Anonymous Pricing of Efficient Allocations in Combinatorial Economies

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Abstract

Auctions and exchanges are important coordination mechanisms for multiagent systems. Most multi-good markets are combinatorial in that the agents have preferences over bundles of goods. We study the possibility of determining prices so as to support (efficient) allocations in combinatorial economies where a seller (or arbitrator) wants to implement an efficient allocation and the prices are required to be anonymous. Conditions on the existence of equilibria are presented and a particularly attractive pricing scheme is studied in detail. The relation of minimal equilibrium prices to Vickrey payments is analyzed. A procedure based on the controlled formation of alliances is suggested that shrinks economies to ensure the existence of prices coherent with the preferred pricing scheme.

1. Introduction

Auctions and exchanges are important coordination mechanisms for multiagent systems. Most multi-good markets are combinatorial in that the agents have preferences over bundles of goods. Combinatorial auctions and combinatorial exchanges have been subjects of intense study in the last few years due to their importance as a solution mechanism for combinatorial resource and task allocation problems involving self-interested, autonomous agents with private information. While the determination of efficient (or approximately efficient) allocations has been studied extensively, the important role of prices for the practical and theoretical implementability of allocations has drawn less attention (notable exceptions include [12, 13, 16, 2]).

In this paper we study different schemes for the pricing of goods and bundles in combinatorial economies where bidders have potentially non-additive preferences on goods, that is, preferences over bundles. Of natural interest are prices that support the computed allocation so that each participating agent will be satisfied with the outcome at the given prices. Different pricing schemes have different impact on the existence of such *equilibrium outcomes*. We study this in detail for a pricing scheme that minimizes the necessity to enforce the correct implementation of an intended outcome and keeps the prices anonymous.

We give a necessary and sufficient condition for the coincidence of Vickrey payments and anonymous minimal equilibrium prices. A procedure is suggested for dealing with non-existence of equilibria. It is based on controlled formation of alliances among consumers. The discussion of future possibilities closes the paper. Related work is discussed throughout the paper.

2. Pricing Schemes

We study the problem of allocating a finite set, $\Omega = \{1, \ldots . m\}$, of m indivisible resources (or *goods*) to a finite set, $N = \{1, \ldots, n\}$, of n competing agents (or *consumers*) so as to maximize the economic efficiency of the allocation, that is the sum of individual utilities of the buyers over all possible allocations. The consumers have utility for bundles of goods, given as a utility function $u_i : 2^{\Omega} \rightarrow \mathbb{Q}_0^+$ ($\mathbb{Q}_0^+$ are the non-negative rational numbers). This information is private to the bidders. No externalities occur (that is, the utilities of a buyer for bundles depend on the bundle he receives only, not on the rest of the allocation or on some information received). All goods belong to a benevolent auctioneer (or *arbitrator*), denoted by 0. A collection $E = (\Omega; u_1, \ldots, u_n)$ of the goods and the utility functions will be called an *economy*. It is the task of the arbitrator to implement an efficient allocation by means of a suitably chosen mechanism. Here, implementation includes the computation of payments, the distribution of the goods to the buyers and the consent of the buyers to payment and distribution (a participation constraint). The instrument of choice,

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to enable the elicitation of utility information and the transfer of utility, is pricing. An *outcome* of a price-based mechanism consists of an allocation and a related vector of payments which determines the amount of money each agent has to pay in order to receive the part of the allocation that is earmarked for him. The arbitrator can only hope to implement a suggested outcome if each agent accepts his part of the outcome. He will do so only if the net utility of doing so is at least as large as the net utility of any other behavioral option (we consider only purchasing decisions as allowed behavioral options). We make the standard assumption that each agent's utility is quasi-linear in money. The net utility of the participation of an agent i, v_i, can be determined as his utility for the received bundle minus the necessary payment. We will further assume that the option to purchase nothing is available for free (that is, the price of the empty bundle is zero). Additionally, an agent can get rid of any allocated good for free (there is *free disposal*). In this context, it is reasonable to restrict attention to price functions which are monotonically increasing in goods, that is, $p(x) \leq p(y)$ if $x \subset y$.

