Numerical Methods within the Ant Colony: The **Illuminating** Case of Multi-Objective Macronutrient Regulation in Eusocial Insects

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Colony-level Macronutrient Regulation
(Dussutour and Simpson 2009)

Nutrient Regulation
Colony Regulation
Quantifying Behavior
Insights
Algorithm
Conclusions

(250 ants)

Feeder 1
(high protein)

Feeder 2
(high carbohydrate)

1p:3c

2p:1c
Colony-level Macronutrient Regulation
(Dussutour and Simpson 2009)

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(250 ants)

Feeder 1
(2p:1c)

Feeder 2
(1p:3c)

Protein (mg/ant)
Carbohydrates (mg/ant)

12.5
10
2
1

Feeder 1
[2p:1c]

Feeder 2
[1p:3c]
Colony-level Macronutrient Regulation
(Dussutour and Simpson 2009)

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Feeder 2
(high carbohydrate)

Feeder 1
(high protein)

Mixture without Larvae
[10p:12.5c]

Mixture with Larvae
[10p:8c]

1p:3c
(250 ants)
(100 larvae)

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(250 ants)

(100 larvae)

Feeder 2
(high carbohydrate)

Feeder 1
(high protein)

Mixture with Larvae
[10p:12c]

Mixture with Larvae
[10p:8c]

Mixture without Larvae
[10p:12c]
Colonies-level Macronutrient Regulation

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**Nutrient Regulation**

**Colony Regulation**

**Quantifying Behavior**

**Insights**

**Algorithm**

**Conclusions**

**Ultimate cause is clear, but what is the mechanism/implementation?**
Colony-level Macronutrient Regulation
Mathematics of Allocation

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![Graph showing protein and carbohydrate consumption]

- Feeder 1 (2p:1c)
- Feeder 2 (1p:3c)

0.58 g
0.45 g

Protein (mg/ant)
Carbohydrates (mg/ant)
Colony-level Macronutrient Regulation
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Protein (mg/ant)
Carbohydrates (mg/ant)

0 0.2 0.4 0.6 0.8
0 0.2 0.4 0.6 0.8

Feeder 1 [2p:1c]
Feeder 2 [1p:3c]

0.58 g protein
0.45 g carbohydrates

x1
x2

Feeder 1 [2p:1c] (ants)
Feeder 2 [1p:3c] (ants)

0 2 4 6 8
0 2 4 6 8

0.58 g protein
0.45 g carbohydrates

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Colony-level Macronutrient Regulation
Mathematics of Allocation

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- Colony Regulation
- Quantifying Behavior
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- Conclusions

\[
\begin{align*}
\frac{a_{p1} x_1^* + a_{p2} x_2^*}{a_{c1} x_1^* + a_{c2} x_2^*} &= c_p \\
0.58 g &= a_{p1} x_1^* + a_{p2} x_2^* \\
0.45 g &= a_{c1} x_1^* + a_{c2} x_2^*
\end{align*}
\]
Colony-level Macronutrient Regulation
Mathematics of Allocation

\[
\begin{align*}
\mathbf{A} \mathbf{x}^* &= \mathbf{c} \\
\begin{cases}
  a_{p1} x_1^* + a_{p2} x_2^* &= c_p \\
  a_{c1} x_1^* + a_{c2} x_2^* &= c_c
\end{cases}
\end{align*}
\]
Colony-level Macronutrient Regulation

Mathematics of Allocation

\[ \begin{align*}
  a_{p1}x_1^* + a_{p2}x_2^* &= c_p \\
  a_{c1}x_1^* + a_{c2}x_2^* &= c_c
\end{align*} \]

\[ \vec{x}^* = A^{-1} \vec{c} \quad \text{– Ants are doing } \text{decentralized} \text{ matrix inversion?} \]
In natural settings, there are more feeders than vital nutrients.
Colony-level Macronutrient Regulation
Mathematics of Allocation

\[
\begin{align*}
\begin{cases}
  a_{11} x_1^* + \cdots + a_{1n} x_n^* &= c_1 \\
  \vdots \\
  a_{m1} x_1^* + \cdots + a_{mn} x_n^* &= c_m \\
\end{cases}
\iff
A_{m \times n} \bar{x}^* = \vec{c}_m
\end{align*}
\]

Solutions may not exist.
Colony-level Macronutrient Regulation

Mathematics of Allocation

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Continuum of solutions! Which solution is best?
Do ants have a pseudoinverse?
Colony-level Macronutrient Regulation
Mathematics of Allocation

minimize \( F(\vec{x}) \)
subject to \( A_{m \times n} \vec{x} \geq \vec{c}_m \)

minimal effort example:
\[
F(\vec{x}) \triangleq x_1 + \cdots + x_n
\]

