

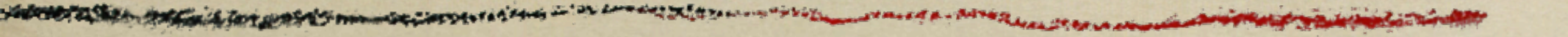
15-780: Graduate AI

Lecture 1. Intro & Logic

Geoff Gordon (this lecture)

Tuomas Sandholm

TAs Byron Boots, Sam Ganzfried



Admin

15-780 Graduate Artificial Intelligence Spring 2009

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School of Computer Science, Carnegie Mellon University



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Class Lectures: Tuesdays and Thursdays 10:30-11:50am in 4623 Wean Hall

Recitations: TBA

This course is targeted at graduate students who need to learn about and perform current-day research in artificial intelligence---the discipline of designing intelligent decision-making machines. Techniques from probability, statistics, game theory, algorithms, operations research and optimal control are increasingly important tools for improving the intelligence and autonomy of machines, whether those machines are robots surveying Antarctica, schedulers moving billions of dollars of inventory, spacecraft deciding which experiments to perform, or vehicles negotiating for lanes on the freeway. This AI course is a review of a selected set of these tools. The course will cover the ideas underlying these tools, their implementation, and how to use them or extend them in your research. Students entering the class should have a pre-existing working knowledge of

<http://www.cs.cmu.edu/~ggordon/780/>

<http://www.cs.cmu.edu/~sandholm/cs15-780S09/>

Website highlights

- Book: Russell and Norvig. *Artificial Intelligence: A Modern Approach*, 2nd ed.
- Grading: 4–5 HWs, “mid”term, project
- Project: proposal, 2 interim reports, final report, poster
- Office hours

Website highlights

- *Authoritative source for readings, HWs*
- *Please check the website regularly for readings (for Lec. 1–3, Russell & Norvig Chapters 7–9)*

Background

- *No prerequisites*
- *But, suggest familiarity with at least some of the following:*
 - *Linear algebra*
 - *Calculus*
 - *Algorithms & data structures*
 - *Complexity theory*

Waitlist, Audits

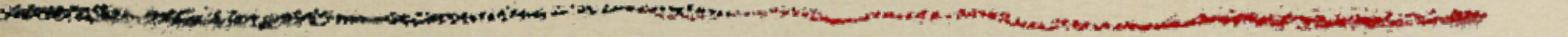
- *If you need us to approve something, send us email*

Course email list

- *15780students@...*
- *domain mailman.srv.cs.cmu.edu*
- *To subscribe/unsubscribe:*
 - *email 15780students-request@...*
 - *word “help” in subject or body*

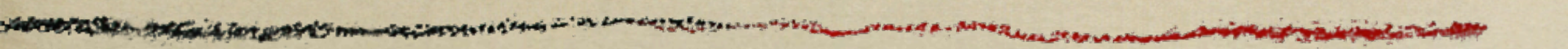
Matlab

- *Should all have access to Matlab via school computers*
 - *Those with access to CS license servers, please use if possible*
 - *Limited number of Andrew licenses*
- *Tutorial TBA soon*
- *HWs: please use C, C++, Java, or Matlab*



Intro

What is AI?



What is AI?

- *Easy part: A*

What is AI?

- *Easy part: A*
- *Hard part: I*

What is AI?

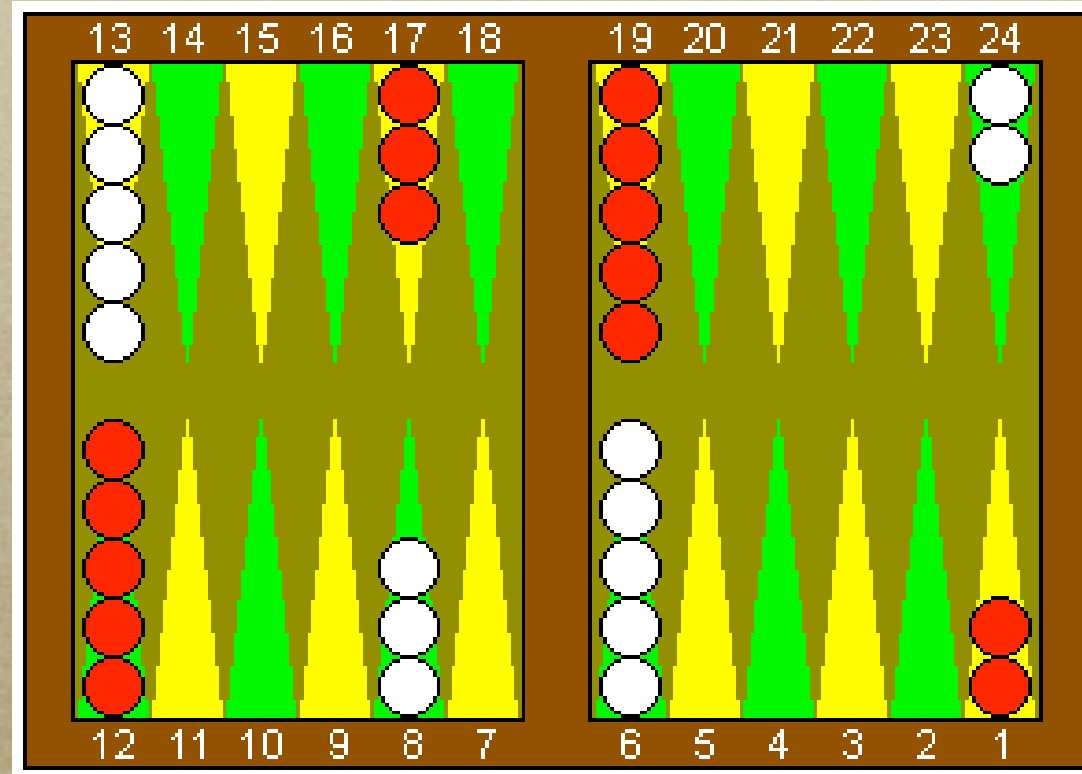
- *Easy part: A*
- *Hard part: I*
 - *Anything we don't know how to make a computer do yet*

What is AI?

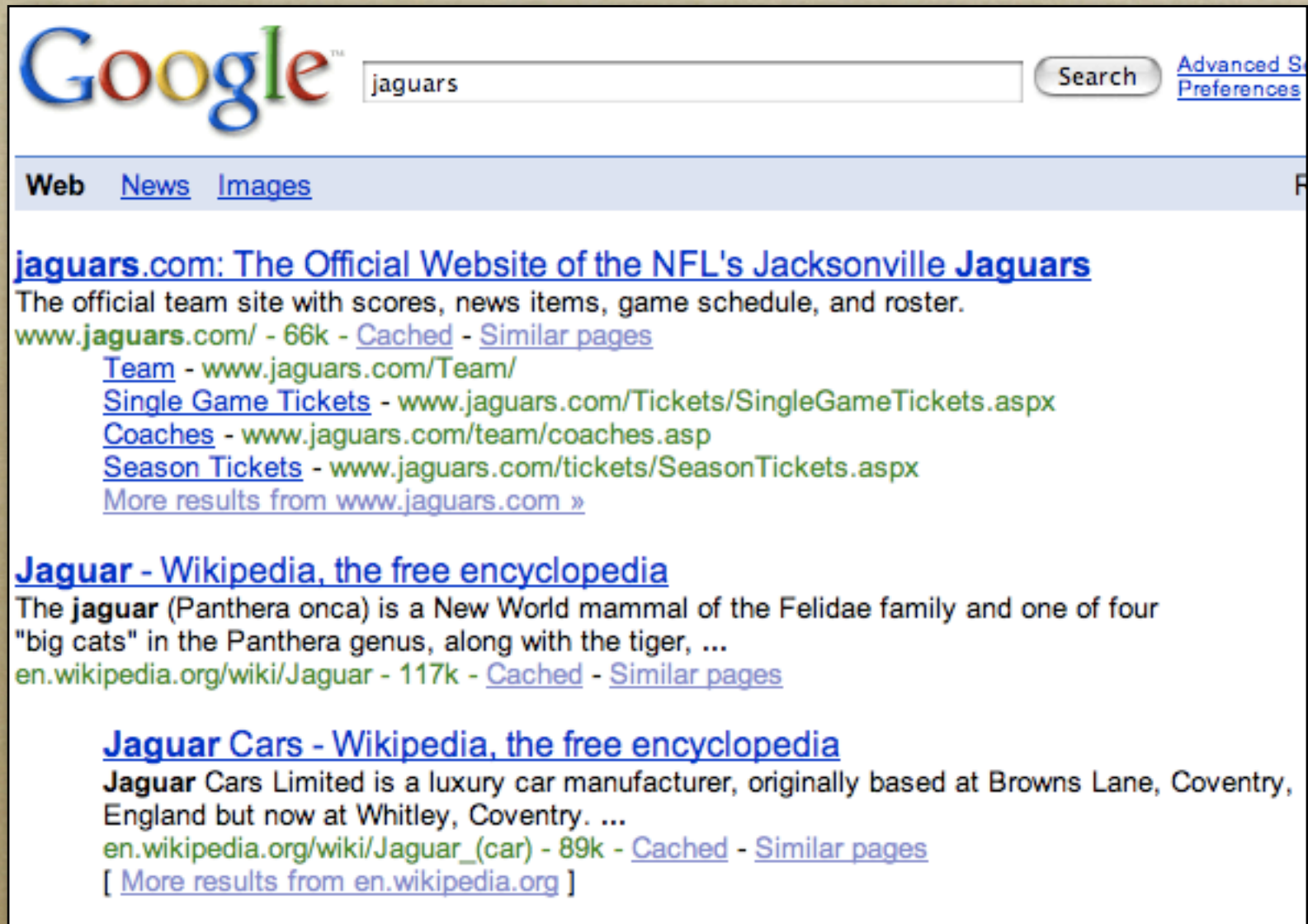
- *Easy part: A*
- *Hard part: I*
 - *Anything we don't know how to make a computer do yet*
 - *Corollary: once we do it, it isn't AI anymore :-)*

Definition by examples

- *Card games*
 - *Poker*
 - *Bridge*
- *Board games*
 - *Deep Blue*
 - *TD-Gammon*
 - *Samuels's checkers player*



Web search



The screenshot shows a Google search interface with the word 'jaguars' entered in the search bar. Below the search bar are links for 'Web', 'News', and 'Images'. The search results are listed below, starting with 'jaguars.com: The Official Website of the NFL's Jacksonville Jaguars'. This result includes a brief description, the URL 'www.jaguars.com/' with statistics (66k), and links for 'Cached' and 'Similar pages'. It also lists specific links for 'Team', 'Single Game Tickets', 'Coaches', and 'Season Tickets', followed by a link to 'More results from www.jaguars.com'. The second result is 'Jaguar - Wikipedia, the free encyclopedia', which describes the jaguar as a New World mammal and lists the URL 'en.wikipedia.org/wiki/Jaguar' with statistics (117k) and links for 'Cached' and 'Similar pages'. The third result is 'Jaguar Cars - Wikipedia, the free encyclopedia', which describes Jaguar Cars Limited as a luxury car manufacturer and lists the URL 'en.wikipedia.org/wiki/Jaguar_(car)' with statistics (89k) and links for 'Cached' and 'Similar pages', followed by a link to 'More results from en.wikipedia.org'.

