HIDDEN MARKOV MODELS IN SPEECH RECOGNITION

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Acknowledgements

Much of this talk is derived from the paper

"An Introduction to Hidden Markov Models",

by Rabiner and Juang

and from the talk

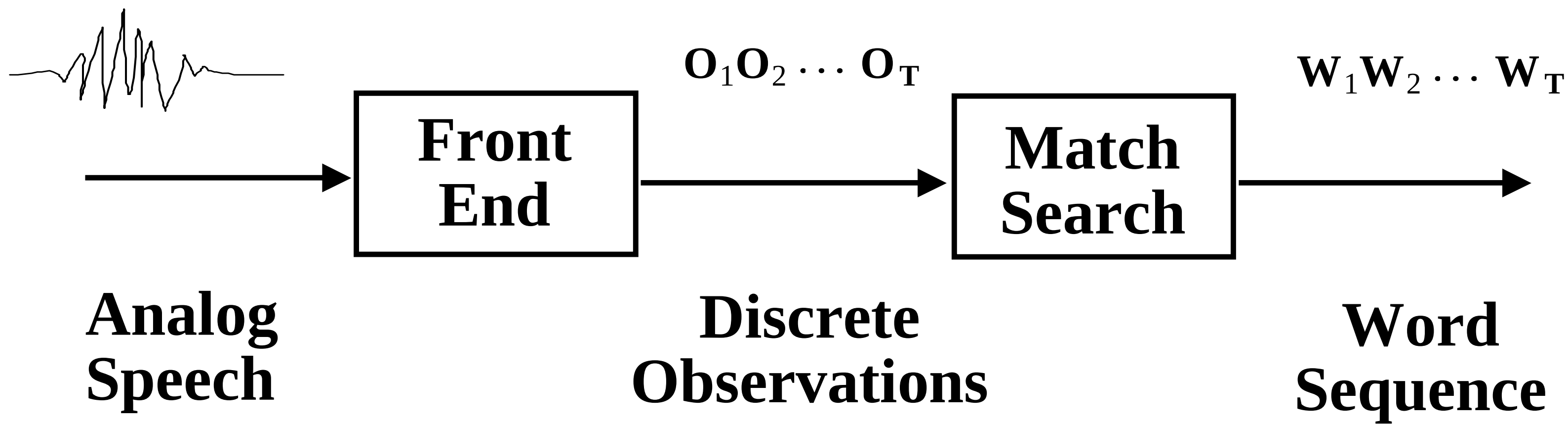
"Hidden Markov Models: Continuous Speech Recognition"

by Kai-Fu Lee

Topics

- **Markov Models and Hidden Markov Models**
- **HMMs applied to speech recognition**
  - **Training**
  - **Decoding**

Speech Recognition



ML Continuous Speech Recognition

Goal:

Given acoustic data $A = a_1, a_2, ..., a_k$

Find word sequence $W = w_1, w_2, ... w_n$

Such that $P(W \mid A)$ is maximized

Bayes Rule:

acoustic model (HMMs) language model

$$P(W \mid A) = \frac{\overbrace{P(A \mid W)}^{\text{acoustic model (HMMs)}} \cdot \overbrace{P(W)}^{\text{language model}}}{P(A)}$$

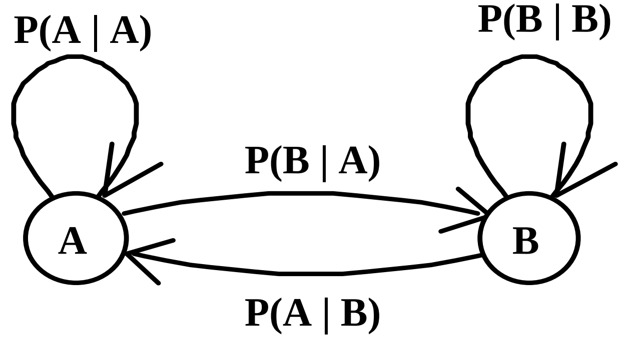
P(A) is a constant for a complete sentence

Markov Models

Elements:

States : $\mathbf{S} = \{\mathbf{S}_0, \mathbf{S}_1, \cdots \mathbf{S}_\mathbf{N}\}$

Transition probabilities : $P(q_t = S_i \mid q_{t-1} = S_j)$



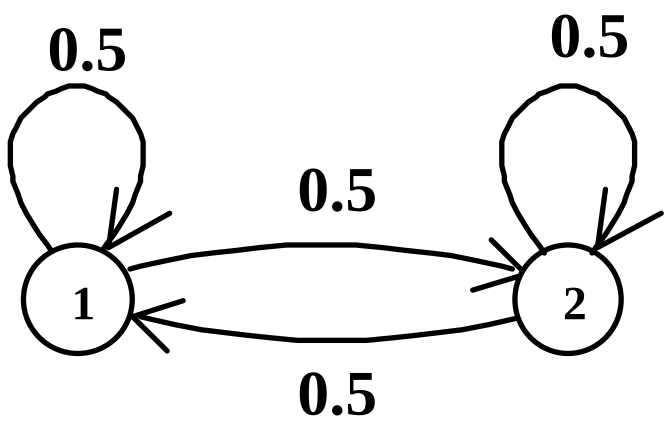
Markov Assumption:

Transition probability depends only on current state

$$P(q_t = S_i \mid q_{t-1} = S_j,\ q_{t-2} = S_k,\ \cdots) = P(q_t = S_i \mid q_{t-1} = S_j) = a_{ji}$$

$$a_{ji} \geq 0 \quad \forall\ j,i \qquad\qquad \sum_{i=0}^{N} a_{ji} = 1 \quad \forall_j$$

