CS 15-892 Foundations of Electronic Marketplaces
Homework 3

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Due December 2nd in the beginning of class.

This homework is self-contained so you will not need any sources beyond the course materials. However, you may use any sources that you want. If you do so, you must cite the sources that you use. Teamwork is not allowed.

1. Review questions. Use at most 10 sentences for each answer. You do not need to write any formal proofs, but you should justify your answers. You are allowed to refer to known results in those justifications.

   - (5 pts) From a game theory perspective, is it possible to build a combinatorial exchange that leads to a Pareto efficient outcome in dominant strategy equilibrium (we require that participation is voluntary, and that the exchange is *ex post* budget-non-negative (that is, the auctioneer does not need to subsidize the exchange))? What about in Bayes-Nash equilibrium? [You can assume that each bidder knows his own valuations on all bundles, and that communication and computation are no problem.]
   - (5 pts) From a game theory perspective, is it possible to build a combinatorial reverse auction that leads to a Pareto efficient outcome in dominant strategy equilibrium (we require that participation is voluntary, and that the exchange is *ex post* budget-non-negative (that is, the auctioneer does not need to subsidize the exchange))? What about in Bayes-Nash equilibrium? [You can assume that each bidder knows his own valuations on all bundles, and that communication and computation are no problem.]
   - (5 pts) From a game theory perspective, is it possible to build a combinatorial auction that leads to a Pareto efficient outcome in dominant strategy equilibrium (we require that participation is voluntary, and that the exchange is *ex post* budget-non-negative (that is, the auctioneer does not need to subsidize the exchange))? What about in Bayes-Nash equilibrium? [You can assume that each bidder knows his own valuations on all bundles, and that communication and computation are no problem.]
   - (5 pts) What is the computational complexity of finding a feasible solution in a combinatorial auction, combinatorial reverse auction, and a combinatorial exchange? How, if at all, do these complexities change if the bidders can submit XOR-constraints between some of their bids?

- (5 pts) How well can the winners in a combinatorial auction, combinatorial reverse auction, and a combinatorial exchange be approximated in polynomial time? How, if at all, do these complexities change if the bidders can submit XOR-constraints between some of their bids?
- (5 pts) What is the computational complexity of finding an optimal solution in a combinatorial auction, combinatorial reverse auction, and a combinatorial exchange? How, if at all, do these complexities change if the bidders can submit XOR-constraints between some of their bids?

2. (30pts) In a combinatorial auction that uses the GVA (that is, Clarke tax mechanism), the payment that bidder i has to make is computed as follows. First, determine the winners. Call the sum of the winning bids of the other agents (except i) a. Then, determine the winners again without i's bids. Call the sum of the winning bids b. Now, agent i pays $b - a$.

   Another generalization of the Vickrey auction to the combinatorial auction setting would determine i's payment differently as follows. For each winning bid S of agent i, let a^i_S be the sum of the prices of the *other* winning bids (by agent i and by the other agents). Then, determine the winners again with bid S removed. Call the sum of the winning bids b^i_S. Now, the "price" of bid S is $b^i_S - a^i_S$. The amount that agent i has to pay overall is

$$\sum_{S \in \text{i's winning bids}} b^i_S - a^i_S$$

   Is this mechanism incentive compatible? If so, prove that. If not, show a manipulation.

3. (**Extra Credit.** 25pts)

   A plutocrat football-team owner has installed a massive luxury suite in his new stadium. In this problem, you will use your knowledge of game theory and mixed-integer programming to get him the highest dollar for season tickets to his luxury suite by designing an optimal auction.

   Here, there are three possible agents interested in the tickets, each of whom has one of two possible valuations:

   - The *A*dvertising Executive. He's got a reasonable shot at needing to entertain a big account. He values the tickets at 400 thousand dollars with probability 1/4, and values the tickets at 100 thousand dollars with probability 3/4.
   - The Lumber *B*aroness. Being a comfortable and secure captain of industry, she is relatively comfortable and secure in her valuations. She values the tickets at 300 thousand or at 250 thousand dollars, each with probability 1/2.
   - The Disgraced *C*EO. She's currently facing federal charges, but there's a small chance she could wiggle her way out of liability. She values the tickets at 500 thousand dollars with probability 1/6, and values the tickets at 75 thousand dollars with probability 5/6.

All values for the agents are drawn independently. Agents know their own valuations, and the probabilities of agents having their valuations are common knowledge, including common knowledge to the auctioneer. Therefore, the revenue-maximizing auction maximizes revenue *in expectation*.

- (5pts) Write out and solve the MIP corresponding to the *revenue-maximizing, individually-rational, efficient, truth promoting in DSE* auction. That is, describe the allocations and payments for all 8 possible inputs.
- (5pts) Solve for the *revenue-maximizing, individually-rational auction that truth promotes in DSE*. What is the expected revenue? How often are the tickets not sold to the highest bidder? How often are the tickets not sold at all? (Recall that you can model the tickets not being sold by introducing a dummy agent into the MIP.)
- (5pts) Solve for the *revenue-maximizing, individually-rational auction that truth promotes in BNE*. What is the expected revenue? How often are the tickets not sold to the highest bidder?
- (10pts) Write out a MIP for the expected revenue of the revenue-maximizing, individually-rational auction that is truth-promoting in BNE, subject to the tickets being sold to the highest bidder (i.e., the auction being efficient) with probability at least p. Plot expected revenue against $p \in [0, 1]$.