Google™ jaguars Search [Advanced Search](#) [Preferences](#)

[Web](#) [News](#) [Images](#)

[jaguars.com: The Official Website of the NFL's Jacksonville Jaguars](#)
The official team site with scores, news items, game schedule, and roster.
[www.jaguars.com/](#) - 66k - [Cached](#) - [Similar pages](#)
[Team](#) - [www.jaguars.com/Team/](#)
[Single Game Tickets](#) - [www.jaguars.com/Tickets/SingleGameTickets.aspx](#)
[Coaches](#) - [www.jaguars.com/team/coaches.asp](#)
[Season Tickets](#) - [www.jaguars.com/tickets/SeasonTickets.aspx](#)
[More results from www.jaguars.com »](#)

[Jaguar - Wikipedia, the free encyclopedia](#)
The **jaguar** (*Panthera onca*) is a New World mammal of the Felidae family and one of four "big cats" in the *Panthera* genus, along with the tiger, ...
[en.wikipedia.org/wiki/Jaguar](#) - 117k - [Cached](#) - [Similar pages](#)

[Jaguar Cars - Wikipedia, the free encyclopedia](#)
Jaguar Cars Limited is a luxury car manufacturer, originally based at Browns Lane, Coventry, England but now at Whitley, Coventry. ...
[en.wikipedia.org/wiki/Jaguar_\(car\)](#) - 89k - [Cached](#) - [Similar pages](#)
[[More results from en.wikipedia.org](#)]

Web search, cont'd



The screenshot shows a job listing on the Flipdog website. The header is blue with the Flipdog logo and a search bar. The search bar contains the text "Job Title, Keywords" and "San Francisco". Below the header, the job title "Senior Marketing Representative" is displayed in large blue font, followed by the company name "Crawley Warren Insurance Services, Inc. (San Francisco, California)". A horizontal dotted line separates the header from the job details. The job details include salary, salary details, position type, ref code, minimum education level, and minimum career level. On the right side, there are two buttons: "Save Job to my monster" and "APPLY NOW".

flipdog™ find local jobs
Job Title, Keywords San Francisco
Powered

Senior Marketing Representative
Crawley Warren Insurance Services, Inc. (San Francisco, California)

Salary: \$20 to \$30
Salary Details: depending on experience
Position Type: Parttime
Ref Code: 60576596
Minimum Education Level:
Some College Coursework Completed
Minimum Career Level: Experienced (Non-Manager)

Save Job to my monster
APPLY NOW

Recommender systems

amazon.com Geoffrey's Amazon.com See all 41 Product Categories Your Account | Cart | Your Lists | Help |

Gift Certificates | International | Hot New Releases | Bestsellers | Today's Deals | Sell Your Stuff

Search Amazon.com GO Gift Certificates Web Search GO

Hello, Geoffrey J Gordon. We have [recommendations](#) for you. (If you're not Geoffrey J Gordon, [click here](#).)

New For You®



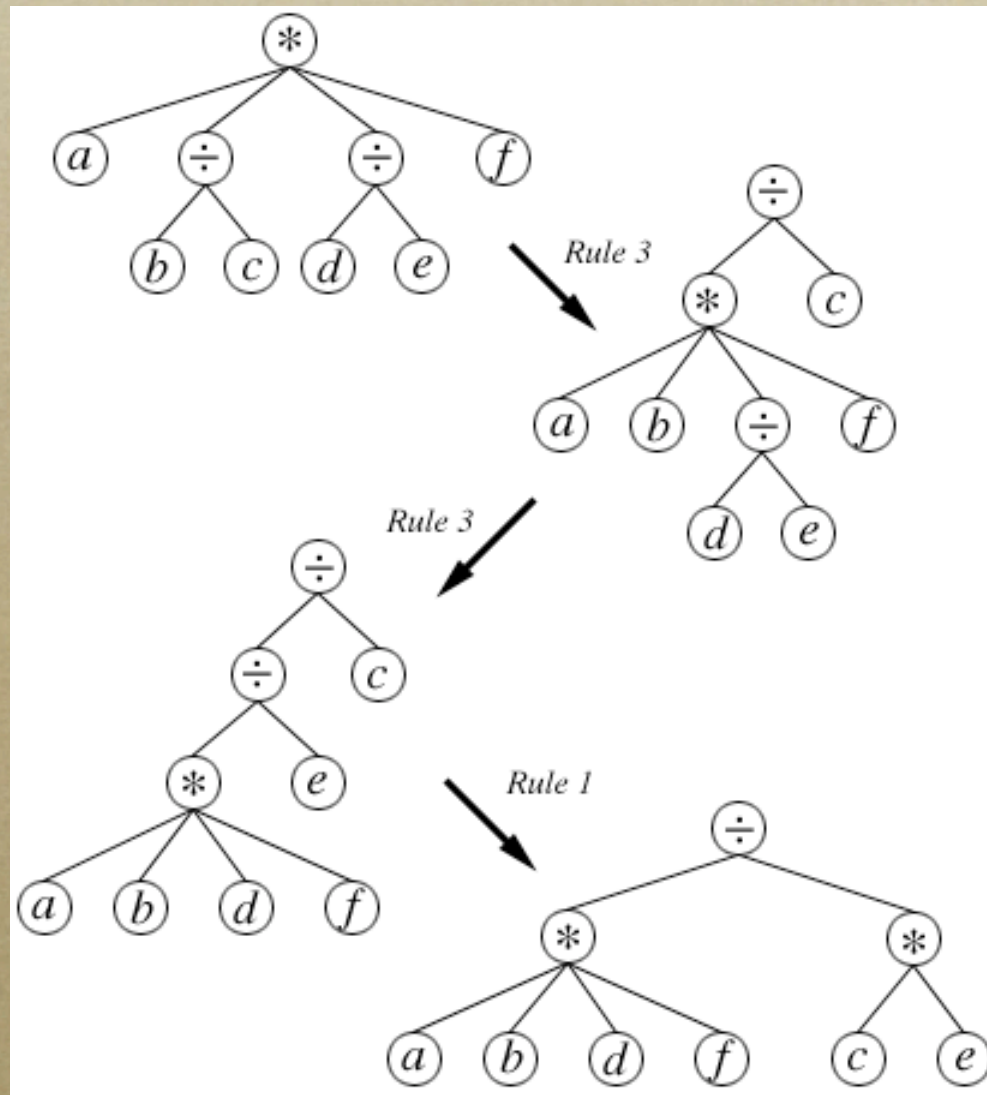
Girl Genius Volume 6: Agatha... Paperback by Phil Foglio
([Why is this recommended for you?](#))



Harry Potter and the Deathly Hallows Hardcover by J. K. Rowling, Mary...
([Why is this recommended for you?](#))

> [See more new releases](#)

Computer algebra systems



from <http://www.math.wpi.edu/IQP/BVCalcHist/calctoc.html>

Grand Challenge road race



Getting from A to B

Round Trip ☐ One Way ☐ Multi-Segment ☐

from or any airport within

to or any airport within

outbound date

return date

travelers

adults (18 to 61)	seniors (62 plus)	youths (12 to 17)	children (2 to 11)	infants in seat (under 2)	infants on lap (under 2)
1	0	0	0	0	0

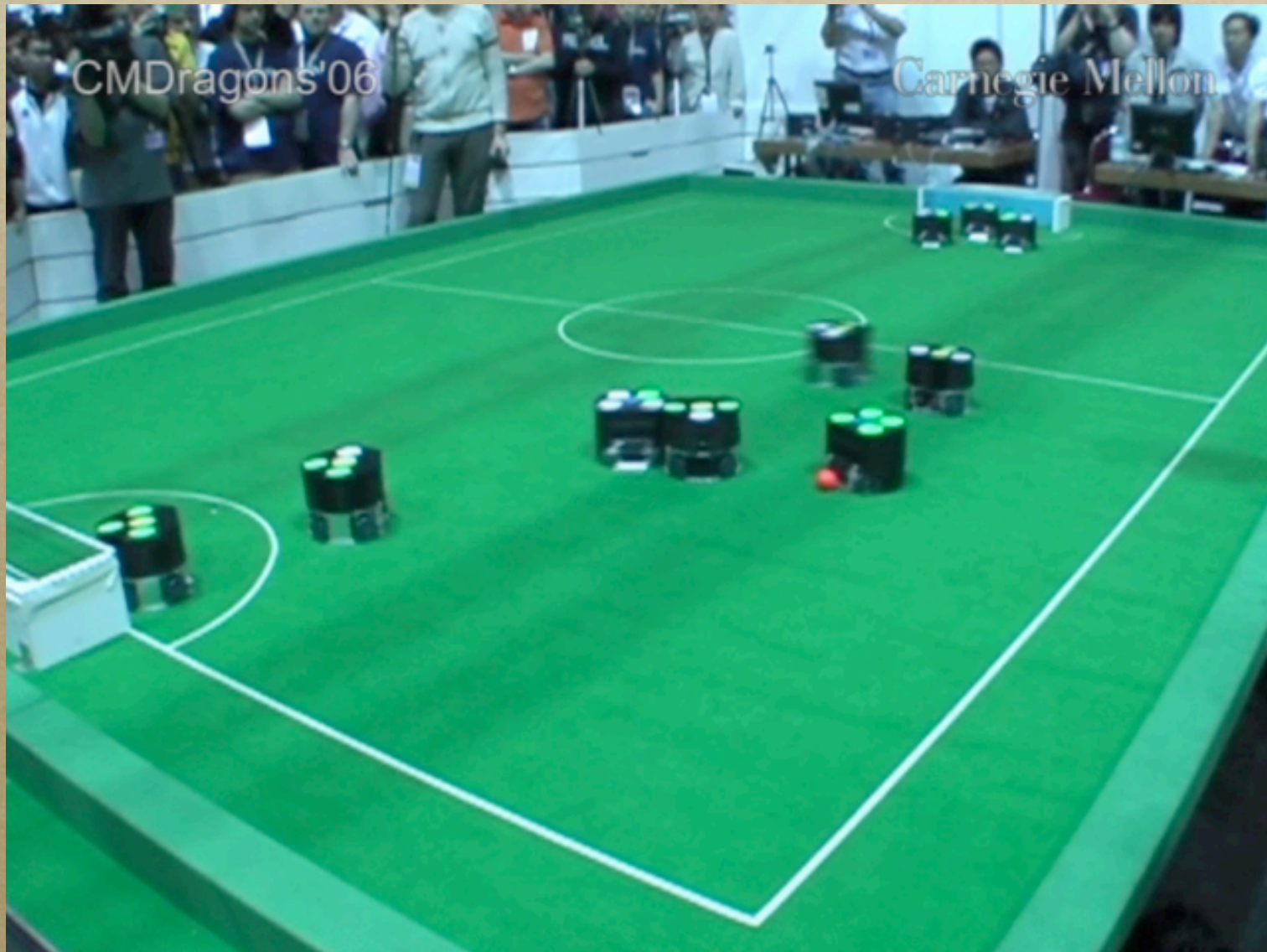
stops ☐ nonstops only ☐ up to 1 stop ☐ up to 2 stops ☒ no limit

sales city
(change only for trips originating outside the United States: [learn more](#))