Single Fair Coin



P(H) = 1.0   P(H) = 0.0

P(T) = 0.0   P(T) = 1.0

Outcome head corresponds to state 1, tail to state 2

Observation sequence uniquely defines state sequence

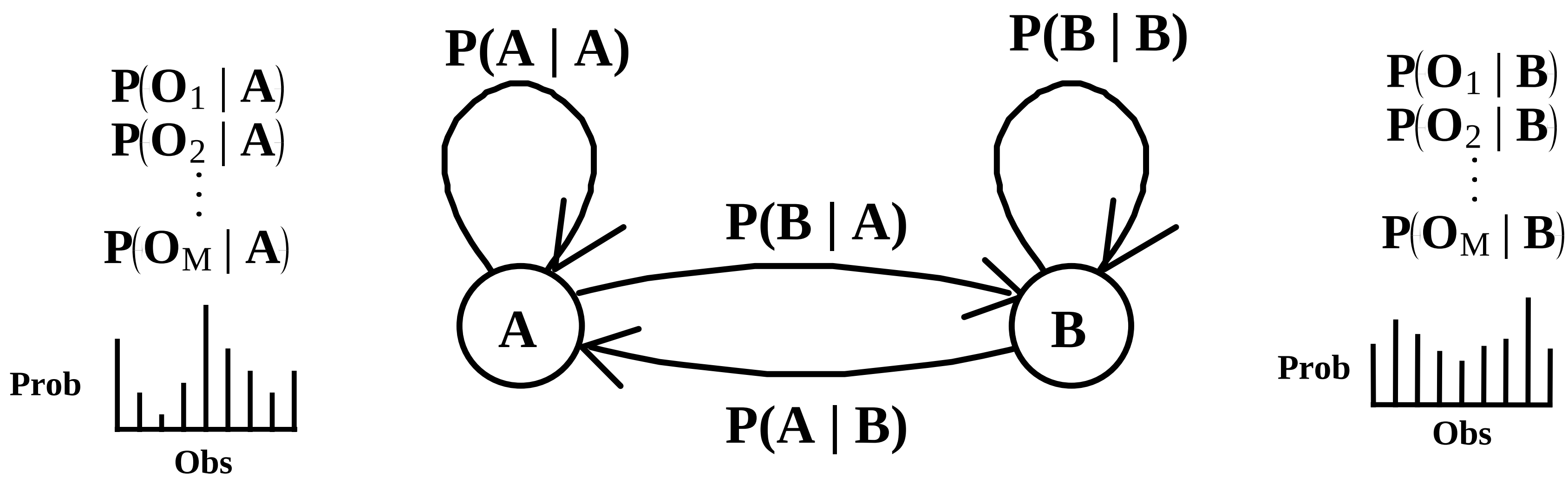
Hidden Markov Models

Elements:

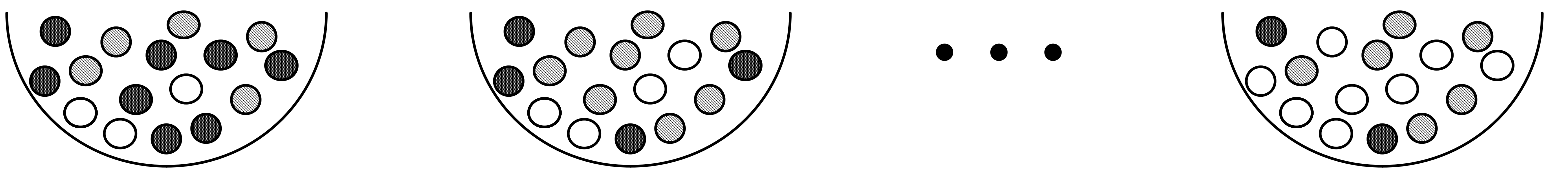
States $S = \{S_0, S_1, \cdots S_N\}$

Transition probabilities $P(q_t = S_i \mid q_{t-1} = S_j) = a_{ji}$

Output prob distributions (at state j for symbol k) $P(y_t = O_k \mid q_t = S_j) = b_j(k)$



