# Introductory Overview Lecture
# The Deep Learning Revolution
# Part III: Learning Deep Generative Models

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# Unsupervised Learning

## Non-probabilistic Models

- Sparse Coding
- Autoencoders
- Others (e.g. k-means)

## Probabilistic Generative) Models

### Tractable Models

- Mixture of Gaussians
- Autoregressive Models
- Normalizing Flows
- Many others

### Non-Tractable Models

- Boltzmann Machines
- Variational Autoencoders
- Helmholtz Machines
- Many others…

Explicit Density p(x)

- Generative Adversarial Networks
- Moment Matching Networks

Implicit Density

# Statistical Generative Models

Data

+

Model family, loss function, optimization algorithm, etc.

Prior Knowledge

Learning

Image x

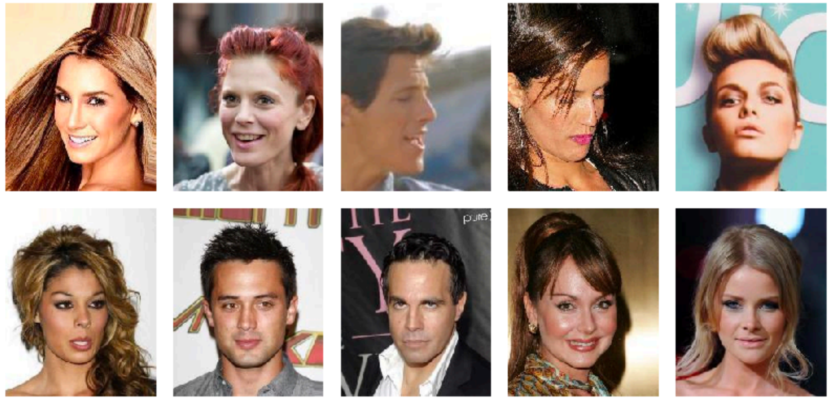
A probability distribution p(x)

probability p(x)

Sampling from p(x) **generates** new images:

Grover and Ermon, DGM Tutorial

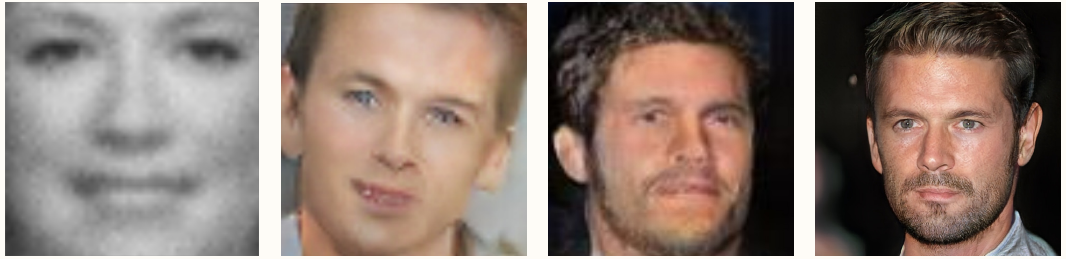
# Statistical Generative Models



Training Data(CelebA)

Model Samples (Karras et.al., 2018)

4 years of progression on Faces



2014 2015 2016 2017

Brundage et al., 2017

# Conditional Generation

- Conditional generative model P(zebra images| horse images)

- Style Transfer

Input Image

Monet

Van Gogh

Zhou el al., Cycle GAN 2017

# Conditional Generation

- Conditional generative model P(zebra images| horse images)

- Failure Case

Zhou el al., Cycle GAN 2017

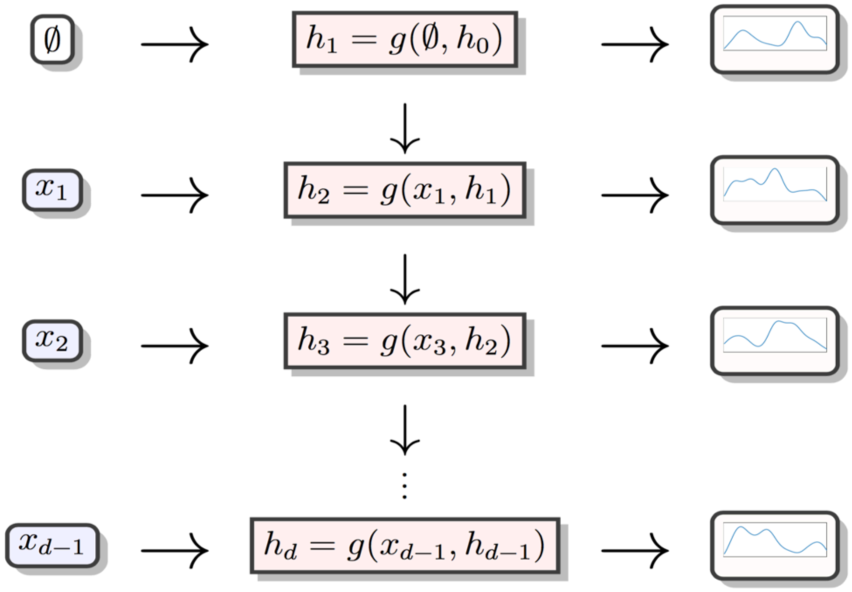
# Talk Roadmap

- Fully Observed Models
- Undirected Deep Generative Models
  - Restricted Boltzmann Machines (RBMs),
  - Deep Boltzmann Machines (DBMs)
- Directed Deep Generative Models
  - Variational Autoencoders (VAEs)
  - Normalizing Flows
- Generative Adversarial Networks (GANs)

# Fully Observed Models

- Density Estimation by Autoregression

$$p(x_1, \ldots, x_d) = \prod_{i=1}^{d} p(x_i | x_{i-1}, \ldots, x_1) \approx \prod_{i=1}^{d} p(x_i | g(x_{i-1}, \ldots, x_1))$$




$p(x_1)$

$p(x_2|x_1)$

$p(x_3|x_2, x_1)$

$p(x_d|x_d, \ldots, x_1)$

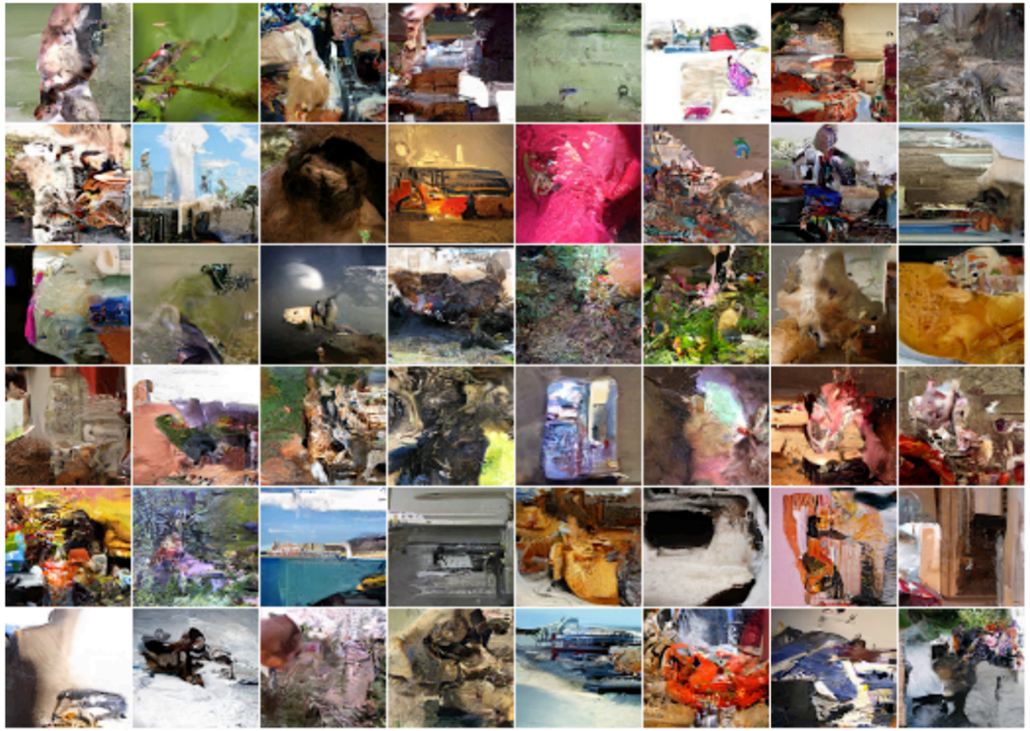
Each conditional can be a deep neural network

- Ordering of variables is crucial

NADE (Uria 2013), MADE (Germain 2017), MAF (Papamakarios 2017), PixelCNN (van den Oord, et al, 2016)

# Fully Observed Models

- Density Estimation by Autoregression

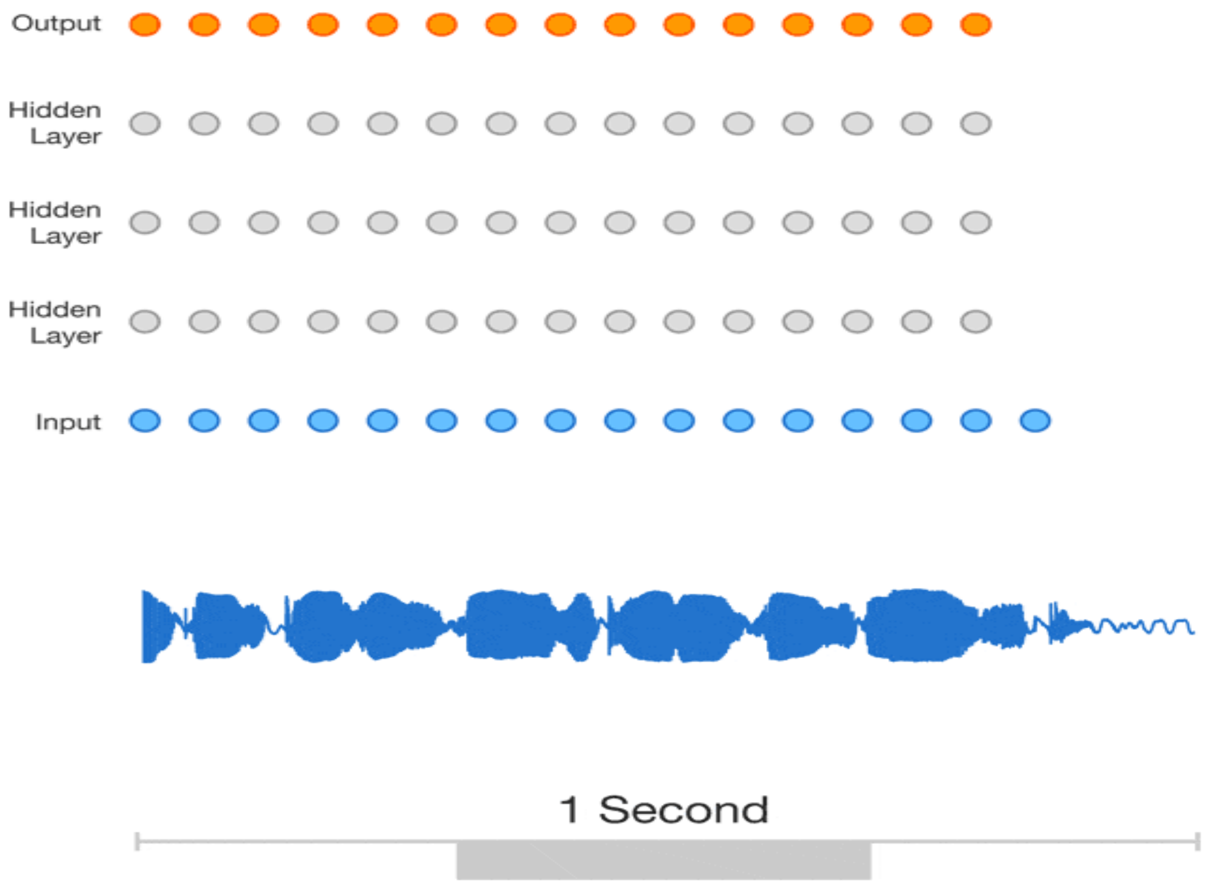


PixelCNN (van den Oord, et al, 2016)

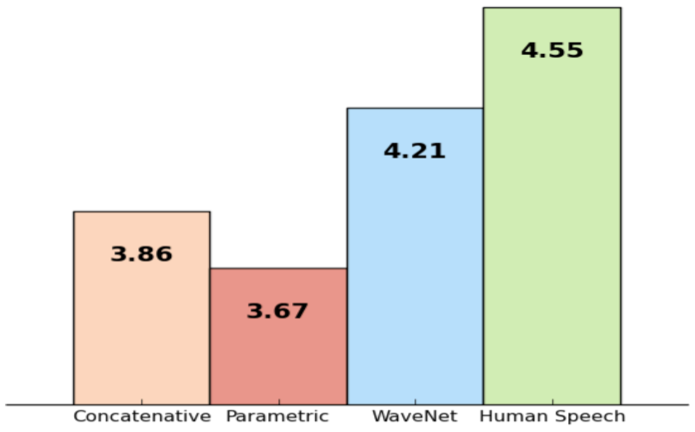
NADE (Uria 2013), MADE (Germain 2017), MAF (Papamakarios 2017), PixelCNN (van den Oord, et al, 2016)

