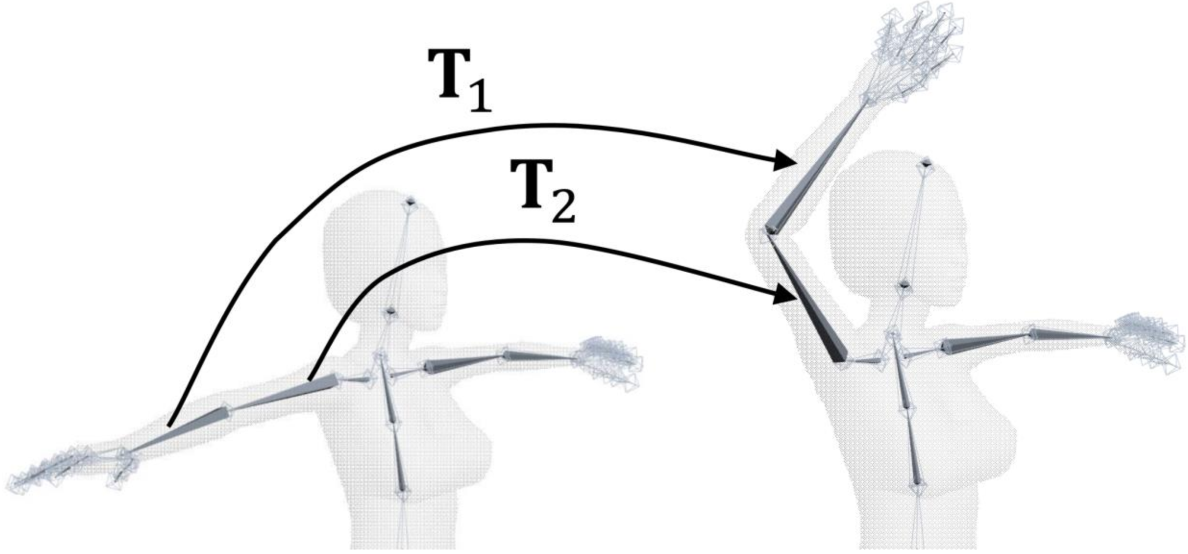
Cage-based mesh editing



can’t precisely control surface properties

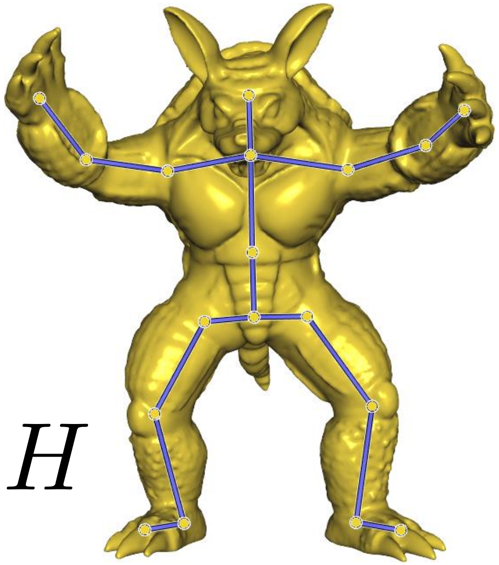
Editing 3D Models

- Options:
  - Directly edit mesh vertices
    - Finding the right level of abstraction is key!
  - Cage-based editing
  - Skeletal Rigs



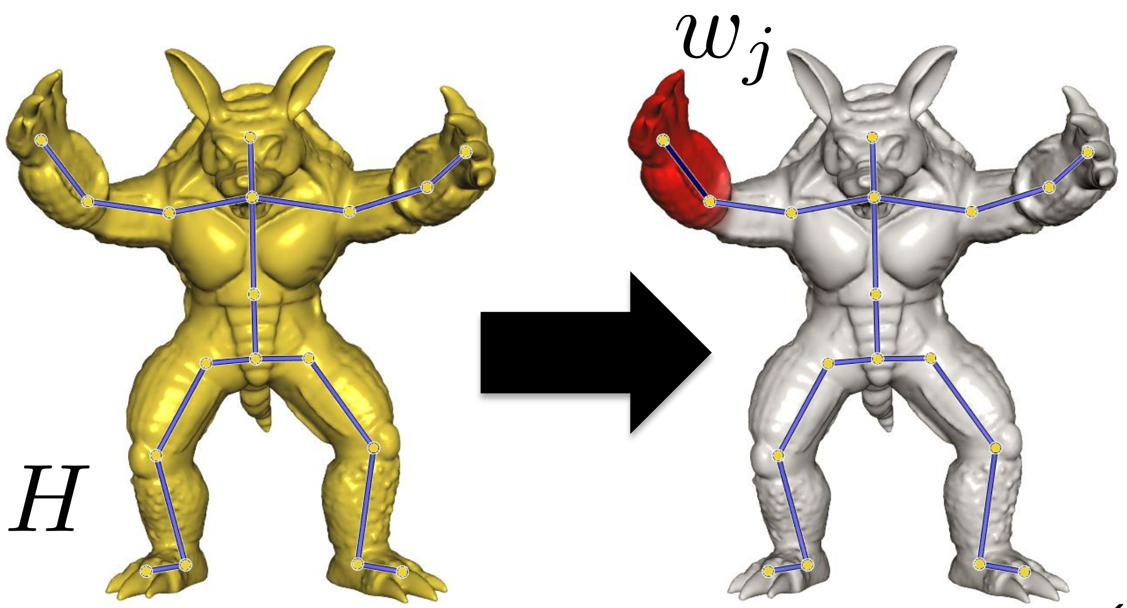
Skeletal Rigs

H



$$\mathbf{v}' = \sum_{j \in H} w_j(\mathbf{v}) \mathbf{T}_j \begin{pmatrix} \mathbf{v} \\ 1 \end{pmatrix}$$

Skeletal Rigs



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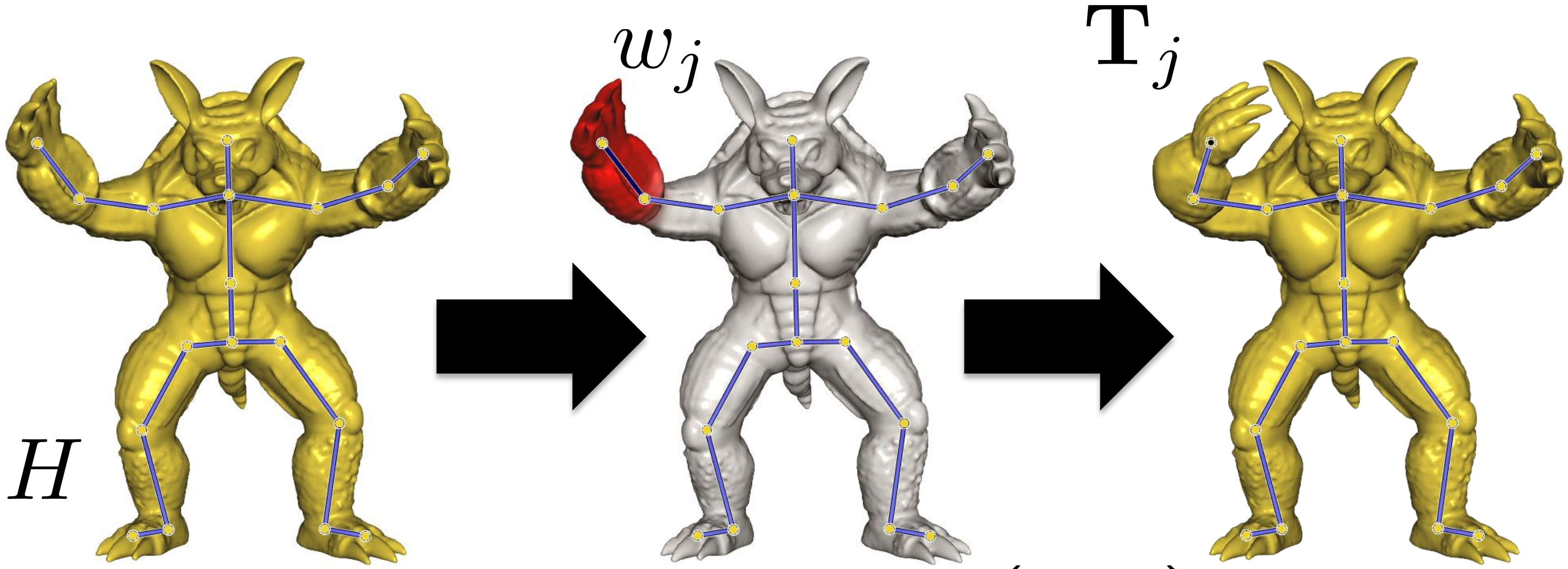

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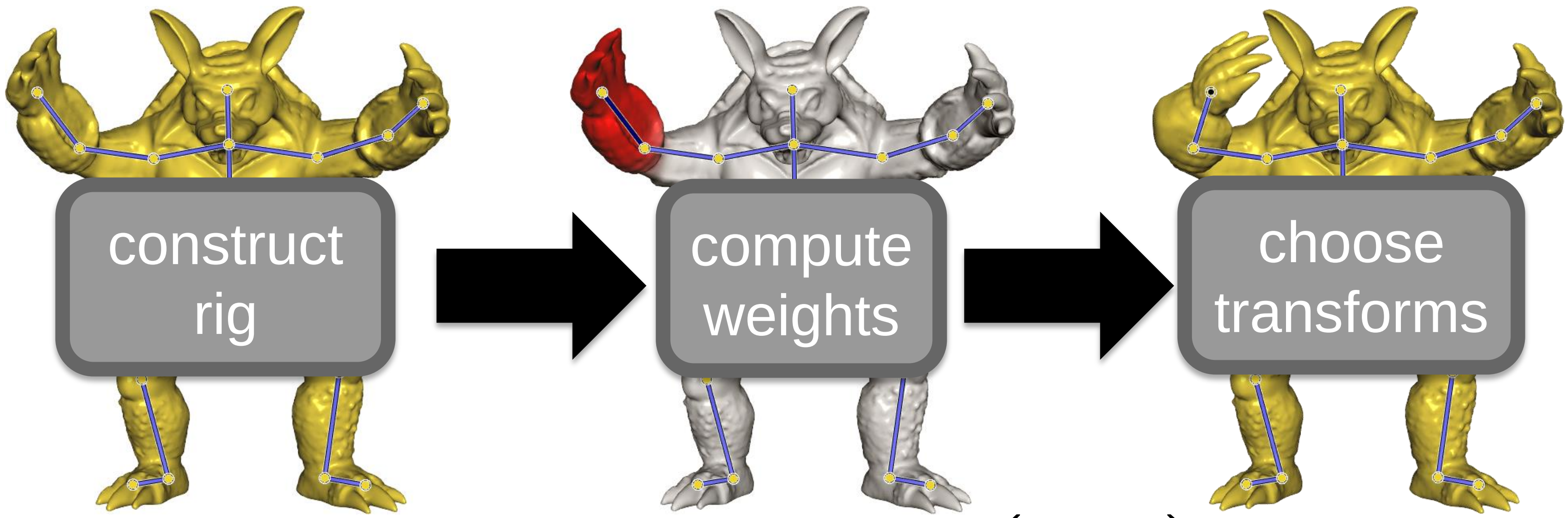
Skeletal Rigs