Discrete Observation HMM



P(R) = 0.31
P(B) = 0.50
P(Y) = 0.19

P(R) = 0.50
P(B) = 0.25
P(Y) = 0.25

P(R) = 0.38
P(B) = 0.12
P(Y) = 0.50

Observation sequence: R B Y Y • • • R

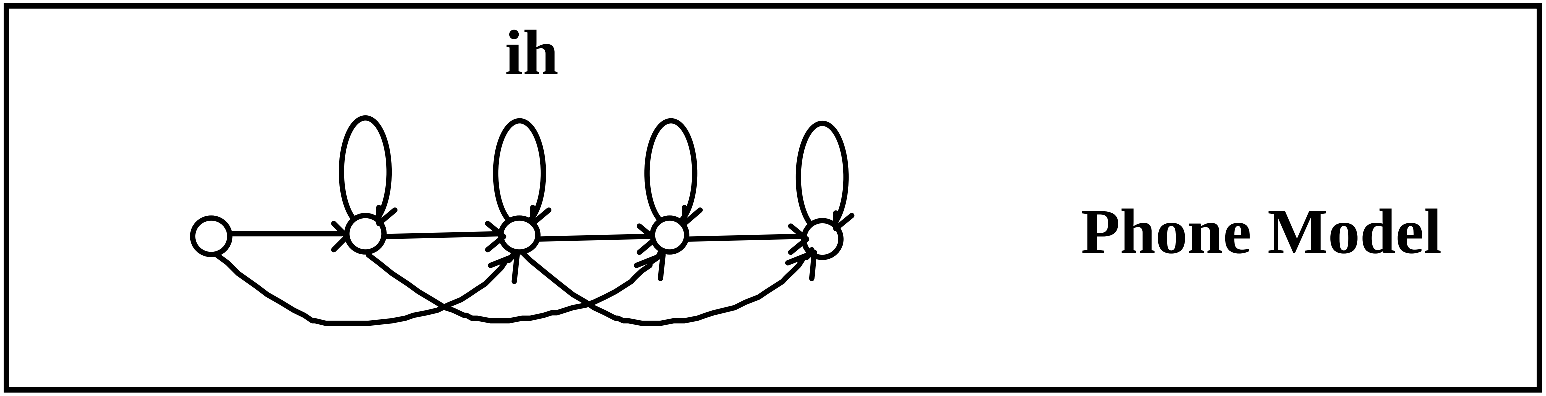
not unique to state sequence

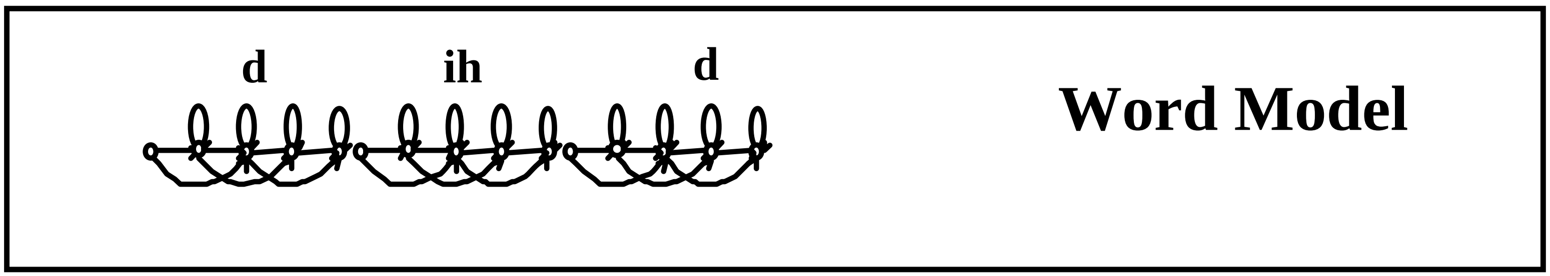
HMMs In Speech Recognition

Represent speech as a sequence of observations

Use HMM to model some unit of speech (phone, word)

Concatenate units into larger units





HMM Problems And Solutions

Evaluation:

- Problem - Compute Probabilty of observation sequence given a model
- Solution - **Forward Algorithm** and **Viterbi Algorithm**

Decoding:

- Problem - Find state sequence which maximizes probability of observation sequence
- Solution - **Viterbi Algorithm**

Training:

- Problem - Adjust model parameters to maximize probability of observed sequences
- Solution - **Forward-Backward Algorithm**

Evaluation

Probability of observation sequence $\mathbf{O} = O_1\, O_2 \cdots O_T$

given HMM model λ is :

$$P(O \mid \lambda) = \sum_{\forall Q} P(O, Q \mid \lambda)$$

$Q = q_0 q_1 \ldots q_T$ is a state sequence

$$= \sum a_{q0q1} b_{q1}(O_1) \times a_{q1q2} b_{q2}(O_2) \cdots a_{qT-1qT} b_{qT}(O_T)$$

Not practical since the number of paths is $O(N^T)$

N = number of states in model
T = number of observations in sequence

The Forward Algorithm

$$\alpha_t(j) = P(O_1\ O_2 \cdots O_t,\ q_t = S_j \mid \lambda)$$

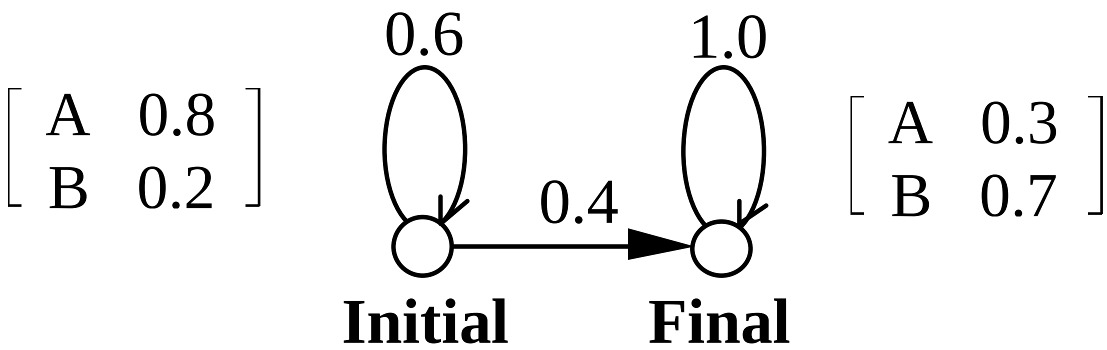
Compute α recursively:

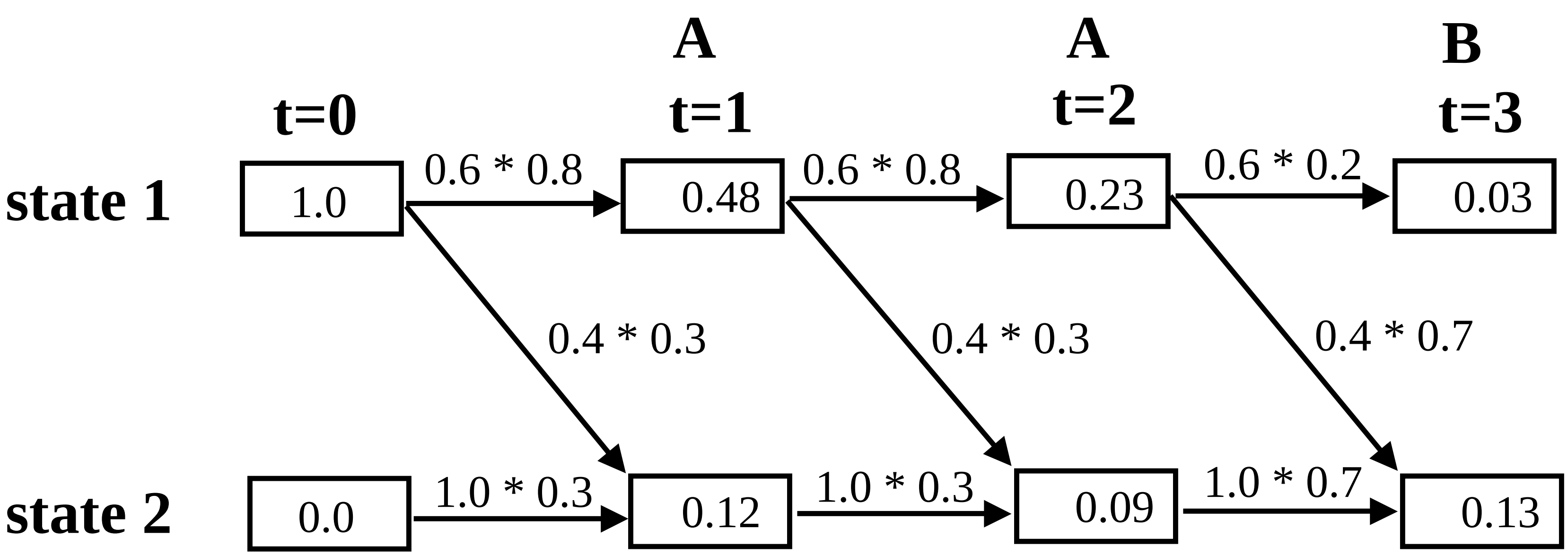
$$\alpha_0(j) = \begin{cases} \mathbf{1} \text{ if j is start state} \\ \mathbf{0} \text{ otherwise} \end{cases}$$