# WaveNet

- Generative Model of Speech Signals




Quality: Mean Opinion Scores




van den Oord et al, 2016

# Talk Roadmap

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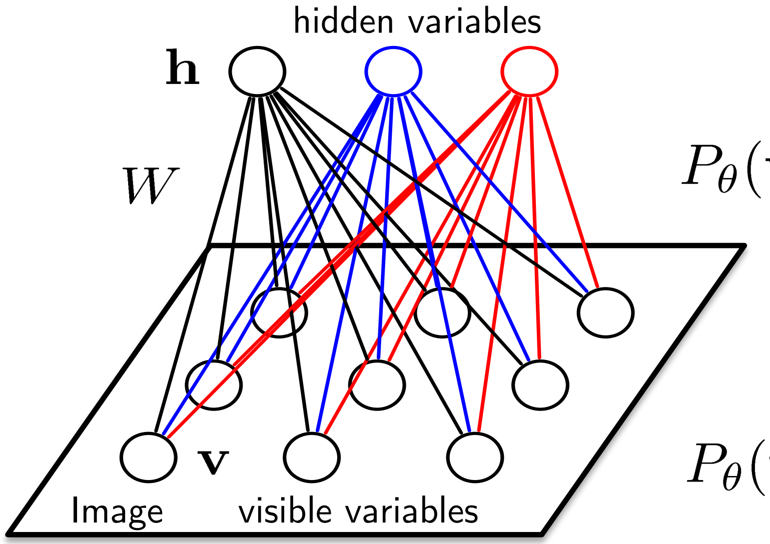
# Restricted Boltzmann Machines




$$P_\theta(\mathbf{v}, \mathbf{h}) = \frac{1}{\mathcal{Z}(\theta)} \exp\left( \underbrace{\sum_{i=1}^{D}\sum_{j=1}^{F} W_{ij} v_i h_j}_{\text{Pairwise}} + \underbrace{\sum_{i=1}^{D} v_i b_i}_{\text{Unary}} + \underbrace{\sum_{j=1}^{F} h_j a_j}_{\text{Unary}} \right)$$

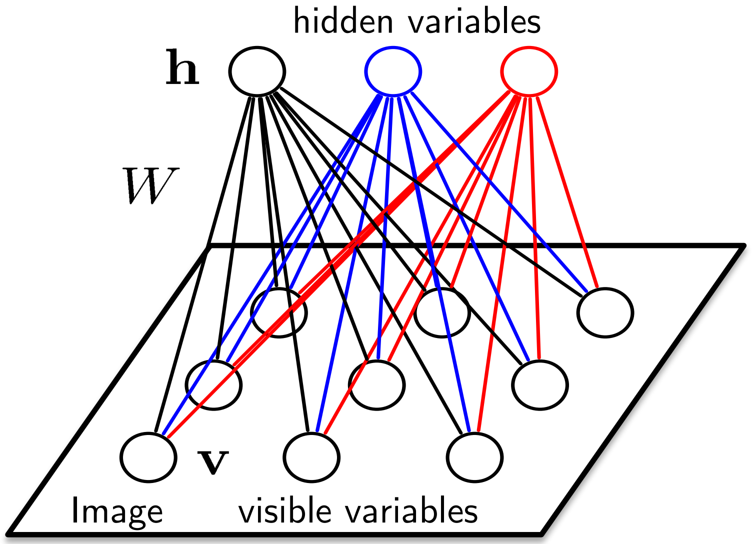
$$\theta = \{W, a, b\}$$

$$P_\theta(\mathbf{v}|\mathbf{h}) = \prod_{i=1}^{D} P_\theta(v_i|\mathbf{h}) = \prod_{i=1}^{D} \frac{1}{1 + \exp(-\sum_{j=1}^{F} W_{ij} v_i h_j - b_i)}$$

- RBM is a Markov Random Field with
  - Stochastic binary visible variables $\mathbf{v} \in \{0, 1\}^D$.
  - Stochastic binary hidden variables $\mathbf{h} \in \{0, 1\}^F$.
  - Bipartite connections

Markov random fields, Boltzmann machines, log-linear models.

# Maximum Likelihood Learning




$$P_\theta(\mathbf{v}) = \frac{P^*(\mathbf{v})}{\mathcal{Z}(\theta)} = \frac{1}{\mathcal{Z}(\theta)} \sum_{\mathbf{h}} \exp\left[\mathbf{v}^\top W\mathbf{h} + \mathbf{a}^\top\mathbf{h} + \mathbf{b}^\top\mathbf{v}\right]$$

- Maximize log-likelihood objective:

$$L(\theta) = \frac{1}{N}\sum_{n=1}^{N} \log P_\theta(\mathbf{v}^{(n)})$$

- Derivative of the log-likelihood:

$$\frac{\partial L(\theta)}{\partial W_{ij}} = \frac{1}{N}\sum_{n=1}^{N} \frac{\partial}{\partial W_{ij}} \log\left(\sum_{\mathbf{h}} \exp\left[\mathbf{v}^{(n)\top} W\mathbf{h} + \mathbf{a}^\top\mathbf{h} + \mathbf{b}^\top\mathbf{v}^{(n)}\right]\right) - \frac{\partial}{\partial W_{ij}} \log \mathcal{Z}(\theta)$$

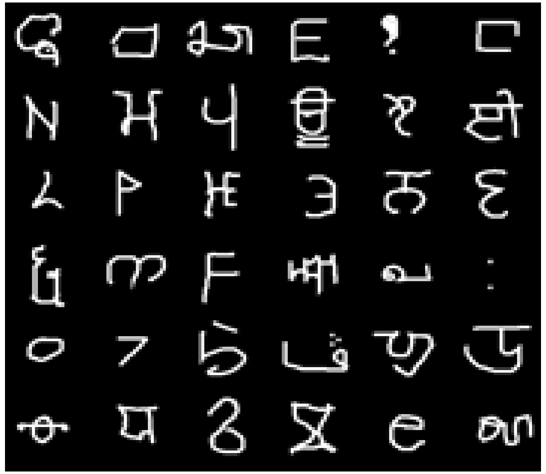
$$= \mathrm{E}_{P_{data}}[v_i h_j] - \underbrace{\mathrm{E}_{P_\theta}[v_i h_j]}_{\text{Difficult to compute: exponentially many configurations}}$$

$$P_{data}(\mathbf{v},\mathbf{h};\theta) = P(\mathbf{h}|\mathbf{v};\theta)P_{data}(\mathbf{v})$$

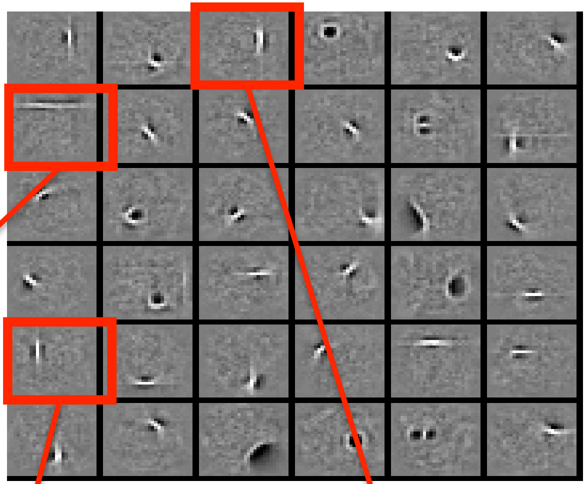
$$P_{data}(\mathbf{v}) = \frac{1}{N}\sum_{n} \delta(\mathbf{v} - \mathbf{v}^{(n)})$$

# Learning Features

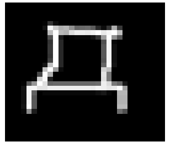
Observed Data
Subset of 25,000 characters

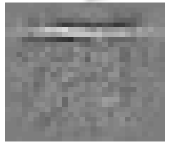
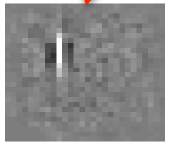
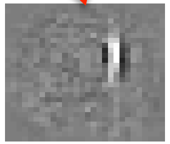


Learned W: "edges"
Subset of 1000 features



New Image:



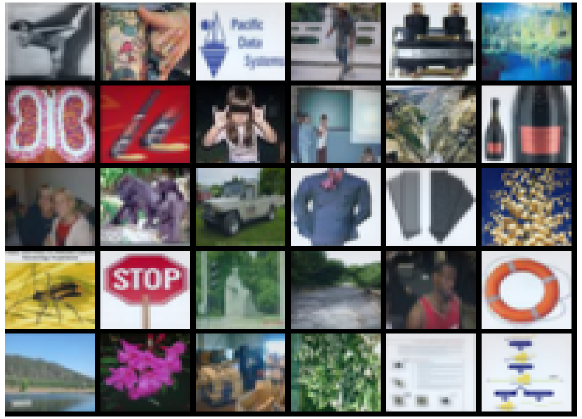
$= \sigma\Big($ $p(h_7 = 1|v)$ $0.99 \times$  $+$ $p(h_{29} = 1|v)$ $0.97 \times$  $+ \; 0.82 \times$  $\ldots\Big)$

$$\sigma(x) = \frac{1}{1+\exp(-x)}$$

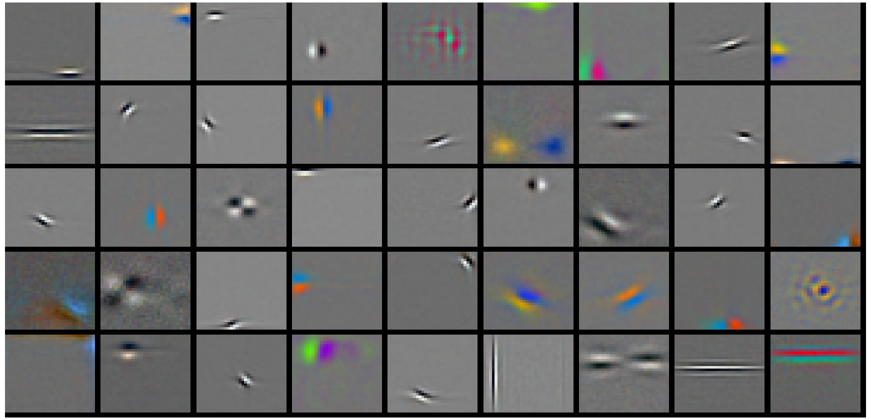
Logistic Function: Suitable for modeling binary images

# RBMs for Real-valued & Count Data

4 million **unlabelled** images



Learned features (out of 10,000)



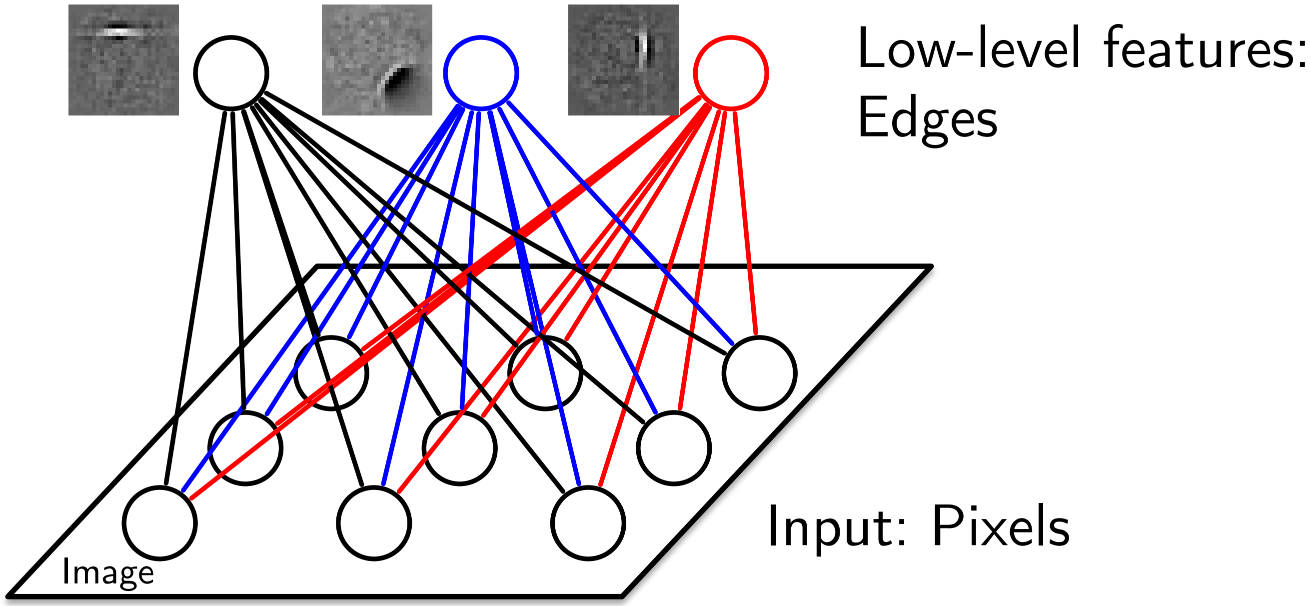
Reuters dataset:
804,414 **unlabeled**
newswire stories

Bag-of-Words

Learned features: ``topics''

| | | | | |
|---|---|---|---|---|
| russian | clinton | computer | trade | stock |
| russia | house | system | country | wall |
| moscow | president | product | import | street |
| yeltsin | bill | software | world | point |
| soviet | congress | develop | economy | dow |

# Deep Boltzmann Machines

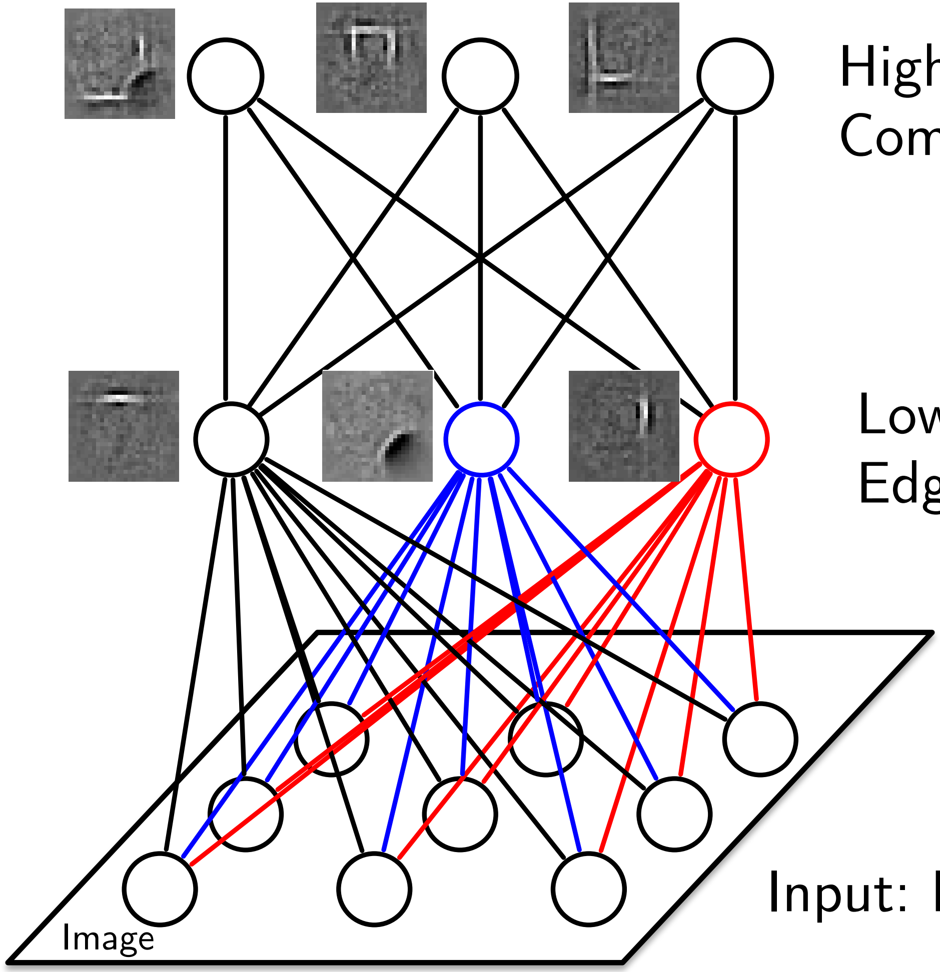



Built from **unlabeled** inputs.