In a price-based mechanism, there is an intimate relation between announced prices and resulting payments. Furthermore, the chosen pricing scheme determines the set of purchasing options to be considered. To see this, consider the following setting. Assume that the arbitrator operates a shop.[1] Each evening the arbitrator runs an elicitation mechanism on his Web site which collects utility information from his customers for his goods and bundles of goods. The mechanism determines an efficient allocation from this information (efficient relative to the reported utilities). Early in the morning he enters his shop and executes one of the following *pricing schemes*:

1. He attaches a price tag to each good (Scheme GOOD).
2. He posts a price list with a price for each possible bundle of goods ((Scheme ALL).
3. He posts a price list with a price for each possible bundle, with the additional rule: "Only one bundle per customer!" (Scheme ANY).
4. He bundles the goods according to the efficient allocation and attaches a price tag to each resulting bundle (Scheme EFF).[2]

Now his customers visit the shop in arbitrary order. Each customer makes his individual purchasing decisions, pays, and leaves the shop. Is it possible for the shop clerk to determine his prices so that the implementation of an outcome with an efficient allocation is self-enforcing?

It is no surprise that the answer depends on the chosen pricing scheme.[3] Before we give the answer, let us study the consequences of the different pricing schemes for the purchasing options that an agent has to consider. Assume that agent 1 enters the shop and that his most preferred bundle, $\{A, B\}$, is still available (in a slight abuse of notation, AB will be used instead of $\{A, B\}$ to denote the bundle). Now, in pricing scheme GOOD, his payment for AB $t_1(AB)$ would be the sum of prices for good A and good B, $p(A) + p(B)$. His net utility of purchasing the bundle will be $v_1(AB) = u_1(AB) - t_1 = u_1(AB) - (p(A) + p(B))$. He will have to compare this to the net utility of any other possible bundle to make an optimal purchasing decision.

In pricing scheme ALL, his calculation will be different: instead of buying the bundle in two transactions (paying $p(A) + p(B)$) he may also choose to buy the bundle directly in one transaction (paying the price $p(AB)$).[4] A utility-maximizing consumer will always look for the *best possible combination of transactions* to determine the potential payment, e.g. the payment that agent 1 will consider for the bundle AB will be $\min\{p(AB), p(A) + p(B)\}$. In pricing scheme ANY, the payment for a bundle that is to be considered is the given price for the bundle, e.g. $t_1(AB) = p(AB)$. Pricing scheme EFF is similar to pricing scheme GOOD, with the notable exception that neither A and B nor AB might be available for purchasing. This would be the case if they have been packaged into bundles containing other goods as well, say AC and BD, so the payment that agent 1 has to consider is the best obtainable price or sum of prices for a bundle or a collection of bundles that contains the considered bundle.

3. Coherent Prices

To make this more precise, some formalization is helpful. We could continue to study the payments that result from the prices, but this would force us to differentiate between prices and payments. A different possibility is to consider pricing scheme ANY only (here, the payment to be considered for any bundle is equal to the given price) and to

1 Admittedly a special kind of shop, because his objective is not to maximize his income but economic efficiency ("welfare") among the bidders.

2 The decision to bundle the goods would allow us to apply each of the pricing schemes GOOD, ALL, and ANY to the new situation. We will only consider the analog of pricing scheme GOOD.

3 In the management science literature, bundling has been studied extensively as a way to optimize the income of a (mostly monopolistic) seller. There, scheme GOOD is called pure unbundling, the other schemes would fall into the category of so-called mixed bundling (for an early reference see [15]). Another "seller strategy" is pure bundling, which refers to bundling all goods into one bundle only. We require a finer-grained distinction.

4 In fact he may also choose to buy bundles A and BC or the bundle ABC–even if C does not add to his utility in the case that this promises a better deal–however, the assumed monotony of prices makes this type of considerations unnecessary.

map the other pricing schemes into *coherence* conditions on the structure of the prices of scheme ANY so that the conditions guarantee that the price for any bundle coincides with the price that an agent would have to consider in a scheme where multiple transactions are possible. First, some terminology will ease the presentation.

