1 Introduction

This note extends the by-value formulation of PCF (Harper, 2018) to account for two aspects of exceptions:

1. Control aspect: normal and exceptional returns.
2. Data aspect: exception values.

The first aspect extends the modal formulation of PCF-by-value to account for both “normal” and “exceptional” returns. The elimination form for the computation type is enriched to account not only for the normal return, but also the exceptional return. The exception return carries with it a value of an unspecified type exn. The second aspect defines the type exn to be the type clsfd of dynamically classified values. In this setting dynamic classifiers label exception values with the form of exception and associate to a value of the corresponding type. Dynamic classification for exceptions is a good choice because it naturally supports modular composition. Depending on how an exception is spelled is anti-modular, because two components might have an exception named error, spelled thus. But in fact those two exceptions, which happen to be called the same thing, should be distinct. To achieve that requires binding and scope so that, by \(\alpha\)-conversion, the spelling of a name never matters.

2 Exception Mechanism

The modal formulation of PCF-by-value is extended with a construct for raising a value of some unspecified type exn, and the elimination form for the modality is extended to account for both the normal return and the exceptional return of a computation. The statics of this extension is given in Figure 1, and the dynamics in Figure 2. Both are formulated with states of the form \(\nu \Sigma \{ e \}\) to account for exception values in the next section. Type safety (progress and preservation) is readily established using methods considered in the text.

3 Exception Values

The type exn of exception values is defined to be the type clsfd of dynamically classified values. The idea is that the class of an exception value determines the type of data associated with that exception class. Exception classes are introduced using an exception declaration; an exception is raised by specifying its class; and an exception is handled by dispatching on the class.
\[
\begin{align*}
\text{RAISE} & \\
\Gamma \vdash_{\Sigma} e : \text{exn} & \quad \text{TRY} \\
\Gamma \vdash_{\Sigma} e_1 : \text{comp}(\tau) & \quad \Gamma, x : \tau \vdash_{\Sigma} e_2 \sim \tau' \quad \Gamma, x : \text{exn} \vdash_{\Sigma} e_3 \sim \tau' \\
\Gamma \vdash_{\Sigma} \text{raise}(e) \sim \tau & \\
\hline
\end{align*}
\]

Figure 1: Modal \textbf{PCF} with Exceptions: Statics

\[
\begin{align*}
\text{RAISE-FINAL} & \\
\nu \Sigma \{ e \} & \quad \text{RAISE-ARG} \\
\nu \Sigma \{ \text{raise}(e) \} & \quad \nu \Sigma \{ e \} \mapsto \nu \Sigma' \{ e' \} \\
\nu \Sigma \{ \text{raise}(e) \} & \quad \nu \Sigma \{ \text{raise}(e) \} \mapsto \nu \Sigma' \{ \text{raise}(e') \} \\
\hline
\text{LET} & \\
\nu \Sigma \{ e_1 \} & \quad \nu \Sigma \{ \text{let} \{ \tau \}(e_1; x.e_2; x.e_3) \} \mapsto \nu \Sigma' \{ \text{let} \{ \tau \}(e'_1; x.e_2; x.e_3) \} \\
\nu \Sigma \{ \text{let} \{ \tau \}(\text{comp}(e); x.e_2; x.e_3) \} & \quad \nu \Sigma \{ \text{let} \{ \tau \}(\text{comp}(e'); x.e_2; x.e_3) \} \\
\hline
\text{LET-COMP} & \\
\nu \Sigma \{ e \} & \quad \nu \Sigma \{ e \} \mapsto \nu \Sigma' \{ e' \} \\
\nu \Sigma \{ \text{let} \{ \tau \}(\text{comp}(e); x.e_2; x.e_3) \} & \quad \nu \Sigma \{ \text{let} \{ \tau \}(\text{comp}(e'); x.e_2; x.e_3) \} \\
\hline
\text{LET-RET} & \\
\nu \Sigma \{ \text{let} \{ \tau \}(\text{ret}(e)); x.e_2; x.e_3 \} & \quad \nu \Sigma \{ \text{let} \{ \tau \}(\text{ret}(e')); x.e_2; x.e_3 \} \\
\hline
\text{LET-RAISE} & \\
\nu \Sigma \{ \text{let} \{ \tau \}(\text{raise}(e)); x.e_2; x.e_3 \} & \quad \nu \Sigma \{ \text{let} \{ \tau \}(\text{raise}(e')); x.e_2; x.e_3 \} \\
\hline
\end{align*}
\]

Figure 2: Modal \textbf{PCF} with Exceptions: Dynamics
Consider following syntactic extension to modal PCF:

$$
\begin{align*}
\text{exc-decl} & \quad \Gamma, x : \text{exccls}(\tau) \vdash \Sigma' e' \approx \tau' \\
\text{exc-raise} & \quad \Gamma \vdash \Sigma e_1 : \text{exccls}(\tau_2) \quad \Gamma \vdash \Sigma e_2 : \tau_2 \\
\text{exc-handle} & \quad \Gamma \vdash \Sigma e_1 : \text{comp}(\tau) \quad \Gamma \vdash \Sigma e_2 : \text{exccls}(\tau_2) \quad \Gamma, x : \tau_2 \vdash \Sigma e_3 \approx \tau \\
\end{align*}
$$