(Salakhutdinov 2008, Salakhutdinov & Hinton 2009)

# Deep Boltzmann Machines

Learn simpler representations, then compose more complex ones




Higher-level features: Combination of edges

Low-level features: Edges

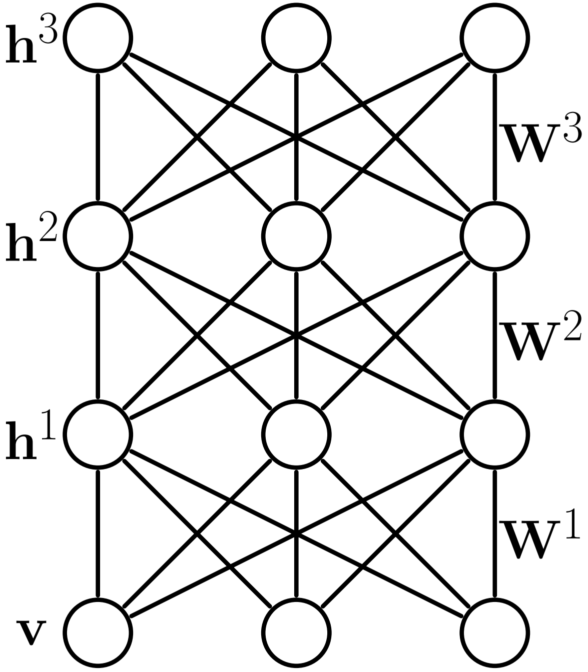
Input: Pixels

Built from **unlabeled** inputs.

(Salakhutdinov 2008, Salakhutdinov & Hinton 2009)

# Model Formation

$$P_\theta(\mathbf{v}, \mathbf{h}^{(1)}, \mathbf{h}^{(2)}, \mathbf{h}^{(3)}) = \frac{1}{\mathcal{Z}(\theta)} \exp\left[\underbrace{\mathbf{v}^\top W^{(1)} \mathbf{h}^{(1)}}_{\textbf{Same as RBMs}} + \mathbf{h}^{(1)\top} W^{(2)} \mathbf{h}^{(2)} + \mathbf{h}^{(2)\top} W^{(3)} \mathbf{h}^{(3)}\right]$$




$\theta = \{W^1, W^2, W^3\}$ model parameters

- Dependencies between hidden variables
- All connections are undirected

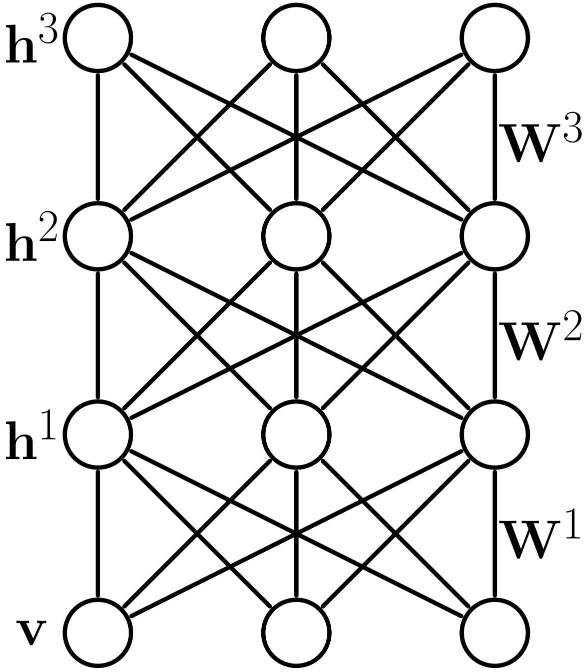
- Maximum Likelihood Learning:

$$\frac{\partial \log P_\theta(\mathbf{v})}{\partial W^1} = \mathbb{E}_{P_{data}}[\mathbf{v}\mathbf{h}^{\mathbf{1}\top}] - \mathbb{E}_{P_\theta}[\mathbf{v}\mathbf{h}^{\mathbf{1}\top}]$$

- Both expectations are intractable

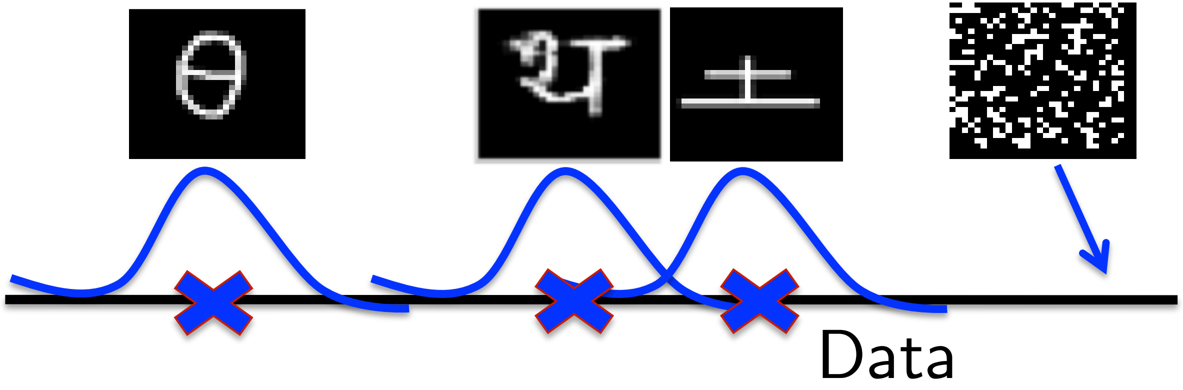
# Approximate Learning

$$P_\theta(\mathbf{v}, \mathbf{h}^{(1)}, \mathbf{h}^{(2)}, \mathbf{h}^{(3)}) = \frac{1}{\mathcal{Z}(\theta)} \exp\left[\mathbf{v}^\top W^{(1)}\mathbf{h}^{(1)} + {\mathbf{h}^{(1)}}^\top W^{(2)}\mathbf{h}^{(2)} + {\mathbf{h}^{(2)}}^\top W^{(3)}\mathbf{h}^{(3)}\right]$$



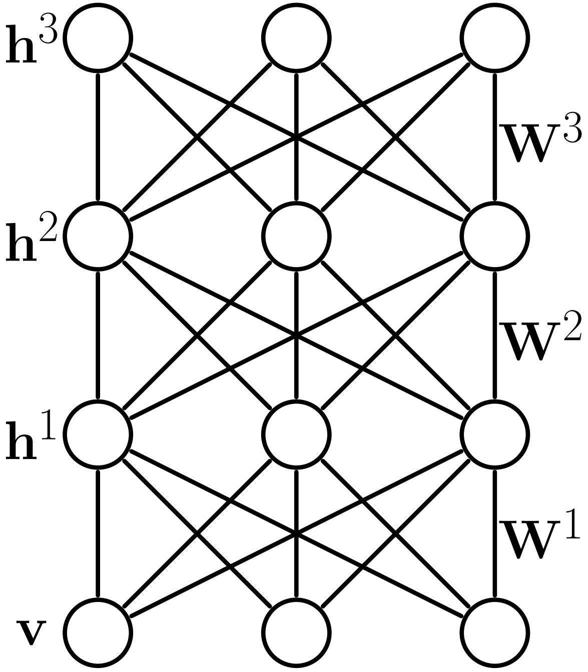

- Maximum Likelihood Learning:

$$\frac{\partial \log P_\theta(\mathbf{v})}{\partial W^1} = \mathbb{E}_{P_{data}}[\mathbf{v}{\mathbf{h}^1}^\top] - \mathbb{E}_{P_\theta}[\mathbf{v}{\mathbf{h}^1}^\top]$$



# Approximate Learning

$$P_\theta(\mathbf{v}, \mathbf{h}^{(1)}, \mathbf{h}^{(2)}, \mathbf{h}^{(3)}) = \frac{1}{\mathcal{Z}(\theta)} \exp\left[\mathbf{v}^\top W^{(1)}\mathbf{h}^{(1)} + \mathbf{h}^{(1)\top} W^{(2)}\mathbf{h}^{(2)} + \mathbf{h}^{(2)\top} W^{(3)}\mathbf{h}^{(3)}\right]$$




- Maximum Likelihood Learning:

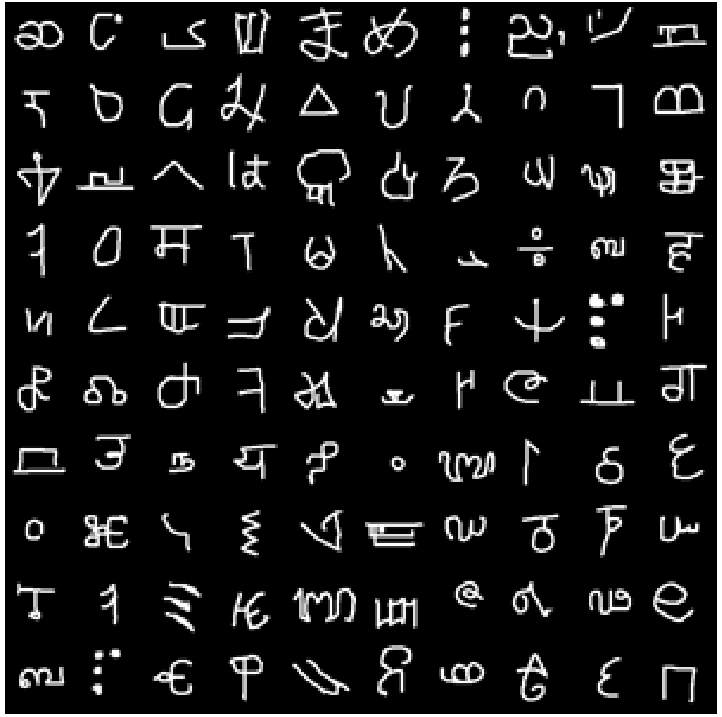
$$\frac{\partial \log P_\theta(\mathbf{v})}{\partial W^1} = \mathbb{E}_{P_{data}}[\mathbf{v}\mathbf{h}^{\mathbf{1}\top}] - \mathbb{E}_{P_\theta}[\mathbf{v}\mathbf{h}^{\mathbf{1}\top}]$$

Variational Inference

Stochastic Approximation (MCMC-based)

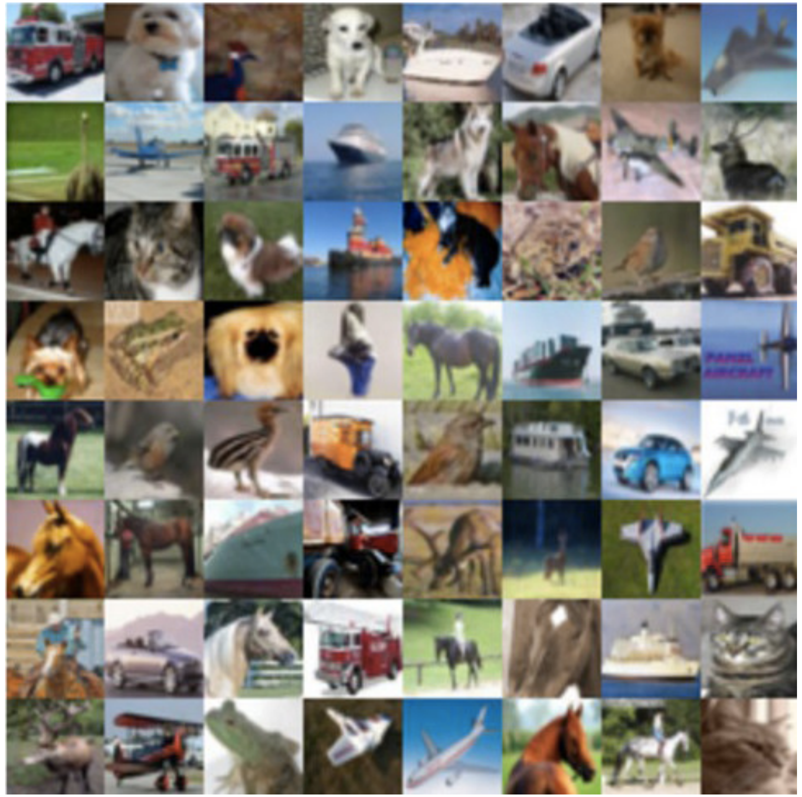
# Good Generative Model?