[more options](#) (cabin, airport changes, seat availability, etc)

- ITA software (<http://beta.itasoftware.com>)

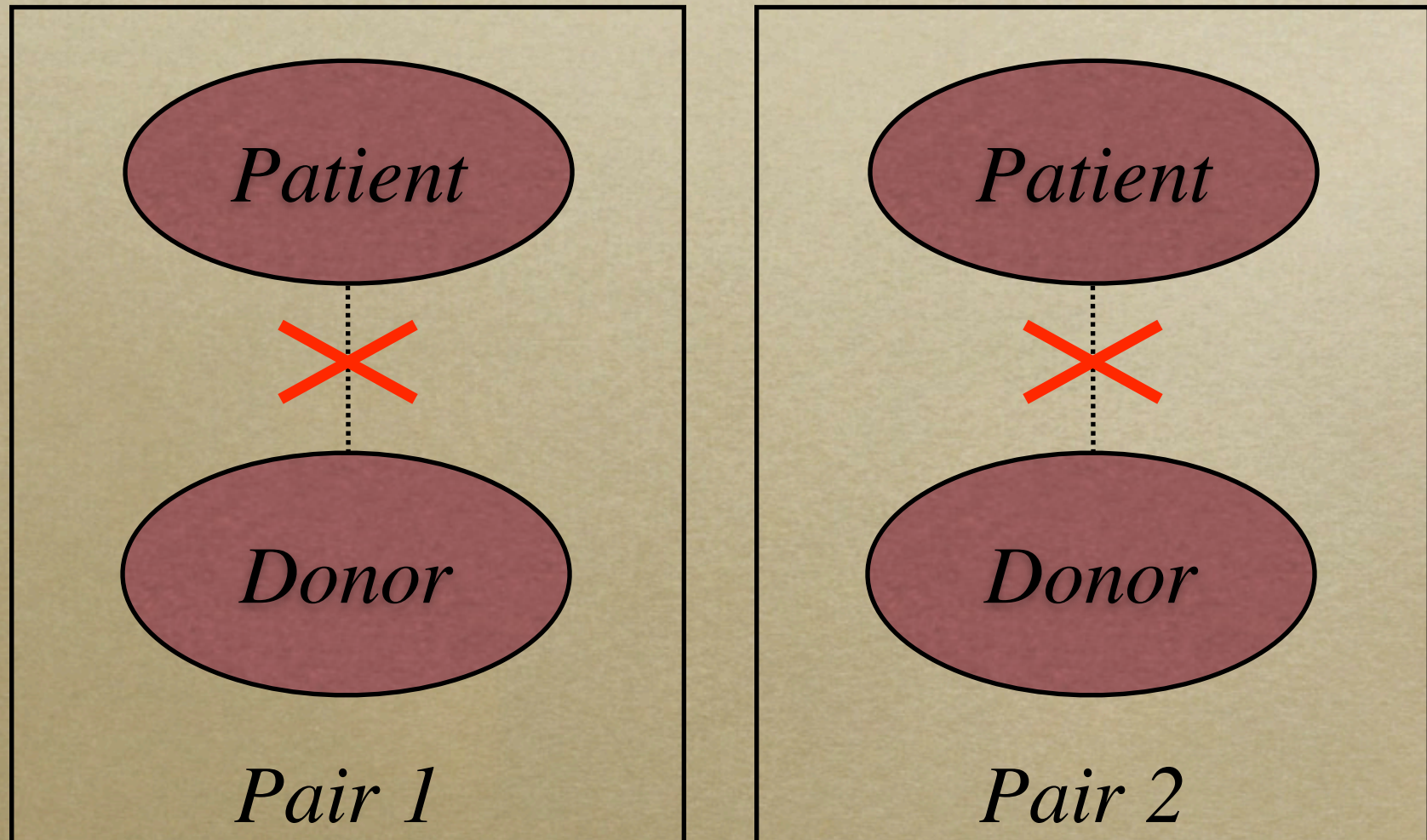
Robocup



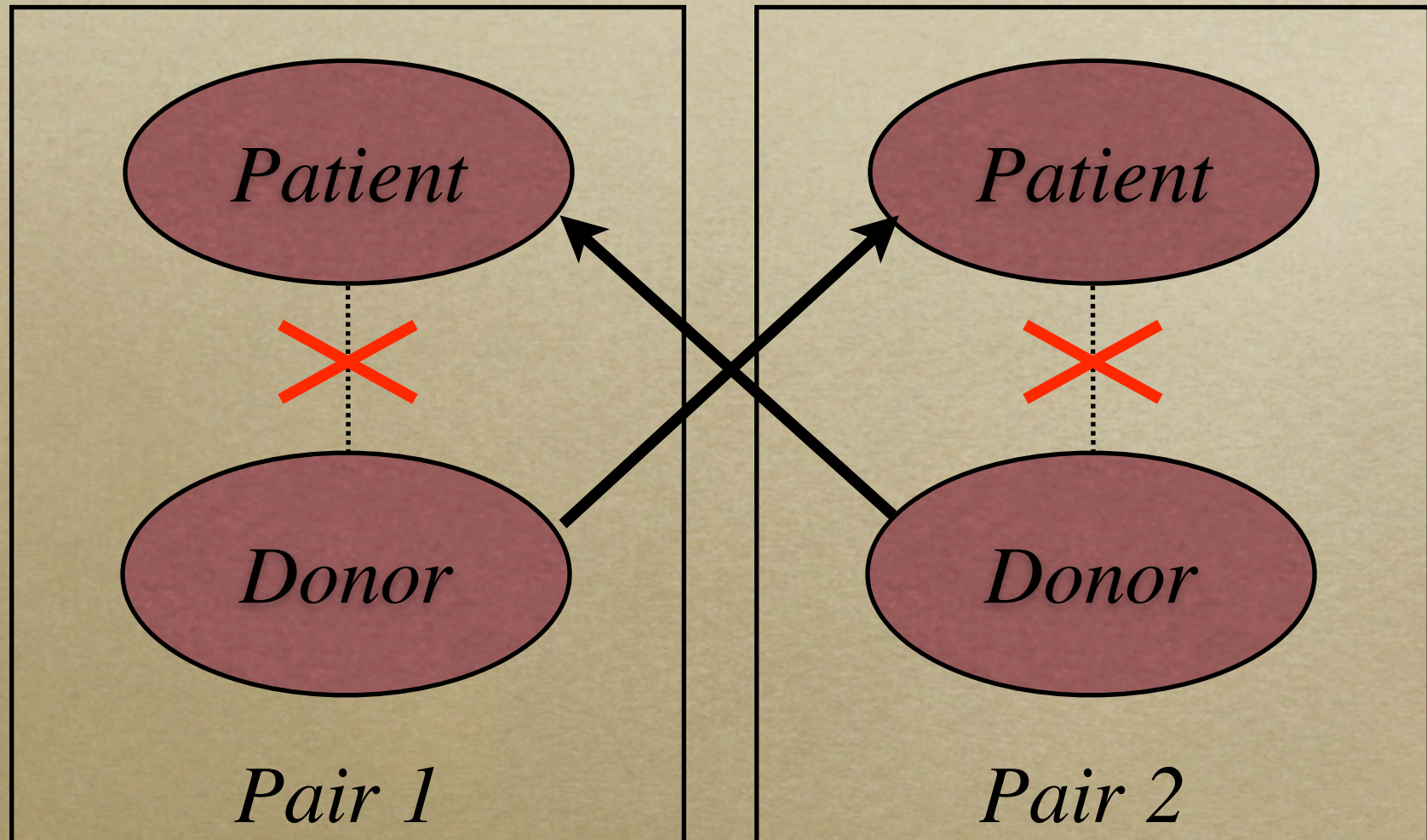
Kidney exchange

- *In US, $\geq 50,000$ /yr get lethal kidney disease*
- *Cure = transplant, but donor must be compatible (blood type, tissue type, etc.)*
- *Wait list for cadaver kidneys: 2–5 years*
- *Live donors: have 2 kidneys, can survive w/ 1*
- *Illegal to buy/sell, but altruists/friends/family donate*