$$\alpha_t(j) = \left[\sum_{i=0}^{N} \alpha_{t-1}(i) a_{ij}\right] b_j(O_t) \qquad t > 0$$

$P(O \mid \lambda) = \alpha_T(S_N)$ Computation is $O(N^2 T)$

Forward Trellis





The Backward Algorithm

$$\beta_t(i) = P(O_{t+1}\ O_{t+2} \cdots O_T \mid q_t = S_i\ , \lambda\ )$$

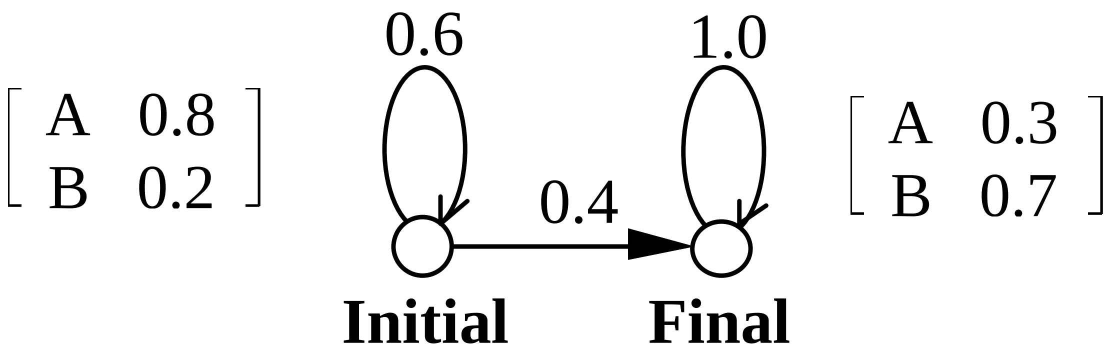
Compute β recursively:

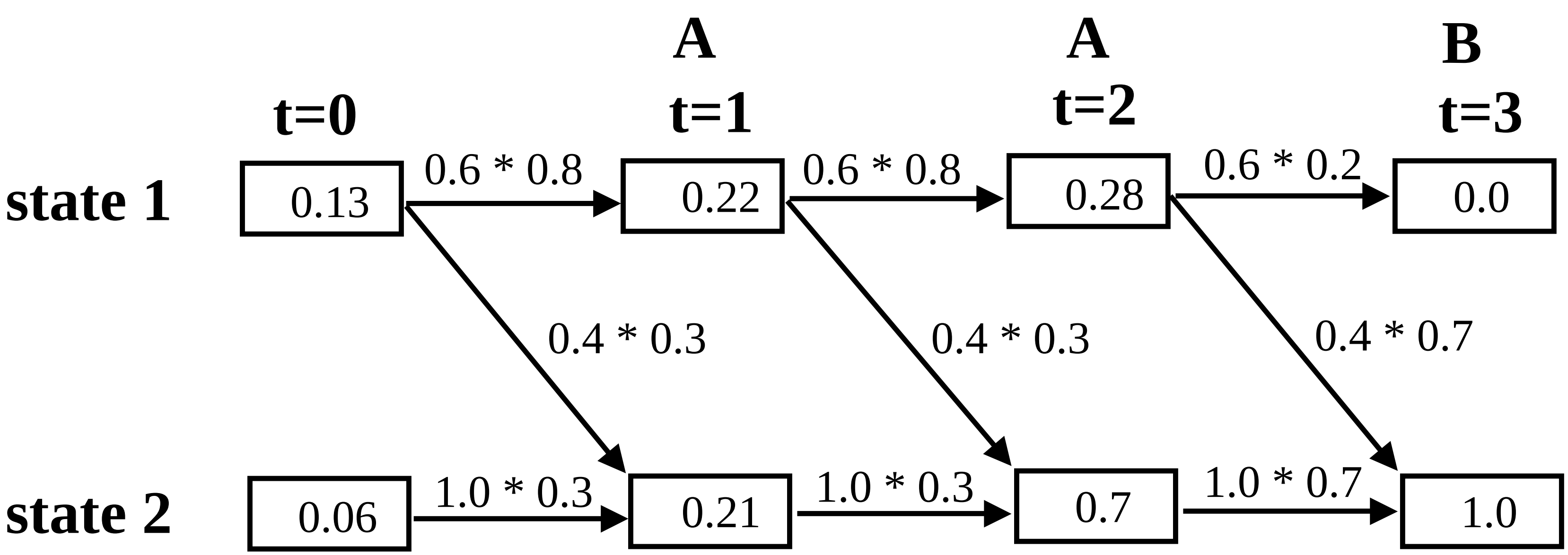
$$\beta_T(i) = \begin{cases} \mathbf{1} \text{ if i is end state} \\ \mathbf{0} \text{ otherwise} \end{cases}$$

$$\beta_t(i) = \sum_{j=0}^{N} a_{ij} b_j(O_{t+1}) \beta_{t+1}(j) \qquad t < T$$

$$P(O \mid \lambda) = \beta_0(S_0) = \alpha_T(S_N) \qquad \text{Computation is } O(N^2 T)$$

Backward Trellis





The Viterbi Algorithm

For decoding:

Find the state sequence **Q** which maximizes **P(O, Q | λ)**

Similar to Forward Algorithm except **MAX** instead of **SUM**

$$VP_t(i) = MAX_{q_0, \cdots q_{t-1}} \; P(O_1 O_2 \cdots O_t, q_t = i \mid \lambda)$$

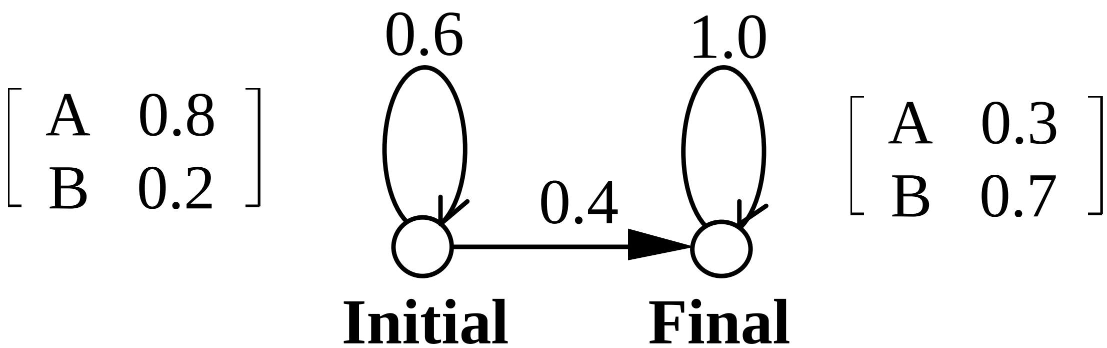
Recursive Computation:

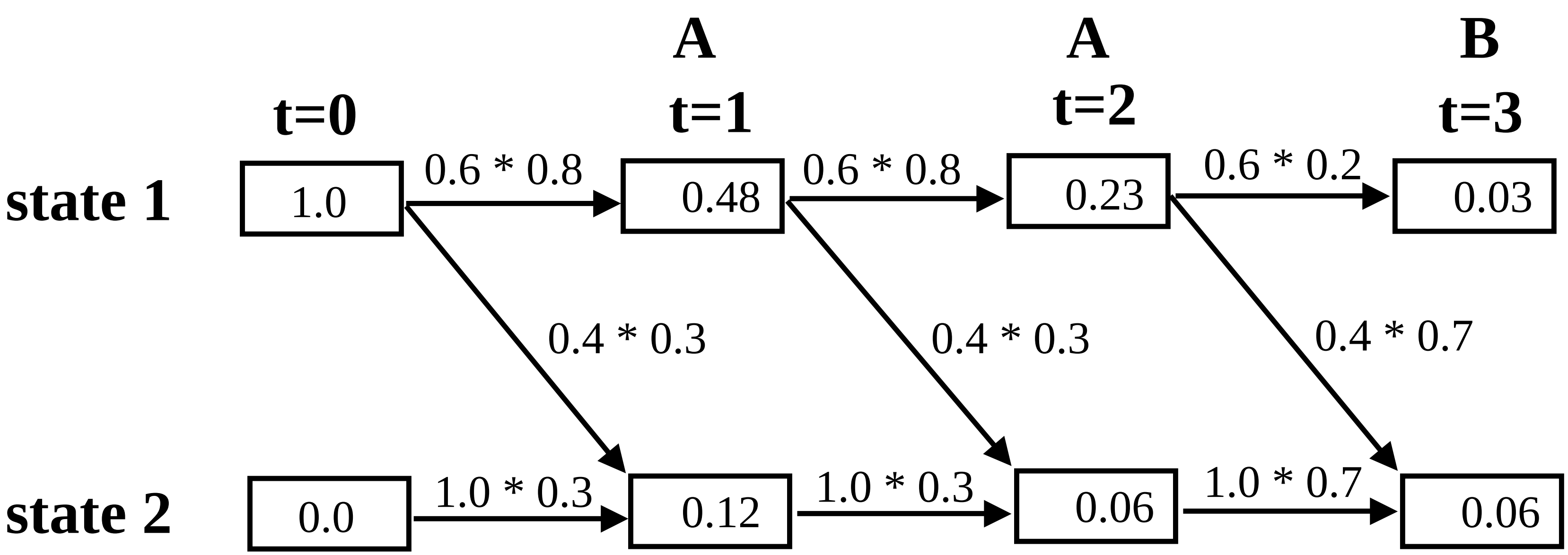
$$VP_t(j) = MAX_{i=0, \cdots, N} \; VP_{t-1}(i) \; a_{ij} b_j(O_t) \qquad t > 0$$

$$P(O, Q \mid \lambda) = VP_T(S_N)$$

Save each maximum for backtrace at end

Viterbi Trellis





Training HMM Parameters

Train parameters of HMM

- Tune λ to maximize $P(O \mid \lambda)$
- No efficient algorithm for global optimum
- Efficient iterative algorithm finds a local optimum

Baum-Welch (Forward-Backward) re-estimation

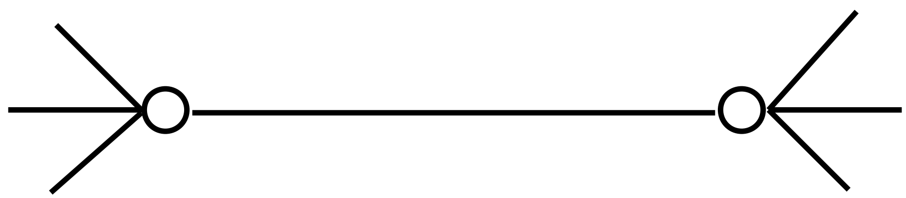
- Compute probabilities using current model λ
- Refine $\lambda \longrightarrow \lambda$ based on computed values
- Use α and β from Forward-Backward

Forward-Backward Algorithm

$\xi_t(i,j) =$ **Probability of transiting from** S_i **to** S_j **at time t given O**

$$= P(\, q_t = S_i,\ q_{t+1} = S_j \mid O,\ \lambda\,)$$

$$= \frac{\alpha_t(i)\ a_{ij}\ b_j(O_{t+1})\ \beta_{t+1}(j)}{P(O \mid \lambda)}$$