- Handwritten Characters

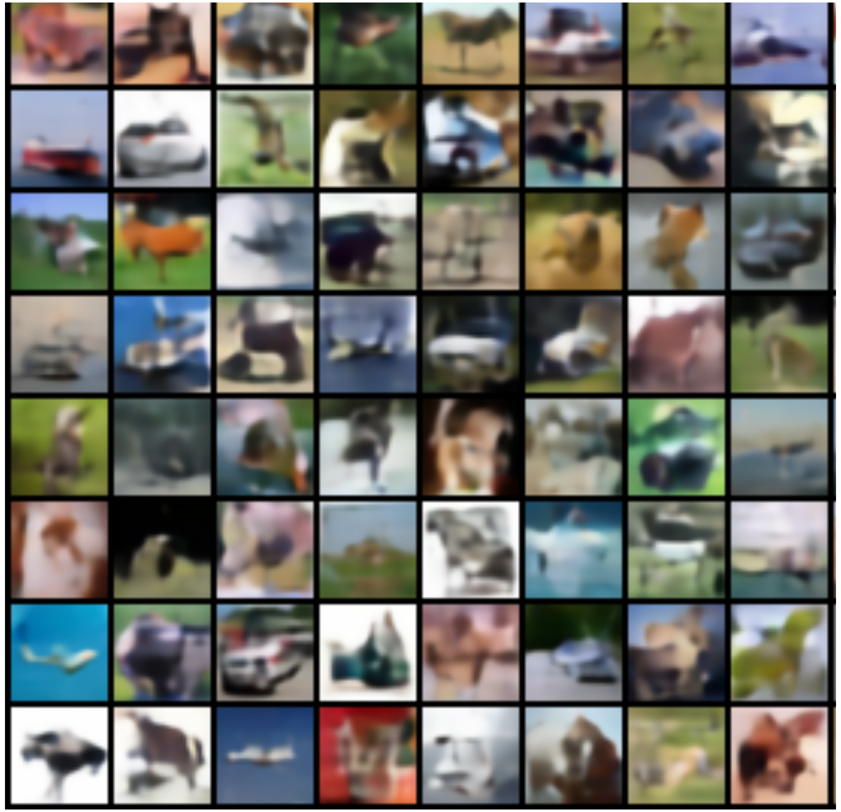


# Good Generative Model?

- CIFAR Dataset



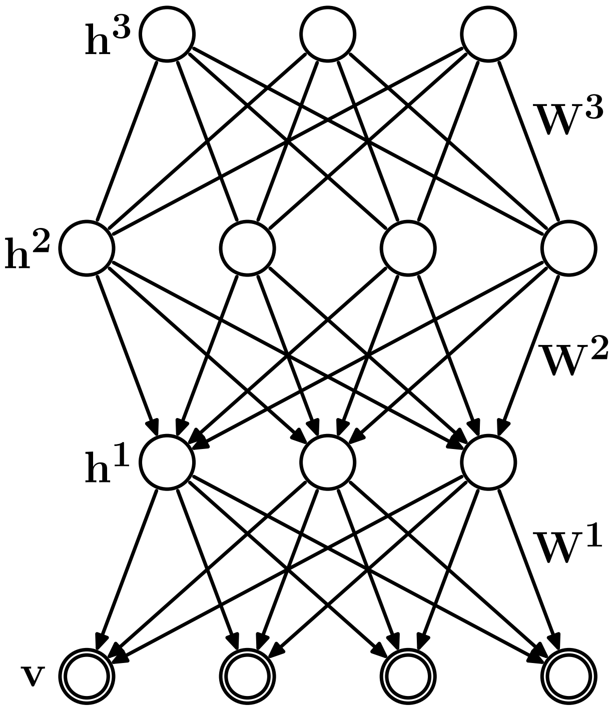
Training

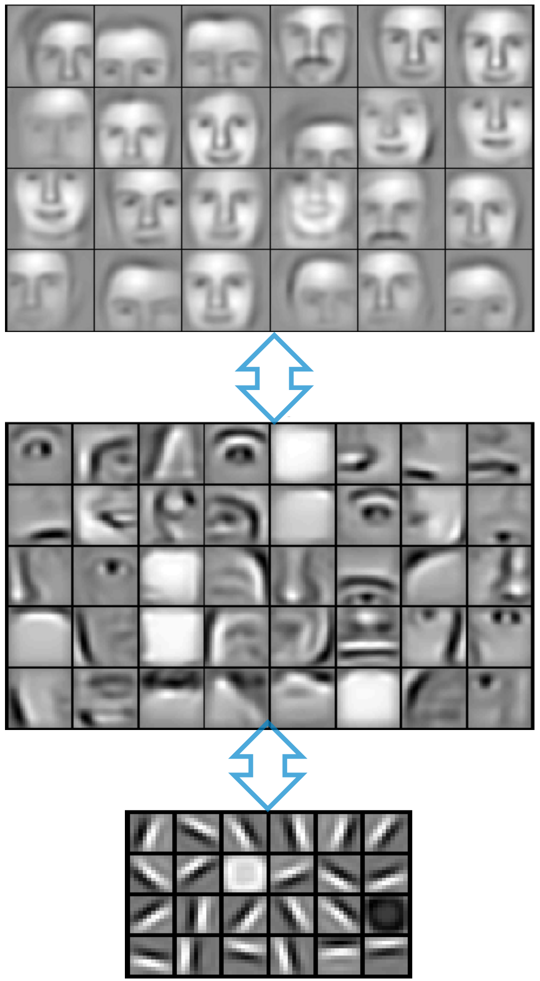


Samples

# Learning Part-Based Representations

Deep Belief Network



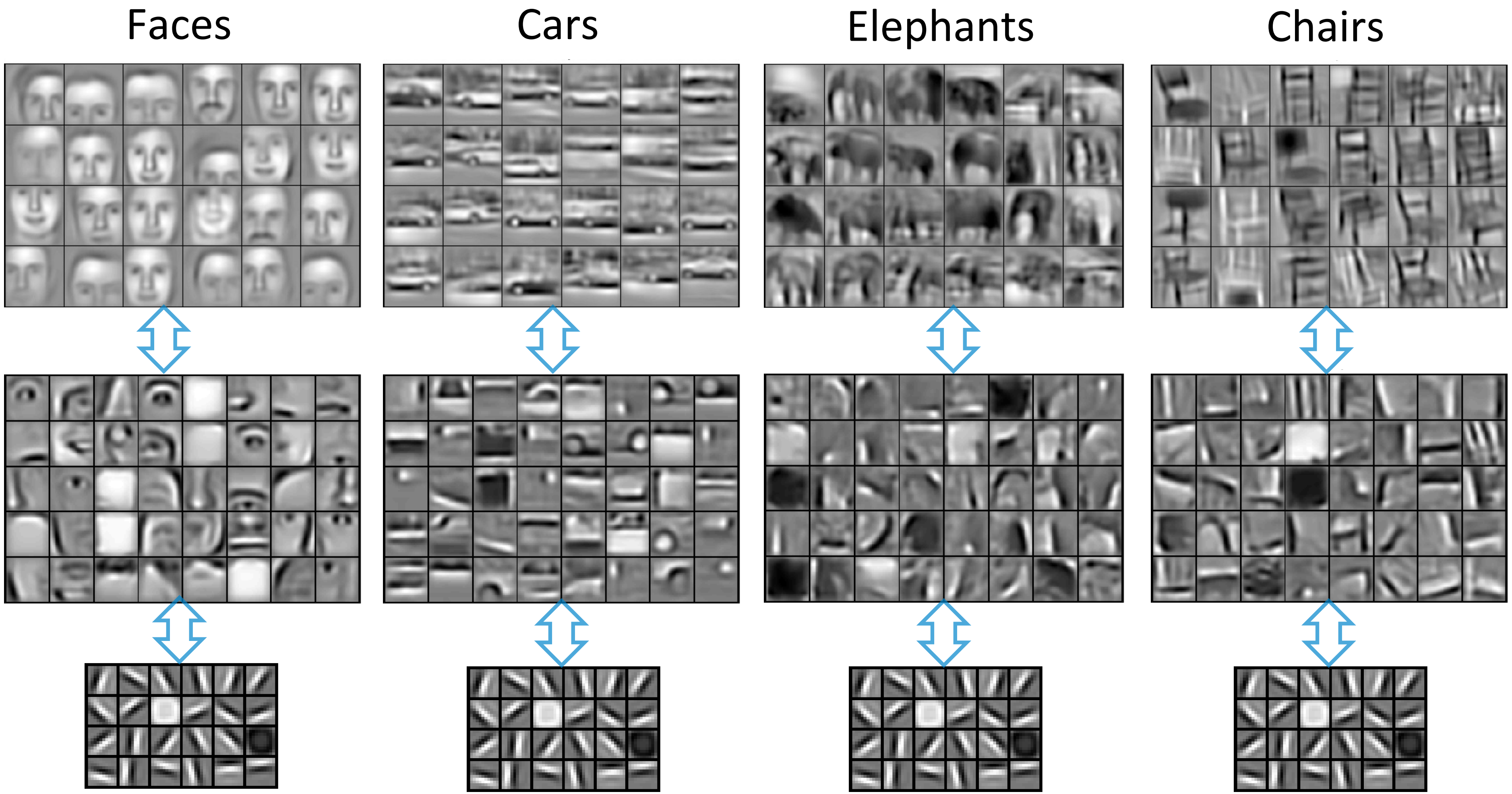
Groups of parts

Object Parts

Trained on face images.

Lee, Grosse, Ranganath, Ng, ICML 2009

# Learning Part-Based Representations




Lee, Grosse, Ranganath, Ng, ICML 2009

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- Fully Observed Models
- Undirected Deep Generative Models
  - Restricted Boltzmann Machines (RBMs),
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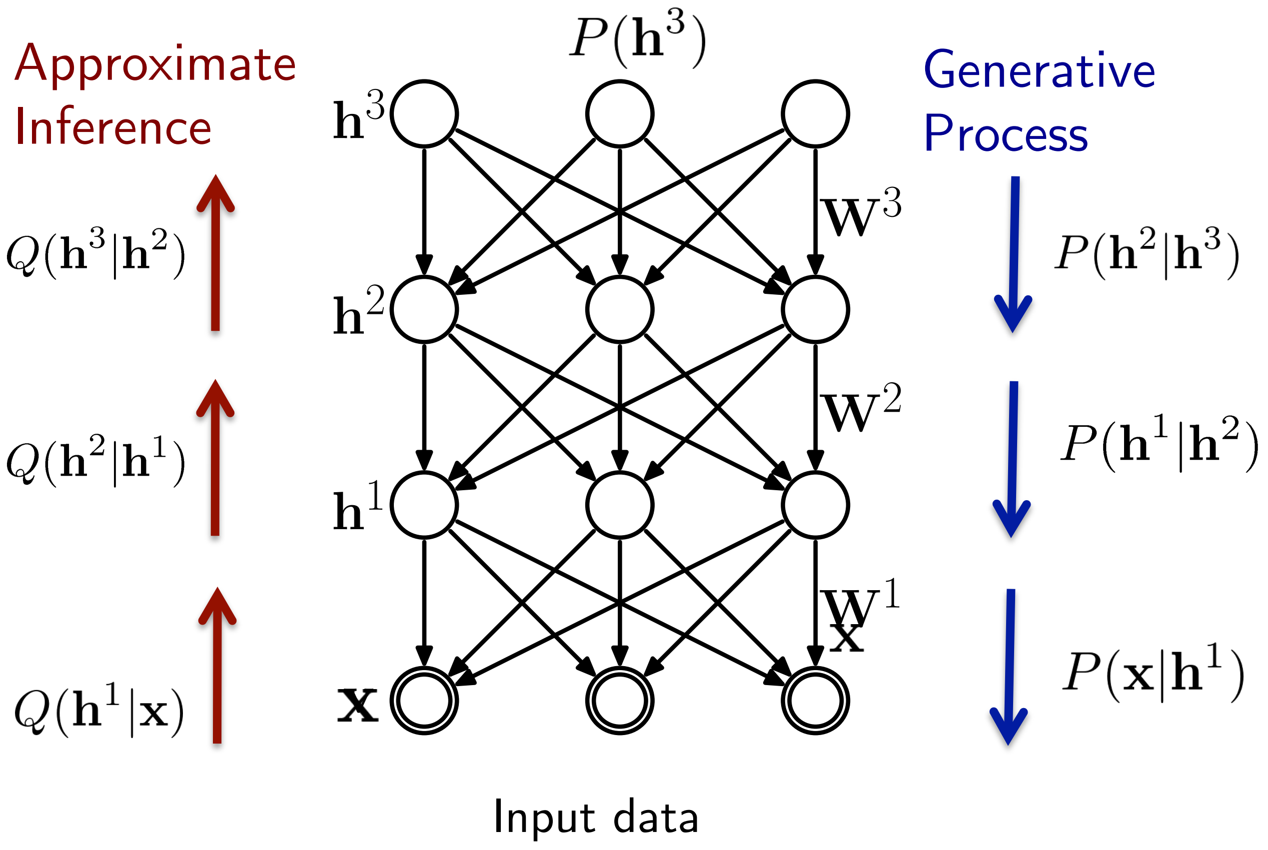
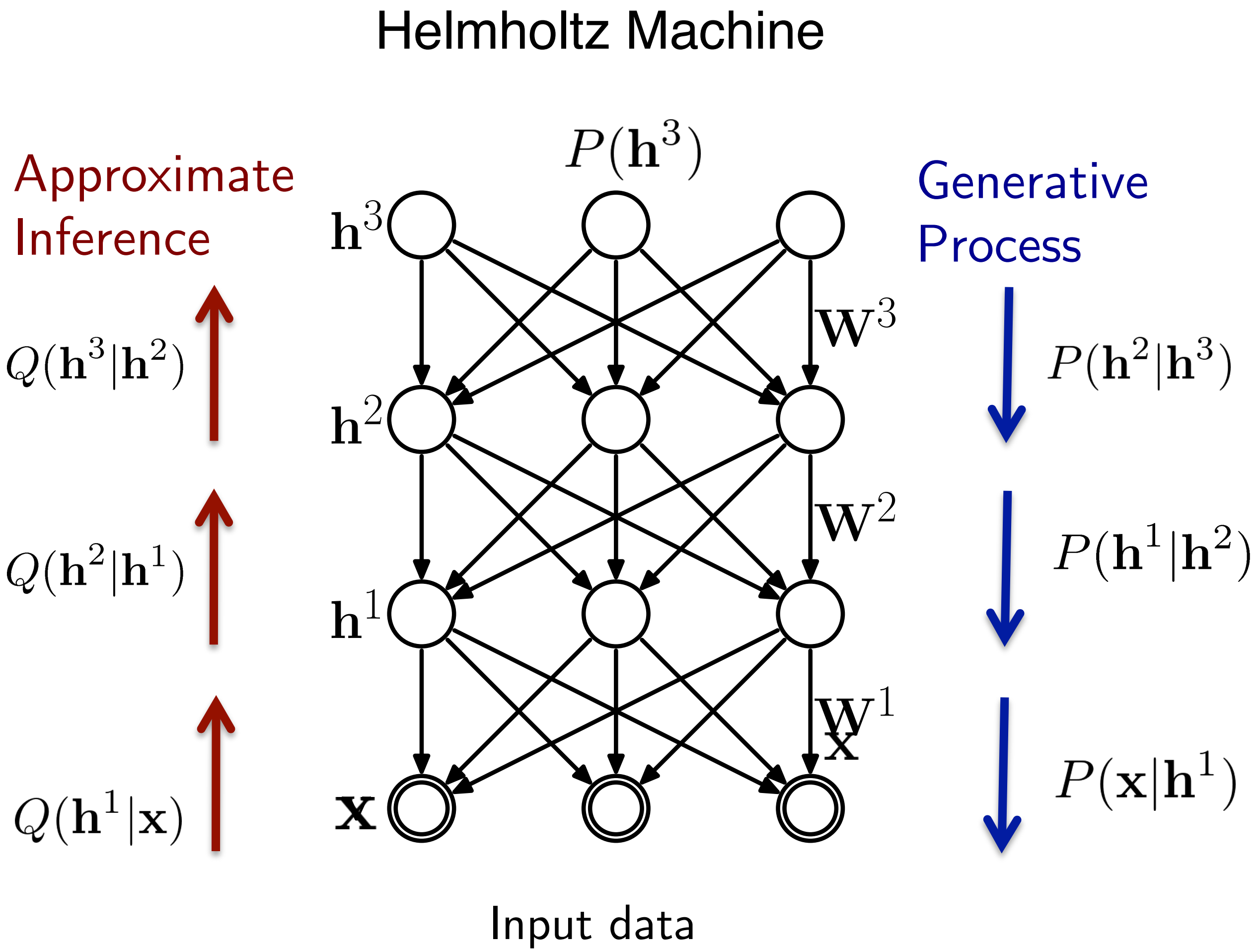
# Helmholtz Machines

