Mechanisms for Dynamic Environments

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Outline

• Prior-Free Online Auction Design:
  - Non-reusable Goods, Finite time horizon.
• General characterization for truthful online auctions
• Prior-Free Online Auction Design:
  - Reusable Goods, infinite time horizon.
• Model-based Online Mechanisms.
• Future Directions.

Related Papers


Example 1: Last-Minute Tickets

Value: $100 $80 $60
Arrival: 11am 11am 12pm
Patience: 2hrs 2hrs 1hr

How should you bid?

“Please bid your value and your patience. A decision will be made by the end of your stated patience.”
Value      $100 $80 $60
Arrival:   11am 11am 12pm
Patience:  2hrs 2hrs 1hr

If truthful, then:
{ <1, $80>,<2, $60> }
However, bidder 1 could
a) reduce bid price to $65
   {<2, $65>, <1, $60>}
   b) delay bid until 12pm
      {<2, $0>, <1, $60>}

Auction: sell one ticket in
each hour (given demand),
to the highest bidder at
second-highest bid price.

Dynamic allocation problems
...are everywhere in computer science
• MoteLab (Berkeley)
  - distributed sensor network testbed
  - researchers compete for the right to sense, aggregate and
    propagate readings
• PlanetLab (Princeton)
  - global overlay network on the Internet
  - supports network research, long-running services
• Grid computing
  - much of science research is now intensively computational
  - globally-distributed computational infrastructure
• Network resource allocation
  - e.g. dynamic negotiation for WiFi bandwidth

Many systems are simultaneously both computational and
economic systems.

Basic Model for Online Auctions
• Valuation \( \theta_i = (a_i, d_i, w_i) \). Discrete time periods.
• Arrival time: \( a_i \). Departure time: \( d_i \). Value, \( w_i \)
• Allocation schedule \( x \in X \).
• \( v_i(x) = w_i \), if \( x(t)=1 \) for some \( t \in [a_i,d_i] \)
• \( 0 \), otherwise
• Quasi-linear utility: \( u_i(x,\text{price}) = v_i(x) - \text{price} \)
• Auction: \( A=\langle f, p \rangle \),
  - allocation rule, \( f : \Theta^n \rightarrow X \)
  - payment rule, \( p : \Theta^n \rightarrow \mathbb{R}^n \)
• Truthful auction: reporting value \( <a_i, d_i, w_i> \) immediately
  upon arrival is a dominant strategy equilibrium.
• Assume: cannot under-report \( a_i \).
Prior-Free: Key Variations

- Limited-supply ($k \geq 1$) of goods, sell in any period before time horizon, $T$.
  - single-unit demand
  - multi-unit demand

- Reusable goods, can sell up to $k$ units in each time period. Finite time horizon, $T$.
  - single-period demand
  - multi-period demand

Prior-Free Auction Design

(c.f. Goldberg, Hartline et al.01)

- $v^{(m)}$ is $m$-th highest value
- $EFF(v) = \sum_{i \leq k} v^{(i)}$ "efficiency"
- $F^{(2)}(v) = \max_{2 \leq i \leq k} \{ i \cdot v^{(i)} \}$ "omniscient revenue"

<table>
<thead>
<tr>
<th>Value</th>
<th>$$500$</th>
<th>$$80$</th>
<th>$$60$</th>
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<tr>
<td>Arrival</td>
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<td>11am</td>
<td>12pm</td>
</tr>
<tr>
<td>Patience</td>
<td>2hrs</td>
<td>2hrs</td>
<td>1hr</td>
</tr>
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EFF: $\$580$
OPT: $\$160$

- $c$-competitive for efficiency if $E[Val(Auc_v)] \geq 1/c \cdot EFF(v)$, for all $v$
- $c$-competitive for revenue if $E[Rev(Auc_v)] \geq 1/c \cdot F^{(2)}(v)$, for all $v$

Limited-Supply Auction

(Lavi & Nisan'00)

- Assume values in $[L,U]$. $k$-unit supply. Let $\phi = (U/L)$.
- Adversarial model: choose values and timing.
- Define a "price schedule": $p(j) = L \cdot \phi^{j/k+1}$, for $j$th unit.
- Sell units while bid value \geq price.