$\alpha_t(i)$ $\quad a_{ij}b_j(O_{t+1})$ $\quad \beta_{t+1}(j)$

Baum-Welch Reestimation

$$\bar{a}_{ij} = \frac{\text{expected number of trans from } S_i \text{ to } S_j}{\text{expected number of trans from } S_i}$$

$$= \frac{\sum_{t=0}^{T-1} \xi_t(i, j)}{\sum_{t=0}^{T-1} \sum_{j=0}^{N} \xi_t(i, j)}$$

$$\bar{b}_j(k) = \frac{\text{expected number of times in state j with symbol k}}{\text{expected number of times in state j}}$$

$$= \frac{\sum_{t:O_{t+1}=k} \sum_{i=0}^{N} \xi_t(i, j)}{\sum_{t=0}^{T-1} \sum_{i=0}^{N} \xi_t(i, j)}$$

Convergence of FB Algorithm

1. Initialize $\lambda = (A, B)$
2. Compute α, β, and ξ
3. Estimate $\bar{\lambda} = (\bar{A}, \bar{B})$ from ξ
4. Replace λ with $\bar{\lambda}$
5. If not converged go to 2

It can be shown that $P(O \mid \bar{\lambda}) > P(O \mid \lambda)$ unless $\bar{\lambda} = \lambda$

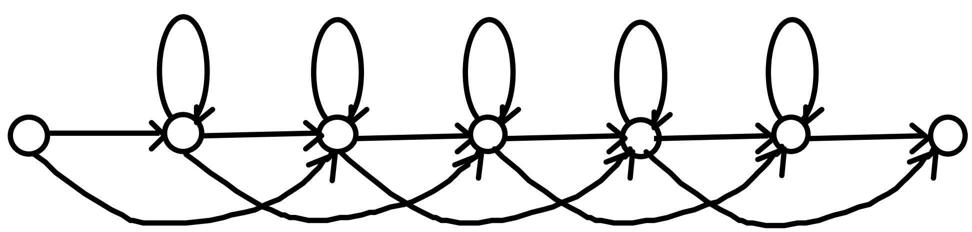
HMMs In Speech Recognition

Represent speech as a sequence of symbols

Use HMM to model some unit of speech (phone, word)