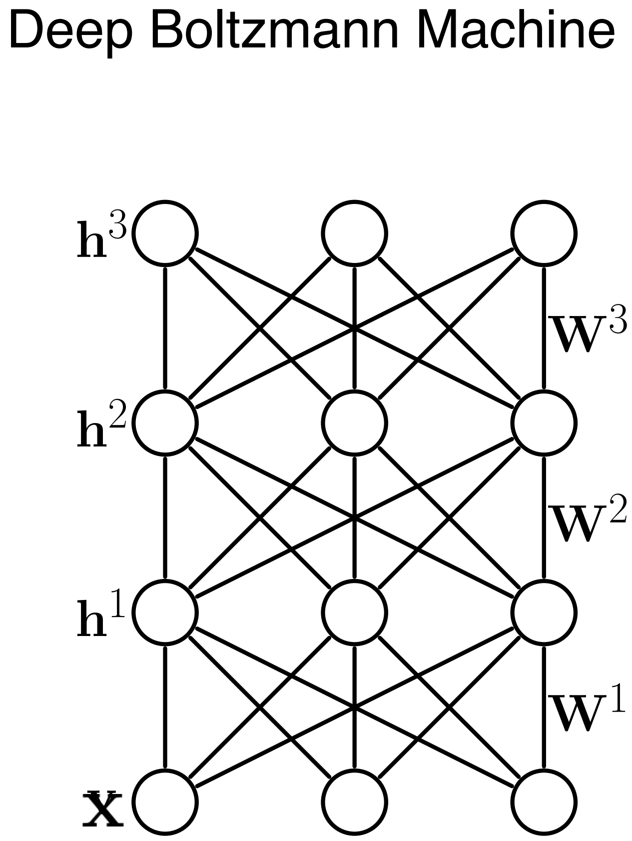
- Hinton, G. E., Dayan, P., Frey, B. J. and Neal, R., Science 1995



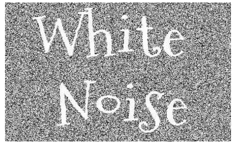

- Kingma & Welling, 2014
- Rezende, Mohamed, Daan, 2014
- Mnih & Gregor, 2014
- Bornschein & Bengio, 2015
- Tang & Salakhutdinov, 2013

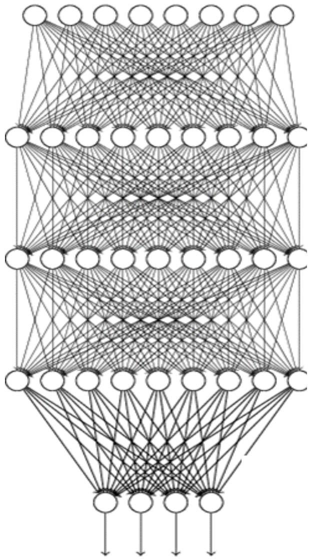
# Helmholtz Machines



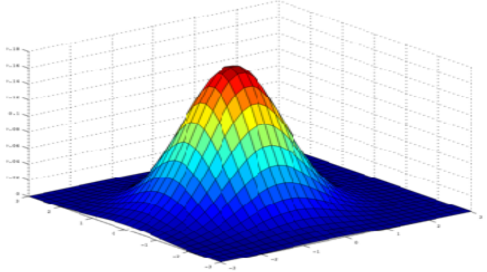



# Deep Directed Generative Models



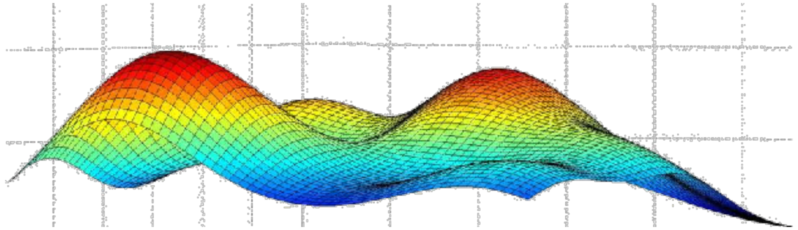



Code Z



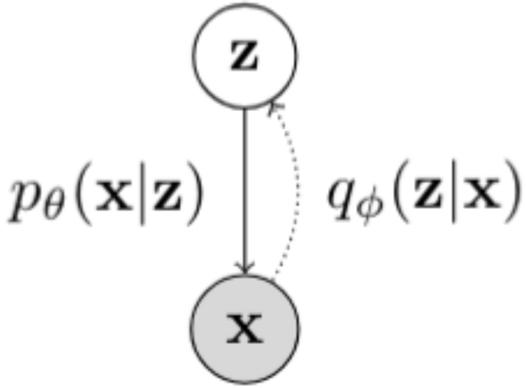
- Recognition
- Bottom-up
- Q(z|x)

- Generative
- Top-Down
- P(x|z)



$D_{real}$

- Latent Variable Models

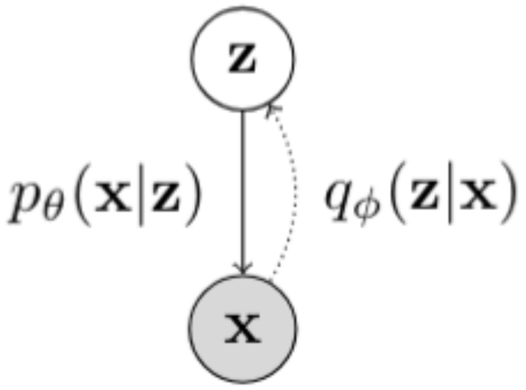



$$\log p_\theta(\mathbf{x}) = \log \int p_\theta(\mathbf{x}, \mathbf{z}) d\mathbf{z}$$

- Conditional distributions are parameterized by deep neural networks

# Directed Deep Generative Models

- Directed Latent Variable Models with Inference Network




- Maximum log-likelihood objective

$$\max_{\theta} \sum_{\mathbf{x} \in \mathcal{D}} \log p_\theta(\mathbf{x})$$

- Marginal log-likelihood is intractable:

$$\log p_\theta(\mathbf{x}) = \log \int p_\theta(\mathbf{x}, \mathbf{z}) \mathrm{d}\mathbf{z}$$

- Key idea: Approximate true posterior p(z|x) with a simple, tractable distribution q(z|x) (inference/recognition network).

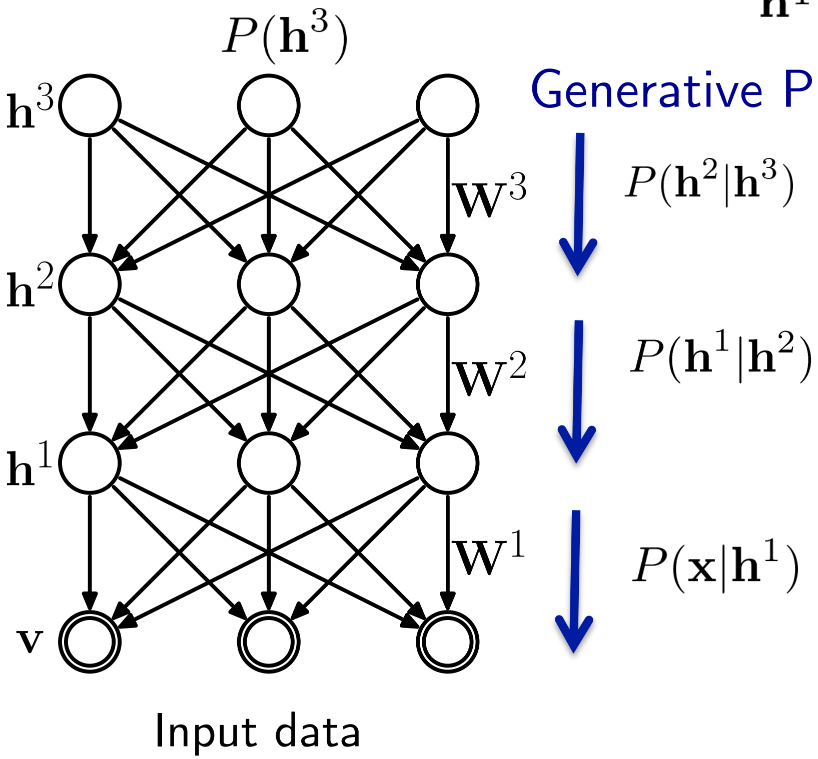
Grover and Ermon, DGM Tutorial

# Variational Autoencoders (VAEs)

- The VAE defines a generative process in terms of ancestral sampling through a cascade of hidden stochastic layers:

$$p(\mathbf{x}|\boldsymbol{\theta}) = \sum_{\mathbf{h}^1,\ldots,\mathbf{h}^L} p(\mathbf{h}^L|\boldsymbol{\theta})p(\mathbf{h}^{L-1}|\mathbf{h}^L,\boldsymbol{\theta})\cdots p(\mathbf{x}|\mathbf{h}^1,\boldsymbol{\theta})$$

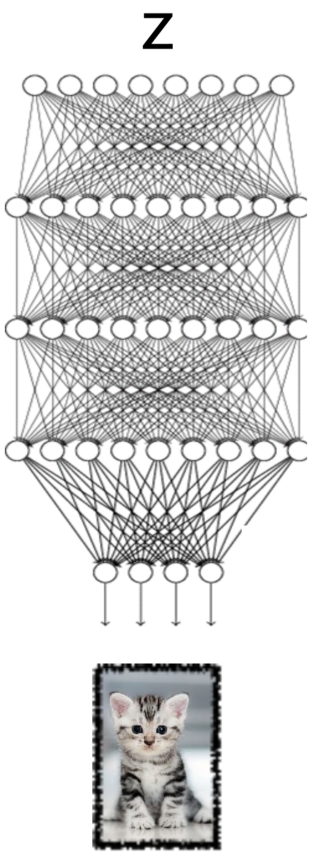
Each conditional term denotes a nonlinear relationship

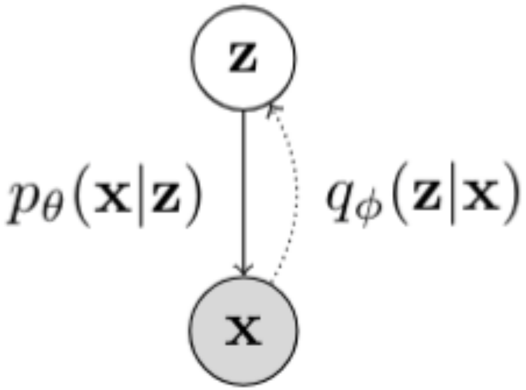



- L is the number of **stochastic** layers
- Sampling and probability evaluation is tractable for each $p(\mathbf{h}^\ell|\mathbf{h}^{\ell+1})$

# Variational Autoencoders (VAEs)

- Single stochastic (Gaussian) layer, followed by many deterministic layers




$$p(\mathbf{z}) = \mathcal{N}(0, I)$$

$$p_\theta(\mathbf{x}|\mathbf{z}) = \mathcal{N}\big(\mu(\mathbf{z}, \theta), \Sigma(\mathbf{z}, \theta)\big)$$

Deep neural network parameterized by θ.
(Can use different noise models)

$$q_\phi(\mathbf{z}|\mathbf{x}) = \mathcal{N}\big(\mu(\mathbf{x}, \phi), \Sigma(\mathbf{x}, \phi)\big)$$

Deep neural network parameterized by ϕ.