Truthful.

$\ln(\phi)$-competitive w.r.t. efficiency and Vickrey revenue, Matching lower-bound, and good average-case performance in simulation.

Our model: Fixed, Unknown Distribution

(Hajiaghayi, Kleinberg, P., ACM'EC04)

- More realistic adversarial model: Lavi & Nisan allowed arbitrary sequencing of arbitrary values
- Instead, we model values as i.i.d. from some unknown distribution.
- Want good performance whatever the distribution.
- Should lead to an auction with better performance in practice.
### Aside: The Online Selection Problem

- Remove incentives, and specialize to the case of disjoint arrival-departure intervals.

#### Reduces to the secretary problem:
- Interview \( n \) job applicants in random order, want to max prob of selecting best applicant (told \( n \))
- Told relative ordering w.r.t. applicants already interviewed, must hire or pass

### Useful info

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>1</th>
<th>1</th>
<th>4</th>
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<td>5</td>
<td>2</td>
<td>7</td>
<td>1,000</td>
<td>3</td>
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E.g., \( n=1, s^*=1, \Pr(\text{succ})=1 \)
\( n=5, s^*=3, \Pr(\text{succ})=0.433 \)
\( n=10, s^*=4, \Pr(\text{succ})=0.399 \)
\( n=20, s^*=8, \Pr(\text{succ})=0.384 \)
\( n=100, s^*=38, \Pr(\text{succ})=0.371 \)
\( n=1000, s^*=369, \Pr(\text{succ})=0.368 \approx 1/e \)
The Secretary Algorithm

- **Theorem** (Dynkin, 1962): The following stopping rule picks the maximum element with probability approaching $1/e$ as $n \to \infty$.
  - Observe the first $\lfloor n/e \rfloor$ elements. Set a threshold equal to the maximum quality seen so far.
  - Stop the next time this threshold is exceeded.

- Asymptotic success probability of $1/e$ is best possible, even if the numerical values of elements are revealed.
  - i.e. optimal competitive ratio in the large $n$ limit

Straw model for an Auction

- **Auction**: $p(t)=\infty$, then set $p(t|\geq \tau) = \max_{i \leq j} w_i$ after $j=\lfloor n/e \rfloor$ bids received. Sell to first subsequent bid with $w_i \geq p(t)$, then set $p(t)=\infty$.
- **Not truthful**: Bidders that span transition, and with high enough values, should delay arrival.

Truthful Auction:
  - At time $\tau$ (for $n/e$ arrival) let $p=q$ be the top two bids yet received.
  - If any agent bidding $p$ has not yet departed, sell to that agent (breaking ties randomly) at price $q$.
  - Else, sell to the next agent whose bid is at least $p$ (breaking ties randomly)

Adaptive Limited-Supply Auction

- At time $\tau$, denoting arrival $j=\lfloor n/e \rfloor$, let $p=q$ be the top two bids yet received.
- If any agent bidding $p$ has not yet departed, sell to that agent (breaking ties randomly) at price $q$.
- Else, sell to the next agent whose bid is at least $p$.

| Agent 1 | $5$ |
| Agent 2 | $2$ |
| Agent 3 | $5$ |
| Agent 4 | $8$ |
| Agent 5 | $4$ |
| Agent 6 | $10$ |

Agent 1 wins, pays $2$
Adaptive Limited-Supply Auction

- At time $\tau$, denoting arrival $j = \lceil n/e \rceil$, let $p \geq q$ be the top two bids yet received.
- If any agent bidding $p$ has not yet departed, sell to that agent (breaking ties randomly) at price $q$.
- Else, sell to the next agent whose bid is at least $p$.