An *allocation* is a vectorized partition $X = (X_1, \ldots, X_n)$ of the goods in Ω, such that $\bigcup_{i \in N} X_i = \Omega$ and $\bigcap_{i \in N} X_i = \emptyset$ (because of the free disposal assumption, we can safely assume that all goods will always be distributed). A $2n$-ary vector $(X_1, \ldots, X_n; t_1, \ldots, t_n)$ will be called *outcome* if $(X_1, \ldots, X_n)$ is an allocation and $t_i \geq 0$ for all $i \in N$ (note that these values are payments to be made, so, in contrast to their sign, they have a negative effect on a consumer's utility). $\sum_{i \in N} u_i(X_i)$ is the *value* of an allocation respectively an outcome. Let $(X_1, \ldots, X_n; t_1, \ldots, t_n)$ be an outcome. Then $v_i = u_i(X_i) - t_i$ is the *net utility* of an implementation of the outcome for consumer $i \in N$.

We assume that each consumer controls his behavior autonomously (cannot be forced to purchase a bundle) and behaves individually rationally (does not pay more for a bundle than it is worth). An outcome is not implementable if $v_i < 0$ for any $i \in N$ (buying nothing at a cost of 0 is always possible). The (rational) objective of each consumers is it to maximize his net utility when presented with a choice of options. As has been said above, we will consider purchasing options only (though this will be extended below to a form of controlled collusion). The available options are determined by the pricing scheme. A (monotonic) price function and the coherence conditions for the different pricing schemes can now be defined as follows.

Let $E = (\Omega; u_1, \ldots, u_n)$ be an economy. We call a function $p : 2^\Omega \rightarrow \mathbb{Q}_0^+$ a *price function*, if $p(\emptyset) = 0$. (Abusing notation slightly, we sometimes speak of a price vector and write p_x instead of $p(x)$.)

Definition 1 (Coherent Prices). *Given a price function $p : 2^\Omega \rightarrow \mathbb{Q}_0$, to be used with pricing scheme* ANY.

- $p(\cdot)$ *is* coherent *with scheme* GOOD *if*

$$p(x) = \sum_{z \in x} p(\{z\}) \quad \forall x \subseteq \Omega, x \neq \emptyset. \qquad (1)$$

- $p(\cdot)$ *is coherent with scheme* ALL *if*

$$p(x) = \min_{Z \in \Pi(x)} \sum_{z \in Z} p(z) \quad \forall x \subseteq \Omega, x \neq \emptyset. \qquad (2)$$

Above, $\Pi(x)$ is the set of all possible partitions of x.[5]

- *Furthermore, every price function is coherent with scheme* ANY.
- *For scheme* EFF*, the coherence is relative to an allocation, that is, $p(\cdot)$ is coherent with pricing scheme* EFF*, if an allocation X exists such that*[6]

$$p(x) = \sum_{z \in X, z \subseteq x} p(z) \quad \forall x \in 2^X, x \neq \emptyset \qquad (3)$$

and

$$p(x) = \min_{z \supset x, z \in 2^X} p(z) \quad \forall x \notin 2^X, x \neq \emptyset \qquad (4)$$

We also say that the prices are coherent *with the allocation X.*

Proposition 2 (GOOD ⇒ EFF ⇒ ALL).
Let $p(\cdot)$ be a price function. If $p(\cdot)$ is coherent with scheme GOOD*, it is also coherent with scheme* EFF*. If $p(\cdot)$ is coherent with scheme* EFF*, it is also coherent with scheme* ALL *(for the sake of brevity, straightforward or non-essential proofs are omitted.)*

Further on, we will assume that the setting is that of scheme ANY (at most one bundle per consumer is allowed).
Pricing schemes GOOD and ANY have been studied extensively in the literature (see [10, 6] for scheme GOOD and [16] for scheme ANY). Both show deficiencies: Prices in scheme GOOD that self-enforce (or, a little bit weaker: support) efficient allocations are only guaranteed to exist under rather strong conditions (gross-substitutes). While supporting prices in scheme ANY do always exist, they require strict means of enforcement to ensure the "correctness" of an implementation—in the above example, enforcing the rule that each consumer may purchase at most one bundle would require to register the customers (this de-anonymization is required to prevent them from making multiple purchases throughout the day). It is also advisable to prevent them from sending in a friend that acts as a buyer and hands his purchase to the original agent. Such enforcement is certainly not viable or desirable in all settings, especially on the Internet where pseudonyms tend to be cheap. We will focus on pricing scheme EFF (we will sometimes write *coherent prices* instead of *prices coherent with scheme* EFF).