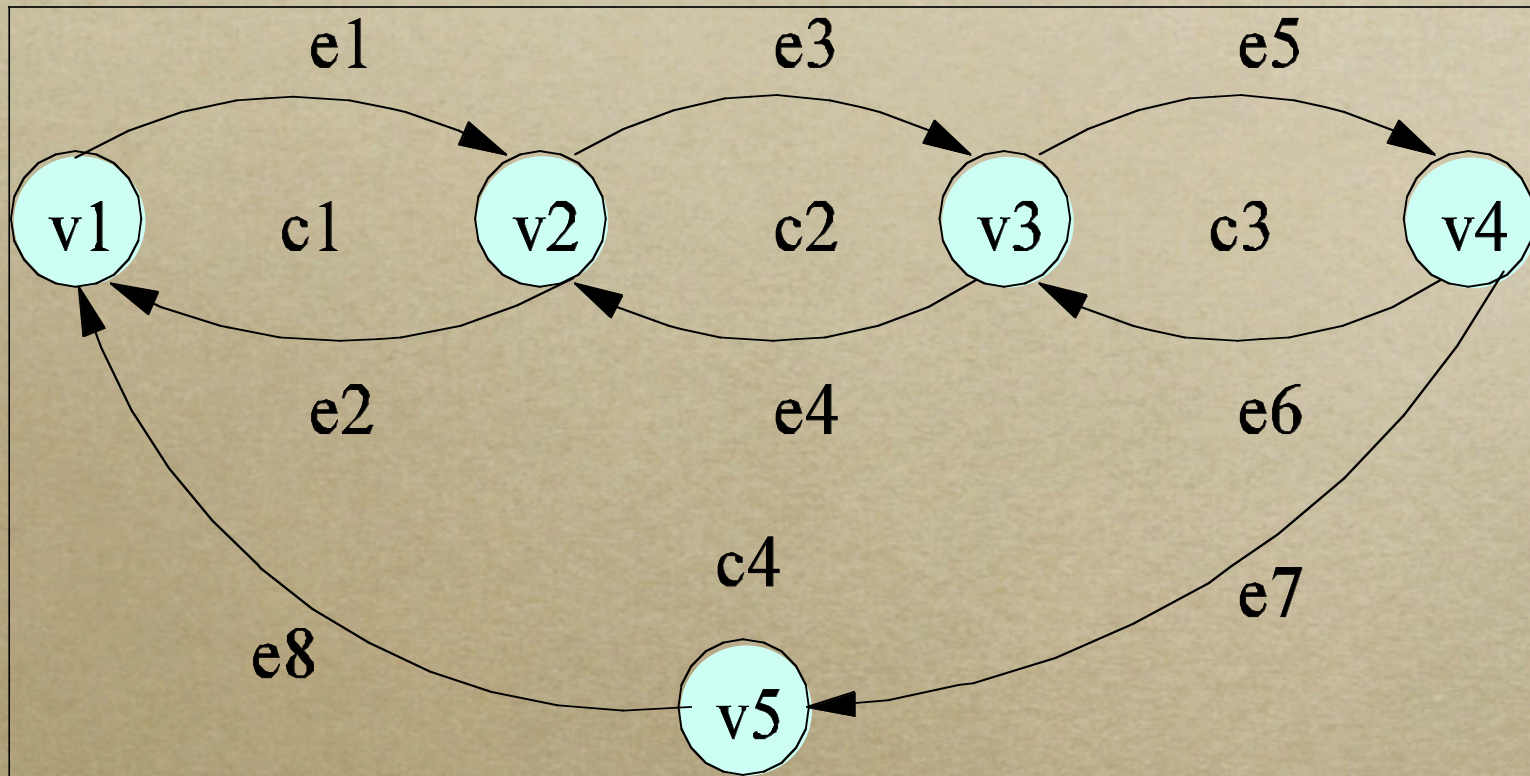
Kidney Exchange



Kidney Exchange



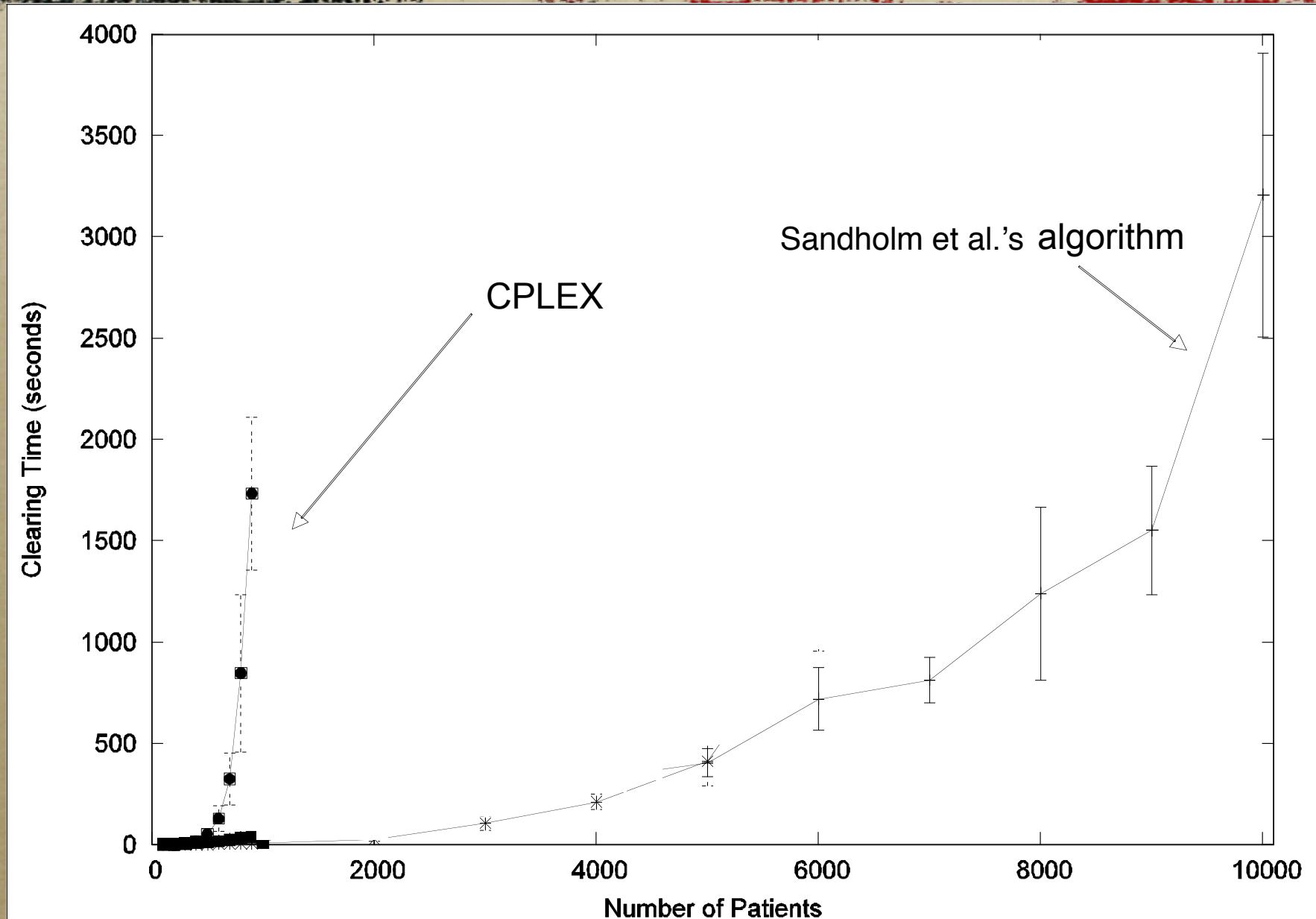
Optimization: cycle cover



*Cycle length constraint => extremely hard (NP-complete)
combinatorial optimization problem*

National market predicted to have 10,000 patients at any one time

Optimization performance



More examples

- *Motor skills: riding a bicycle, learning to walk, playing pool, ...*
- *Vision*

More examples

- *Valerie and Tank, the Roboceptionists*
- *Social skills: attending a party, giving directions, ...*



More examples

- *Natural language*
- *Speech recognition*

Common threads

- *Finding the needle in the haystack*
 - *Search*
 - *Optimization*
 - *Summation / integration*
- *Set the problem up well (so that we can apply a standard algorithm)*

Common threads

- *Managing uncertainty*
 - *chance outcomes (e.g., dice)*
 - *sensor uncertainty (“hidden state”)*
 - *opponents*
- *The more different types of uncertainty, the harder the problem (and the slower the solution)*

Classic AI

- *No uncertainty, pure search*
 - *Mathematica*
 - *deterministic planning*
 - *Sudoku*
- *This is the topic of Part I of the course*

<http://www.cs.ualberta.ca/~aixplore/search/IDA/Applet/SearchApplet.html>

Classic AI

- *No uncertainty, pure search*
 - *Mathematica*
 - *deterministic planning*
 - *Sudoku*
- *This is the topic of Part I of the course*

SuDoku Puzzle

		6	3			4	7	
		5	8		7			
1							2	3
	6		1	9				
4	9							
						1	9	8
6					3	5		
		8		5				2
	7	4			6		8	

<http://www.cs.ualberta.ca/~aixplore/search/IDA/Applet/SearchApplet.html>

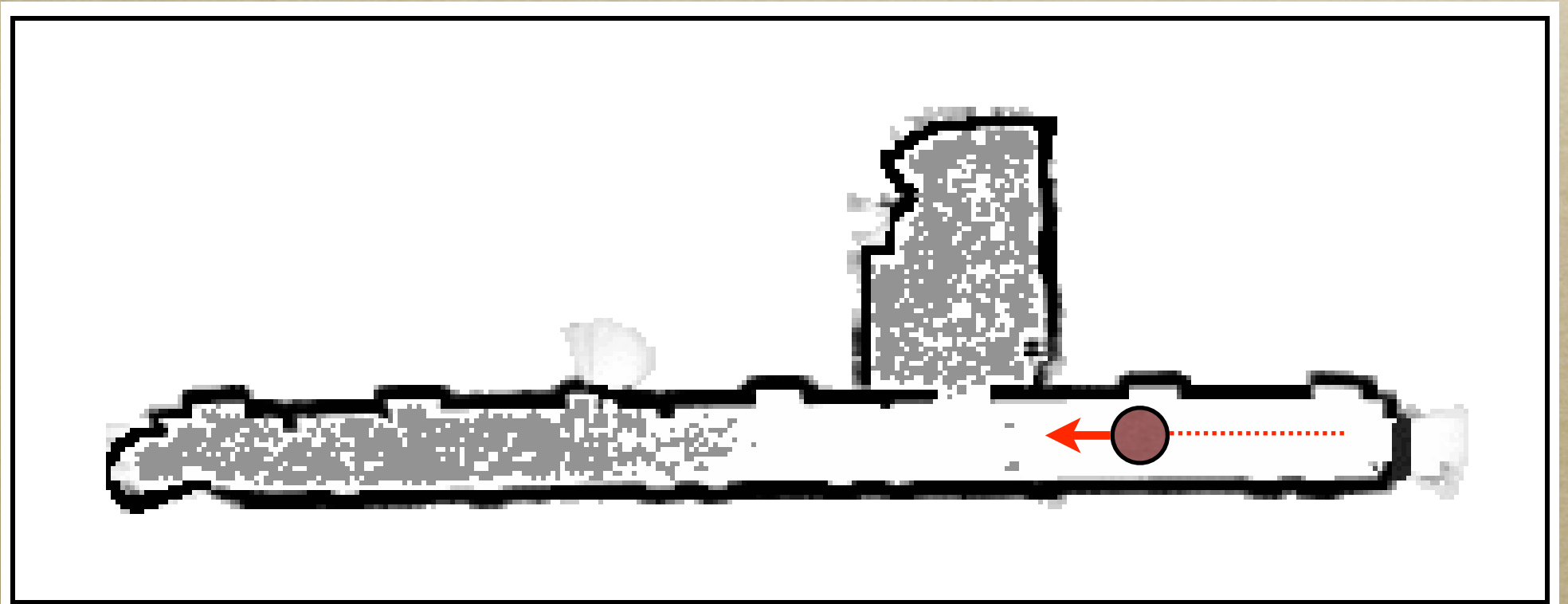
Outcome uncertainty

- *In backgammon, don't know ahead of time what the dice will show*
- *When driving down a corridor, wheel slippage causes unexpected deviations*
- *Open a door, find out what's behind it*
- *MDPs (later)*

Sensor uncertainty

- *For given set of immediate measurements, multiple world states may be possible*
- *Image of a handwritten digit $\rightarrow 0, 1, \dots, 9$*
- *Image of room $\rightarrow$ person locations, identities*
- *Laser rangefinder scan of a corridor $\rightarrow$ map, robot location*

Sensor uncertainty example

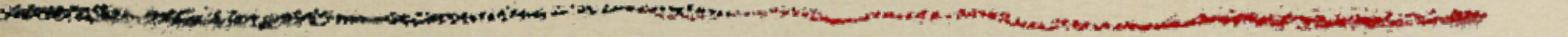


Opponents cause uncertainty

- *In chess, must guess what opponent will do; cannot directly control him/her*
- *Alternating moves (game trees)*
 - *not really uncertain (Part I)*
- *Simultaneous or hidden moves: game theory (later)*