Recent research aims to automate each step



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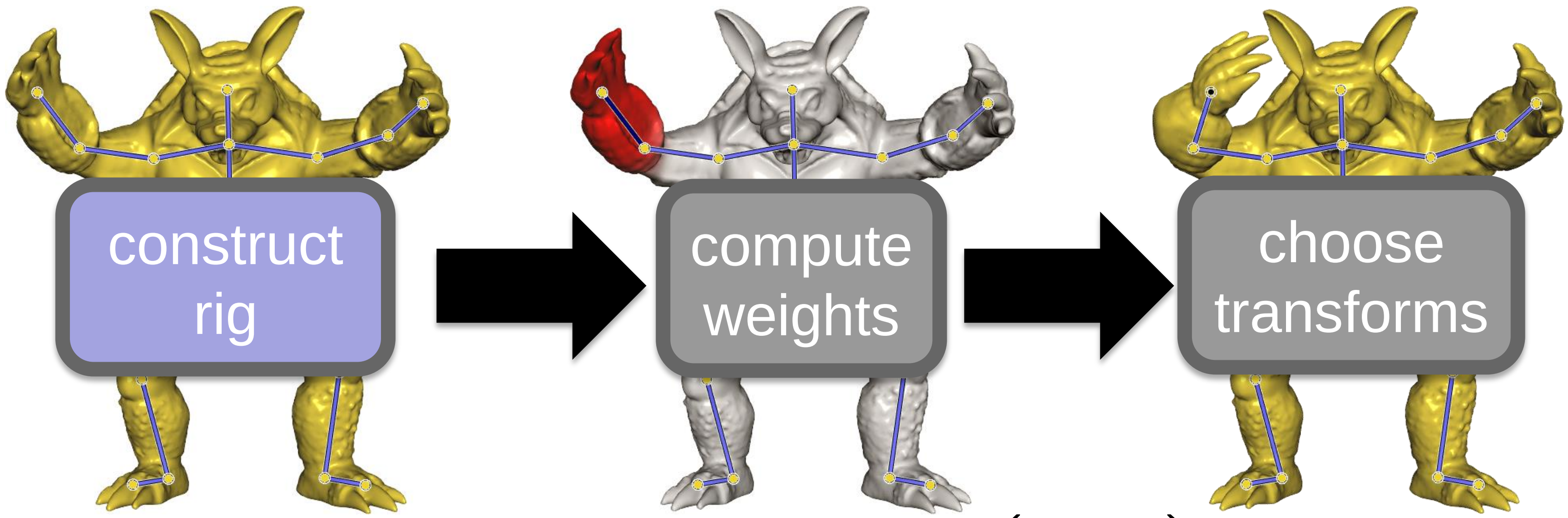
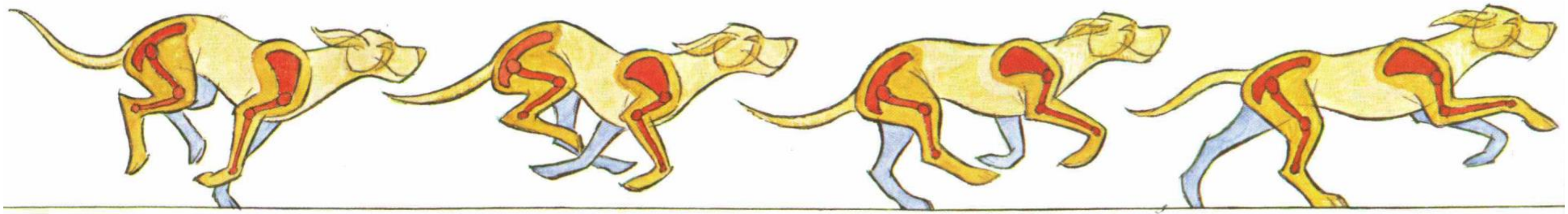


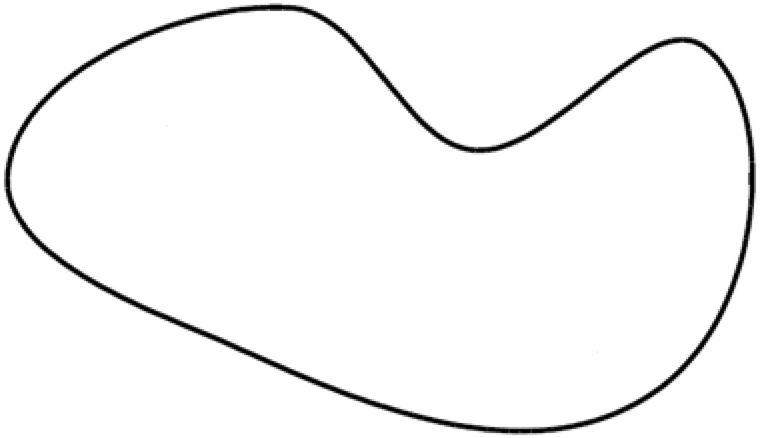
$$\mathbf{v}' = \sum_{j \in H} w_j(\mathbf{v}) \mathbf{T}_j \begin{pmatrix} \mathbf{v} \\ 1 \end{pmatrix}$$

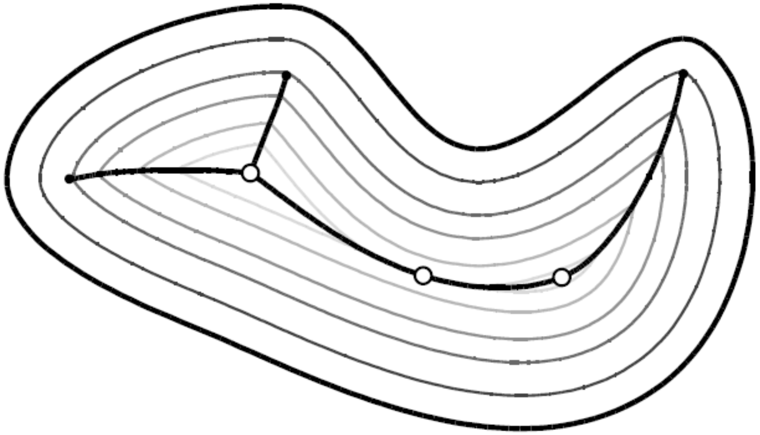
From shape to skeleton

Skeleton implies shape…



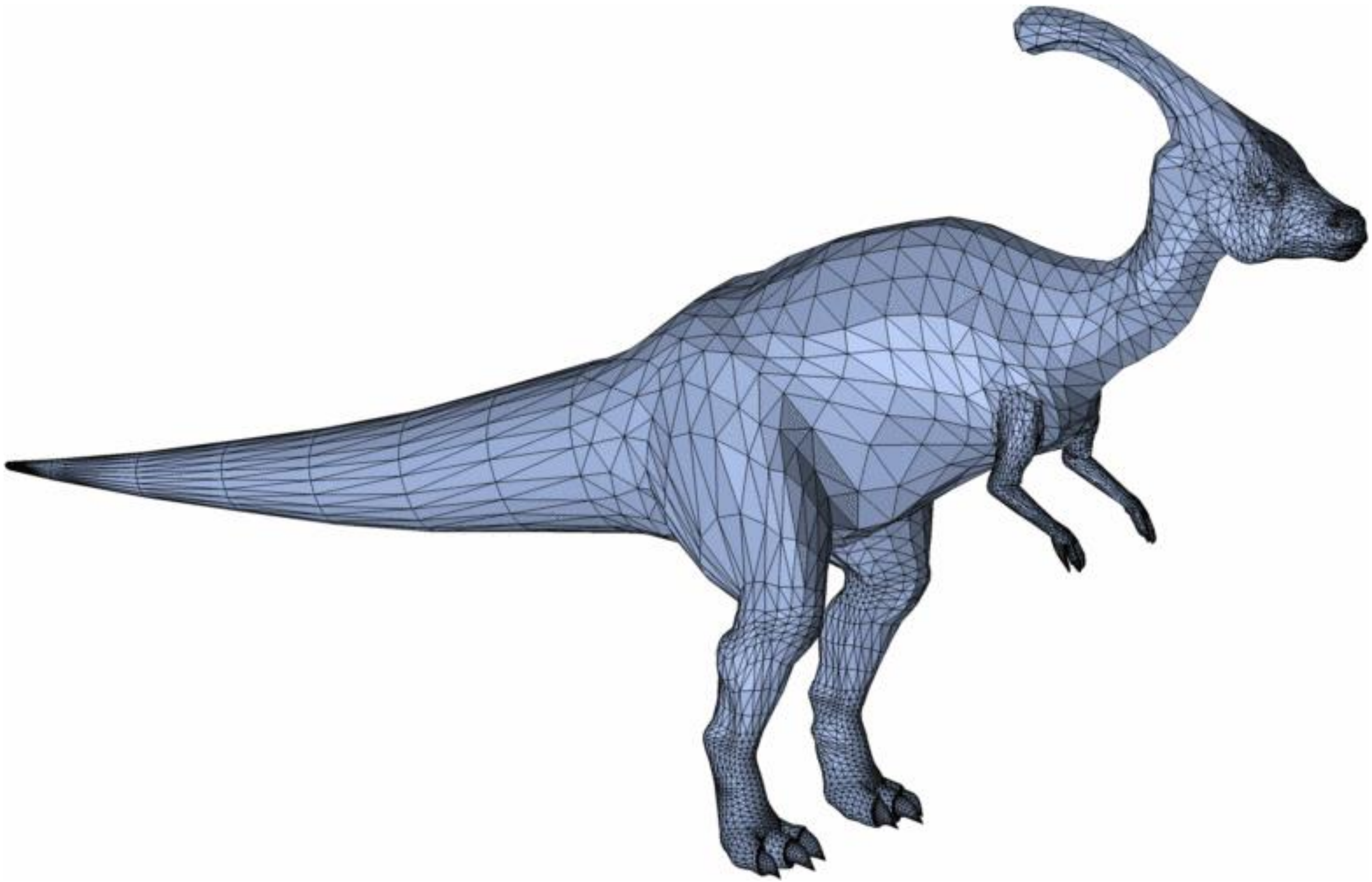
…and shape implies skeleton



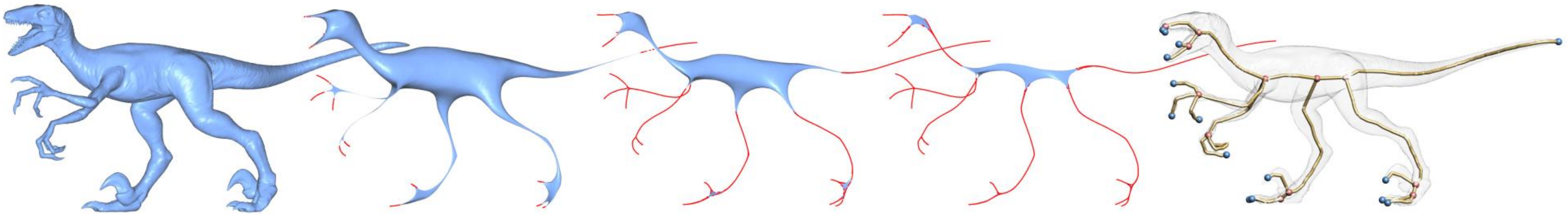


Medial Axis (topological skeleton)