Now, as the dependency of the purchasing decision on the pricing scheme is hidden in the coherence condition, we can use the prices directly in the definition of the net utilities that are to be considered. So, given a suitable price func-

5 The above condition is also used regularly in the management science literature, see for example [8].

6 To understand the following notation note that *power set* and *element-of operator* are used here on a sequenced partition X (ie., an allocation) in a canonical extension of their usual meaning. The *elements* of X are the sets $X_i \subseteq \Omega$. The *power set* 2^X consists of all combinations of the *elements* of X. We write $x \in 2^X$ for a $x \subseteq \Omega$ if a partition of x exists such that every element of the partition (itself a subset of Ω) is an element of X (in other words: the partition is an element of 2^X).

tion $p(\cdot)$, $v_i^p(x) = u_i(x) - p(x)$ denotes the net utility of agent i for bundle x.

Let us return to the above example and consider the first customer, say i, entering the store. He is faced with the whole range of purchasing options. Let us restrict our attention to prices that are coherent with scheme EFF. To make his purchasing decision, he will have to pick the optimal way to purchase each bundle. *Because of the coherence conditions, it is not necessary to consider multiple transactions* – the one-transaction price given for a bundle is already minimal. He will have to compare the obtainable net utility with the net utility related to any other bundle, that is, he has to solve the problem $\arg\max_{x\subseteq\Omega} v_i^p(x)$ at the given prices p (where the price of a bundle that is not directly available has to be computed as outlined at the end of Section 2).

4. Equilibria

To ease the handling of race conditions and indifference, we will switch now from a shop environment to a distribution environment where goods and bundles are presented in a catalog. Once the arbitrator has determined an efficient allocation from certain valuation information, he will determine prices coherent with the chosen pricing scheme. He will then send the price list (and the additional condition of scheme ANY) to the participating consumers. Each consumer will determine a set B containing all bundles that maximize his net utility at the given prices. He will then submit a list of mutually exclusive orders of individual bundles, containing all bundles from B. Once the arbitrator has received the orders, he will distribute the goods to the customers so as to maximize efficiency. If a customer receives one of the requested bundles, he will clearly be satisfied with the outcome. If every customer receives a requested bundle and if the objective of the arbitrator is fulfilled by the resulting allocation, the outcome determines an equilibrium.

Definition 3 (Satisfied, Supports). *A consumer i is* satisfied *with an allocation X at given prices $p(\cdot)$, iff the bundle X_i he receives maximizes his net utility, that is*

$$v_i^p(X_i) \geq v_i^p(z) \quad \forall z \subseteq \Omega.$$

The price function supports *an allocation X if every agent $i \in N$ is satisfied with X.*

Definition 4 (Equilibrium). *Let E be an economy, X an allocation, and $p(\cdot)$ a price function. The pair $(X; p(\cdot))$ is an* equilibrium *, if every participant is individually satisfied with the induced outcome. In the considered situation, this corresponds to*

Arbitrator: *$(X_1, \ldots, X_n)$ is an efficient allocation.*

Consumer: *Every consumer $i \in N$ is satisfied with X_i.*

If such an outcome exists for an economy and a given price function, the price function will be called equilibrium price function.

It might be surprising that one of the standard results, the first theorem of Welfare economics, has been turned into an ingredient of the definition – namely that every equilibrium[7] is efficient. This is due to the situation under study: the key property of an equilibrium is that all agents are individually satisfied with the result. Here, one of the agents (the arbitrator), has preferences for complete allocations, which explains the fact that a global social criterion (efficiency) coincides with a criterion for individual satisfaction.

A key question for the arbitrator now is if, for every resource allocation problem and a given pricing scheme, a price function exists that supports an efficient allocation. A price function supports the efficient allocation if each buyer receives the bundle that maximizes his net utility over all bundles for the given price function. This is the case for pricing scheme ANY:

Proposition 5. *For any economy E under pricing scheme* ANY*, an outcome with an efficient allocation and a supporting price function exists.*

This has been shown in [16] as a consequence of results presented in [11]. Such a result does not hold for scheme GOOD, as the following simple example demonstrates:

		A	B	AB
Utility	Agent 1	0	0	**3**
	Agent 2	2	2	2
Prices	GOOD	≥ 2	≥ 2	$p(A)+p(B) \leq 3$
	ALL,ANY	2.1	2.1	2.5
	EFF	2.5	2.5	2.5

From the conditions that follow from the necessity to satisfy both agents, a contradiction follows immediately. The prices given for the other schemes are, in contrast, equilibrium prices.