Output Probabilities - Prob of observing symbol in a state

Transition Prob - Prob of staying in or skipping state

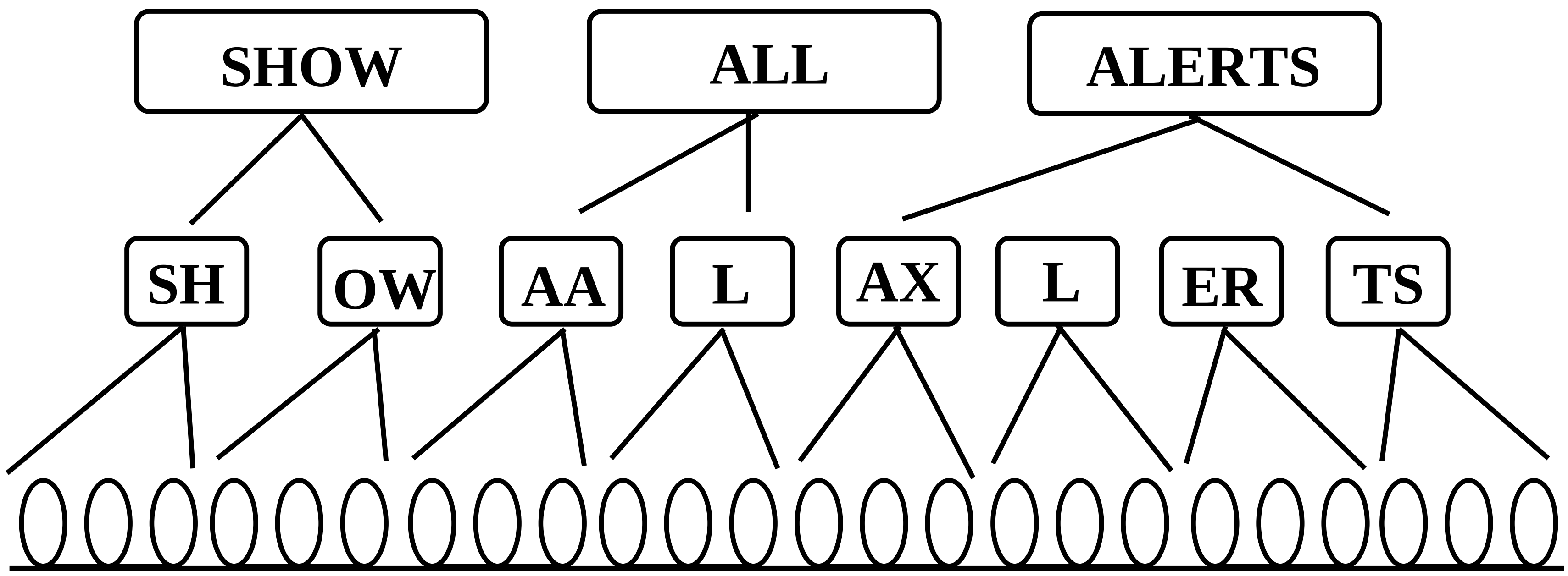


Phone Model

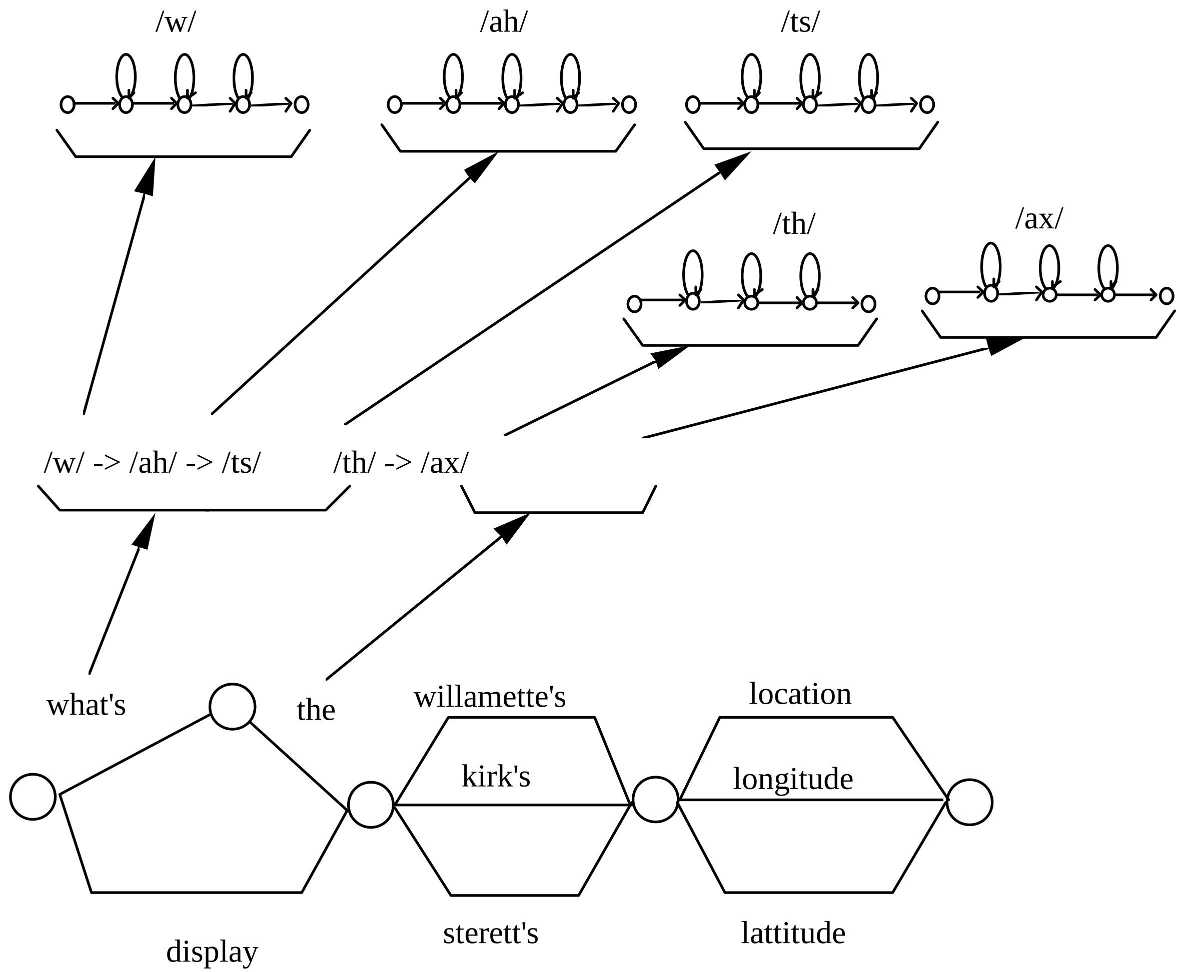
Training HMMs for Continuous Speech

- Use only orthograph transcription of sentence
  - no need for segmented/labelled data
- Concatenate phone models to give word model
- Concatenate word models to give sentence model
- Train entire sentence model on entire spoken sentence