Surface flow degenerates to skeleton approximating medial axis



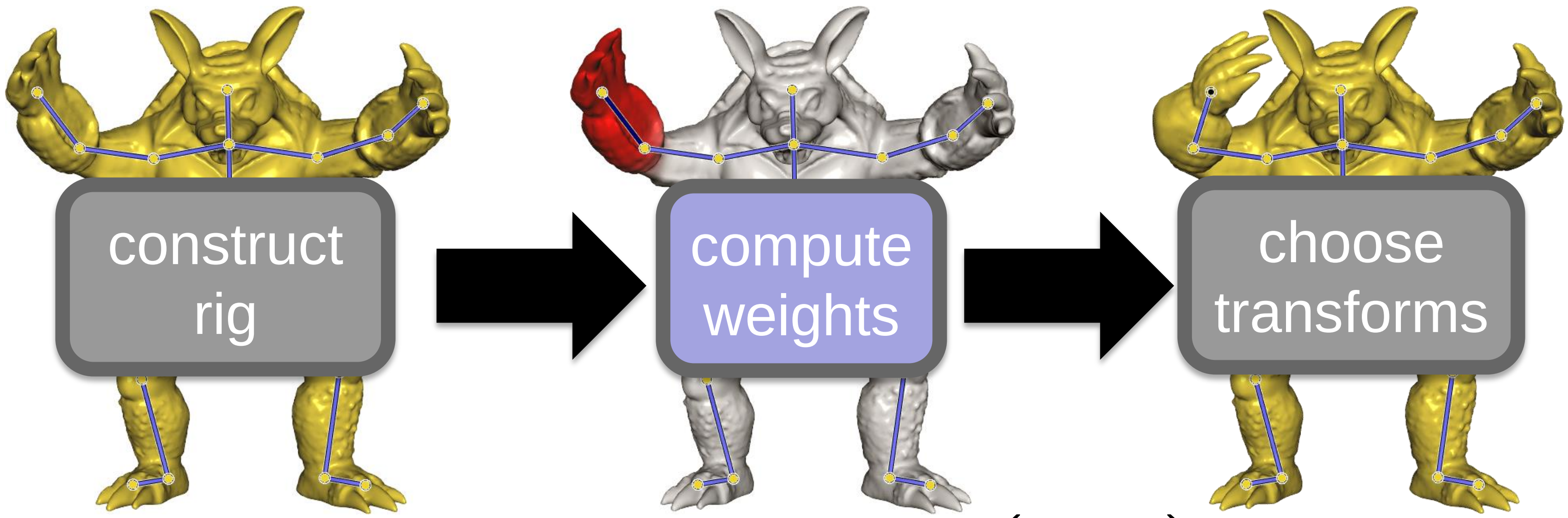
Medial axis



[Au et al. 2008]

Skeletal Rigs

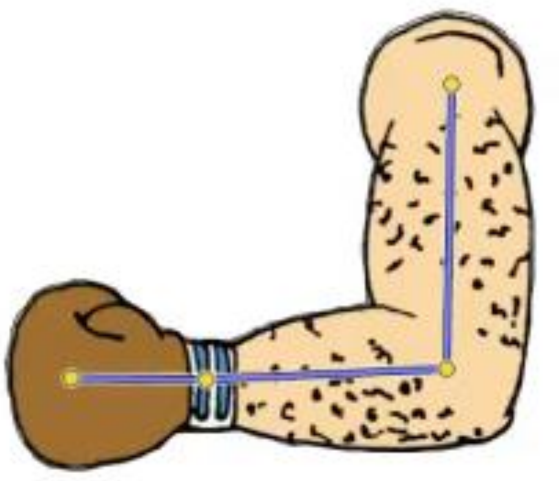
Recent research aims to automate each step



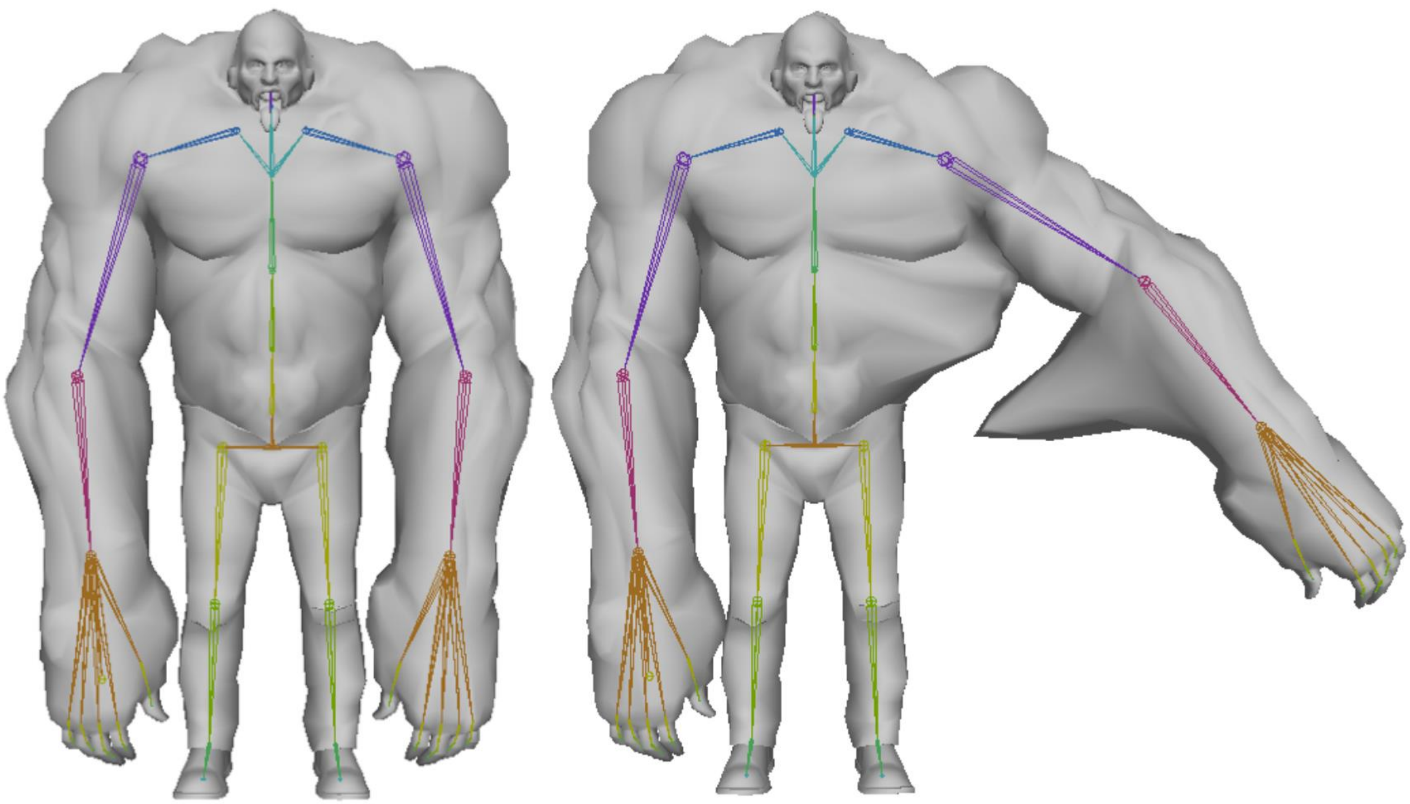
$$\mathbf{v}' = \sum_{j \in H} w_j(\mathbf{v}) \mathbf{T}_j \begin{pmatrix} \mathbf{v} \\ 1 \end{pmatrix}$$

Weight computation

The closer to a bone a vertex is, the larger the weight should be…

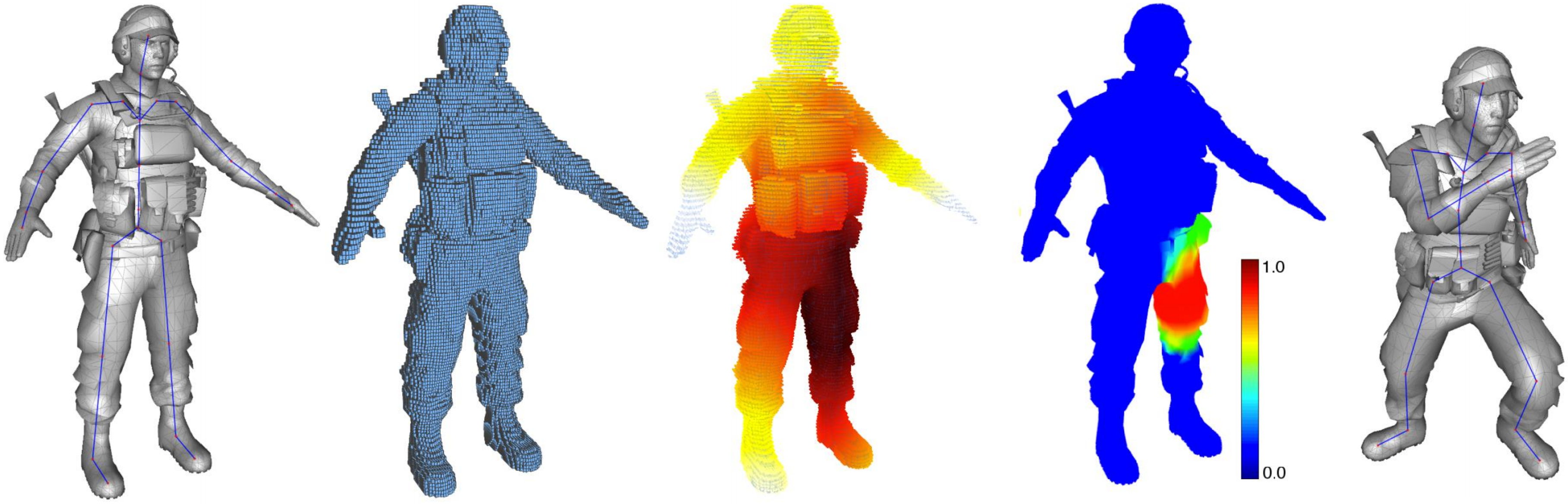


Weight computation

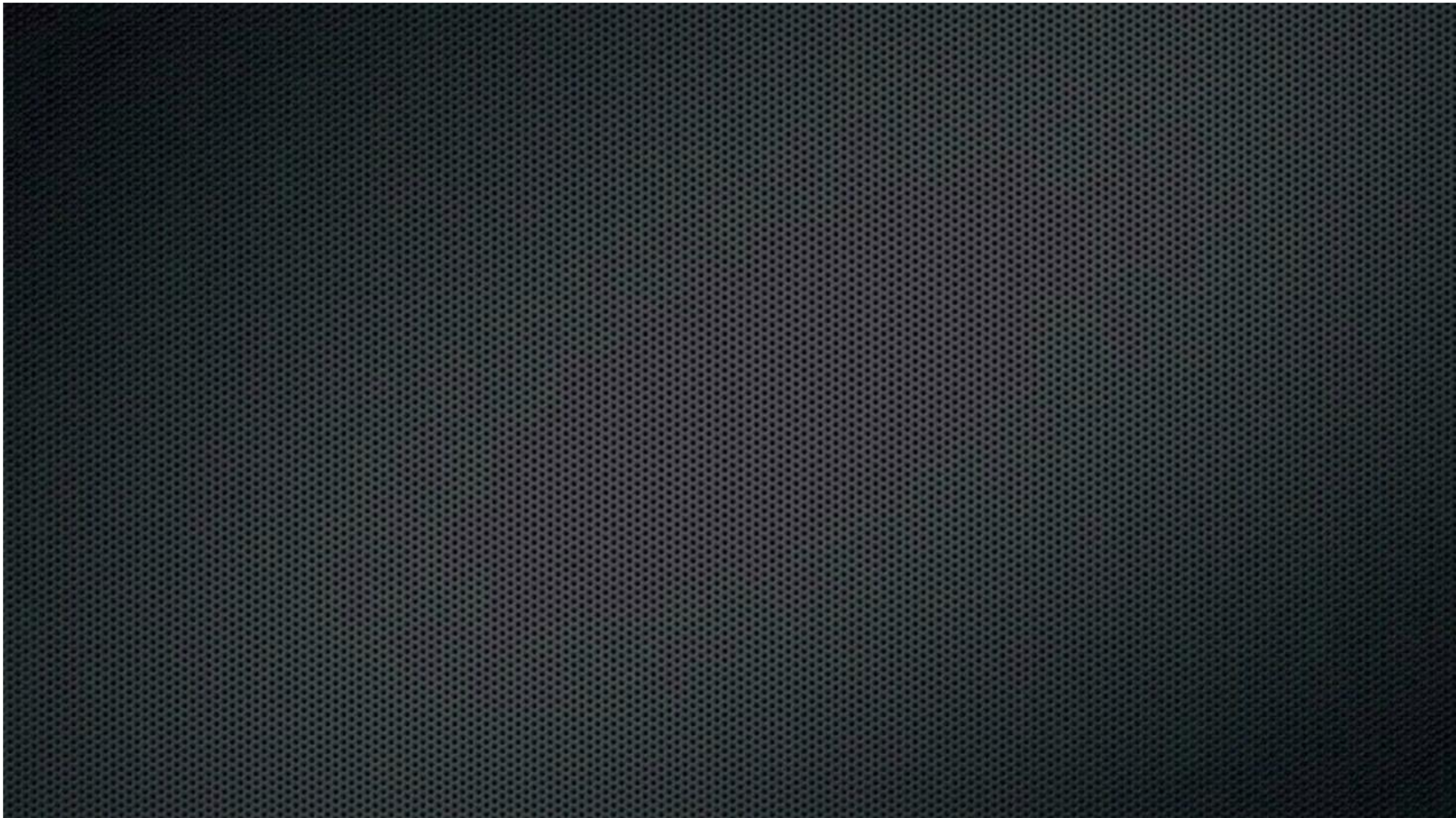


Weight computation

Want a measure of distance that is shape-aware!