As we stated above, we consider pricing scheme EFF the scheme that combines a significant design flexibility (it allows us to solve an extended set of allocation problems compared to scheme GOOD) with a reduced necessity for enforcement (in contrast to scheme ANY). However, scheme EFF does not solve all existence problems that are due to combinatorial (that is, non-additive) preferences immediately, as the following example demonstrates:

7 Note that this does not refer to *strategic equilibria*. An interesting study of such equilibria with respect to buyer-induced bundling (where buyers in equilibrium restrict their reports to a certain set of bundles) is [9].

		A	B	AB
Utility	Agent 1	**5**	5	5
	Agent 2	0	**3**	3
	Agent 3	0	0	7
Prices	GOOD,ALL,EFF	$\leq p_B$	≤ 3	$p_A + p_B \geq 7$
	ANY	1	2	7.1

We will, however, demonstrate below that the initial economy can be modified without an impact on efficiency such that equilibrium prices coherent with scheme EFF exist. The basic idea is to shrink the economy by creating alliances of agents that submit joint bids. Reconsider the above example with an alliance of agents 1 and 2.

Utility	A	B	AB
Agent (1+2)	5	5	**8**
Agent 3	0	0	7
Scheme EFF prices	7	7	7

To better understand the situations for which no equilibrium exists, consider the following: Assume that an efficient allocation has been determined from truthfully reported utilities. In this allocation, each consumer receives a certain bundle of goods. We will now create new goods by packaging each of these bundles permanently together, for example A, B, and C may form a new good $\overline{ABC}$. Doing so will also thin out the utility function, the utilities for now unavailable goods become irrelevant.

This condensed economy has a simple assignment solution: each consumer receives exactly one good. To find this efficient allocation, it would be sufficient to receive bids for the goods only–no utilities for non-singleton bundles are required. This would correspond to a situation where all goods are perfect substitutes and the utility for a bundle equals the maximum utility of the goods contained in it.

Now the catch is the following: equilibrium prices for this simple assignment economy always exist [5] and the minimal equilibrium prices coincide with Vickrey payments [11] for the reduced assignment economy.[8] If we consider the reduced combinatorial economy with all utilities for the bundles of the newly created goods, all results of Gul and Stacchetti [6] can be applied to it. For example, a sufficient condition for the existence of equilibrium prices is the gross substitutes condition of Kelso and Crawford [10] (or, with monotonic utilities, the no-complementarities condition [6]). Note that the equilibrium prices of scheme GOOD and the corresponding existence conditions carry over to our consideration of scheme EFF in the original economy–that is:

Theorem 6. *An coherent equilibrium in scheme* EFF *for an economy exists if and only if an equilibrium in scheme* GOOD *for one of the reduced combinatorial economies that correspond to one of the efficient allocations exists.*

The proof is straightforward and is based on two observations. First, the reduced combinatorial economy leaves out details of the utility functions that do not impact efficiency. If no equilibrium for this reduced set of information exists, there can be no equilibrium for the complete set of information. Second, each equilibrium for the reduced combinatorial economy can be extended to an equilibrium for the original economy: the aggregated goods can be allocated as bundles of the original goods (which leaves the supported efficient allocation essentially untouched), and the prices for the goods and bundles in the reduced economy can be extended to prices for the goods and bundles in the original economy without violating the equilibrium conditions or the coherence conditions.

The extension works as follows. X_i is the bundle that is allocated to i in the chosen efficient allocation. X_i corresponds to one aggregated good in the reduced combinatorial economy. The equilibrium price for this aggregated good in the reduced combinatorial economy is taken as the price for the bundle X_i, that is $p(X_i) = p^r(X_i)\ \forall i \in N$. Each bundle that can be formed from bundles in the efficient allocation is priced additively, that is $p(z) = \sum_{x \in z} p(x)\ \forall z \in 2^X$. Each remaining bundle is obtainable for the minimal price of a covering bundle, that is $p(y) = \min_{y \subset z, z \in 2^X} p(z)\ \forall y \notin 2^X$. It is immediate to see that a violation of the equilibrium conditions would imply a violation of efficiency and would thus contradict the efficiency assumption.

