Due Tuesday, 4/20/2004 in the beginning of class. In this homework, you may use any sources that you want but you must cite the sources that you use. Teamwork is not allowed. If you have questions, the instructor’s office hour is Tuesdays 3–3:30 pm in Wean Hall 4606 (that is, three doors down from the classroom, right after class).

1. (33 pts) Throughout this question, you may restrict your analysis to pure strategies, that is, you do not have to consider mixed strategies. At the root of the tree below, it is agent 1’s turn to move. At the node marked “2”, it is agent 2’s turn to move. At the end of every branch, the payoffs are shown (agent 1’s payoff is listed first).

   (a) Draw the strategic form (aka. normal form aka. matrix form) of the following game tree.

   (b) Name the dominant strategy equilibria of this game, if there are any.

   (c) Name the Nash equilibria of this game, if there are any.

   (d) Name the subgame perfect Nash equilibria of this game, if there are any.

   (e) Name the Pareto efficient outcomes of this game, if there are any.

   (f) Name the social welfare maximizing outcomes of this game, if there are any.

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\begin{array}{c}
3, 3 \\
/ \ \\
/ \ \\
/ \ \\
1 \ \\
/ \\
/ \\
/ \\
2 \ \\
/ \\
/ \\
2, 7
\end{array}
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2. **Game of chicken.** Let there be two teenagers who play the following risky game.

They head toward each other in separate cars. Just before collision, each one has the choice of continuing straight, or avoiding collision by turning right. If both continue, they will die in the collision. If one continues and the other turns, both survive but the former becomes the hero and the latter is humiliated. If both turn, both survive but both are moderately humiliated. Let the game be represented by the following normal form:

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                  straight       turn
Agent 2
---------------------------
straight      | -3, -3 2, 0 |
Agent 1       |         |
turn          | 0, 2    1, 1 |
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a. (8 pts) Does this game have pure strategy Nash equilibria? If so, what are they?
b. (20pts) What are the mixed strategy Nash equilibria of this game?
c. (5pts) In each equilibrium, what is the probability that the youngsters will die?

3. (33pts) Consider a private-values auction of one good when bidders have quasilinear utility functions and know their own valuations. Prove that bidding truthfully is a dominant strategy in the Vickrey auction. (There might be other equally good strategies, but do not worry about those.) [Prove this from first principles; do not use the fact that the Vickrey auction is a special case of the Clarke tax mechanism.]

4. (34pts) Write a short essay (max 1 typed page) on proxy bidder agents in auctions. Specifically, discuss what the implications are for truth-telling and for social welfare. Discuss these two questions both for private-value auctions and for correlated-value auctions.