1. Review questions. Use at most 10 sentences for each answer.

- (5 pts) From a game theory perspective, is it possible to build a combinatorial exchange that leads to a Pareto efficient outcome in dominant strategy equilibrium (we require that participation is voluntary, and that the exchange is ex post budget-non-negative (that is, the auctioneer does not need to subsidize the exchange))? What about in Bayes-Nash equilibrium? [You can assume that each bidder knows his own valuations on all bundles, and that communication and computation are no problem.]

- (5 pts) From a game theory perspective, is it possible to build a combinatorial reverse auction that leads to a Pareto efficient outcome in dominant strategy equilibrium (we require that participation is voluntary, and that the exchange is ex post budget-non-negative (that is, the auctioneer does not need to subsidize the exchange))? What about in Bayes-Nash equilibrium? [You can assume that each bidder knows his own valuations on all bundles, and that communication and computation are no problem.]

- (5 pts) From a game theory perspective, is it possible to build a combinatorial auction that leads to a Pareto efficient outcome in dominant strategy equilibrium (we require that participation is voluntary, and that the exchange is ex post budget-non-negative (that is, the auctioneer does not need to subsidize the exchange))? What about in Bayes-Nash equilibrium? [You can assume that each bidder knows his own valuations on all bundles, and that communication and computation are no problem.]

- (5 pts) What is the computational complexity of finding a feasible solution in a combinatorial auction, combinatorial reverse auction, and a combinatorial exchange? How, if at all, do these complexities change if the bidders can submit XOR-constraints between some of their bids?

- (5 pts) How well can the winners in a combinatorial auction, combinatorial reverse auction, and a combinatorial exchange be approximated in polynomial
time? How, if at all, do these complexities change if the bidders can submit XOR-constraints between some of their bids?

• (5 pts) What is the computational complexity of finding an optimal solution in a combinatorial auction, combinatorial reverse auction, and a combinatorial exchange? How, if at all, do these complexities change if the bidders can submit XOR-constraints between some of their bids?

2. (30 pts) In a combinatorial auction that uses the GVA (that is, Clarke tax mechanism), the payment that bidder \( i \) has to make is computed as follows. First, determine the winners. Call the sum of the winning bids of the other agents (except \( i \)) \( a \). Then, determine the winners again without \( i \)’s bids. Call the sum of the winning bids \( b \). Now, agent \( i \) pays \( b - a \).

Another generalization of the Vickrey auction to the combinatorial auction setting would determine \( i \)’s payment differently as follows. For each winning bid \( S \) of agent \( i \), let \( a^i_S \) be the sum of the prices of the other winning bids (by agent \( i \) and by the other agents). Then, determine the winners again with bid \( S \) removed. Call the sum of the winning bids \( b^i_S \). Now, the “price” of bid \( S \) is \( b^i_S - a^i_S \). The amount that agent \( i \) has to pay overall is

\[
\sum_{S \in i’s \ winning \ bids} b^i_S - a^i_S
\]

Is this mechanism incentive compatible? If so, prove that. If not, show a manipulation.