Forward-Backward Training for Continuous Speech



Recognition Search



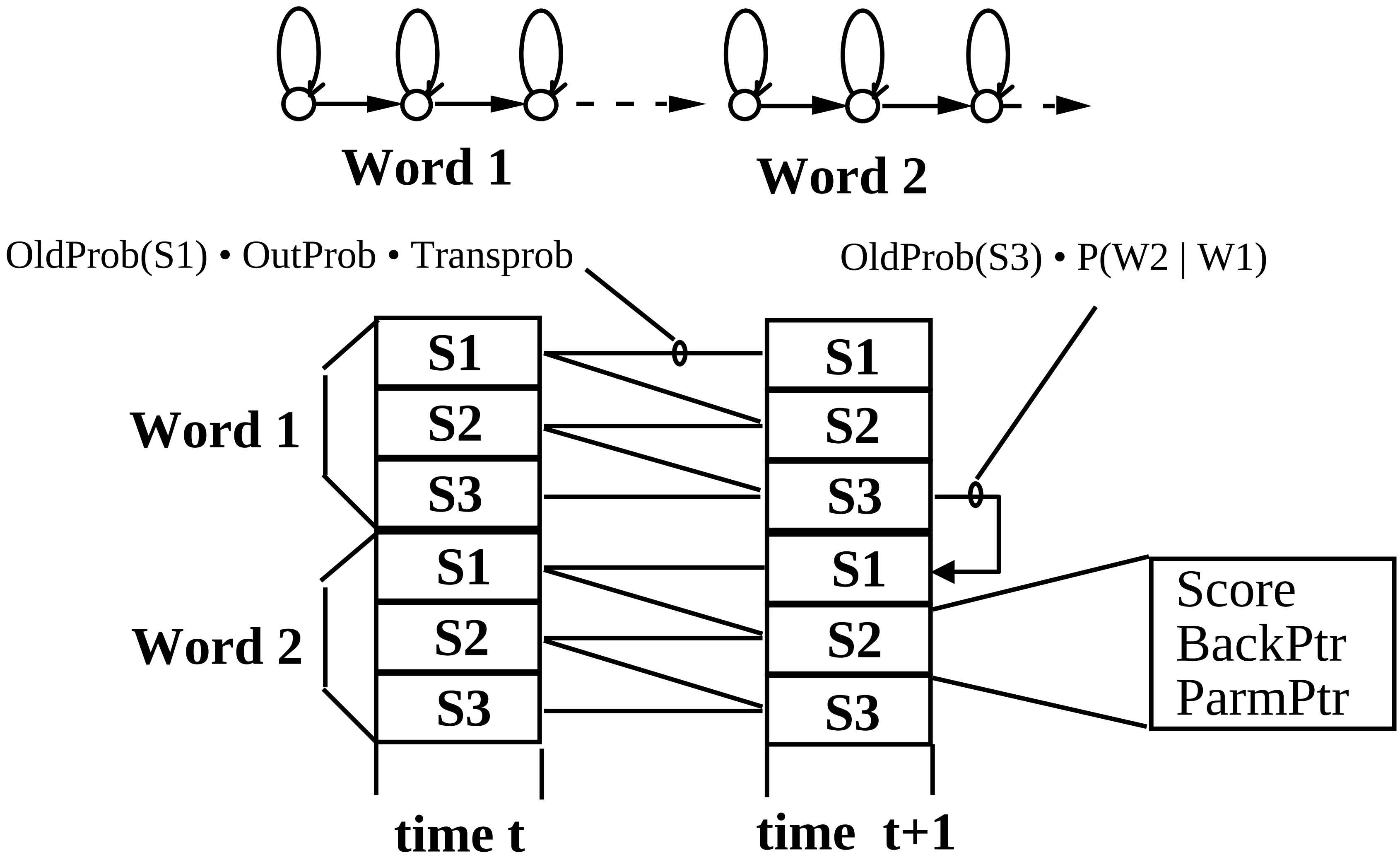
Viterbi Search

- **Uses Viterbi decoding**
    - **Takes MAX, not SUM**
    - **Finds optimal state sequence $P(O, Q \mid \lambda)$ not optimal word sequence $P(O \mid \lambda)$**
- **Time synchronous**
    - **Extends all paths by 1 time step**
    - **All paths have same length (no need to normalize to compare scores)**

Viterbi Search Algorithm

0. Create state list with one cell for each state in system
1. Initialize state list with initial states for time t= 0
2. Clear state list for time t+1
3. Compute within-word transitions from time t to t+1
   - If new state reached, update score and BackPtr
   - If better score for state, update score and BackPtr
4. Compute between word transitions at time t+1
   - If new state reached, update score and BackPtr
   - If better score for state, update score and BackPtr
5. If end of utterance, print backtrace and quit
6. Else increment t and go to step 2

Viterbi Search Algorithm



Viterbi Beam Search

Viterbi Search

All states enumerated

Not practical for large grammars

Most states inactive at any given time

Viterbi Beam Search - prune less likely paths

States worse than threshold range from best are pruned

From and To structures created dynamically - list of active states

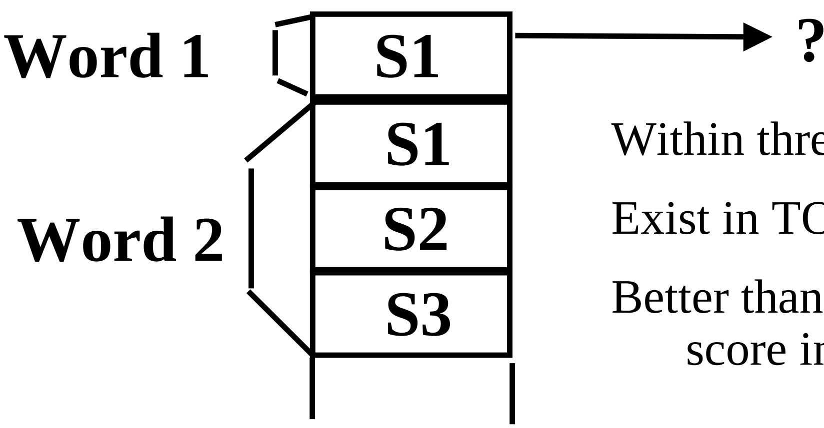
Viterbi Beam Search

FROM BEAM

States within threshold from best state

TO BEAM

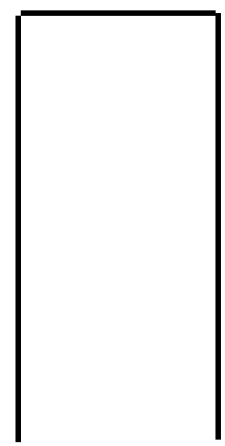
Dynamically constructed



Within threshold ?

Exist in TO beam?

Better than existing score in TO beam?



time t

time t+1

Continuous Density HMMs

Model so far has assumed discete observations, each observation in a sequence was one of a set of M discrete symbols

Speech input must be Vector Quantized in order to provide discrete input.

VQ leads to quantization error

The discrete probability density $b_j(k)$ can be replaced with the continuous probability density $b_j(\mathbf{x})$ where $\mathbf{x}$ is the observation vector

Typically Gaussian densities are used

A single Gaussian is not adequate, so a weighted sum of Gaussians is used to approximate actual PDF

Mixture Density Functions

$b_j(\mathbf{x})$ is the probability density function for state j

$$\boxed{b_j(x) = \sum_{m=1}^{M} c_{jm} N\left[x, \mu_{jm}, U_{jm}\right]}$$

$\mathbf{x}$ = Observation vector $x_1, x_2, \cdots, x_D$

M = Number of mixtures (Gaussians)

c_{jm} = Weight of mixture m in state j where $\sum_{m=1}^{M} c_{jm} = 1$

N = Gaussian density function

μ_{jm} = Mean vector for mixture m, state j

U_{jm} = Covariance matrix for mixture m, state j

Discrete Hmm vs. Continuous HMM

- **Problems with Discrete:**
  - **quantization errors**
  - **Codebook and HMMs modelled separately**

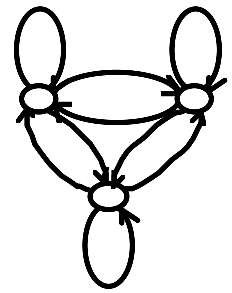
- **Problems with Continuous Mixtures:**
  - **Small number of mixtures performs poorly**
  - **Large number of mixtures increases computation and parameters to be estimated**

    c_{jm}, μ_{jm}, U_{jm} for $j = 1, \cdots, N$ and $m = 1, \cdots, M$

- **Continuous makes more assumptions than Discrete, especially if diagonal covariance pdf**
- **Discrete probability is a table lookup, continuous mixtures require many multiplications**

Model Topologies

**Ergodic - **Fully connected, each state has transition to every other state



**Left-to-Right - **Transitions only to states with higher index than current state. Inherently impose temporal order. These most often used for speech.