Other agents cause uncertainty

- *In many AI problems, there are other agents who aren't (necessarily) opponents*
 - *Ignore them & pretend part of Nature*
 - *Assume they're opponents (pessimistic)*
 - *Learn to cope with what they do*
 - *Try to convince them to cooperate (paradoxically, this is the hardest case)*
- *More later*



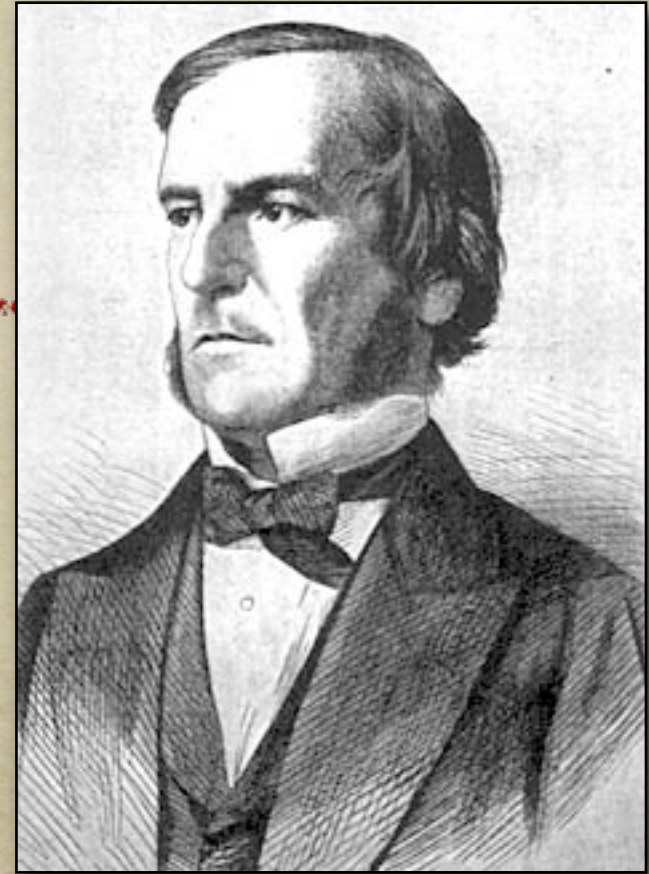
Logic

Why logic?

- *Search: for problems like Sudoku, can write compact description of rules*
- *Reasoning: figure out consequences of the knowledge we've given our agent*
- *... and, logical inference is a special case of probabilistic inference*

Propositional logic

- *Constants: T or F*
- *Variables: x, y (values T or F)*
- *Connectives: $\wedge, \vee, \neg$*
 - *Can get by w/ just NAND*
 - *Sometimes also add others:*
 $\oplus, \Rightarrow, \Leftrightarrow, \dots$



George Boole
1815–1864

Propositional logic

- *Build up expressions like $\neg x \Rightarrow y$*
- *Precedence: $\neg, \wedge, \vee, \Rightarrow$*
- *Terminology: variable or constant with or w/o negation = **literal***
- *Whole thing = **formula or sentence***

Expressive variable names

- *Rather than variable names like x , y , may use names like “rains” or “happy(John)”*
- *For now, “happy(John)” is just a string with no internal structure*
 - *there is no “John”*
 - *happy(John) $\Rightarrow$ $\neg$ happy(Jack) means the same as $x \Rightarrow \neg y$*

But what does it mean?

- *A formula defines a mapping*
 $(\text{assignment to variables}) \mapsto \{T, F\}$
- *Assignment to variables = **model***
- *For example, formula $\neg x$ yields mapping:*

x	$\neg x$
T	F
F	T

Questions about models and sentences

- *How many models make a sentence true?*
 - *A sentence is **satisfiable** if it is True in some model*
 - *If not satisfiable, it is a **contradiction** (False in every model)*
 - *A sentence is **valid** if it is True in every model (called a **tautology**)*

Questions about models and sentences

- *How is the variable X set in $\{\text{some}, \text{all}\}$ true models?*
- *This is the most frequent question an agent would ask: given my assumptions, can I conclude X ? Can I rule X out?*

More truth tables

x	y	$x \wedge y$
T	T	T
T	F	F
F	T	F
F	F	F

x	y	$x \vee y$
T	T	T
T	F	T
F	T	T
F	F	F

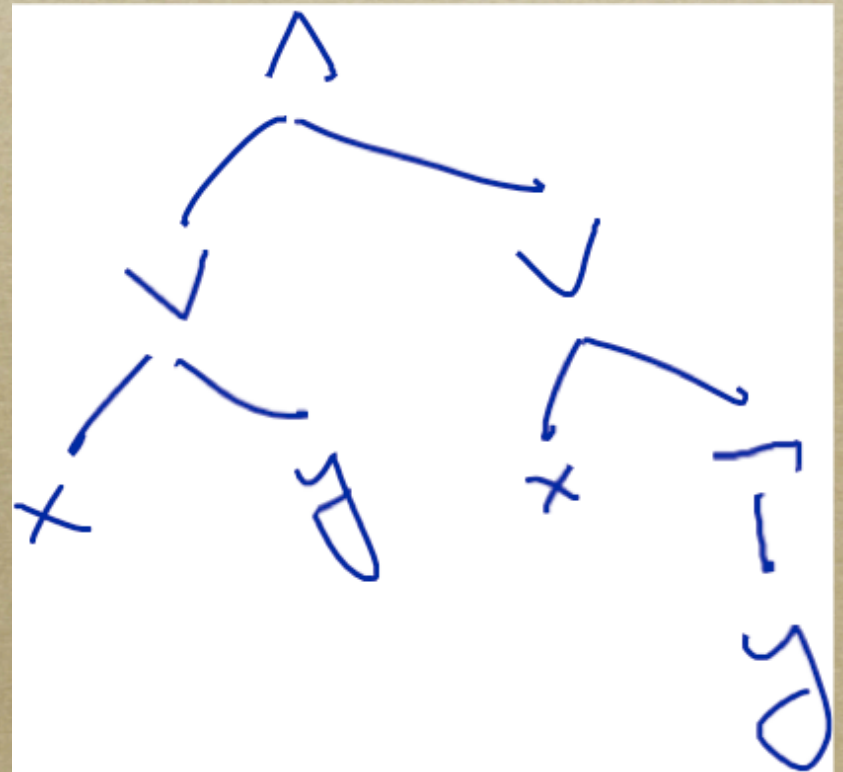
Truth table for implication

- $(a \Rightarrow b)$ is logically equivalent to $(\neg a \vee b)$
- If a is True, b must be True too
- If a False, no requirement on b
- E.g., “if I go to the movie I will have popcorn”: if no movie, may or may not have popcorn

a	b	$a \Rightarrow b$
T	T	T
T	F	F
F	T	T
F	F	T

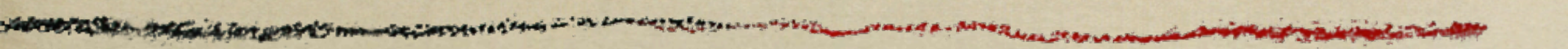
Complex formulas

- *To evaluate a bigger formula*
 - $(x \vee y) \wedge (x \vee \neg y)$ when $x = F, y = F$
- *Build a parse tree*
- *Fill in variables at leaves using model*
- *Work upwards using truth tables for connectives*



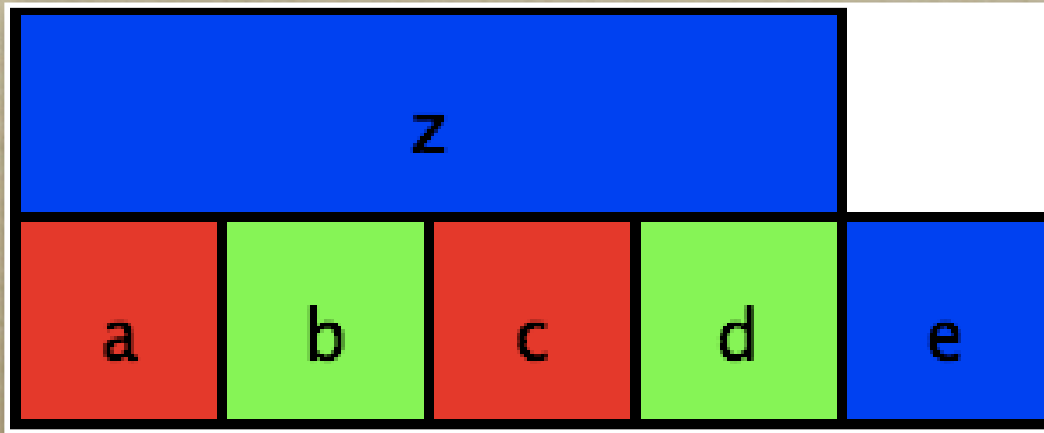
Another example

$$(x \vee y) \Rightarrow z \quad x = F, y = T, z = F$$



Example

3-coloring



Sudoku

SuDoku Puzzle

		6	3			4	7	
		5	8		7			
1							2	3
	6		1	9				
4	9							
						1	9	8
6					3	5		
		8		5				2
	7	4			6		8	

<http://www.cs.qub.ac.uk/~I.Spence/SuDoku/SuDoku.html>

Minesweeper

0	0	1	v1		
0	0	1	v2		
0	0	1	v3		
1	1	2	v4		
v8	v7	v6	v5		

$V = \{ v1, v2, v3, v4, v5, v6, v7, v8 \}, D = \{ B \text{ (bomb)}, S \text{ (space)} \}$

$C = \{ (v1,v2) : \{ (B, S), (S,B) \}, (v1,v2,v3) : \{ (B,S,S), (S,B,S), (S,S,B) \}, \dots \}$

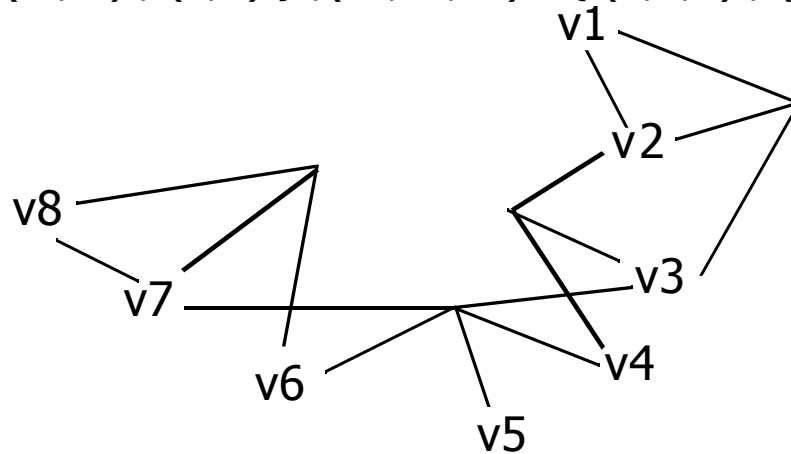


image courtesy Andrew Moore

Propositional planning

init: have(cake)

goal: have(cake), eaten(cake)

eat(cake):

pre: have(cake)

eff: -have(cake), eaten(cake)

bake(cake):

