A (Logical) Framework for Collaboration

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Frank and I have been colleagues since 1988!
Many projects, many students, many interactions over many years.

A central theme: **logical frameworks**.

Frank has long taken the lead on developing and extending these ideas.

- **Substructurality**: linear and concurrent LF’s.
- **Implementation**: unification, argument synthesis, theorem proving.
- **Meta-theory**: normalization, decidability.

Frank’s work is exemplary in integrating **theory** and **practice**, the ethos of the Principles of Programming Group at CMU.
A major theme of Frank’s work has been the design and development of Twelf.

- What’s in a name? First there was Elf, then there was Zwölf Twelf.
- See https://twelf.org for code, examples, and ask The Twelf Elf.

Many components, variations, experiments:

- Type checking: higher-order abstract syntax, judgments-as-types.
- Unification-based argument synthesis, definition mechanism.
- Logic programming interpretation: backchaining (cf λ-Prolog)
- Coverage and totality checking: a prover for ∀∃ formulae.

The tool of choice for language definition, both in-the-small and in-the-large.
Language Definition in Twelf

Higher-order Abstract Syntax:
• Syntactic categories: $tp : Type$, $tm : Type$.
• Hierarchical structure: $arr : tp → tp → tp$.
• Binding and scope: $lam : (tm → tm) → tm$.

Statics via Judgments-as-Types:
• Typing: $of : tm → tp → Type$.
• Example: $arr-I : (x : tm → of x T_1 → of (F x) T_2) → of (lam F) (arr T_1 T_2)$.

Dynamics via Plotkin’s SOS:
• Transition: $step : tm → tm → Type$.
• Example: $beta : step (ap (lam F) M) (F M)$.
Type preservation:

- Judgment: \( \text{pres} : \text{of } e \ T \rightarrow \text{step } e \ e' \rightarrow \text{of } e' \ T \rightarrow \text{Type} \).
- Proofs relate derivations to derivations:

\[
\text{beta-pres} : \text{pres} (\text{arr-E}(\text{arr-I } DF \ DA) (\beta) (DF \ DA)).
\]

Preservation Thm: \( \forall Dst : \text{step } e \ e' \ \forall Dof : \text{of } e \ T \ \exists Dof : \text{of } e' \ T \).

- Worlds specification: \( \%\text{worlds()} (\text{pres } \_\_ ) \).
- Mode specification: \( \%\text{mode pres }+Dst +Dof -Dof \).
- Totality check: \( \%\text{total } D (\text{pres } D \_\_ ) \).

The proof is entirely readable and maintainable! (cf ML programming)
Remarkably, Twelf scales to handle all of Standard ML!

- **Elaboration**: human-oriented external language to well-behaved internal language.
- **Statics of IL**: standard typing rules plus some tricks for state.
- **Dynamics of IL**: SOS rules with state.
- **Progress and Preservation**: well-elaborated programs are well-behaved.

**Key points:**

- **Pair-programmed** weekly over about six months.
- **Proofs are readable**: enumeration of cases + coverage and totality.
- **Size**: about 30,000 lines of Twelf in total.

But is it the same language? (Long discussion ensues.)
Meta-Theory of LF Itself

*On Equivalence and Canonical Forms in the LF Type Theory* is memorable.

- Reduction-free algorithm for equality checking with canonical “witness.”
- Soundness and completeness by type-directed logical relations.
- (Later extended by Crary to families.)

Referees objected to there being no proof of “strong normalization” (of what?).

Mechanized independently by Urban in Nominal Isabelle.

- **Mistake** in the treatment of canonical families.
- We suggested two fixes, **both worked**.
- I chaired presentation at conference.
I have been very fortunate to be included as an honorary member of Frank’s family.

- Nancy, his wife, and her parents, brothers, and sister.
- Their children, Andreas, Marina, and Nils.
- Their nieces, nephews, in-laws, and friends.
- Many family occasions and events, both joyous and otherwise.

I am enormously grateful for the friendship and support that Frank and Nancy have given me from (literally) the day I arrived in Pgh through many years of ups and downs, both personal and professional.
Happy Birthday, Frank!

Thanks for many years of collaboration and friendship!