1. (30pts) Consider a private-values auction of one good when bidders have quasilinear utility functions and know their own valuations. Prove that bidding truthfully is a (weakly) dominant strategy in the Vickrey auction. [Prove this from first principles; do not use the fact that the Vickrey auction is a special case of the Groves mechanism.]

2. (40pts) Consider designing a mechanism (where participation is ex post individually rational) for the following setting. You have one company (Profit & Gamble) to sell. You don’t care about keeping it or getting rid of it. There are two bidders with quasilinear utility functions. Bidder 1’s valuation is drawn from a uniform distribution on [0,1] (1 = one billion dollars). Bidder 2’s valuation is independently drawn from a uniform distribution on [1,4].

   (a) Design a mechanism that uses take-it-or-leave-it offers (at most one offer to each bidder) and attempts to maximize revenue subject to that. What is your expected revenue in your mechanism? What is your worst-case revenue in your mechanism? Is your mechanism Pareto efficient? Justify your answer.

   (b) What is the optimal (i.e., revenue-maximizing) auction for the setting? What is your expected revenue in that auction? What is your worst-case revenue in that auction? Is that auction Pareto efficient? Justify your answer.