CS 15-892 Foundations of Electronic Marketplaces
Homework 1

Fall Semester 2011

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Due October 4th, in the beginning of class. In this homework, you may use any sources that you want but you must cite the sources that you use. Teamwork is not allowed. If you have questions, please visit the instructor's office hours.

1. (10 pts) Throughout this question, you may restrict your analysis to pure strategies.

   a. Draw the strategic (normal) form of the following game tree.

   b. Name the dominant strategy equilibria of this game, if there are any.

   c. Name the iterated dominance solutions of this game, if there are any.

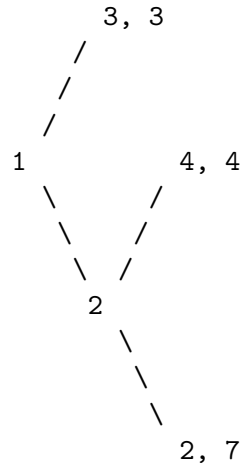
   d. Name the Nash equilibria of this game, if there are any.

   e. Name the Nash equilibria, if there are any, which do not involve the play of weakly dominated strategies.

   f. Name the subgame perfect Nash equilibria of this game, if there are any.

   g. Name the Pareto efficient outcomes of this game, if there are any.

   h. Name the social welfare maximizing outcomes of this game, if there are any.



2. **Game of chicken.** Let there be two teenagers who play the following risky game. They head toward each other in separate cars. Just before collision, each one has the choice of continuing straight, or avoiding collision by turning right. If both continue, they will die in the collision. If one continues and the other turns, both survive but the former becomes the hero and the latter is humiliated. If both turn, both survive but both are moderately humiliated. Let the game be represented by the following normal form:

		Agent 2	
		straight	turn
Agent 1	straight	-3, -3	2, 0
	turn	0, 2	1, 1

a. (5 pts) Does this game have pure strategy Nash equilibria? If so, what are they?

b. (15pts) What are the mixed strategy Nash equilibria of this game?

c. (2pts) In each equilibrium, what is the probability that the youngsters will die?

3. **Iterated elimination of dominated strategies as a solution concept for games.** An agent's strategy is *strictly dominated* if that agent has another strategy that gives strictly higher payoff to the agent no matter what strategies other agents choose. An agent's strategy is *weakly dominated* if that agent has another strategy that gives at least equally high payoff to the agent no matter what strategies other agents choose, and strictly higher payoff to the agent for at least one choice of strategies by the others. Now, to try to solve a game, we can iteratively eliminate dominated strategies until all the remaining strategies are undominated.

   a. (10pts) Show that iterated elimination of weakly dominated strategies is path dependent, i.e., the order in which strategies are eliminated can affect the outcome.

   b. (10pts) Prove that iterated elimination of strictly dominated strategies is path independent, i.e., the order in which strategies are eliminated cannot affect the outcome. For simplicity, you do not have to consider mixed strategies.

   c. (10pts) Show that a pure strategy can be strictly dominated by a mixed strategy in a game where the pure strategy is not even weakly dominated by any pure strategy.

   d. (10pts) Show that an agent's mixed strategy can be strictly dominated in a game where none of the agent's pure strategies are even weakly dominated.