

# Lateralization and detection of low-frequency binaural stimuli: Effects of distribution of internal delay

Richard M. Stern and Glenn D. Shear

*Department of Electrical and Computer Engineering and Biomedical Engineering Program,  
Carnegie Mellon University, Pittsburgh, Pennsylvania 15213*

(Received 20 August 1992; revised 8 December 1995; accepted 1 April 1996)

The position-variable model [R. M. Stern, Jr., and H. S. Colburn, *J. Acoust. Soc. Am.* **64**, 127–140 (1978)], which had been developed to describe binaural phenomena at 500 Hz, is extended to describe and predict binaural lateralization and detection phenomena at frequencies of 250–1200 Hz. The major modification made to the model is the use of a frequency-dependent form of the function that describes the relative number of binaural coincidence detectors in the model as a function of their internal delay. This function is fitted to describe the lateralization of pure tones with a fixed ITD over a range of frequencies, and the ratio of  $N_0S_\pi$  to  $N_\pi S_0$  binaural detection thresholds. The extended model describes without further modification the general dependence of the lateral position of pure tones on ITD and frequency, the lateralization of amplitude-modulated tones, and the “dominant region” lateralization phenomenon. © 1996 Acoustical Society of America.

PACS numbers: 43.66.Ba, 43.66.Pn, 43.66.Qp [HSC]

## INTRODUCTION

This paper describes modifications of the position-variable model of binaural interaction (Colburn, 1973; Stern and Colburn, 1978) that extend the range of stimulus frequencies over which the model can be applied to the range of 250–1200 Hz. We discuss in this paper the motivation for changing some of the components of the model, and we present and discuss new comparisons of theoretical predictions to experimental measurements for several binaural stimuli below 1200 Hz, including amplitude-modulated tones and stimuli associated with the binaural “dominant region” phenomenon (which implies that frequency components in a midrange close to 700 Hz are more salient in the lateralization process).

The position-variable model was originally developed as a means to unify binaural lateralization, discrimination, and detection phenomena within the confines of a single theoretical framework. An important attribute of this model is its use of the display of binaural information proposed by Colburn (1973) that records coincidences in firing times from auditory-nerve fibers of comparable characteristic frequency from the two ears, after a small fixed interaural time delay. This display effectively estimates the interaural cross-correlation function of binaural stimuli after peripheral frequency analysis, and it may be regarded as an implementation of the running cross-correlation operation proposed by Sayers and Cherry (1957), using a neural mechanism of the type proposed by Jeffress (1948). This display of interaural cross-correlation after peripheral filtering is a common feature of most binaural processing models [e.g., Colburn (1973), Blauert and Cobben (1978), Stern and Colburn (1978), Lindemann (1986), Shamma *et al.* (1989), Shackleton *et al.* (1992)], as well as modern models of dichotic pitch perception and associated phenomena [e.g., Bilsen and Goldstein (1974), Raatgever and Bilsen (1986)].

In this paper we consider changes to the model that are

needed to enable it to describe phenomena in the frequency range of 200–1200 Hz. We begin with a detailed consideration of the results of Schiano *et al.* (1986), who demonstrated that the lateral position of a pure tone with a fixed interaural time difference (ITD) remains approximately the same, even though the frequency of the tone is varied. The most important modification to the model at the level of binaural processing is a redefinition of the function specifying the relative number of coincidence-counting units as a function of internal delay and characteristic frequency. Since this distribution function was originally developed by Colburn (1973, 1977) to describe ratios of binaural detection thresholds as a function of target frequency, it is also important to redefine it in a fashion that does not significantly degrade the model’s ability to describe these detection phenomena.

In Sec. I we describe several recent modifications to the model that characterizes the peripheral auditory-nerve response to pure tones and bandpass noise. These changes were needed to correct several general problems with the auditory-nerve model that have been identified since the model was last formally described in Stern and Colburn (1978, 1985). We describe in Sec. II the specific modifications to the function that specifies the distribution of binaural coincidence-counting units with respect to internal delay and characteristic frequency that enable the model to describe the perception of binaural stimuli over a wider range of frequencies, including the data of Schiano *et al.* (1986). In Sec. III we describe and discuss predictions of the model for other types of low-frequency lateralization phenomena, including pure tones at other frequencies and ITDs, and low-frequency amplitude-modulated tones. Finally, we demonstrate in Sec. III that the form of data by Raatgever (Raatgever, 1980; Raatgever and Bilsen, 1986) implying the existence of a low-frequency “dominant region” of lateralization is also a consequence of the function specifying the distribution of inter-

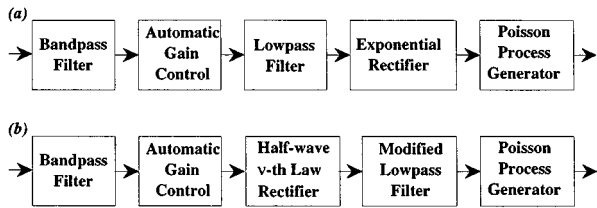


FIG. 1. Block diagrams of (a) the original auditory-nerve model as specified by Colburn (1973) and (b) the new version of the auditory-nerve model used for the present work.

nal delays. In a companion publication, Stern and Shear (1996) summarize the considerations leading to the reformulation of the function describing the distribution of binaural coincidence-counting units with respect to internal delay and characteristic frequency. This publication also contains a complete specification of the position-variable model as implemented to describe these data.