Weight computation



Skeletal Rigs

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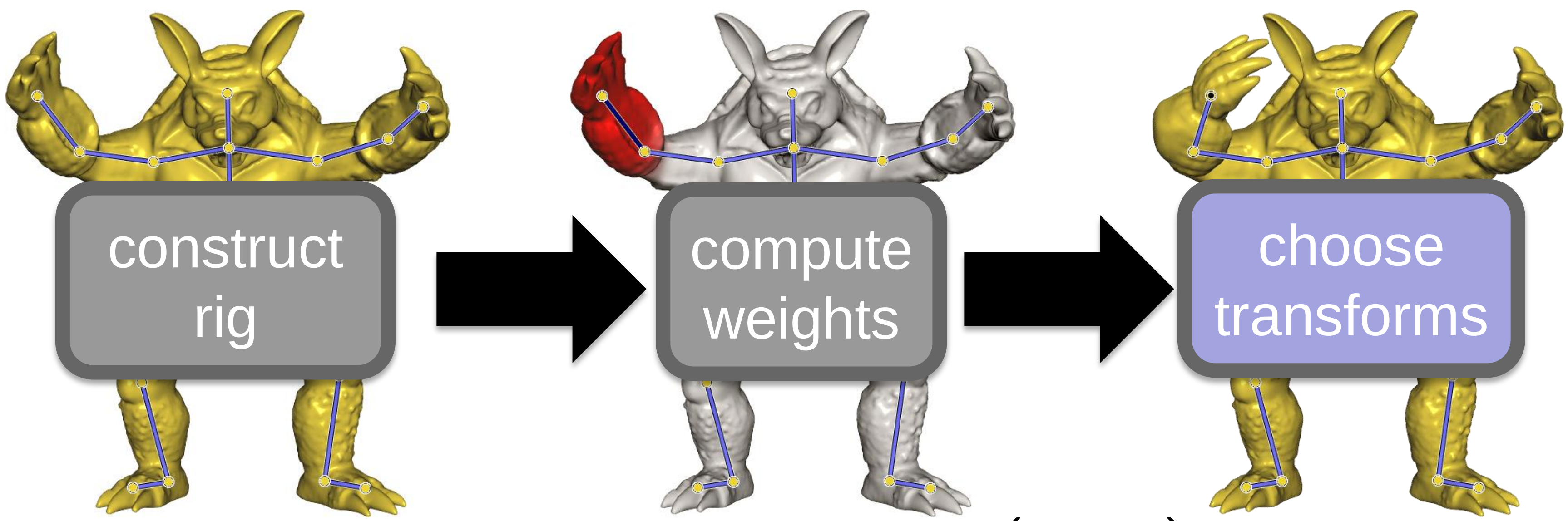

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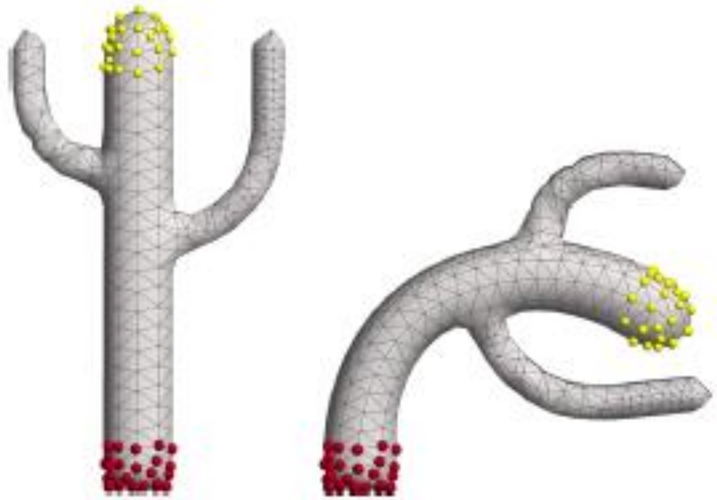
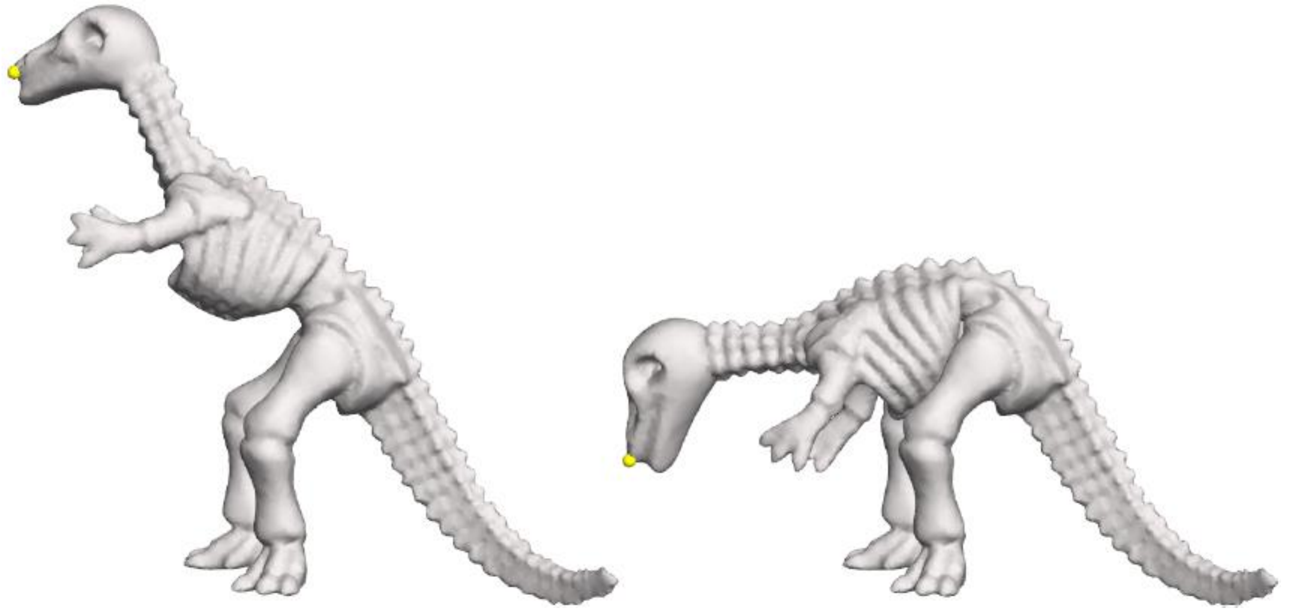
Editing 3D Models

- Options:
  - Directly edit mesh vertices
    - Finding the right level of abstraction is key!
  - Cage-based editing
  - Skeletal Rigs
  - High-level surface editing



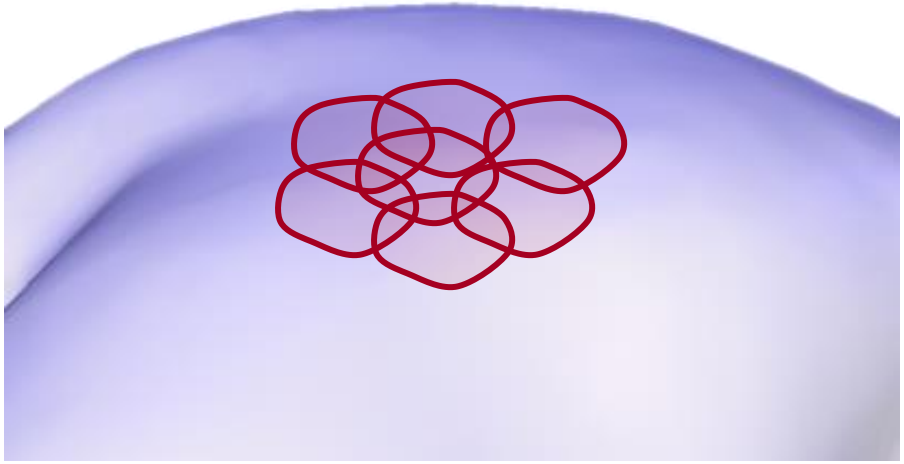
As-Rigid-As-Possible Surface Modeling (Sorkine & Alexa, 2007)

- What do we want?
  - Smooth deformations
  - Precise control over specific features
  - As-rigid-as-possible deformations for details
  - Ease of use



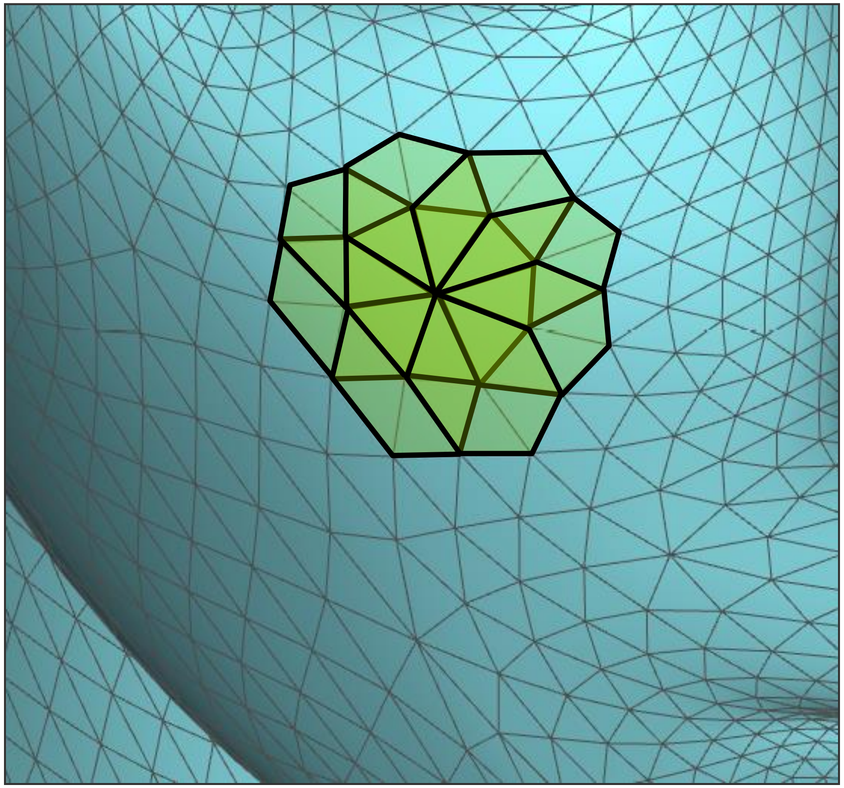
As-Rigid-As-Possible Surface Modeling (Sorkine & Alexa, 2007)