# Variational Bound

- VAE is trained to maximize the variational lower bound:

$$
\begin{aligned}
\log p_\theta(\mathbf{x}) &= \log \int p_\theta(\mathbf{x}, \mathbf{z}) d\mathbf{z} = \log \int q_\phi(\mathbf{z}|\mathbf{x}) \frac{p_\theta(\mathbf{x}, \mathbf{z})}{q_\phi(\mathbf{z}|\mathbf{x})} d\mathbf{z} \\
&= \log \mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})} \left[ \frac{p_\theta(\mathbf{x}, \mathbf{z})}{q_\phi(\mathbf{z}|\mathbf{x})} \right] \\
&\geq \mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})} \log \left[ \frac{p_\theta(\mathbf{x}, \mathbf{z})}{q_\phi(\mathbf{z}|\mathbf{x})} \right] \\
&= \log p_\theta(\mathbf{x}) - \mathrm{KL}\big(q_\phi(\mathbf{z}|\mathbf{x}) || p_\theta(\mathbf{z}|\mathbf{x})\big) = \mathcal{L}(\mathbf{x})
\end{aligned}
$$

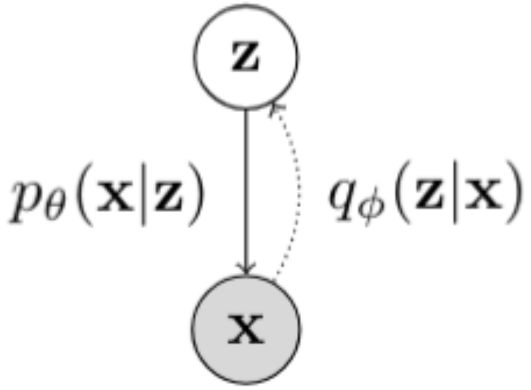
Tightness Condition:

$$q_\phi(\mathbf{z}|\mathbf{x}) = p_\theta(\mathbf{z}|\mathbf{x})$$

- Trading off the data log-likelihood and the KL divergence from the true posterior
- Hard to optimize the variational bound with respect to the q recognition network (high variance)
- Key idea of Kingma and Welling is to use reparameterization trick

# Reparameterization

- Assume that the recognition distribution is Gaussian:




$$q_\phi(\mathbf{z}|\mathbf{x}) = \mathcal{N}\big(\mu(\mathbf{x},\phi), \Sigma(\mathbf{x},\phi)\big)$$

- Alternatively, we can express this in term of auxiliary variable:

$$\mathbf{z}(\epsilon, \mathbf{x}, \phi) = \Sigma(\mathbf{x},\phi)^{1/2}\epsilon + \mu(\mathbf{x},\phi), \quad \epsilon \sim \mathcal{N}(0, I)$$

- The recognition distribution can be expressed as a deterministic mapping

$$\underbrace{\mathbf{z}(\epsilon, \mathbf{x}, \phi)}_{\text{Deterministic Encoder}}$$

- Distribution of ε does not depend on ϕ

# Computing Gradients

- The gradients of the variational bound w.r.t the recognition (similar w.r.t the generative) parameters:

Autoencoder

$$\begin{aligned}\nabla_\phi \mathcal{L}(\mathbf{x}) &= \nabla_\phi \mathbb{E}_{\mathbf{z}\sim q_\phi(\mathbf{z}|\mathbf{x})}\left[\log \frac{p_\theta(\mathbf{x},\mathbf{z})}{q_\phi(\mathbf{z}|\mathbf{x})}\right] \\ &= \nabla_\phi \mathbb{E}_{\epsilon\sim\mathcal{N}(0,I)}\left[\log \frac{p_\theta(\mathbf{x},\mathbf{z}(\epsilon,\mathbf{x},\phi))}{q_\phi(\mathbf{z}(\epsilon,\mathbf{x},\phi)|\mathbf{x})}\right] \\ &= \mathbb{E}_{\epsilon\sim\mathcal{N}(0,I)}\left[\nabla_\phi \log \frac{p_\theta(\mathbf{x},\mathbf{z}(\epsilon,\mathbf{x},\phi))}{q_\phi(\mathbf{z}(\epsilon,\mathbf{x},\phi)|\mathbf{x})}\right]\end{aligned}$$

Gradients can be computed by backprop

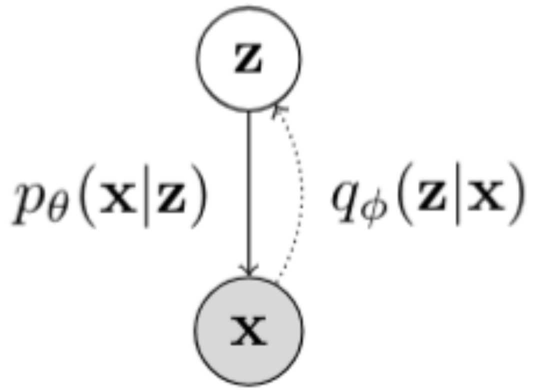
The mapping **z** is a deterministic neural net for fixed ε

# Importance Weighted Autoencoder

- Improve VAE by using the following k-sample importance weighting of the log-likelihood:

$$\mathcal{L}_k(\mathbf{x}) = \mathbb{E}_{\mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_k \sim q_\phi(\mathbf{z}|\mathbf{x})}\left[\log \frac{1}{k}\sum_{i=1}^{k} \frac{p_\theta(\mathbf{x},\mathbf{z}_i)}{q_\phi(\mathbf{z}_i|\mathbf{x})}\right]$$

$$= \mathbb{E}_{\mathbf{z}_1,\mathbf{z}_2,\ldots,\mathbf{z}_k \sim q_\phi(\mathbf{z}|\mathbf{x})}\left[\log \frac{1}{k}\sum_{i=1}^{k} w_i\right]$$




- where multiple z are sampled from the recognition network.

unnormalized importance weights

- Can improve the tightness of the bound.

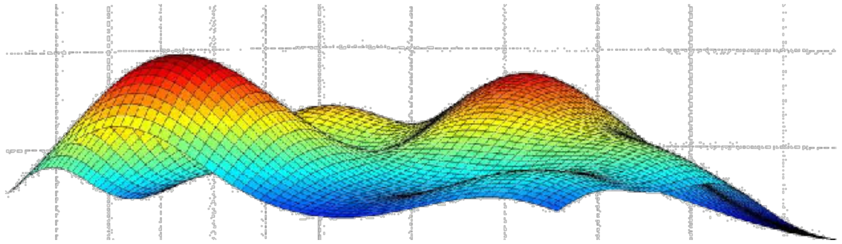
Burda, Grosse, Salakhutdinov, ICLR 2016, Mnih & Rezende, ICML 2016

# Talk Roadmap

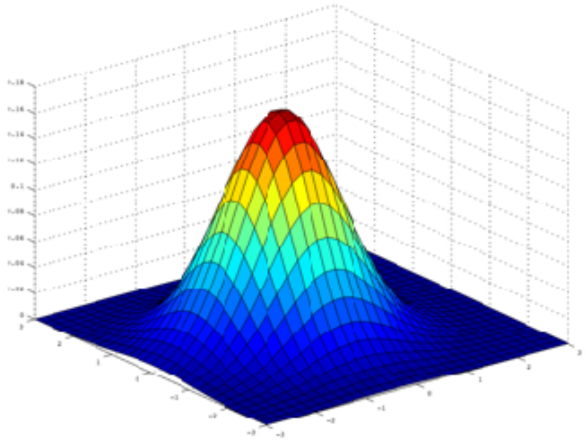
- Fully Observed Models
- Undirected Deep Generative Models
  - Restricted Boltzmann Machines (RBMs),
  - Deep Boltzmann Machines (DBMs)
- Directed Deep Generative Models
  - Variational Autoencoders (VAEs)
  - Normalizing Flows
- Generative Adversarial Networks (GANs)

# Generative Adversarial Networks (GAN)

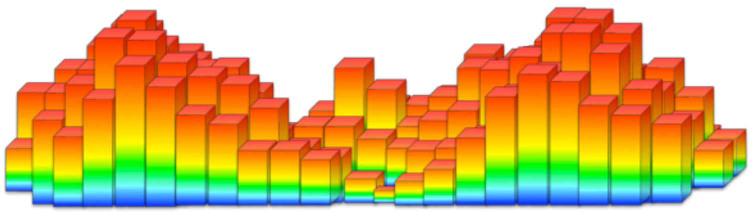
- Implicit generative model for an unknown target density $p(x)$
- Converts sample from a known noise density $p_Z(z)$ to the target $p(x)$



Unknown target density $p(x)$ of data over domain $\mathcal{X}$, e.g. $\mathbb{R}^{32\times 32}$



Noise density $p_Z(z)$ over space $\mathcal{Z}$



Distribution of generated samples should follow target density $p(x)$

[Slide Credit: Manzil Zaheer]

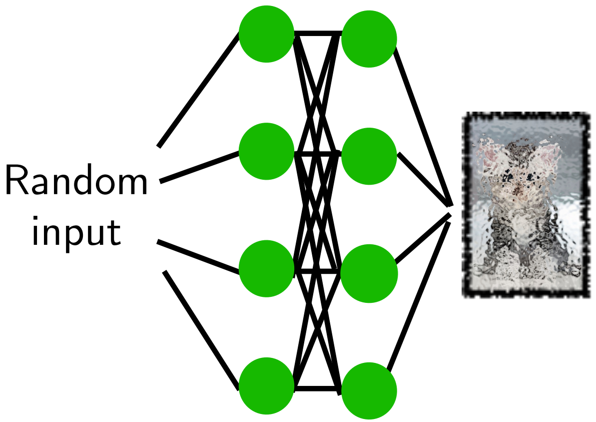
Goodfellow et al, 2014

# GAN Formulation

- GAN consists of two components

**Generator**

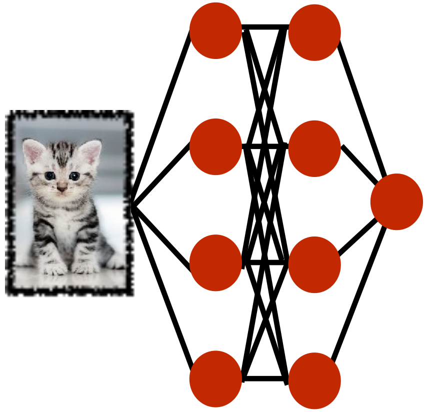
$$G : \mathcal{Z} \to \mathcal{X}$$




Goal: Produce samples indistinguishable from true data

**Discriminator**

$$D : \mathcal{X} \to \mathbb{R}$$



Goal: Distinguish true and generated data apart

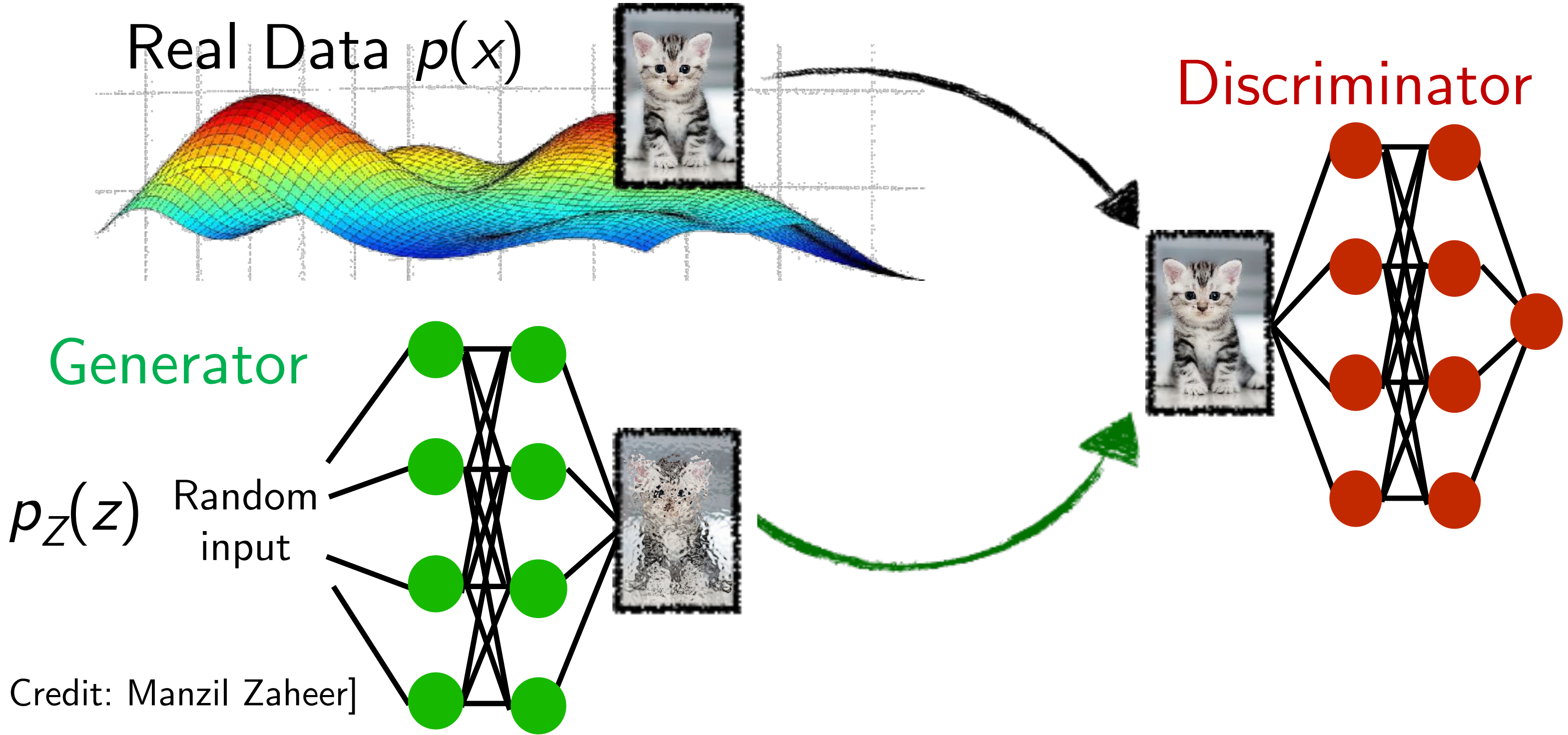
[Slide Credit: Manzil Zaheer]