Agent 1: $5$
Agent 2: $2$
Agent 3: $5$
Agent 4: $8$
Agent 5: $4$
Agent 6: $10$

Analysis: Truthfulness

- If agent $i$ wins, the price charged to her does not depend on her reported valuation.
- $P_r(\text{agent } i \text{ wins})$ is (weakly) increasing in $w_i$, hence no incentive to understate $w_i$.
- Reporting $w_i' > w_i$ cannot increase the probability that agent $i$ wins at a price $\leq w_i$, hence no incentive to overstate $w_i'$.
- Price facing agent $i$ is never influenced by $d_i$, so no incentive to misstate $d_i$.
- Just need to check effect of arrival time.

Analysis: Truthfulness

- Claim: Given two arrival times $a_i < a_i'$, it’s always better to report $a_i$ if possible.
- Let $r, s$ be the $\lceil n/e \rceil$-th and $\lceil n/e \rceil$-th arrival times excluding agent $i$. 

Agent 1: $5$
Agent 2: $2$
Agent 3: $5$
Agent 4: $8$
Agent 5: $4$
Agent $i$: $10$
Analysis: Truthfulness

• Stating true arrival, agent 2 defines transition. Offered price $5 on transition.

Analysis: Truthfulness

• Stating arrival time in \((a_i, r]\) changes nothing. Offered price $5 on transition.

Analysis: Truthfulness

• Stating arrival time in \([a_i, r]\) changes nothing.
• Stating arrival time in \((r, s)\) influences the transition time \(\tau\) but not the pricing. Still offered price $5.

Analysis: Truthfulness

• Stating arrival time in \([a_i, r]\) changes nothing.
• Stating arrival time in \((r, s)\) influences the transition time \(\tau\) but not the pricing.
• Stating arrival time \(\geq s\) influences the transition, but price not improved.
Analysis: Competitive Ratio

- **Claim**: Competitive ratio for efficiency is $e + o(1)$, assuming all valuations are distinct.
- **Case 1**: Item sells at time $t$. Winner is highest bidder among first $\lfloor n/e \rfloor$. With probability $\approx 1/e$, this is also the highest bidder among all $n$ agents.
- **Case 2**: Otherwise, the auction picks the same outcome as the secretary algorithm, whose success probability is $\approx 1/e$.

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Analysis: Competitive Ratio

- **Claim**: Competitive ratio for revenue (wrt Vickrey) is $e^2 + o(1)$, assuming all valuations are distinct.
- Estimate probability of selling to highest bidder at second-highest price. Use same two cases as before.
  - **Case 1**: Probability $\approx 1/e^2$.
    - (prob $1/e$ that second highest also is in first half)
  - **Case 2**: Probability $(1/e)(1/e)$.
    - (prob. that highest in first-half is the second-highest overall is $1/e$ conditioned on highest in second-half; prob. that choose highest in case 2 is $1/e$)

- $4 + o(1)$-competitive for revenue (and also efficiency), by setting transition time at $n/2$.
- Lower-bounds of 2-competitive for efficiency, 1.5-competitive for revenue (in our model).

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General approach -- Two phase

- "Learning phase"
  - use a sequence of bids to set price for rest of auction

Transition:
  - be sure that remains truthful for agents on transition

- "Accepting phase"
  - exploit information, retain truthfulness

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Multi-Item Online Auction ($k>1$)

- Adopt a variation on the Dual-Price Sampling Optimal Threshold (DSOT) auction (Goldberg, Hartline et al'01; also Segal'03).
  - (Learning) Choose pivotal bidder, $j \sim \text{Binom}(n, \frac{1}{2})$.
  - (Transition) Sell up to $s = \lceil k/2 \rceil$ items at time $\tau$, to agents present and bidding above $(s+1)$-st bid price so far.
  - (Accepting) After $\tau$, set price to be the revenue-optimizing fixed price, $p^{opt}$ for bids in first half. Sell item to bid $\geq p^{opt}$ while supply.