Unfortunately, the minimal equilibrium prices do not necessarily coincide with the Vickrey payments for the relevant (ie., the allocated) bundles. This also has been shown in [6] and carries over to the extended equilibrium. In fact, the Vickrey payments are lower bounds for the prices of the allocated bundles.

In view of the desired property that no incentive should be given to misreport utilities, it is of importance to identify the situations where Vickrey payments can be extended to (minimal) equilibrium prices. Only in such cases can we hope to achieve our initial goal to determine efficient allocations with the help of anonymous price that can be implemented without enforcement.

This necessary and sufficient condition can be formulated directly as follows:

Proposition 7 (Vickrey payments as prices). *Vickrey payments and minimal equilibrium prices in an economy E coincide if and only if an efficient allocation X exists such that the condition*

$$\sum_{X_j \in z} u_j(X_j) - u_i(z) \geq\ V(E_{-i}) - \sum_{X_j \in z} V(E_{-j}) \quad (5)$$

8 Also note that these prices can be determined at polynomial cost once an efficient bundling is known, for example with an Linear Program with integral solutions, see [11].

holds for all agents $i \in N$ and all bundles $z \in 2^X$. Here, $V(E_{-l})$ is the value of the best allocation if agent l is removed from the economy E.

Proof: Below, we use $t(l)$ to denote the Vickrey payment of agent l. We use E^{-B} to denote the removal of a bundle B from an economy E. Now, starting with equilibrium conditions, (add $\forall z \in 2^X, i \in N$ to each line)

$$u_i(X_i) - t(i) \geq u_i(z) - \sum_{X_j \in z} t(j)$$

$$\Leftrightarrow\ V(E) - V(E_{-i}) \geq\ u_i(z) - \sum_{X_j \in z} \left(V(E_{-j}) - V(E_{-j}^{-X_j})\right)$$

$$\Leftrightarrow \sum_{X_j \in z} u_j(X_j) - u_i(z) \geq\ V(E_{-i}) - \sum_{X_j \in z} V(E_{-j})$$

□

This result sets the limits for what can be achieved with price-based mechanisms that stick to an anonymous, enforcement-free price structure and want to guarantee efficiency (which requires incentive compatibility and participation incentives, and thus depends on the above condition).

If no (minimal) equilibrium prices exist, the arbitrator may choose to ask selected agents to cooperate (“collude”). Besides the bundling of goods, this “bundling” of agents is another possibility to make complementarities irrelevant. This will be analyzed in the next section.[9]

5. Shrinking the Economy

The basic idea is as follows: once an efficient allocation has been determined, maximal equilibrium prices based on the bids for the efficient bundles (the bundles of goods that are allocated to agents in an efficient allocation) can be determined easily, for example with the (dual) LP given in [11]. Recall the above reduction of the economy to a simple assignment economy. These prices cannot be increased without violating the equilibrium conditions for the goods (not to speak of the bundles). Now the arbitrator can check whether the equilibrium conditions are violated for the bundles that can be formed by bundling the pre-packaged efficient bundles. Assume that this is the case for a bundle, say $\overline{AB}\ \overline{CD}$. Due to competition for the included goods $\overline{AB}$ and $\overline{CD}$, neither the prices of $\overline{AB}$ nor $\overline{CD}$ can be raised. Now, if all agents that receive a good in the bundle (here the agent receiving $\overline{AB}$ and the agent receiving $\overline{BD}$) would form an alliance and would jointly bid their added utilities, the prices that would result from computing maximal equilibrium prices for the now shrunken economy could not violate the equilibrium condition for the new good $\overline{\overline{AB}\,\overline{CD}}$ anymore (or the assumed efficiency would be contradicted).

9 A formalization is available in an extended version.

Of course, agents could refuse to form an alliance, or they may be unable to arrange for an efficient distribution of the aggregated good. Note however that, if the agents accept and create efficient alliances (ie., alliances that split the bundle they receive so as to maximize efficiency among the agents in the alliance), it is immediate that the resulting allocation is efficient and that the price vector after the final iteration of the shrinking process outlined above is an equilibrium price vector for the shrunken set of agents and the shrunken set of goods. A crucial operation for each alliance is of course the split of the payment – this will either depend on the announced utility of the agents (violating incentive compatibility) or the agents will run the risk (if it is constant and non-zero) that they run into a loss (violating participation constraints). So, as a consequence, a mechanism that utilizes the formation of alliances cannot have both properties.