Goodfellow et al, 2014

# GAN Formulation: Discriminator

- Discriminator's objective: Tell real and generated data apart like a classifier

$$\max_{D} \mathbb{E}_{x\sim p}\left[\log D(x)\right] + \mathbb{E}_{z\sim p_Z}\left[\log\left(1 - D(G(z))\right)\right]$$




$D$ outputs:

$D(x) = 1$ real

$D(x) = 0$ generated

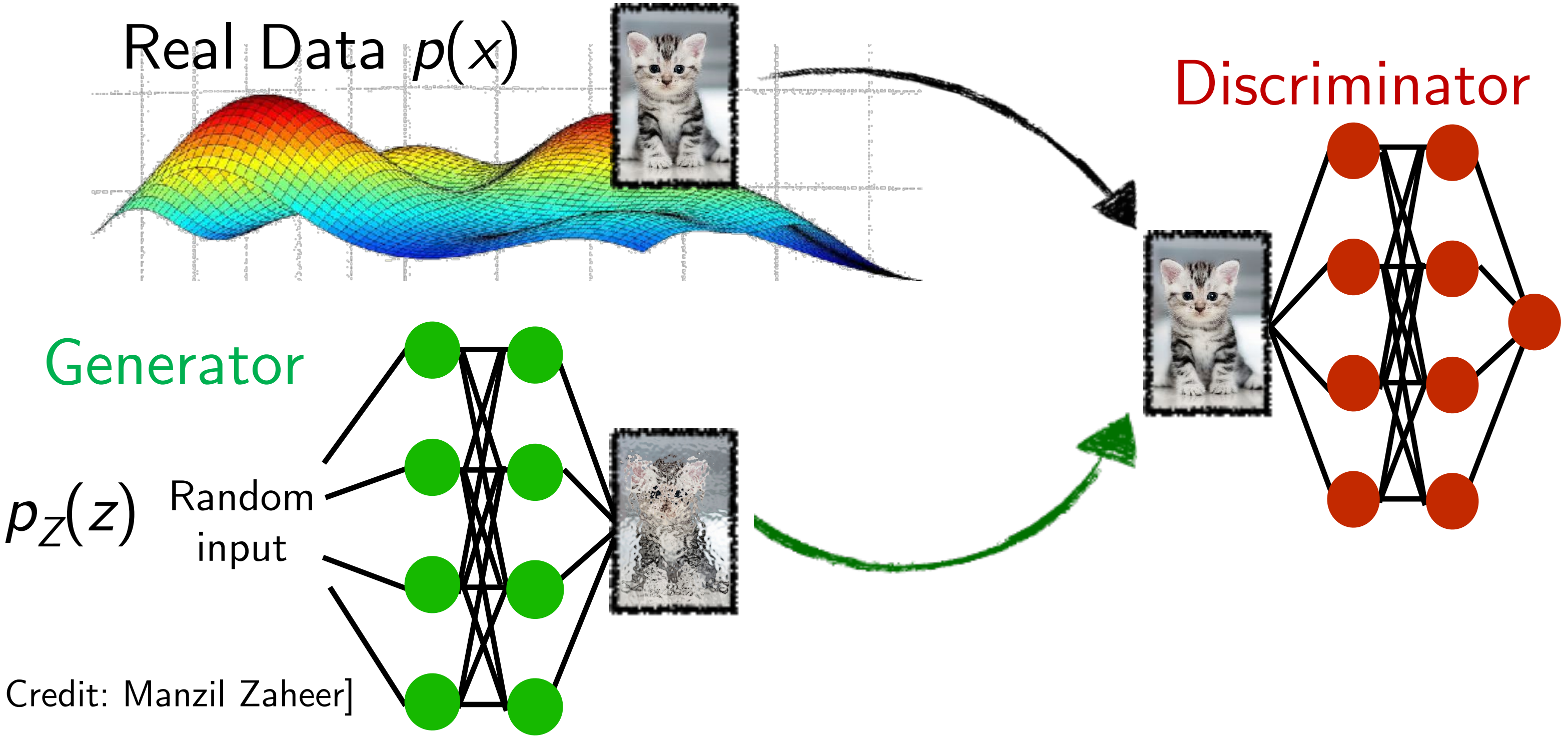
[Slide Credit: Manzil Zaheer]

Goodfellow et al, 2014

# GAN Formulation: Generator

- Generator's objective: Fool the best discriminator

$$\min_{G} \max_{D} \mathbb{E}_{x\sim p}\left[\log D(x)\right] + \mathbb{E}_{z\sim p_Z}\left[\log\left(1 - D(G(z))\right)\right]$$




$D$ outputs:

$D(x) = 1$ real

$D(x) = 0$ generated

[Slide Credit: Manzil Zaheer]

Goodfellow et al, 2014

# GAN Formulation: Optimization

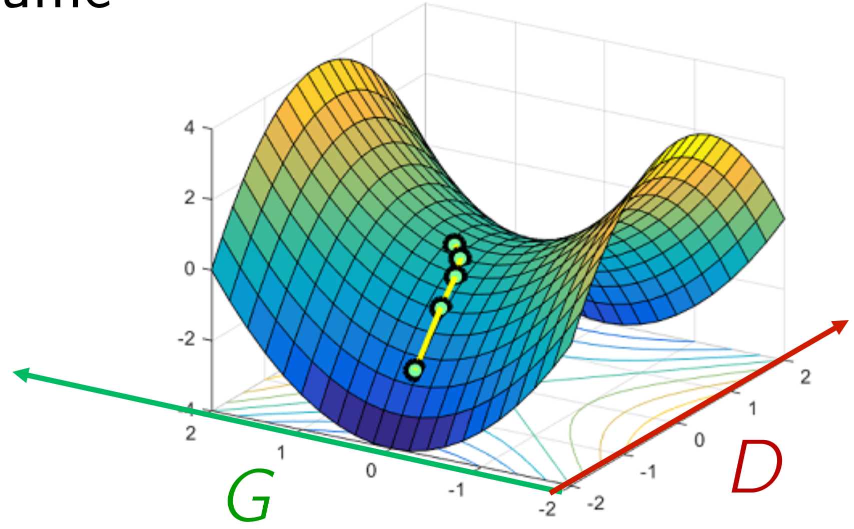
- Overall GAN optimization

$$\min_{G} \max_{D} V(G, D) = \mathbb{E}_{x \sim p}\left[\log D(x)\right] + \mathbb{E}_{z \sim p_Z}\left[\log\left(1 - D(G(z))\right)\right]$$

- The generator-discriminator are iteratively updated using SGD to find "equilibrium" of a "min-max objective" like a game

$$G \leftarrow G - \eta_G \nabla_G V(G, D)$$

$$D \leftarrow D - \eta_D \nabla_D V(G, D)$$




[Slide Credit: Manzil Zaheer]

# Wasserstein GAN

- WGAN optimization

$$\min_{G} \max_{D} W(G, D) = \mathbb{E}_{x \sim p}\left[D(x)\right] - \mathbb{E}_{z \sim p_Z}\left[D(G(z))\right]$$

- Difference in expected output on real vs. generated images
  - Generator attempts to drive objective ≈ 0
- More stable optimization

Compare to training DBMs

$$\frac{\partial \log P_\theta(\mathbf{v})}{\partial W^1} = \mathbb{E}_{P_{data}}[\mathbf{v}\mathbf{h}^{\mathbf{1}\top}] - \mathbb{E}_{P_\theta}[\mathbf{v}\mathbf{h}^{\mathbf{1}\top}]$$

$D$ outputs:

$D(x) = 1$ real

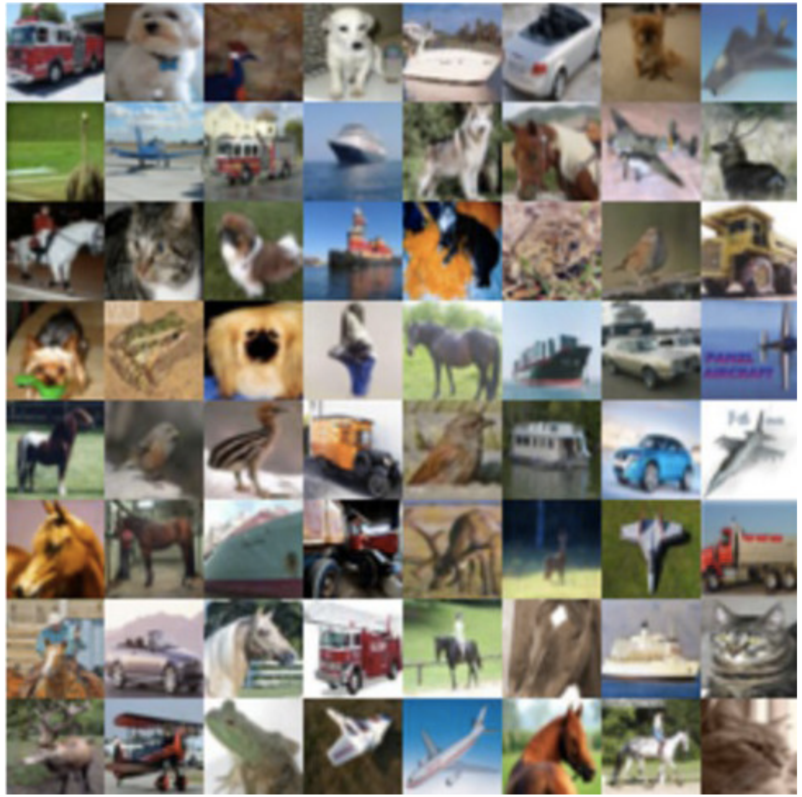
$D(x) = 0$ generated

Arjovsky et al., 2017

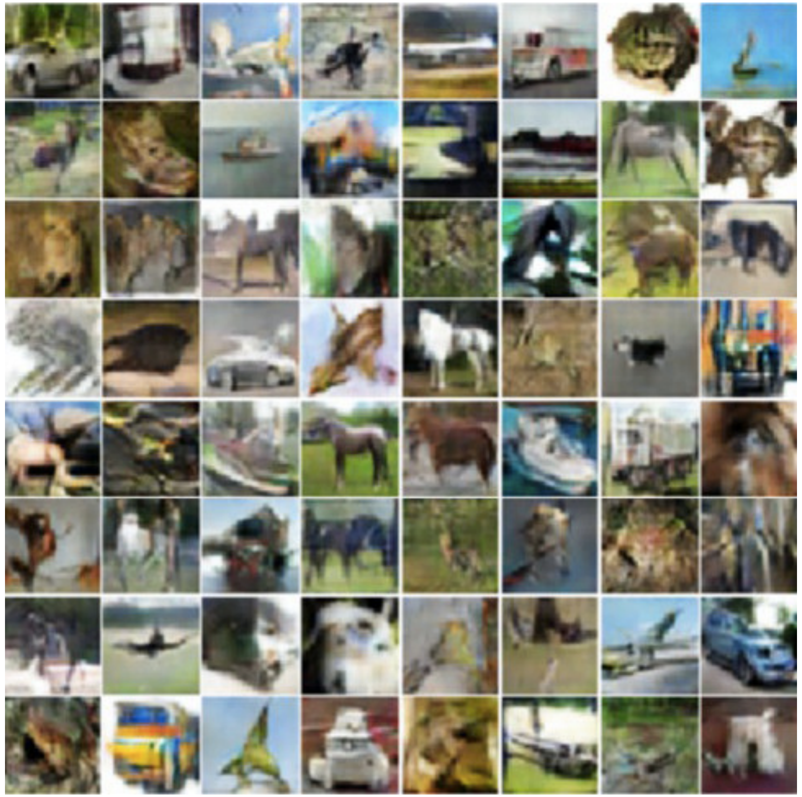
# LSUN Bedroom: Samples

Radford et al., 2015

# CIFAR Dataset



Training



Samples

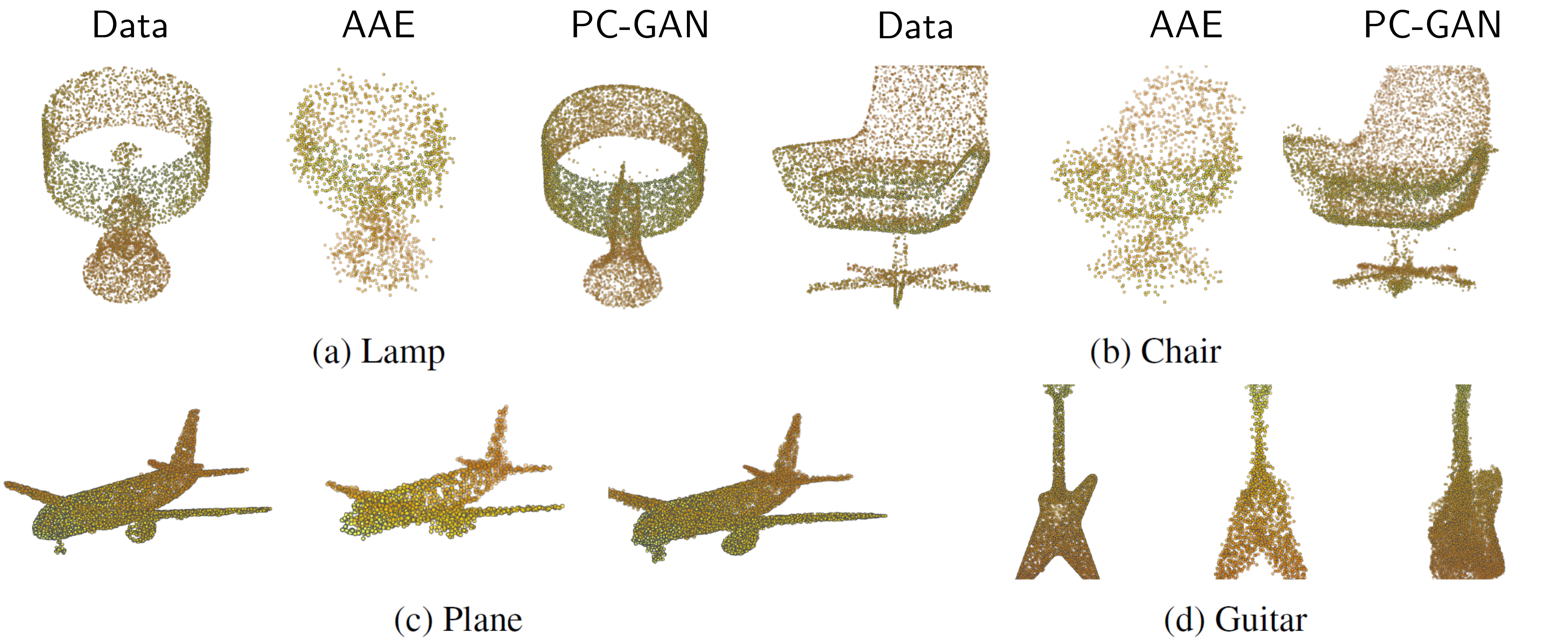
Salimans et. al., 2016

# ImageNet: Cherry-Picked Samples

- Open Question: How can we quantitatively evaluate these models!