- Truthfulness: have $p(s+1) \leq p^{opt}$
- Constant-competitive with $\mathcal{F}(2)$ for revenue.
- Constant-competitive for efficiency (and also revenue), by setting $s = \lceil k/3 \rceil$, and adopting $p(t) = (s+1)$-st bid in accepting phase. (i.e. a lower price.)
Characterization of Truthful auctions

Definition. Allocation rule \( f: \Theta^n \to \{0,1\}^n \) is monotonic if for every agent \( i \) and every \((\theta, \theta') \in \Theta^n\) with \([a'_i, d'_i] \subseteq [a_i, d_i]\), and \( w_i \geq w'_i \), we have \( f_i(\theta) \geq f_i(\theta') \).

Definition. The “critical value” price is:
\[
\psi_i(a_i, d_i, \theta_{-i}) = \min w'_i \text{ s.t. } f_i(a_i, d_i, w'_i, \theta_{-i}) = 1 \quad \infty, \quad \text{if no such } w'_i \text{ exists}
\]

Definition. The “critical period” is the first \( t \in [a_i, d_i] \) with minimal \( \psi_i(a_i, t, \theta_{-i}) \).

Theorem. An online auction is truthful if and only if the allocation rule, \( f \), is monotonic, sets payment equal to critical value, and assigns item after the critical period.

Via an Agent-independent Price Schedule

• Define an agent-independent price schedule, \( \psi_i(t, \theta_{-i}) \) for allocation in period \( t \)
• Allocate good to agent if and only if \( \psi_i(t', \theta_{-i}) \leq w_i \) for some \( t' \in [a_i, d_i] \), at price \( \psi_i(a_i, d_i, \theta_{-i}) = \min_{t' \in [a_i, d_i]} \psi_i(t', \theta_{-i}) \).
• Allocate no earlier than period \( t' \) for which \( \psi_i(t', \theta_{-i}) \) is minimal in \([a_i, d_i] \).

Prior-Free: Key Variations

• Limited-supply (\( k \geq 1 \)) of goods, sell in any period before time horizon, \( T \).
  - single-unit demand
  - multi-unit demand

• Reusable goods, can sell up to \( k \) units in each time period. Finite time horizon, \( T \).
  - single-period demand
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• Allocate no earlier than period \( t' \) for which \( \psi_i(t', \theta_{-i}) \) is minimal in \([a_i, d_i] \).

Example: single-unit auction. Let \( j = \lfloor n/e \rfloor \), and use “outside bid” refer to a bid from an agent \( \neq i \).
\[
\psi_i(t, \theta_{-i}) = \begin{cases} 
\infty & \text{for } < j - 1 \text{ outside bids} \\
\frac{b^{(0)}_{j-1}}{i} & \text{for } j - 1 \text{ outside bids} \\
\frac{b^{(1)}_{j-1}}{i} & \text{for } \geq j - 1 \text{ outside bids, before item sells} \\
\infty & \text{otherwise}
\end{cases}
\]
Formal Model: Re-usable Goods

- One good in each time slot (can extend to \(k > 1\)).
- Agent value \(<a_i, d_i, w_i>\). Value for one time slot in \([a_i, d_i]\).
- No-late departures (i.e. \([a'_i, d'_i] \subseteq [a_i, d_i]\))
  - (WiFi) suppose can verify presence, and fine an agent that reports \(d'_i > d_i\) but leaves at \(d_i\).
  - (Grid) reasonable to hold result until time \(d'\) with some small probability.
- Necessary to assume NLD to achieve a bounded competitive ratio on efficiency (Lavi & Nisan’05)

**Theorem.** Online auction is truthful if and only if the allocation rule, \(f\), is monotonic, sets payment equal to critical value. Can assign at any time in interval w/ NLD.