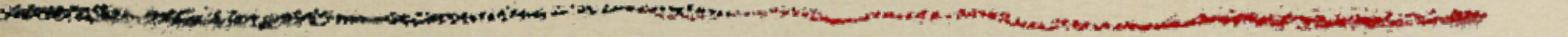
pre: -have(cake)

eff: have(cake)

Other important logic problems

- *Scheduling (e.g., of factory production)*
- *Facility location*
- *Circuit layout*
- *Multi-robot planning*

- *Important issue: handling uncertainty*



Working with formulas

Truth tables get big fast

x	y	z	$(x \vee y) \Rightarrow z$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Truth tables get big fast

x	y	z	a	$(x \vee y \vee a) \Rightarrow z$
T	T	T	T	
T	T	F	T	
T	F	T	T	
T	F	F	T	
F	T	T	T	
F	T	F	T	
F	F	T	T	
F	F	F	T	
T	T	T	F	
T	T	F	F	
T	F	T	F	
T	F	F	F	
F	T	T	F	
F	T	F	F	
F	F	T	F	
F	F	F	F	

Definitions

- Two sentences are *equivalent*, $A \equiv B$, if they have same truth value in every model
 - $(rains \Rightarrow pours) \equiv (\neg rains \vee pours)$
 - reflexive, transitive, commutative
- *Simplifying* = transforming a formula into a shorter*, equivalent formula

Transformation rules

$$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge$$

$$(\alpha \vee \beta) \equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee$$

$$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge$$

$$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee$$

$$\neg(\neg\alpha) \equiv \alpha \quad \text{double-negation elimination}$$

$$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee$$

$$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge$$

α, β, γ are arbitrary formulas

More rules

$$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition}$$

$$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta) \quad \text{implication elimination}$$

$$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination}$$

$$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta) \quad \text{de Morgan}$$

$$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) \quad \text{de Morgan}$$

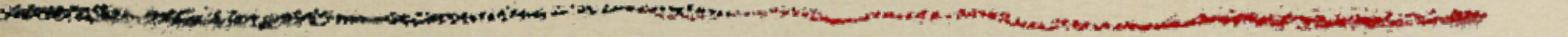
α, β are arbitrary formulas

Still more rules...

- ... can be derived from truth tables
- For example:
 - $(a \vee \neg a) \equiv \text{True}$
 - $(\text{True} \vee a) \equiv \text{True}$
 - $(\text{False} \wedge a) \equiv \text{False}$

Example

$$(a \vee \neg b) \wedge (a \vee \neg c) \wedge (\neg(b \vee c) \vee \neg a)$$



Normal Forms

Normal forms

- *A normal form is a standard way of writing a formula*
- *E.g., conjunctive normal form (CNF)*
 - *conjunction of disjunctions of literals*
 - $(x \vee y \vee \neg z) \wedge (x \vee \neg y) \wedge (z)$
 - *Each disjunct called a **clause***
- *Any formula can be transformed into CNF w/o changing meaning*

CNF cont'd

$$\begin{aligned} & \text{happy}(\text{John}) \wedge \\ & (\neg \text{happy}(\text{Bill}) \vee \text{happy}(\text{Sue})) \wedge \\ & \text{man}(\text{Socrates}) \wedge \\ & (\neg \text{man}(\text{Socrates}) \vee \text{mortal}(\text{Socrates})) \end{aligned}$$

- *Often used for storage of knowledge database*
 - *called **knowledge base** or **KB***
- *Can add new clauses as we find them out*
- *Each clause in KB is separately true (if KB is)*

Another normal form: DNF

- *DNF = disjunctive normal form = disjunction of conjunctions of literals*
- *Doesn't compose the way CNF does: can't just add new conjuncts w/o changing meaning of KB*
- *Example:*

$$\begin{aligned} & (rains \vee \neg pours) \wedge fishing \\ & \equiv \\ & (rains \wedge fishing) \vee (\neg pours \wedge fishing) \end{aligned}$$

Transforming to CNF or DNF

- *Naive algorithm:*
 - *replace all connectives with $\wedge \vee \neg$*
 - *move negations inward using De Morgan's laws and double-negation*
 - *repeatedly distribute over $\wedge$ over $\vee$ for DNF ($\vee$ over $\wedge$ for CNF)*

Example

- *Put the following formula in CNF*

$$(a \vee b \vee \neg c) \wedge \neg(d \vee (e \wedge f)) \wedge (c \vee d \vee e)$$

$$(a \vee b \vee \neg c) \wedge \neg(d \vee (e \wedge f)) \wedge (c \vee d \vee e)$$

Example

- *Now try DNF*

$$(a \vee b \vee \neg c) \wedge \neg(d \vee (e \wedge f)) \wedge (c \vee d \vee e)$$

Discussion

- *Problem with naive algorithm: it's exponential! (Space, time, size of result.)*
- *Each use of distributivity can almost double the size of a subformula*

A smarter transformation

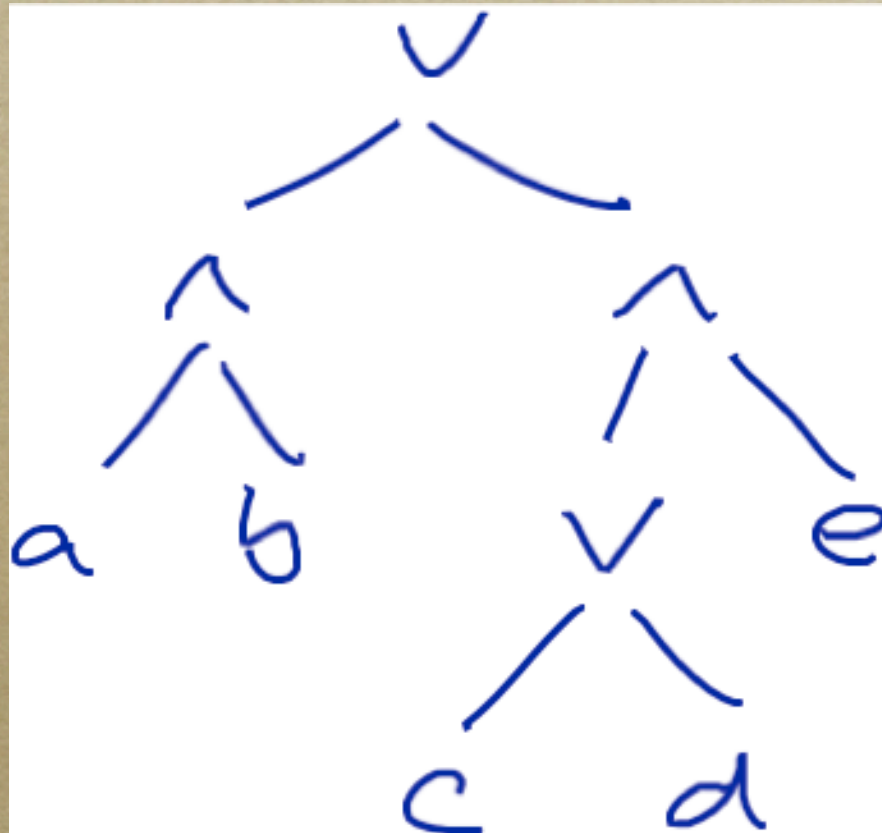
- *Can we avoid exponential blowup in CNF?*
- *Yes, if we're willing to introduce new variables*
- *G. Tseitin. On the complexity of derivation in propositional calculus. Studies in Constrained Mathematics and Mathematical Logic, 1968.*

Tseitin example

- *Put the following formula in CNF:*

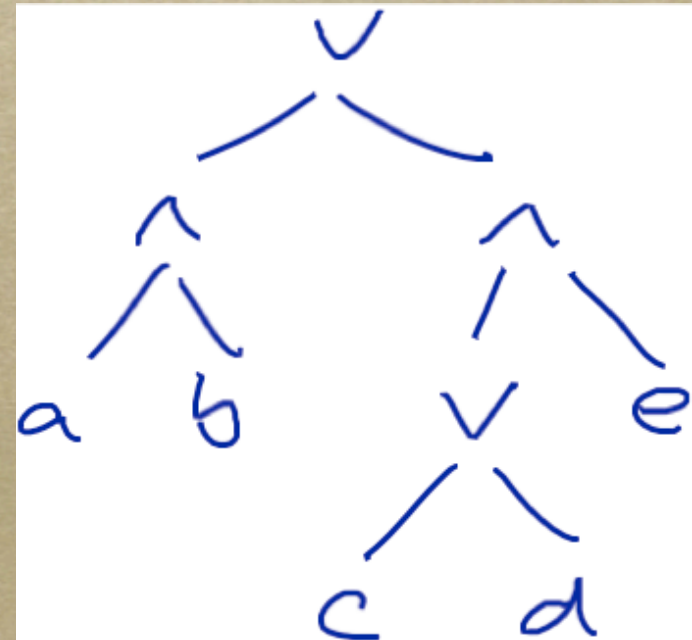
$$(a \wedge b) \vee ((c \vee d) \wedge e)$$

- *Parse tree:*



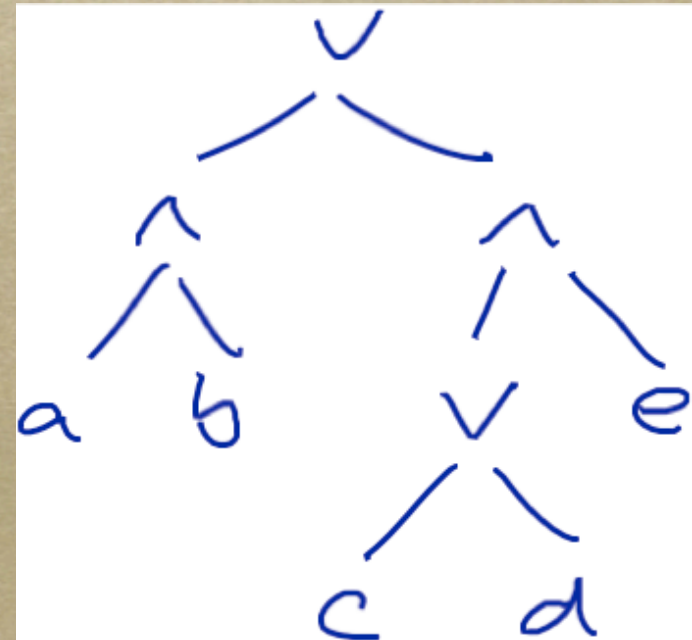
Tseitin transformation

- *Introduce temporary variables*
 - $x = (a \wedge b)$
 - $y = (c \vee d)$
 - $z = (y \wedge e)$