- Main idea:
  - Preserve shape of surface patches as much as possible (subject to user constraints)
  - Surface patches should overlap to prevent bending



As-Rigid-As-Possible Surface Modeling (Sorkine & Alexa, 2007)

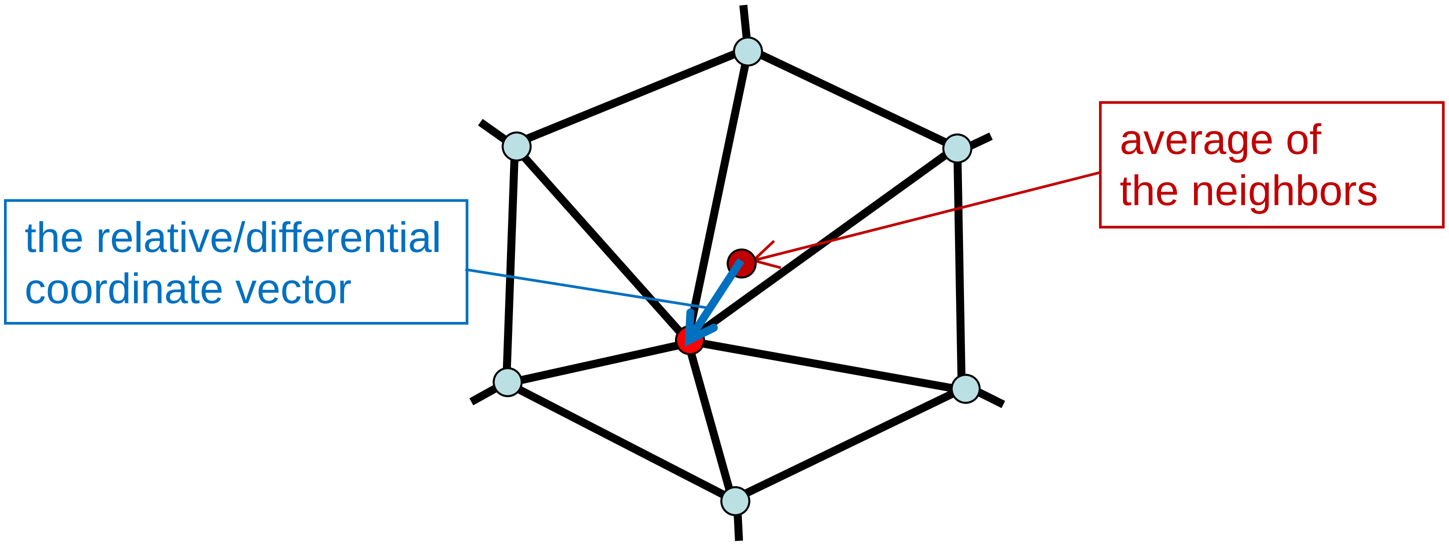
- Surface patches
  - 1 ring neighbor for each vertex



As-Rigid-As-Possible Deformations

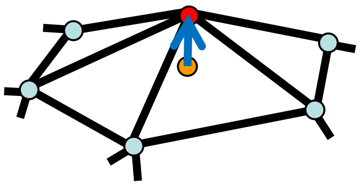
- Approach 1: differential coordinates

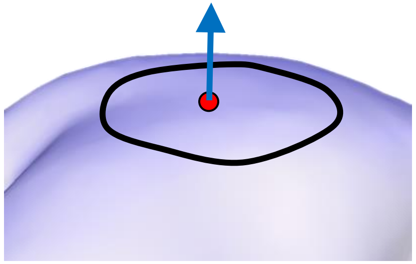
$$\boldsymbol{\delta}_\mathbf{i} = \mathbf{v}_\mathbf{i} - \frac{1}{d_i} \sum_{j \in N(i)} \mathbf{v}_\mathbf{j}$$



Why differential coordinates?

- They represent the local detail / local shape description
  - The direction approximates the normal
  - The size approximates the mean curvature





Reconstruction

- Reconstruct position of each vertex using differential coordinates and neighborhood positions:

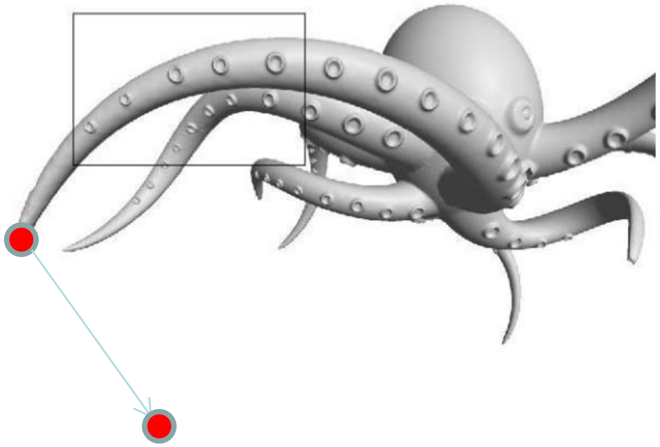
Find v_i such that $\frac{1}{d_i} \sum_{\mathbf{v} \in N(i)} (\mathbf{v_i} - \mathbf{v})$ as close as possible to $\boldsymbol{\delta}_\mathbf{i}$

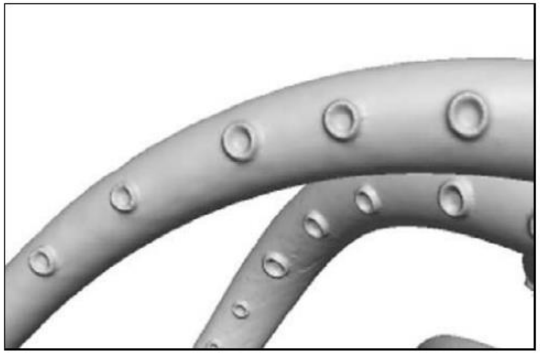
Reconstruction

- Or, in matrix form:

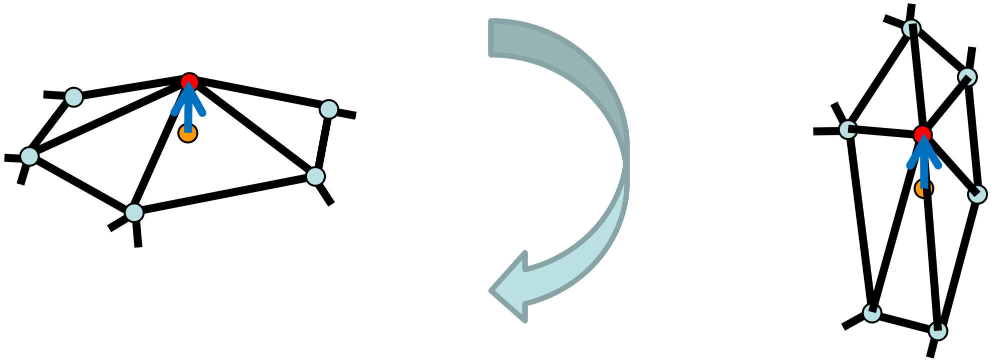
$$\left\| M \cdot P - \delta \right\|^2$$

Reconstruction





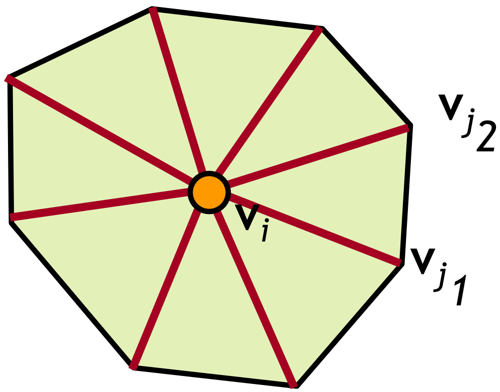
Problems?



Not rotation invariant!!! Can we do better?

Rotation-invariant deformation energy

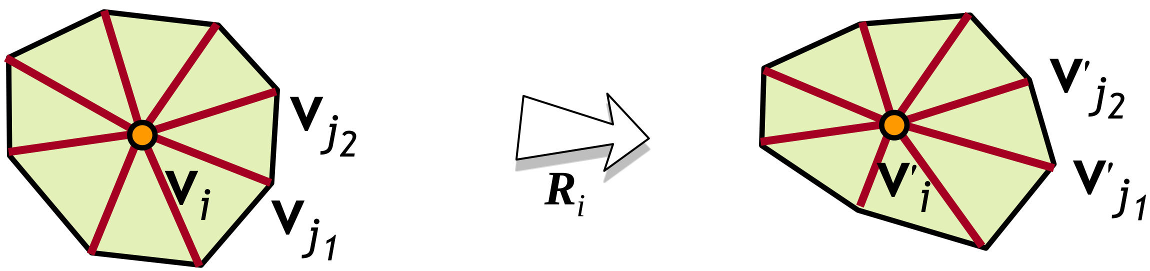
- Ask each 1-ring neighborhood to transform rigidly up to some rotation R



$$\min \sum_{j \in N(i)} \left\| (\mathbf{v}'_i - \mathbf{v}'_j) - R_i(\mathbf{v}_i - \mathbf{v}_j) \right\|^2$$

Deformations and Rotations

- If $\mathbf{v}, \mathbf{v}'$ are known then $\boldsymbol{R}_i$ is uniquely defined



- Compute deformation gradient F s.t. V’ = F V
- Polar Decomposition: F = R D
  - $F = U\Sigma W^T$; $R = UW^T$

Positive semi-definite, symmetric matrix representing deformations (stretch, shear)

Energy formulation

- Can formulate overall energy as:

$$\min_{\mathbf{v}'} \sum_{i=1}^{n} \sum_{j \in N(i)} \left\| (\mathbf{v}'_i - \mathbf{v}'_j) - R_i (\mathbf{v}_i - \mathbf{v}_j) \right\|^2$$

$$s.t.\ \mathbf{v}'_j = \mathbf{c}_j,\ j \in C$$

- $\mathbf{v}'$ and $\boldsymbol{R}$ treated as separate sets of variables

Iterative solver

- Alternating iterations
  - Given initial guess $\mathbf{v}'_0$, find optimal rotations $\boldsymbol{R}_i$
    - Easy to parallelize
  - Keep $\boldsymbol{R}_i$ fixed, minimize the energy by finding new $\mathbf{v}'$

$$\min_{\mathbf{v}'} \sum_{i=1}^{n} \sum_{j \in N(i)} \left\| (\mathbf{v}'_i - \mathbf{v}'_j) - R_i(\mathbf{v}_i - \mathbf{v}_j) \right\|^2$$

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$$L\mathbf{v}' = \mathbf{b}$$

Uniform mesh Laplacian

Iterative solver

- Each iteration decreases the energy (or at least guarantees not to increase it)
- The matrix L stays fixed
  - Precompute factorization
  - back-substitute in each iteration (+ the SVD computations)

