

# Corrigendum: On Equivalence and Canonical Forms in the LF Type Theory

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A gap in the proof of Theorem 5.2 of the above-named paper is filled, and an improved proof is sketched.

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There is a gap in the proof of part (3) of Theorem 5.2 in [Harper and Pfenning 2005] wherein it is stated that algorithmically equivalent families of a kind are definitionally equivalent. The proof requires an additional rule of definitional equality expressing extensionality of families of higher kind:

$$\frac{\Gamma, x:C \vdash A x = B x : K}{\Gamma \vdash A = B : \Pi x:C.K}$$

This rule was omitted in [Harper and Pfenning 2005], but is valid with respect to the logical relation interpretation given in the paper. With the addition of this rule the proof of Theorem 5.2 goes through as suggested in the paper, and no other results are disturbed by its addition.

It is also possible to avoid reliance on extensionality of families by making the following adjustments to the proof given in the paper:

- (1) Replace the three rules for kind-directed family equality by the following rule:

$$\frac{\Delta \vdash A \longleftrightarrow B : K}{\Delta \vdash A \Longleftrightarrow B : K}$$

This modification simplifies the algorithm by reducing algorithmic equivalence of families to structural equivalence, regardless of their kind.

- (2) Add the following rule of structural equivalence of families to compensate for

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the preceding change:

$$\frac{\Delta \vdash A_1 \longleftrightarrow B_1 : \text{type}^- \quad \Delta, x : A_1^- \vdash A_2 \longleftrightarrow B_2 : \text{type}^-}{\Delta \vdash \Pi x : A_1.A_2 \longleftrightarrow \Pi x : B_1.B_2 : \text{type}^-}$$

- (3) Correspondingly, change the definition of the logical relations in Section 4.1 so that conditions (3) and (4) are replaced by

$$\Delta \vdash A = B \in \llbracket K \rrbracket \text{ iff } \Delta \vdash A \longleftrightarrow B : K.$$

The remainder of the proof may then be adjusted accordingly without disrupting the main result.

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#### REFERENCES

- HARPER, R. AND PFENNING, F. 2005. On equivalence and canonical forms in the  $\lambda$ f type theory. *ACM Transactions on Computational Logic* 6, 1 (January).

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