Example: Grid scheduling

<table>
<thead>
<tr>
<th>Value</th>
<th>$100</th>
<th>$80</th>
<th>$60</th>
</tr>
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<tbody>
<tr>
<td>Arrival</td>
<td>11am</td>
<td>11am</td>
<td>12pm</td>
</tr>
<tr>
<td>Patience</td>
<td>2hrs</td>
<td>2hrs</td>
<td>1hr</td>
</tr>
<tr>
<td>Duration</td>
<td>1hr</td>
<td>1hr</td>
<td>1hr</td>
</tr>
</tbody>
</table>

Allocation rule: In each period, \(t\), allocate the good to the highest unassigned bid.

Payment rule: Pay smallest amount could have bid and still received good (in some period).

monotone: smaller \([a'_i, d'_i]\), smaller \(w'_i\), cannot help.

reduces to seq. of Vickrey for impatient bidders.

Efficiency: Competitive Analysis

2-competitive wrt efficiency, (maximum-weighted matching in bipartite graph).

(Tight. But, 1.618 poss. without incentives!)

Extends to \(k > 1\) case (still 2-competitive).

Revenue Analysis: Consider VCG

\(n\) slots, \(n+1\) bids

\(n-1\) high bidders
Revenue Analysis: VCG

VCG: \( V^* = 2(n-1) + 1 \)

- \( V = 2(n-2) + 2 \)
- \( V - 3 = 2(n-2) + 2 \)
- \( V - 4 = 2(n-2) + 2 \)

n slots, n+1 bids

n-1 high bidders
Revenue Analysis: VCG

VCG: 

V* = 2(n-1) + 1
V - 2 = 2(n-2) + 2
V - 3 = 2(n-2) + 2
V - 4 = 2(n-2) + 2
V - 5 = 2(n-2) + 2
V - 6 = 2(n-1) + 1

n slots, n+1 bids

Revenue(VCG) = 1(n-1) + 1

Revenue: Competitive Analysis

VCG: 

V* = 2(n-1) + 1
V - 2 = 2(n-2) + 2
V - 3 = 2(n-2) + 2
V - 4 = 2(n-2) + 2
V - 5 = 2(n-2) + 2
V - 6 = 2(n-1) + 1

n slots, n+1 bids

Revenue(VCG) = 1(n-1) + 1
Revenue(Auc) = 1
\Rightarrow \text{competitive ratio can be arbitrarily bad!}
• Actually, have a general negative result available for the revenue-competitiveness of a deterministic online auction for this problem.

Can achieve $O(\log_2(\phi))$ competitive with a randomized auction, for $\phi=(U/L)$, even with unknown bounds.

Prior-Free Online Auction Design:
- Non-reusable Goods, Finite time horizon.
- General characterization for truthful online auctions
- Prior-Free Online Auction Design:
- Reusable Goods, infinite time horizon.
- Model-based Online Mechanisms
- Future directions.

Model-Based Online Mechanisms
(P. & Singh'03, P., Singh & Yanovsky'04)

- Agents, and the auctioneer, have a common prior.
- $\theta$ iid from distribution $g(\theta)$.
- Mechanism makes a sequence of decisions $\{k_1,k_2,\ldots\}$
- Agents $\theta_i=[a_i,d_i,v_i]$. $v_i(k)\geq 0$.
- Goal: maximize the expected sequential value.

As a Markov Decision Process

- State: $h_t=(\theta_1,\ldots,\theta_t; k_1,\ldots,k_{t-1})$. Time horizon $T$.
- Model: $\Pr(h_{t+1}|h_t,k_t): R(h_t,k_t)=\sum_i[v_i(k_t)-v_i(k_{t+1})]$.
- Policy: $\pi=(\pi_1,\ldots,\pi_T)$, $\pi_t: H_t \rightarrow K_t$
- $V(\pi(h_t))=E_\pi[R(h_t,\pi(h_t))+R(h_{t+1},\pi(h_{t+1}))+\ldots+R(h_T,\pi(h_T))$]

- Efficient policy, $\pi^*$, maximizes MDP value in all states; value $V^*(h_t)$.
- Solve via dynamic programming, policy iteration, linear programming, etc.