Slide Credit: Ian Goodfellow

# Modelling Point Cloud Data

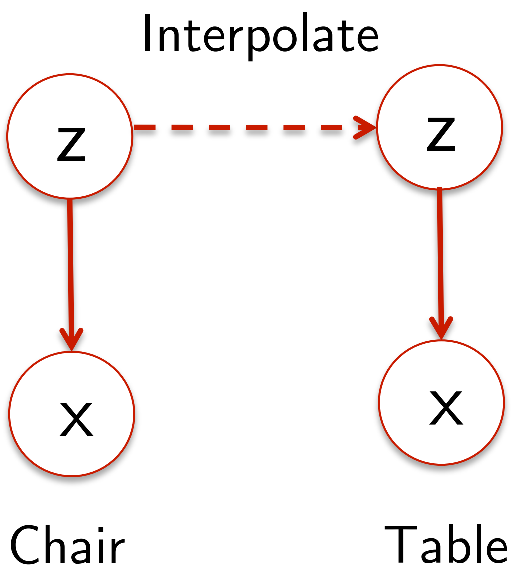


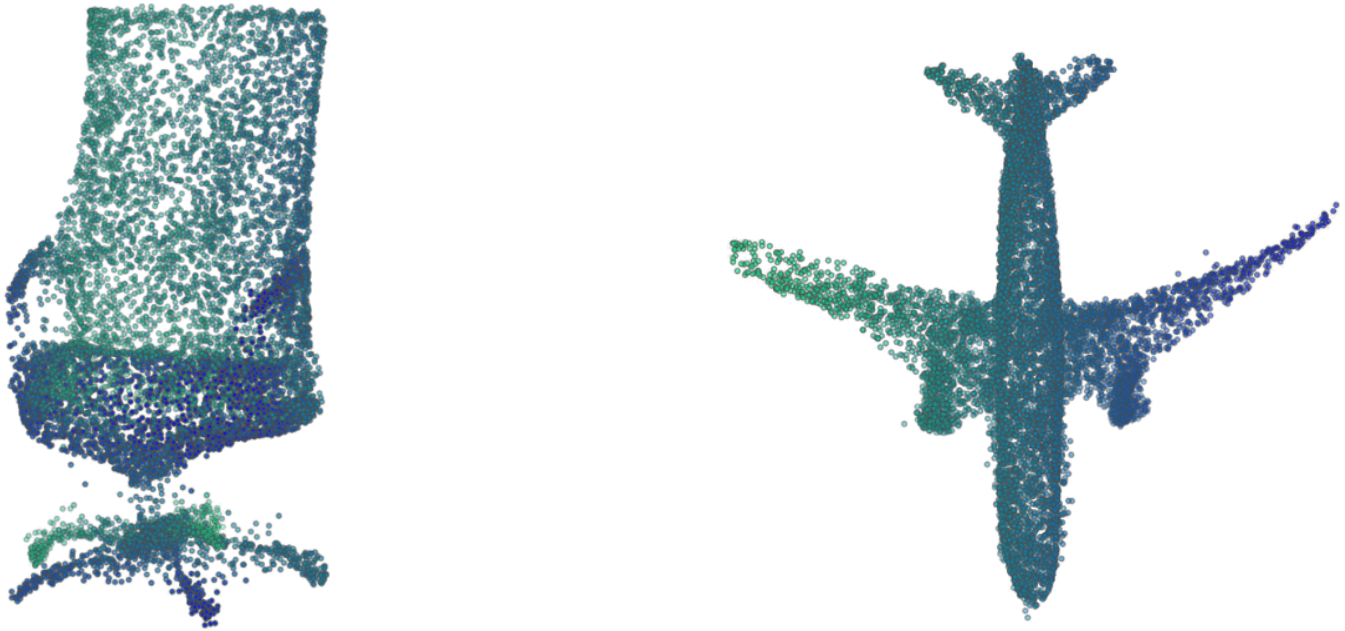

(a) Lamp (b) Chair

(c) Plane (d) Guitar

Zaheer et al. Point Cloud GAN 2018

# Interpolation in Latent Space

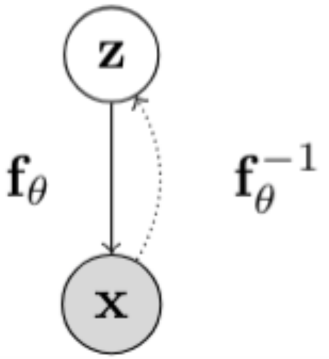


Zaheer et al. Point Cloud GAN 2018

# Normalizing Flows

- Directed Latent Variable Invertible models




- The mapping between x and z is deterministic and invertible:

$$\begin{aligned} \mathbf{x} &= \mathbf{f}_\theta(\mathbf{z}) \\ \mathbf{z} &= \mathbf{f}_\theta^{-1}(\mathbf{x}) \end{aligned}$$

- Use change-of-variables to relate densities between z and x

$$p_X(\mathbf{x};\theta) = p_Z(\mathbf{z}) \left| \det \frac{\partial \mathbf{f}_\theta^{-1}(\mathbf{x})}{\partial X} \right|_{X=\mathbf{x}}$$

Grover and Ermon DGM Tutorial, NICE (Dinh et al. 2014), Real NVP (Dinh et al. 2016)

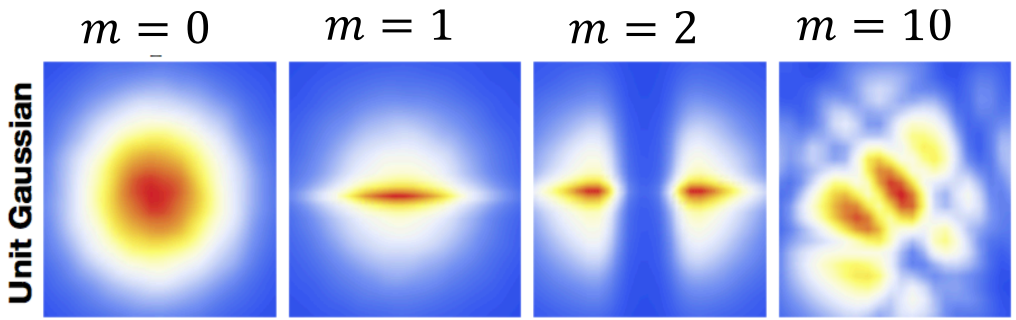
# Normalizing Flows

- Invertible transformations can be composed:

$$\mathbf{x} = \mathbf{f}_\theta^M \circ \cdots \circ \mathbf{f}_\theta^1(\mathbf{z}^0); \qquad p_X(\mathbf{x};\theta) = p_{Z^0}(\mathbf{z}^0) \prod_{m=1}^{M} \left| \det \frac{\partial (\mathbf{f}_\theta^m)^{-1}}{\partial Z^m} \right|_{Z^m = \mathbf{z}^m}$$

- Planar Flows

$$f(\mathbf{z}) = \mathbf{z} + \mathbf{u} g(\mathbf{w}^\top \mathbf{z} + b)$$




Rezendre and Mohamed, 2016

Rezendre and Mohamed, 2016, Grover and Ermon DGM Tutorial

# Normalizing Flows

- Maximum log-likelihood objective

$$\max_{\theta} \log p_X(\mathcal{D};\theta) = \sum_{\mathbf{x}\in\mathcal{D}} \left( \log p_Z(\mathbf{z}) - \log \left| \det \frac{\partial (\mathbf{f}_\theta)^{-1}}{\partial X} \right|_{X=\mathbf{x}} \right)$$

- Exact log-likelihood evaluation via inverse transformations
- Sampling from the model

$$\mathbf{z} \sim p_Z(\mathbf{z}), \quad \mathbf{x} = \mathbf{f}_\theta(\mathbf{z})$$

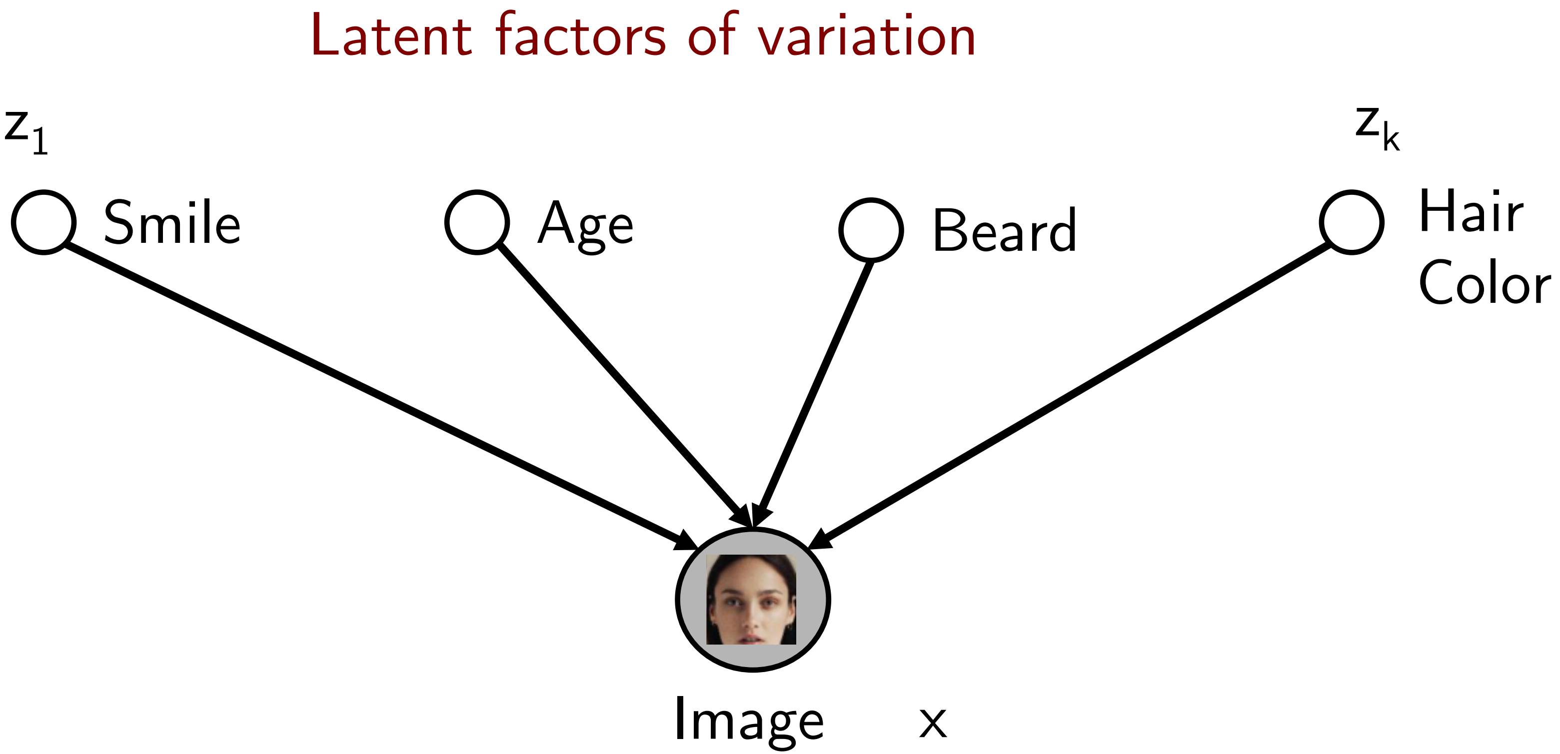
- Inference over the latent representations:

$$\mathbf{z} \quad = \quad \mathbf{f}_\theta^{-1}(\mathbf{x})$$

Rezendre and Mohamed, 2016, Grover and Ermon DGM Tutorial

# Example: GLOW

- Generative Flow with Invertible 1x1 Convolutions
  https://blog.openai.com/glow/




Kingma, Dhariwal, 2018

# Example: GLOW

Input

Smile

Add Beard

Increase Age

https://blog.openai.com/

Remove Beard

Decrease Age

# Thank you