Fundamentals of Algorithms Fall 2012 Homework 1

Due: Monday August 27 at 10 AM

Instructions: Prove By Induction that:

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| Problem #1 | |
| Original Problem | 1 + 2 +3 + … +n = (1/2)(n)(n+1) |
| Base Case | 1 = (1/2)(1)(1+1)  1 = (1/2)(2)  Therefore,  1 = 1 |
| Inductive Hypothesis | n = k  1 + 2 +3 + … +k = (1/2)(k)(k+1) |
| Prove | 1 + 2 +3 + … +k+(k+1) = (1/2)(k+1)(k+2) |
| Solution | 1 + 2 +3 + … +k+(k+1) = (1/2)(k)(k+1) +(k+1)  =(1/2)((k)(k+1)+(2)(k+1))  =(1/2)(k^2+3k+2)  =(1/2)(k+1)(k+2) QED |

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| Problem #2 | |
| Original Problem | 1^2 + 2^2 + 3^2 + …+n^2 = (1/6)(n)(n+1)(2n+1) |
| Base Case | 1^2 = (1/6)(1)(1+1)(2(1)+1)  1 = (1/6)(1)(2)(3)  1 = (1/6)(6)  Therefore,  1 = 1 |
| Inductive Hypothesis | n = k  1^2 + 2^2 + 3^2 + …+k^2 = (1/6)(k)(k+1)(2k+1) |
| Prove | 1^2 + 2^2 + 3^2 + …+k^2 +(k+1)^2= (1/6)(k+1)(k+2)(2(k+1)+1)  =(1/6)((k^2+3k+2)(2k+3))  =(1/6)(2k^3+k^2+2k^2+k+6k^2+12k+1)  =(1/6)(2k^3+9k^2+13k+6) |
| Solution | 1^2 + 2^2 + 3^2 + …+k^2 +(k+1)^2=(1/6)(k)(k+1)(2k+1) +(k+1)^2  =(1/6)((k^2+k)(2k+1)+6(k^2+2k+1))  =(1/6)(2k^3+k^2+2k^2+k+6k^2+12k+1)  =(1/6)(2k^3+9k^2+13k+6) QED |

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| Problem #3 | |
| Original Problem | 1^3 +2^3 + 3^3 + … +n^3 = (1/4)(n^2)(n+1)^2 |
| Base Case | 1^3 = (1/4)(1^2)(1+1)^2  1 = (1/4)(1)(4)  Therefore,  1 = 1 |
| Inductive Hypothesis | n = k  1^3 +2^3 + 3^3 + … +k^3 = (1/4)(k^2)(k+1)^2 |
| Prove | 1^3 +2^3 + 3^3 + … +k^3 +(k+1)^3= (1/4)((k+1)^2)(k+2)^2  = (1/4)(k^4 + 6k^3+13k^2+12k+4) |
| Solution | 1^3 +2^3 + 3^3 + … +k^3 +(k+1)^3 = (1/4)(k^2)(k+1)^2 +(k+1)^3  =(1/4)(k^4+2k^3+k^2)+4(k+1)^3  = (1/4)(k^4 + 6k^3+13k^2+12k+4)  =(1/4)((k+1)^2)(k+2)^2 QED |
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| Problem #4 | |
| Original Problem | 1 + 3 + 5 + … + (2k-1) = k^2 |
| Base Case | 1 = 1^2  Therefore,  1 = 1 |
| Inductive Hypothesis | k = r  1 + 3 + 5 + … + (2r-1) = r^2 |
| Prove | 1 + 3 + 5 + … + (2r-1)+(2(r+1)-1) =( r+1)^2 |
| Solution | 1 + 3 + 5 + … + (2r-1)+(2(r+1)-1) = r^2 +(2(r+1)-1)  =r^2 + 2r+1  =(r+1)(r+1)  =(r+1)^2 QED |

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| Problem #5 | |
| Original Problem | 1 + 2 + 4 + … + 2^k = 2^(k+1) - 1 |
| Base Case | 1 + 2 = 2^(1+1) – 1  1 + 2 = 4-1  Therefore,  3 = 3 |
| Inductive Hypothesis | k = r  1 + 2 + 4 + … + 2^r = 2^(r+1) - 1 |
| Prove | 1 + 2 + 4 + … + 2^r+2^(r+1) = 2^(r+2) - 1 |
| Solution | 1 + 2 + 4 + … + 2^r+2^(r+1) = 2^(r+1) – 1 + 2^(r+1)  = (2)( 2^(r+1)) – 1)  = 2^(r+1+1)-1  = 2^(r+2)-1 QED |