An interesting variant of the shrinking mechanism is the following: Instead of leaving the splitting of bundles and payments to the agents, the arbitrator, once it has received the bids necessary to compute an efficient allocation[10], could compute payments for all agents as if they would have accepted all alliance formations and distribute the efficient bundles (without aggregating them) directly to the correct agent. Of course, participation and honesty would depend on the rule for the computation of the payments. However from a practical perspective, the arbitrator can keep the formation of alliances and aggregated bundles largely invisible to the individual agents: every agent can be provided with an individualized[11] equilibrium price vector that supports the bundle he receives. In cases where it is difficult or expensive for the agent to obtain information about the utilities, bids and/or price vectors of the other agents, the agent has no practical handle to plan his bidding strategically, especially if the arbitrator guarantees that in the case that condition (5) holds for the reported utilities, minimal equilibrium prices will be determined:

Instead of stopping as soon as the existence of an equilibrium is established, the shrinking process could be continued until the equilibrium prices coincide with Vickrey payments (again if considered for the shrunken set of agents where “colluding” agents are represented by a new *alliance agent*). In case of truthful reporting, this would perfectly internalize the negative effect of each alliance on the other participants, thus no incentive to introduce false names (compare [17]) would remain for the alliances.

Note that the shrinking procedure that is outlined above will remain incentive compatible as long as (5) is fulfilled (because no alliances will be formed). However, the participating agents cannot know beforehand that this will be

10 For example with a partially revealing combinatorial auction as described in [4].

11 This breaks anonymous pricing.

the case, so, to be sure, they would have to collect information about the competitors to finally recognize that this information is not necessary (unless they can be used for bribing, [14]). In principle, this will be a significant problem for all the mechanisms that rely on conditions that the agents' utility functions have to fulfill (like the English auction in [7] or the ascending auction in [1])–neither the participants nor the arbitrator can be sure of the properties that the other agents' utility functions exhibit. Note that, for example, a violation of the individualistic gross substitutes condition can be detected easily from the *reported utilities*, but, if an agent that violates it participates in an auction that *requires* it, the agent will misreport his valuations accordingly. This will compromise efficiency even though the auction pretended to be incentive compatible for cases where the condition is fulfilled–but this will generally not be known a priori.

6. Conclusions and Future Research

We introduced the notion of coherent equilibria to concisely capture the differences between pricing schemes. We suggested a pricing scheme for which enforcement-free, anonymous equilibrium prices exist in a wider range of situation than in the classic prices-for-basic-goods-only scheme. The problematic complementarities can be neglected for sub-bundles of the bundles in the efficient allocation. We have also discussed the existence of equilibrium prices in various price schemes. We introduced a necessary and sufficient condition for the coincidence of (extended) Vickrey payments and minimal anonymous equilibrium prices. If threshold problems foreclose the existence of equilibrium prices, a procedure that shrinks the economy by forming alliances can be applied. This procedure may iterate and eventually produces an economy for which coherent anonymous prices exist.

Some of the results can be extended directly to a setting with income-maximizing sellers and an arbitrator interested in implementing an efficient allocation. For example, the process of forming alliances can symmetrically be applied to the seller side (which can be used to split the surplus from selling a bundle that consists of goods from different sellers). Two-sided markets with an arbitrator that can control the flow of information between buyers and sellers will be analyzed in future work.

In situations that call for anonymous, enforcement-free prices for outcomes to be implementable, prices that are coherent with scheme EFF are especially attractive compared to the traditional prices of scheme GOOD or the prices obtained for the more recently suggested scheme ANY. Prices for scheme ANY are not truly anonymous. If anonymity is not an issue, the results presented in [12, 3] (which may require enforcement) become relevant. A controlled shrinking of the economy, and the suggested partial differentiation of prices between sellers and buyers, may have an interesting impact on their results if enforcement is not an option. However, the strategic impact of the formation of alliances will have to be considered carefully (in general, no incentive compatibility can be expected). Although, positive analytic results will be difficult to obtain in the general situation, studying behavioral options in selected application domains will give new insights and remains for future work.

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