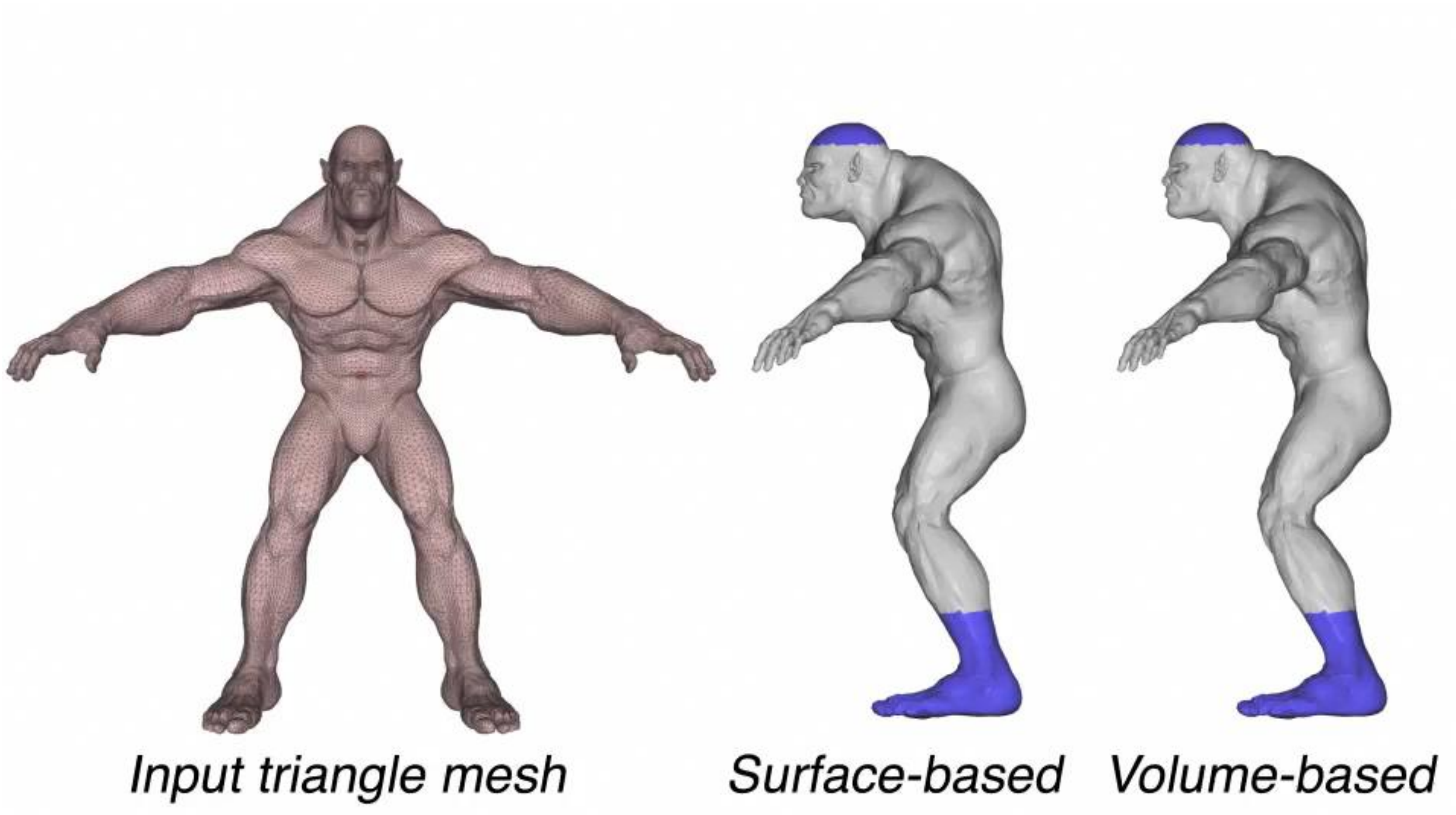
$$L\mathbf{v}' = \mathbf{b}$$

As-Rigid-As-Possible Surface Modeling

As-Rigid-As-Possible
Surface Modeling

Olga Sorkine   Marc Alexa
TU Berlin

Surface vs volumetric models



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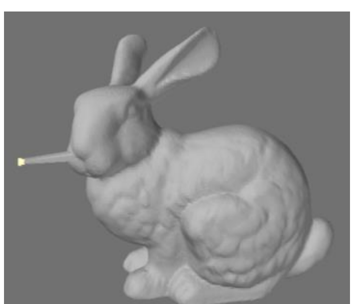
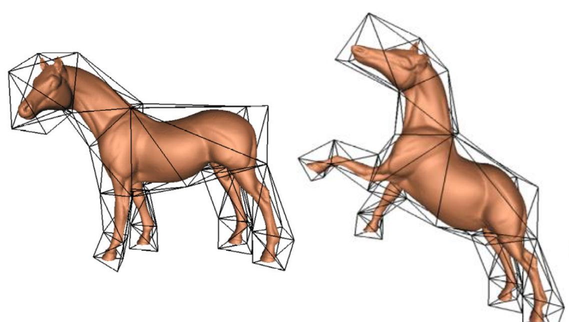
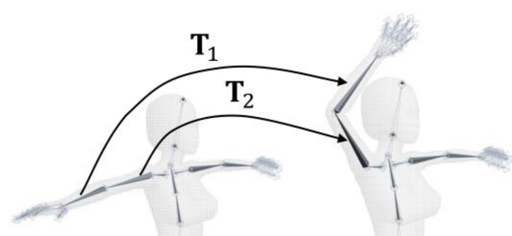
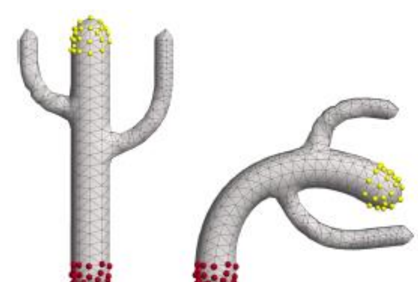
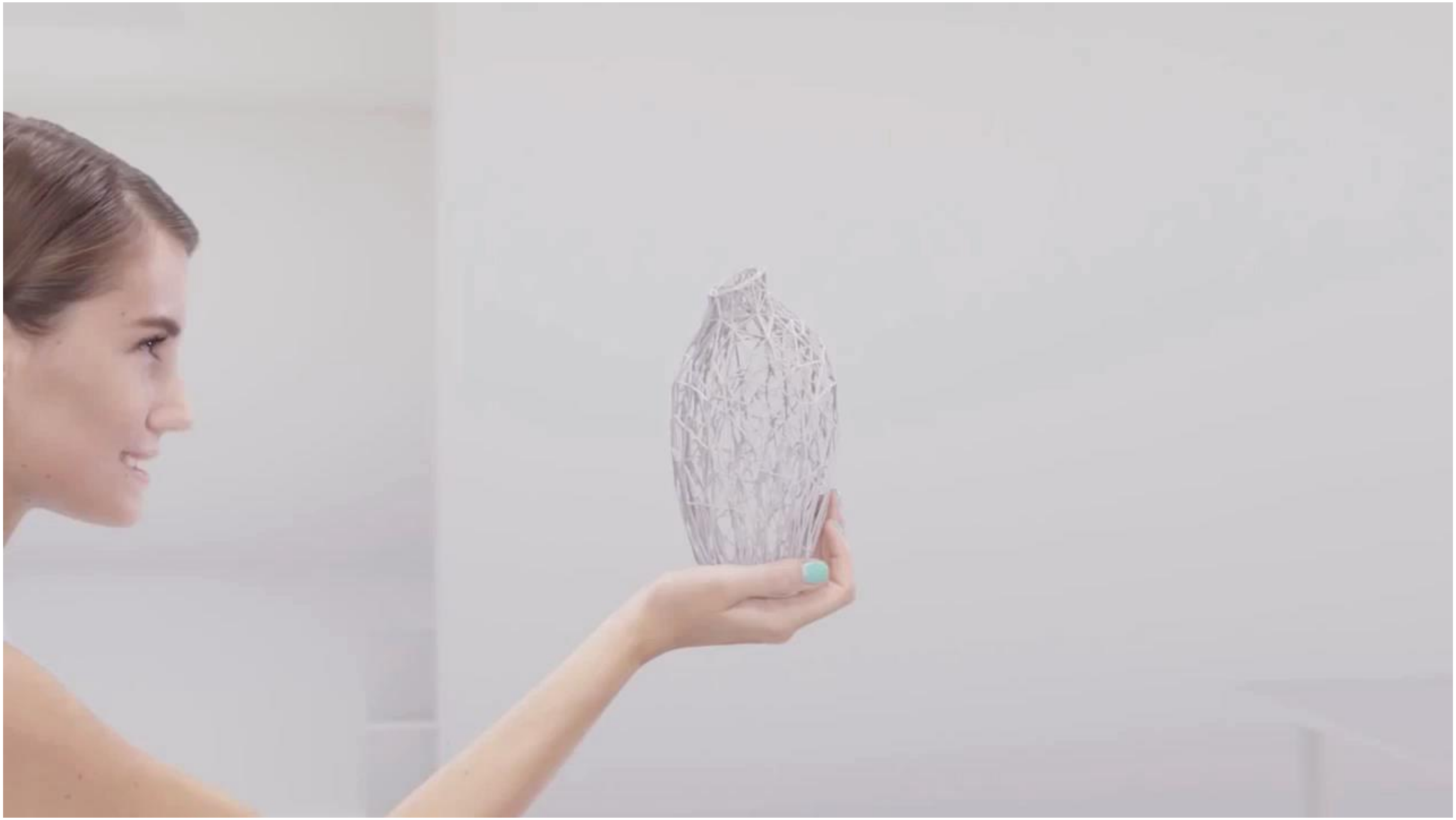