“Stalling” == “Action space rich enough that cannot improve policy by delaying the arrival of an agent.”

- How to handle self-interest?
An Online VCG Mechanism

- Receive reports. Implement \( \pi^*(h'_i) \).
- Payment: \( p_i = v_i'(k^*) - \{V^*(h_{a_i}) - V^*(h_{a'_i})\} \)

Theorem. Given a correct model, and a policy with stalling, the online VCG is Bayes-Nash IC and implements the efficient policy.

EU(\( \theta_i' \)) = \( v_i(\pi^*(h_{a_i})) + V^*(h_{a_i}) - v_i'(\pi^*(h_{a_i})) - V^*(h_{a'_i}) \)

Remarks.

- BNIC not DSIC. Correctness of \( \pi^* \) requires correct model \( f(\theta) \), which requires other agents play equilibrium.
- c.f. offline VCG, where the center can make the value-maximizing choice (based on reports), whatever the reports.
Remarks.

• BNIC not DSIC. Correctness of $\pi^*$ requires correct model $f(\theta)$, which requires other agents play equilibrium.
• c.f. offline VCG, where the center can make the value-maximizing choice (based on reports), whatever the reports.

• ex post individual-rational given "value monotonicity", i.e. addition of an agent has a (weakly) +ve effect on total MDP value.
• ex ante no-deficit given "no positive externalities", i.e. addition of an agent has a (weakly) -ve effect on MDP value to others.

Algorithmic Remark: Sparse-Sampling:

$V^{ss}(h) = \max_k \{R(k) + \mathbb{E}_{\text{child}} V^{ss}(\text{child})\}$

Policy $\pi^{ss}$, estimate $V^{ss}(h)$:

$|V'(h) - V^{ss}(h)| \leq \varepsilon$

$|V'(h) - \mathbb{E}(V^{ss}(h))| \leq \varepsilon$

in time $O((K \cdot w)^T)$, with $w = poly(K, 1/\varepsilon, R_{max}, T)$, for $R_{max}$ bound on reward in a state.

Example: Eff, Rev in WiFi problem

Eff, Rev in WiFi problem (P., Singh & Yanovsky'04)

5 channels

Revenue and value normalized by the unlimited supply value.
Future Direction: Introduce Learning.

- What if center has only a distribution on priors, and a MLE of the model, denoted $f'(\theta)$?
- Would like to converge to optimal $\pi^*$ over time.

Main problems:
(A1) retaining incentive-compatibility with respect to time despite the adaptiveness of the policy.
(A2) retaining incentive-compatibility despite an approximate policy.

Remark: the online VCG mechanism is not BNIC with an approximate model.
Future Direction: Introduce Learning.

- What if center has only a distribution on priors, and a MLE of the model, denoted $f'(0)$?
- Would like to converge to optimal $\pi^*$ over time.
- Main problems:
  (A1) retaining incentive-compatibility with respect to time despite the adaptiveness of the policy.
  (A2) retaining incentive-compatibility despite an approximate policy.
- Remark: the online VCG mechanism is not BNIC with an approximate model.
- Current work: focus on a “single-minded domain”. In that domain, optimal policies are monotonic, whatever the model ⇒ can get a positive result.
- General problem of learning + MDPs is open.

Summary

- Many computational systems present dynamic resource allocation problems.
- Need to extend MD to handle dynamics.
- Two styles of analysis.
- Prior-free: DSIC mechanisms with online competitive results for non-reusable and reusable-good scenarios.
- Model-based: BNIC mechanisms to implement optimal MDP policies.
- Future direction: Allow for learning.