Figure 3: Derived Exceptions: Statics

The exception declaration $\text{dclexc}(\tau)(x.e)$ declares a new exception class, $x$, with associated type $\tau$ for use within $e$. Notice that $x$ is a variable that will be replaced by a class reference within the scope $e$; because it is bound within its, its name is immaterial. The underlying class name is implicitly bound within $e$, and is not accessible. But, being bound, it, too, is different from any other exception class allocated anywhere in a program.

The raise expression $\text{raiseexc}(e_1; e_2)$ raises an exception value constructed by applying the class reference given by $e_1$ to the associated value given by $e_2$ to obtain a value of type $\text{clsfd}$, the exn type. The handle expression $\text{hdlexc}(e_1; e_2; y.e_3)$ evaluates the computation given by $e_1$. If it returns normally, that is the value returned by the expression; if it raises an exception of class given by $e_2$, then it binds the associated value to $y$ within $e_3$; if it raises any other exception value, then the exception value is re-raised to the surrounding context.

The statics of these constructs is given in Figures 3. It is possible to give a dynamics as well, but instead the dynamics is specified by defining these constructs in terms of dynamic classification, class references, and the control mechanism given in the previous section. These definitions are given in Figure 4. The exception declaration introduces a new class, $a$, with the associated type given, and propagates a reference to it into the body of the declaration, a computation of some type. Raising an exception constructed from a class reference and a value of its associated type results creates a classified value to be raised in the sense of Section 2. Handling an exception uses the composite sequencing construct of lax logic to evaluate the given computation, returning its value in the case of a normal return. In the case of an exceptional return the exception value is tested against the given class reference, either propagating the associated value to the handler, or re-raising the exception, according to whether the test succeeds or fails.

4 Summary

There are two aspects of an exception mechanism, the control aspect and the data aspect. The control aspect is neatly accounted for in by-value PCF by considering that an encapsulated computation can complete in one of two ways, by returning a value, or by raising an exception. The elimination form for the computation type accounts for both of these outcomes in a natural way by providing both a normal continuation and an exceptional continuation.
\[
\text{exn} \triangleq \text{clsfd}
\]
\[
\text{exccls}(\tau) \triangleq \text{cls}(\tau)
\]
\[
\text{dclexc}\{\tau\}\{x.e'\} \triangleq \text{new}\{\tau\}\{a.[\text{cls}[a]/x]e'\}
\]
\[
\text{raiseexc}(e_1;e_2) \triangleq \text{raise}(\text{inref}(e_1;e_2))
\]
\[
\text{hdlexc}(e_1;e_2;x.e_3) \triangleq \text{let}\{\tau\}\{e_1;y.\text{ret}(y);y.\text{isof}(y;e_2;x.e_3;\text{raise}(y))\}
\]

Figure 4: Derived Exceptions: Dynamics

In contrast the control aspect is problematic in by-name PCF, because using exceptions relies on a clear understanding control flow, which is absent in by-name languages. For example, \(s(\text{raise}(e))\) is a value in by-name PCF. The encapsulated exception is raised only when the predecessor is evaluated, and not otherwise. It lies in wait like a land mine, waiting to be triggered by an unsuspecting part of the program that is in no way responsible for its origin. Worse, application of a function to an argument that may raise an exception strews the argument throughout the body of the function wherein it may never be evaluated, or evaluated more than once. It is a morass from which there is no escape.

The data aspect accounts for the type of value to be raised when an exception is to be signalled. The most useful choice for the exception value type is the type \text{clsfd} of classified values whose elements are values of an arbitrary type \(\tau\) labeled with a dynamically generated class, the class of exception being raised. By the miracle of \(\alpha\)-equivalence a dynamically generated class is an unguessable secret known only to those program components that have the constructor and destructor associated with it. In particular, when exception values are dynamically classified, the class of the exception is a shared secret between raisers and handlers that have access to that class. Thus, exception raising is not, \textit{pace} the methodology literature that denigrates it, an uncontrolled transfer of control, but the exact opposite. By controlling who has access to the destructor for the class, the raiser can limit the possible handlers for it; correspondingly, by controlling the components that have access to the constructor for the class, the handler can control which components may raise an exception handled by it.

Achieving secrecy requires dynamic generation of exception classes, a computational effect. Thus, pure languages cannot provide the guarantees provided by the secrecy of exception classes. The effect of exception class generation is essential for modularity; without it, two components may accidentally use the same exception class, resulting in chaos. The generation effect is essential, and cannot be sacrificed.

References