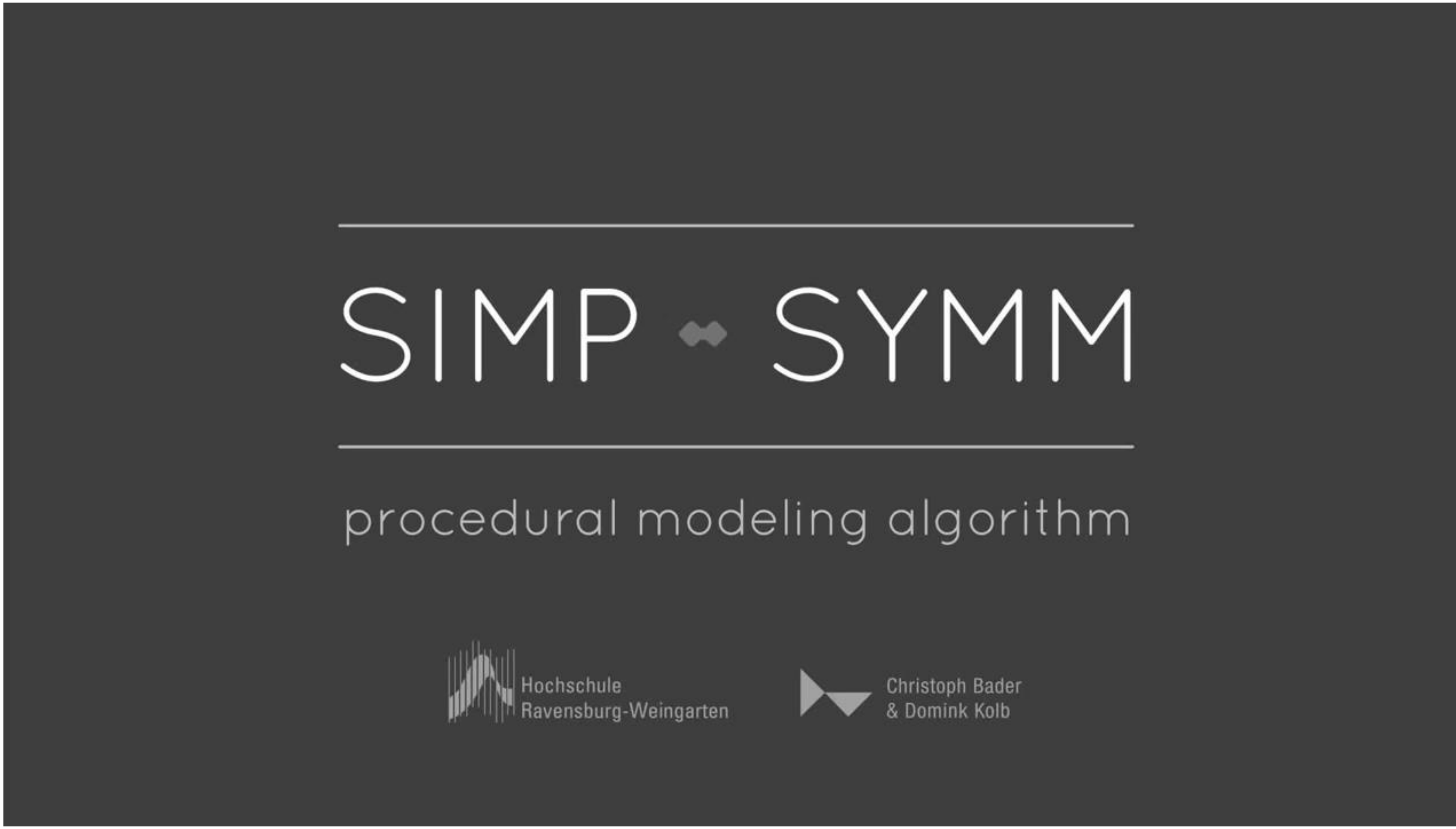




Creating new models



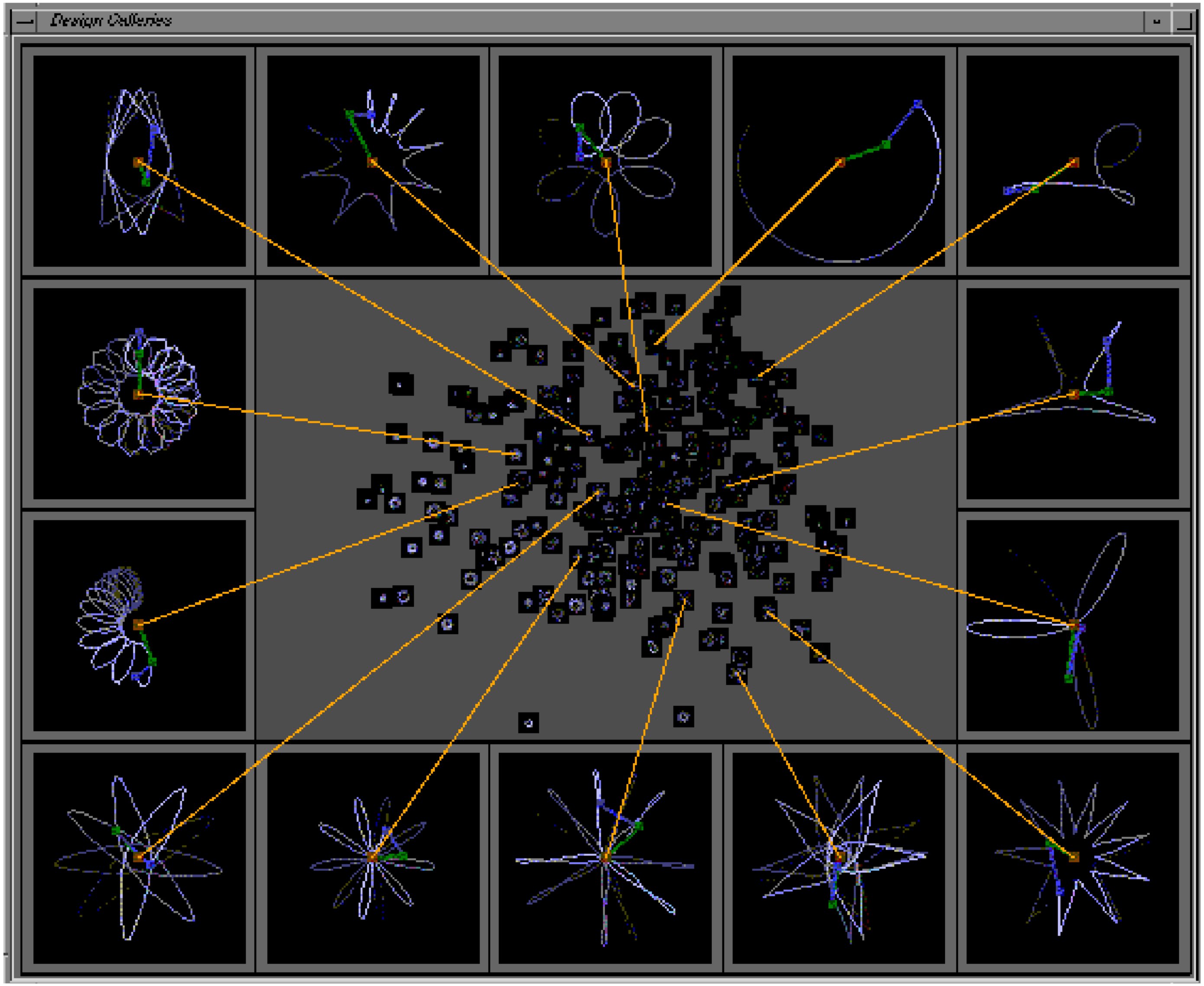
Creating new models



Creating new models

- Finding the right level of abstraction is key
  - Restrict design space to some extent
  - Trade-off between flexibility and ease of use
- Alternatives?
  - Explore design space

Design Space Exploration

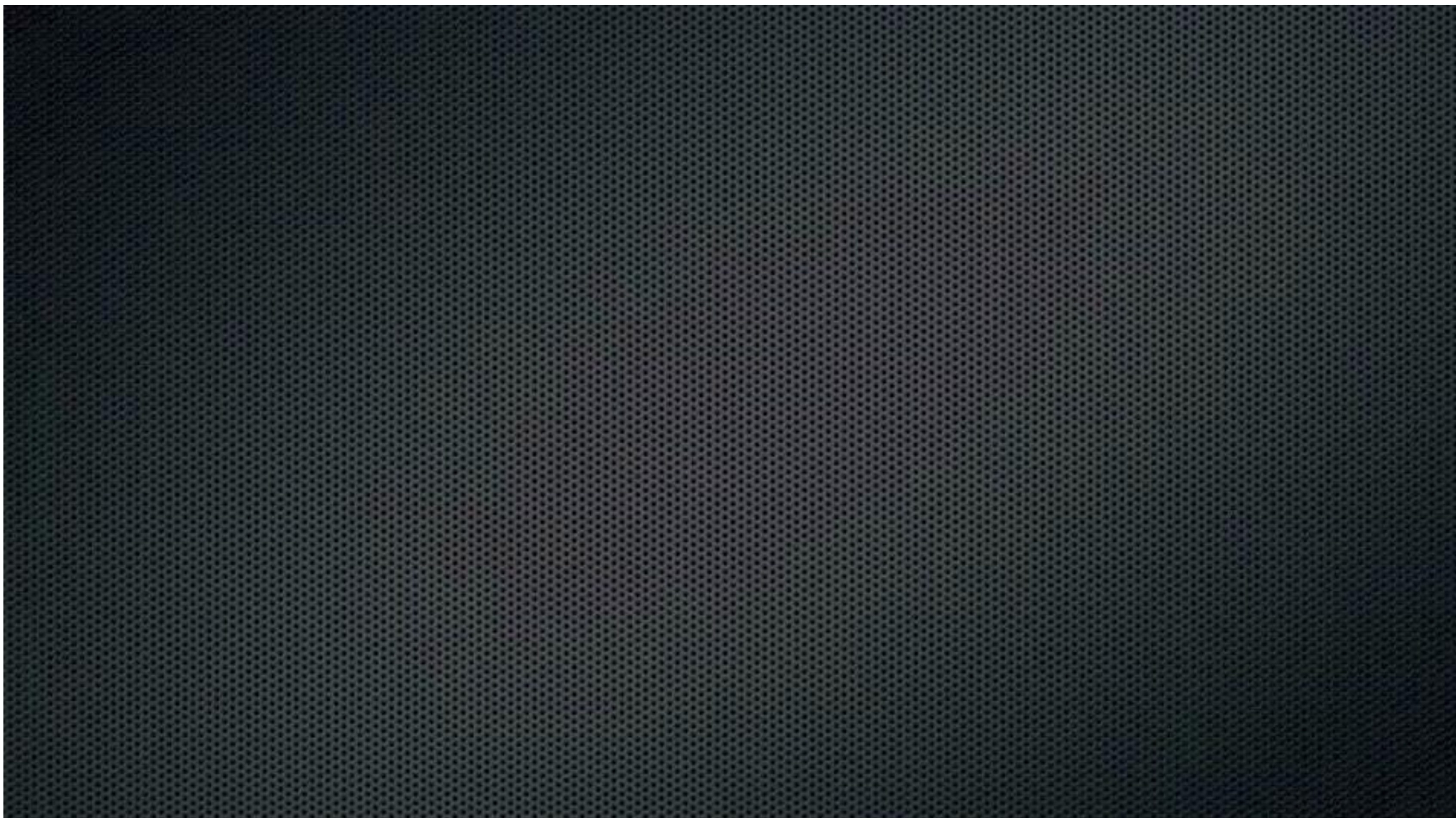


Design Space Exploration

- Sample parameter space
  - Poisson sampling
- Present design space in a manageable way
  - Cluster similar designs
  - Visualize designs exhibiting greatest variation
  - Hierarchical refinement

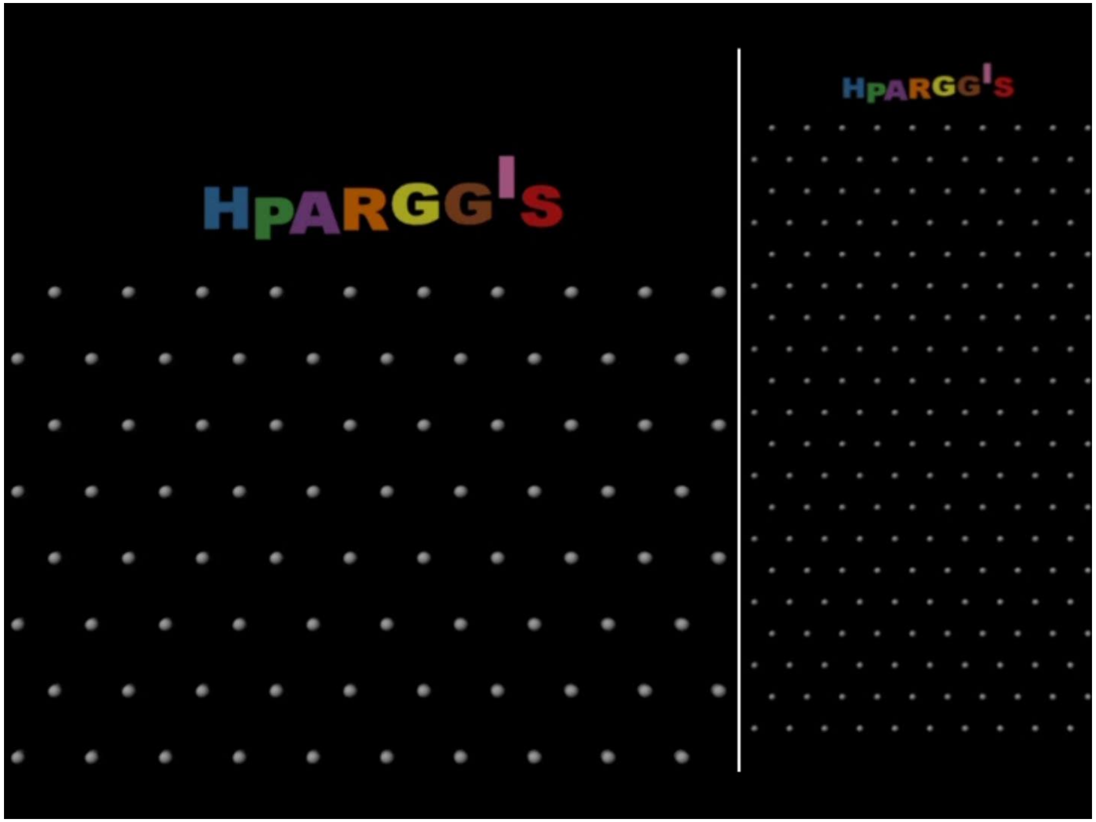
Design Space Exploration - Examples

Eric Brochu, Tyson Brochu and Nando de Freitas. ***A Bayesian Interactive Optimization Approach to Procedural Animation Design***. ACM SIGGRAPH/Eurographics Symposium on Computer Animation, 2010