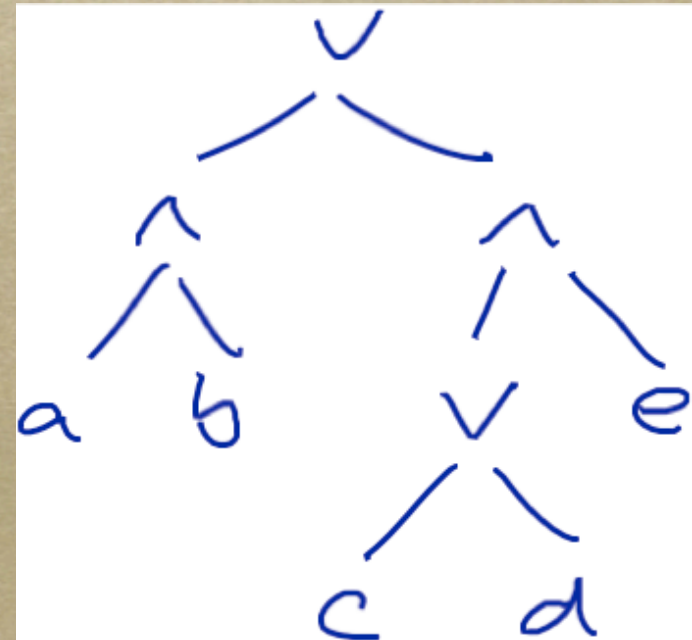
Tseitin transformation

- To ensure $x = (a \wedge b)$, want
 - $x \Rightarrow (a \wedge b)$
 - $(a \wedge b) \Rightarrow x$



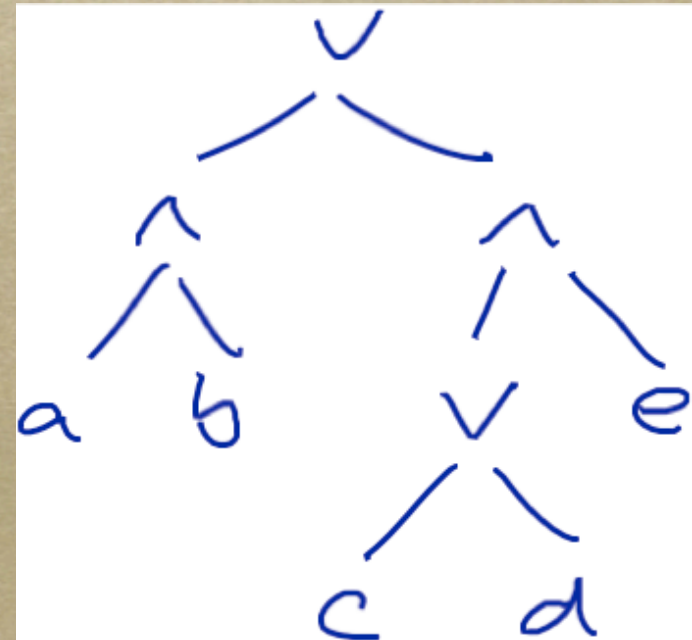
Tseitin transformation

- $x \Rightarrow (a \wedge b)$
- $(\neg x \vee (a \wedge b))$
- $(\neg x \vee a) \wedge (\neg x \vee b)$



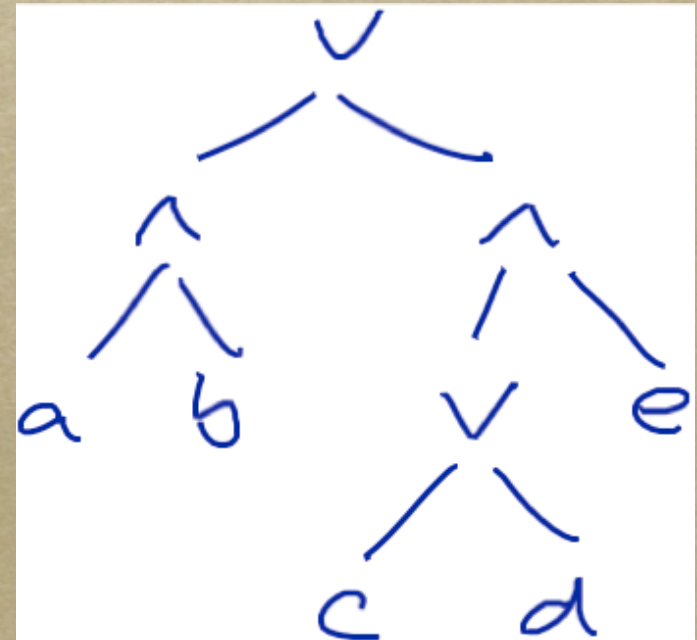
Tseitin transformation

- $(a \wedge b) \Rightarrow x$
- $(\neg(a \wedge b) \vee x)$
- $(\neg a \vee \neg b \vee x)$



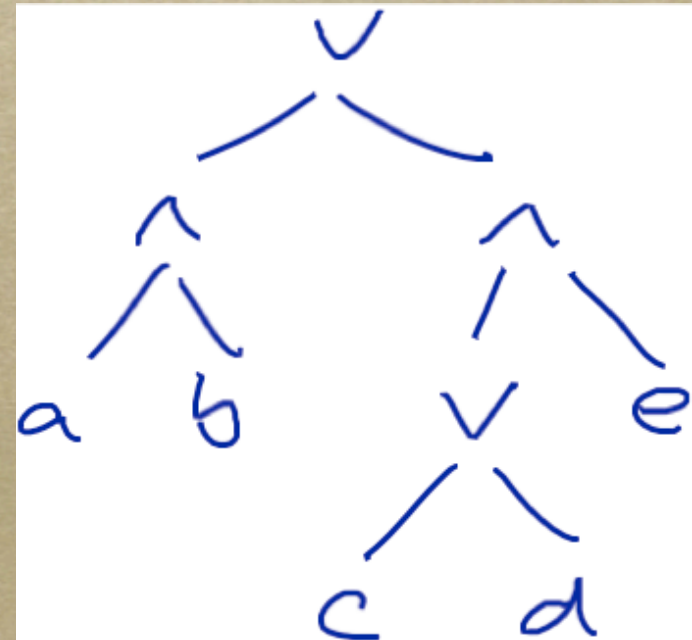
Tseitin transformation

- *To ensure $y = (c \vee d)$, want*
 - $y \Rightarrow (c \vee d)$
 - $(c \vee d) \Rightarrow y$



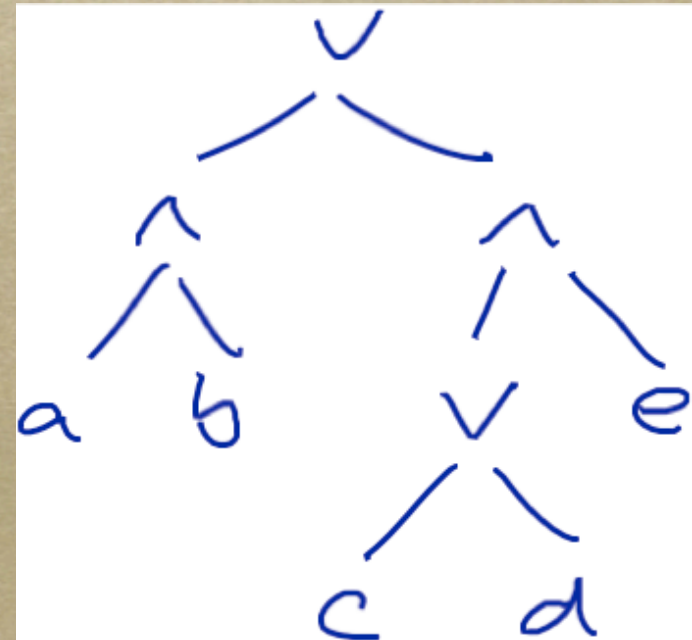
Tseitin transformation

- $y \Rightarrow (c \vee d)$
- $(\neg y \vee c \vee d)$
- $(c \vee d) \Rightarrow y$
- $((\neg c \wedge \neg d) \vee y)$
- $(\neg c \vee y) \wedge (\neg d \vee y)$

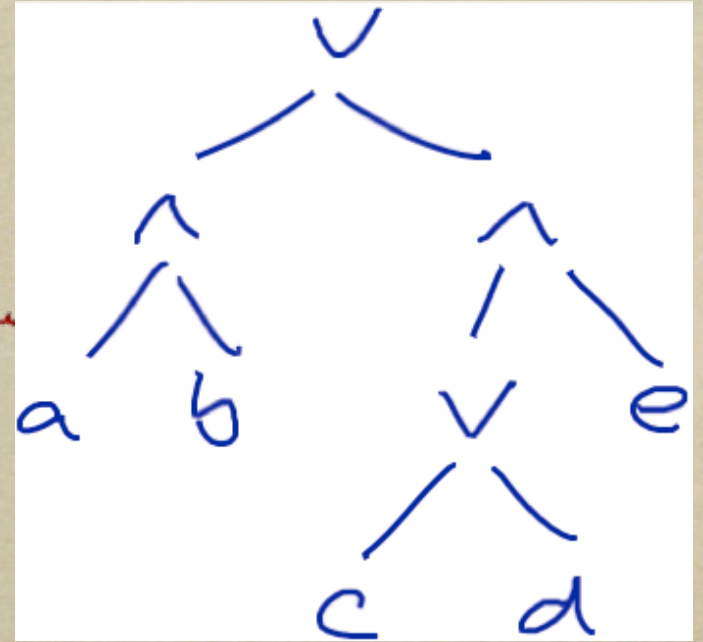


Tseitin transformation

- *Finally*, $z = (y \wedge e)$
- $z \Rightarrow (y \wedge e) \equiv (\neg z \vee y) \wedge (\neg z \vee e)$
- $(y \wedge e) \Rightarrow z \equiv (\neg y \vee \neg e \vee z)$



Tseitin end result



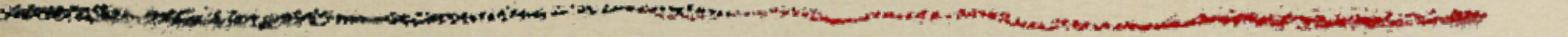
$$(a \wedge b) \vee ((c \vee d) \wedge e) \equiv$$

$$(\neg x \vee a) \wedge (\neg x \vee b) \wedge (\neg a \vee \neg b \vee x) \wedge$$

$$(\neg y \vee c \vee d) \wedge (\neg c \vee y) \wedge (\neg d \vee y) \wedge$$

$$(\neg z \vee y) \wedge (\neg z \vee e) \wedge (\neg y \vee \neg e \vee z) \wedge$$

$$(x \vee z)$$



Proofs

Entailment

- *Sentence A entails sentence B , $A \models B$, if B is True in every model where A is*
 - *same as saying that $(A \Rightarrow B)$ is valid*

Proof tree

- *A tree with a formula at each node*
- *At each internal node, children $\models$ parent*
- *Leaves: assumptions or premises*
- *Root: consequence*
- *If we believe assumptions, we should also believe consequence*