Alternative model: Optimization under constraints
Solutions exist and are unique!
minimize $F(\vec{x})$
subject to $A_{m \times n} \vec{x} \geq \vec{c}_m$

minimal effort example:
$F(\vec{x}) \triangleq x_1 + \cdots + x_n$

Alternative model: **Optimization under constraints**
Decentralized solver implementable on ants?
minimize $F(\vec{x})$
subject to $A_{m \times n}\vec{x} \geq \vec{c}_m$

“minimal effort:” $F(\vec{x}) \triangleq x_1 + \cdots + x_n$
minimize $F(\vec{x})$

subject to $A_{m \times n} \vec{x} \geq \vec{c}_m$

“minimal effort:” $F(\vec{x}) \triangleq x_1 + \cdots + x_n$
minimize $F(\vec{x})$
subject to $A_{m \times n} \vec{x} \geq \vec{c}_m$

"minimal effort:"
$F(\vec{x}) \triangleq x_1 + \cdots + x_n$
minimize $F(\vec{x})$
subject to $A_{m \times n} \vec{x} \geq \vec{c}_m$

"minimal effort:"

$F(\vec{x}) \triangleq x_1 + \cdots + x_n$
Existing IFD-inspired dynamic resource allocation strategies in engineering

- AAV cooperative control (Finke and Passino 2007; Moore et al. 2009)
- Water distribution (Ramirez-Llanos and Quijano 2010)
- Temperature control (Pantoja et al. 2011)
Algorithmic Insights

Social Foraging – Ideal Free Distribution (Fretwell and Lucas 1969; Fretwell 1972)

Nutrient Regulation

Insights

Lighting

Social Foraging

Algorithm

Conclusions

$x_1 = 3$

$x_2 = 2$

$x_3 = 5$

$s_i(x_i)$

minimize $\max\{s_i(x_i)\}$

subject to $x_1 + \cdots + x_n = N$

$s_1(x_1^*) \approx s_2(x_2^*) \approx s_3(x_3^*)$
Algorithmic Insights

Social Foraging – Ideal Free Distribution (Fretwell and Lucas 1969; Fretwell 1972)

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\[
x_1 = 3 \\
x_2 = 2 \\
x_3 = 5
\]

\[
s_i(x_i) \\
\minimize \sum \int_0^{x_i} \frac{1}{s_i(\tau)} d\tau
\]

subject to \( x_1 + \cdots + x_n \geq N \)

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\[
\begin{align*}
  x_1 &= 3 \\
  x_2 &= 2 \\
  x_3 &= 5 \\

  s_i(x_i) \\

  x_1 + x_2 + x_3 &= N \\
  s_1(x_1^*) &\approx s_2(x_2^*) &\approx s_3(x_3^*) \\

  \text{minimize} \quad F(\bar{x}) \\
  \text{subject to} \quad a_1 x_1 + \cdots + a_n x_n \geq c
\end{align*}
\]
MultiIFD discrete-time realization with speed–accuracy tradeoff parameter $\delta$:

A violation of constraint $j \in \{1, 2, \ldots, m\}$ induces marginal IFD:

$$\bar{x}_{\text{next}} - \bar{x}_{\text{prev}} \propto \left[ \frac{a_{j1}}{\nabla_1 F(x)}, \frac{a_{j2}}{\nabla_2 F(x)}, \ldots, \frac{a_{jn}}{\nabla_n F(x)} \right]^T.$$
MultiIFD discrete-time realization with speed–accuracy tradeoff parameter $\delta$:  

- A violation of constraint $j \in \{1, 2, \ldots, m\}$ induces marginal IFD: 
  
  $$\vec{x}_{\text{next}} - \vec{x}_{\text{prev}} \propto \begin{bmatrix} a_{j1} \nabla_1 F(\vec{x}) & a_{j2} \nabla_2 F(\vec{x}) & \cdots & a_{jn} \nabla_n F(\vec{x}) \end{bmatrix}^T.$$ 

- For each patch $i \in \{1, 2, \ldots, n\}$, animals regularly deallocate: 
  
  $$x_{i,\text{next}} - x_{i,\text{prev}} = -\delta.$$
**“MultiIFD” Asynchronous Distributed Solver**