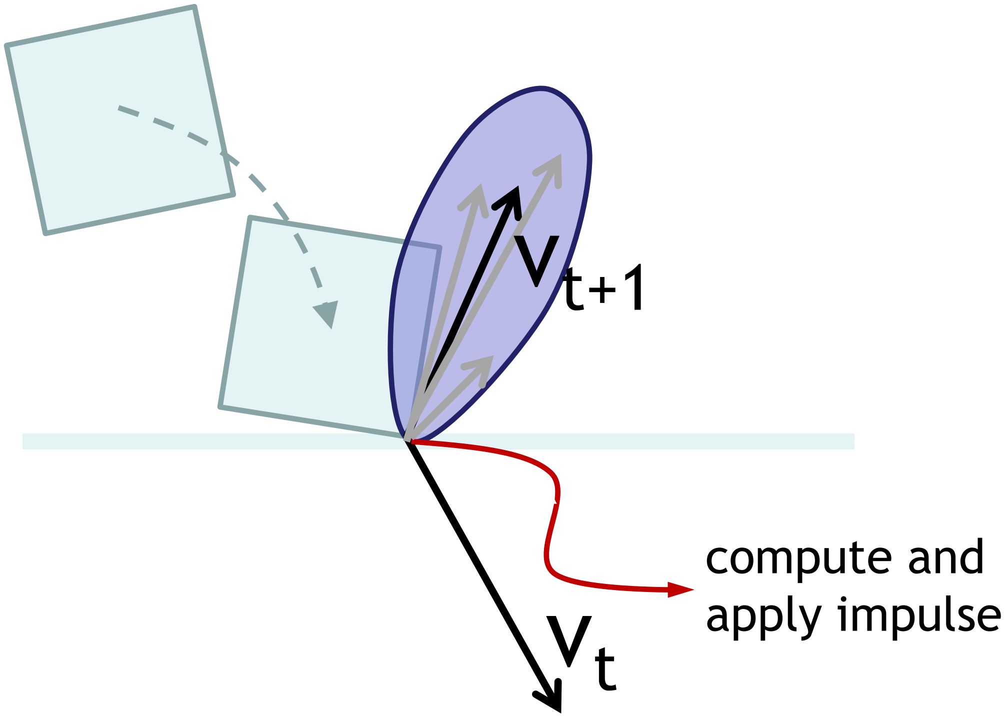
Design Space Exploration - Examples

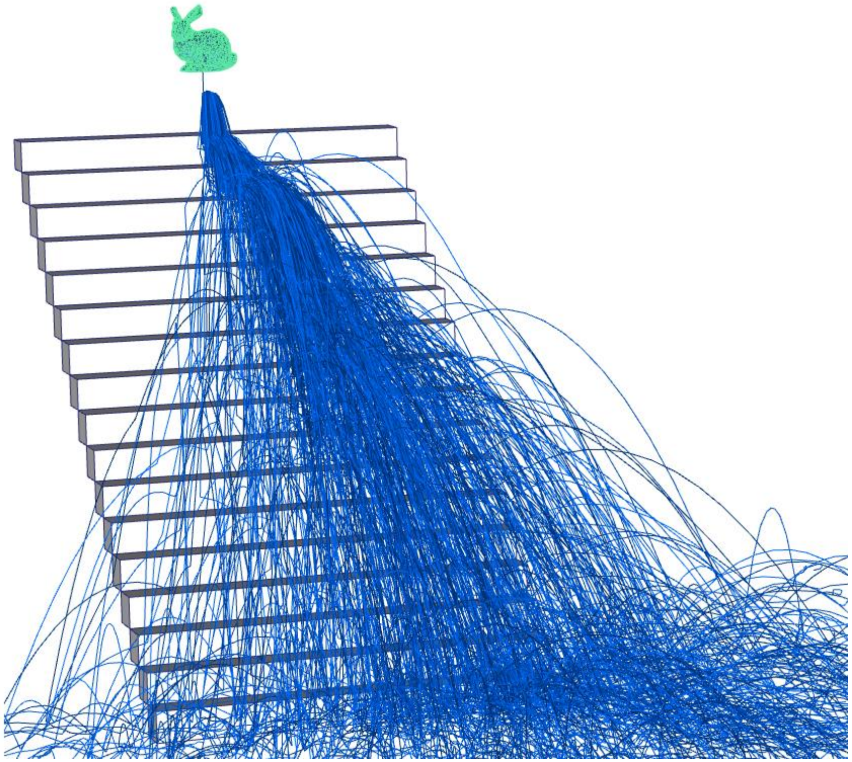
Many-Worlds Browsing for Control of Multibody Dynamics Twigg and James, SIGGRAPH 2007



Many Worlds Browsing

Sampling Plausible Worlds (parameter choices)

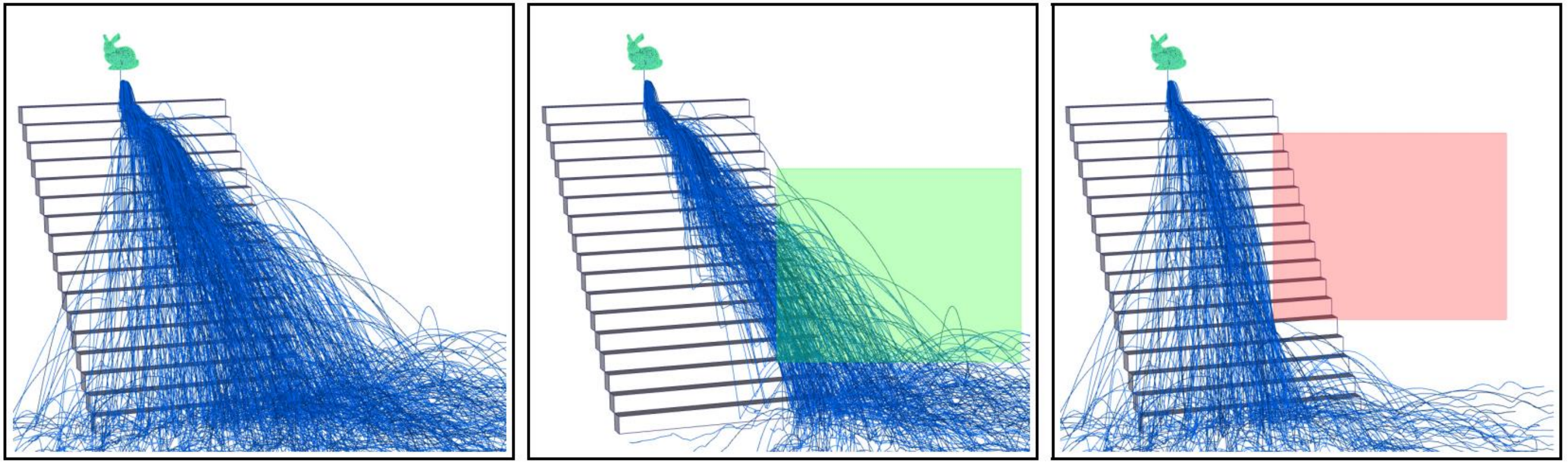




[O'Sullivan et al., 2003]

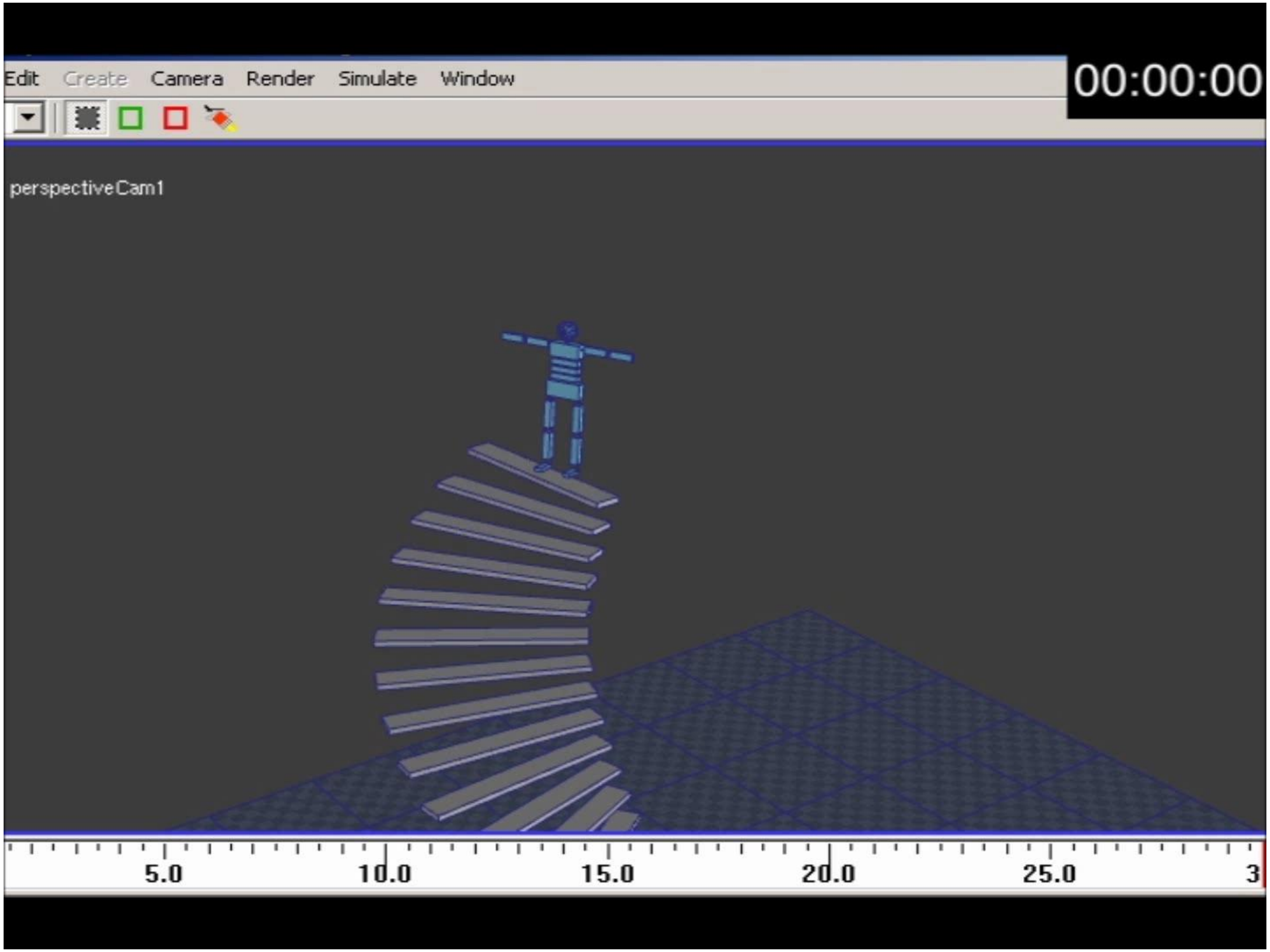
Many Worlds Browsing

Interactive Browsing – various criteria



Input   Positive   Negative

Many Worlds Browsing



Questions?