Proof tree example

$rains \Rightarrow pours$

$pours \wedge outside \Rightarrow rusty$

$rains$

$outside$

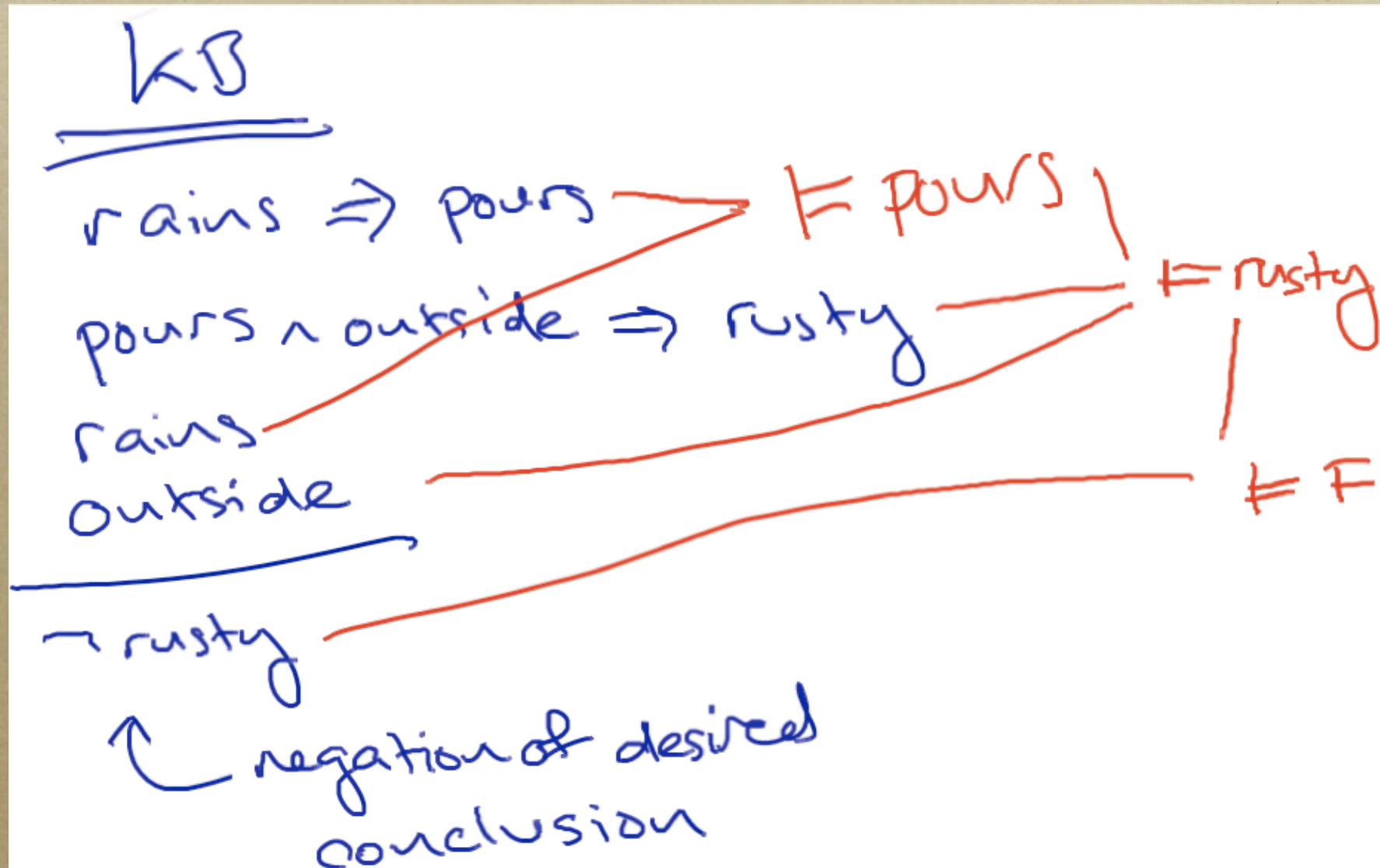
Proof by contradiction

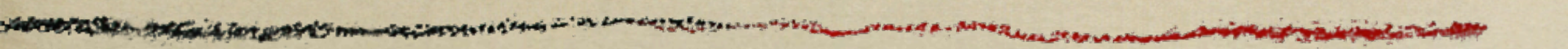
- *Assume opposite of what we want to prove, show it leads to a contradiction*
- *Suppose we want to show $KB \models S$*
- *Write KB' for $(KB \wedge \neg S)$*
- *Build a proof tree with*
 - *assumptions drawn from clauses of KB'*
 - *conclusion = F*
 - *so, $(KB \wedge \neg S) \models F$ (contradiction)*

Proof by contradiction

KB
 $\text{rains} \Rightarrow \text{pours}$
 $\text{pours} \wedge \text{outside} \Rightarrow \text{rusty}$
 rains
 outside
—
 $\neg \text{rusty}$
↖ negation of desired
conclusion

Proof by contradiction





Inference rules

Inference rule

- *To make a proof tree, we need to be able to figure out new formulas entailed by KB*
- *Method for finding entailed formulas = **inference rule***
- *We've implicitly been using one already*

Modus ponens

$$\frac{(a \wedge b \wedge c \Rightarrow d) \quad a \quad b \quad c}{d}$$

- *Probably most famous inference rule: all men are mortal, Socrates is a man, therefore Socrates is mortal*

- *Quantifier-free version:*

man(Socrates) $\wedge$

(man(Socrates) $\Rightarrow$ mortal(Socrates))

Another inference rule

$$\frac{(a \Rightarrow b) \quad \neg b}{\neg a}$$

- *Modus tollens*
- *If it's raining the grass is wet; the grass is not wet, so it's not raining*

One more...

$$\frac{(a \vee b \vee c) \quad (\neg c \vee d \vee e)}{a \vee b \vee d \vee e}$$

- *Resolution*
- *Combines two sentences that contain a literal and its negation*
- *Not as commonly known as modus ponens / tollens*

Resolution example

- *Modus ponens / tollens are special cases*

- *Modus tollens:*

$$(\neg \textit{raining} \vee \textit{grass-wet}) \wedge \neg \textit{grass-wet} \models \neg \textit{raining}$$

Resolution

$$\frac{(a \vee b \vee c) \quad (\neg c \vee d \vee e)}{a \vee b \vee d \vee e}$$

- *Simple proof by case analysis*
- *Consider separately cases where we assign $c = \text{True}$ and $c = \text{False}$*

Resolution

$$(a \vee b \vee c) \wedge (\neg c \vee d \vee e)$$

◦ *Case $c = \text{True}$*

$$(a \vee b \vee T) \wedge (F \vee d \vee e)$$

$$= (T) \wedge (d \vee e)$$

$$= (d \vee e)$$

Resolution

$$(a \vee b \vee c) \wedge (\neg c \vee d \vee e)$$

◦ *Case $c = \text{False}$*

$$(a \vee b \vee F) \wedge (T \vee d \vee e)$$

$$= (a \vee b) \wedge (T)$$

$$= (a \vee b)$$

Resolution

$$(a \vee b \vee c) \wedge (\neg c \vee d \vee e)$$

- *Since c must be True or False, conclude*

$$(d \vee e) \vee (a \vee b)$$

as desired

Soundness and completeness

- *An inference procedure is **sound** if it can only conclude things entailed by KB*
 - *common sense; haven't discussed anything unsound*
- *A procedure is **complete** if it can conclude everything entailed by KB*

Completeness

- *Modus ponens by itself is **incomplete***
- *Resolution is **complete** for propositional logic*
 - *not obvious—famous theorem*

Variations

- *Horn clause inference (faster)*
- *Ways of handling uncertainty (slower)*
- *CSPs (sometimes more convenient)*
- *Quantifiers / first-order logic*

Horn clauses

- *Horn clause: $(a \wedge b \wedge c \Rightarrow d)$*
- *Equivalently, $(\neg a \vee \neg b \vee \neg c \vee d)$*
- *Disjunction of literals, **at most one** of which is positive*
- *Positive literal = **head**, rest = **body***

Use of Horn clauses

- *People find it easy to write Horn clauses (listing out conditions under which we can conclude head)*

$$\text{happy}(\text{John}) \wedge \text{happy}(\text{Mary}) \Rightarrow \text{happy}(\text{Sue})$$

- *No negative literals in above formula; again, easier to think about*

Why are Horn clauses important

- *Modus ponens alone is complete*
- *So is modus tollens alone*
- *Inference in a KB of propositional Horn clauses is linear*

Forward chaining

- *Look for a clause with all body literals satisfied*
- *Add its head to KB (modus ponens)*
- *Repeat*
- *See RN for more details*

Handling uncertainty

- *Fuzzy logic / certainty factors*
 - *simple, but don't scale*
- *Nonmonotonic logic*
 - *also doesn't scale*
- *Probabilities*
 - *may or may not scale—more later*
 - *Dempster-Shafer (interval probability)*

Certainty factors

- *KB assigns a **certainty factor** in $[0, 1]$ to each proposition*
- *Interpret as “degree of belief”*
- *When applying an inference rule, certainty factor for consequent is a function of certainty factors for antecedents (e.g., minimum)*

Problems w/ certainty factors

- *Hard to separate a large KB into mostly-independent chunks that interact only through a well-defined interface*
- *Certainty factors are not probabilities (i.e., do not obey Bayes' Rule)*

Nonmonotonic logic

- *Suppose we believe all birds can fly*
- *Might add a set of sentences to KB*

bird(Polly) $\Rightarrow$ flies(Polly)

bird(Tweety) $\Rightarrow$ flies(Tweety)

bird(Tux) $\Rightarrow$ flies(Tux)

bird(John) $\Rightarrow$ flies(John)

...

Nonmonotonic logic

- *Fails if there are penguins in the KB*
- *Fix: instead, add*

$$\text{bird}(\text{Polly}) \wedge \neg \text{ab}(\text{Polly}) \Rightarrow \text{flies}(\text{Polly})$$

$$\text{bird}(\text{Tux}) \wedge \neg \text{ab}(\text{Tux}) \Rightarrow \text{flies}(\text{Tux})$$

...

- *$\text{ab}(\text{Tux})$ is an “abnormality predicate”*
- *Need separate $\text{ab}_i(x)$ for each type of rule*

Nonmonotonic logic

- *Now set as few abnormality predicates as possible*
- *Can prove $\text{flies}(\text{Polly})$ or $\text{flies}(\text{Tux})$ with no $\text{ab}(x)$ assumptions*
- *If we assert $\neg \text{flies}(\text{Tux})$, must now assume $\text{ab}(\text{Tux})$ to maintain consistency*
- *Can't prove $\text{flies}(\text{Tux})$ any more, but can still prove $\text{flies}(\text{Polly})$*

Nonmonotonic logic

- *Works well as long as we don't have to choose between big sets of abnormalities*
 - *is it better to have 3 flightless birds or 5 professors that don't wear jackets with elbow-patches?*
 - *even worse with nested abnormalities: birds fly, but penguins don't, but superhero penguins do, but ...*