- **MultiIFD** discrete-time realization with speed–accuracy tradeoff parameter $\delta$:
  - A violation of constraint $j \in \{1, 2, \ldots, m\}$ induces **marginal** IFD:
    
    $$
    \vec{x}_{\text{next}} - \vec{x}_{\text{prev}} \propto \begin{bmatrix}
    a_{j1} \nabla_1 F(\vec{x}) \\
    a_{j2} \nabla_2 F(\vec{x}) \\
    \vdots \\
    a_{jn} \nabla_n F(\vec{x})
    \end{bmatrix}^T .
    $$
  
  - For each patch $i \in \{1, 2, \ldots, n\}$, animals regularly deallocate:
    
    $$
    x_{i,\text{next}}^* - x_{i,\text{prev}}^* = -\delta.
    $$

---

*Figure:* A diagram illustrating the allocation and deallocation of resources between two patches, highlighting the impact of constraint violations and speed–accuracy tradeoff on the system's dynamics.
**MultiIFD** discrete-time realization with speed–accuracy tradeoff parameter $\delta$:

- A violation of constraint $j \in \{1, 2, \ldots, m\}$ induces **marginal** IFD:

  $$\vec{x}^\text{next} - \vec{x}^\text{prev} \propto \begin{bmatrix}
a_{j1} \nabla_1 F(\vec{x}) \\
a_{j2} \nabla_2 F(\vec{x}) \\
\vdots \\
a_{jn} \nabla_n F(\vec{x})
\end{bmatrix}^\top.$$

- For each patch $i \in \{1, 2, \ldots, n\}$, animals regularly deallocate:

  $$\vec{x}^\text{next}_i - \vec{x}^\text{prev}_i = -\delta.$$
MultiIFD discrete-time realization with speed-accuracy tradeoff parameter $\delta$: A violation of constraint $j \in \{1, 2, \ldots, m\}$ induces marginal IFD:

$$\vec{x}_{\text{next}} - \vec{x}_{\text{prev}} \propto \left[ a_j^1 \nabla_1 F(\vec{x}), a_j^2 \nabla_2 F(\vec{x}), \ldots, a_j^n \nabla_n F(\vec{x}) \right]^\top.$$ 

For each patch $i \in \{1, 2, \ldots, n\}$, animals regularly reallocate:

$$x_{\text{next}}^i - x_{\text{prev}}^i = -\delta.$$
Validation with Animal Models

Simulated trajectory

Feeder 1 [2p:1c] (ants)

Feeder 2 [1p:3c] (ants)

0.58 g protein
0.45 g carbohydrates

\[ x_1 + x_2 = 17 \]
\[ x_1 + x_2 = 15 \]
\[ x_1 + x_2 = 13 \]
\[ x_1 + x_2 = 11 \]
Validation with Animal Models

Simulated trajectory
Outcomes from simulated experiments

Feeder 1 [2p:1c] (ants)

Feeder 2 [1p:3c] (ants)

0.58 g protein

0.45 g carbohydrates
Simulated trajectory
Outcomes from simulated experiments

Outcomes from Dussutour and Simpson (2009)

Feeder 1 [2p:1c] (ants)
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0.58 g protein

0.45 g carbohydrates

$x_1 + x_2 = 17$
$x_1 + x_2 = 15$
$x_1 + x_2 = 13$
$x_1 + x_2 = 11$
$x_1 + x_2 = 9$
$x_1 + x_2 = 7$
$x_1 + x_2 = 5$

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Validation on Small-scale Lighting Testbed
Real-time Shoebox

- Real-time control hardware
- Asynchronous events
- Auto-commissioning period

*nb: CDMA and VLC*

*(Linnartz et al. 2008)*
Validation on Small-scale Lighting Testbed
Centralized Solver Results
Validation on Small-scale Lighting Testbed
MultiIFD Results – Slide Along Constraint
Summary and Conclusions

- **Summary:**
  - Social-insect colonies regulate macronutrient intake
  - Colonies *somehow* solve non-separable allocation problem
  - Optimization under multiple constraints is useful conceptual tool
  - Lighting analogy suggests new experiments

- **MultiIFD principles:**
  - *Stigmergic* coordination – colony nutrients are a *shared memory*
  - Decentralized implementation is robust and adaptive

- **Ongoing work:** *Temnothorax* as model system
  - Very high resolution possible
  - Measurement of small quantities of ingested food is challenging

- **Future work:** Stoichiometry; *Camponotus; Solenopsis; Paratrechina*
Summary:
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Thanks!

Thanks! Questions?
Thanks!

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Questions? Comments?