## I. MODIFICATIONS TO THE MODEL OF AUDITORY-NERVE ACTIVITY

The position-variable model develops its predictions by comparisons of the outputs of a model for auditory-nerve activity originally specified by Colburn (1973). This model of auditory-nerve activity develops analytical expressions for the mean and variance of the putative neural response to sounds, based on the characterization of the peripheral auditory-nerve activity as a nonhomogeneous Poisson process. This class of Poisson-based auditory-nerve models has the advantage that quantitative predictions can be developed for a number of interesting stimuli without resorting to simulation, but it suffers from two limitations: some of the assumptions used to specify the model are known to be incorrect, and it is possible to develop the necessary analytical expressions only for a limited set of stimuli. Because of these difficulties, more computationally oriented (and more physiologically accurate) models of the peripheral auditory response to sound are now becoming increasingly popular [e.g., Carney (1993), Payton (1988), Meddis *et al.* (1990)]. The Meddis model of auditory nerve activity, for example, has been incorporated into the binaural processing model of Shackleton *et al.* (1992).

The auditory-nerve model used by Stern and Colburn (1978) consisted of a bandpass filter followed by an automatic gain control device, a single-pole low-pass filter, an exponential rectifier, and a mechanism that generates event times according to a nonhomogeneous Poisson process with a rate that is proportional to the instantaneous output of the exponential rectifier. For various reasons first described by Shear (1987), we have found it necessary to modify this model of the auditory periphery. Figure 1 compares the functional blocks of the original formulation by Colburn with the model for auditory-nerve activity used in the calculations described in this paper. There are three differences between the two models: the nonlinear rectifier and the lowpass filter have been changed in form, and the order of the filter and rectifier was interchanged. We discuss the rationale for these three changes below.

## A. The nonlinear rectifier

The exponential rectifier used by Colburn (1973, 1977) and Stern and Colburn (1978) was replaced by a half-wave cube-law rectifier of the form

$$R_\nu[z] = \begin{cases} z^\nu, & \text{for } z \geq 0, \\ 0, & \text{for } z < 0, \end{cases} \quad (1)$$

where  $\nu$  is set equal to 3 for reasons discussed in Stern and Shear (1996).

The change in the type of rectifier was motivated by observations of Colburn (1977) who noted that the original model predicts much greater variability in the instantaneous firing rate for noise stimuli than is actually observed physiologically. This problem is caused by the conjunction of two properties of the original model. First, the original auditory nerve model has no provision to describe the observed refractory period between neural firings. Second, the statistics of the predicted response of the original model to noise are dominated by the rare times at which the sound waveform has extremely large amplitude. These large amplitudes are exponentiated by the original rectifier, producing extremely large output values. While one remedy to this problem would be to incorporate refractoriness into the model for auditory-nerve activity, this greatly complicates the statistical analysis [e.g., Teich and Lachs (1979)]. We chose instead to specify a rate function for the Poisson process which does not cause large input values to produce as many events per second as the original model does, by replacing the exponential rectifier by a half-wave  $\nu$ th-law rectifier (with  $\nu$  small), which exhibits less expansive transfer characteristics for large input values. While the new model still does not describe the refractory nature of neural firing, the statistics of its response to noise stimuli are more reasonable.

## B. The low-pass filter

The single-pole low-pass filter introduced by Colburn (1973) was replaced by a low-pass filter with the transfer function

$$G(f) = \begin{cases} 1, & \text{for } 0 \leq f \leq 1200 \text{ Hz}, \\ \frac{1 - (f/5600)}{1 - (1.2/5.6)}, & \text{for } 1200 < f \leq 5600 \text{ Hz}, \\ 0, & \text{for } f > 5600 \text{ Hz}. \end{cases} \quad (2)$$

This filter is the component of the auditory-nerve model that represents the loss of synchrony of the predicted firing patterns with respect to stimulus fine structure as frequency increases. Equation (2) above is based on data of Johnson (1980) that describe the relative amount of synchrony observed in the response of fibers of the cat's auditory nerve as stimulus frequency increased. Johnson's results were unavailable to Colburn at the time that he formulated the original auditory-nerve model, and Eq. (2) describes the dependence of synchrony as a function of stimulus frequency more accurately than the corresponding expression originally proposed by Colburn (1973).

### C. Ordering of the functional elements

The order of the low-pass filter and the nonlinear rectifier has been interchanged in the modified auditory-nerve model. This configuration is preferred over the original one because it enables the detection of low-frequency envelopes of high-frequency stimuli, which helps the model describe the lateralization of certain high-frequency stimuli [e.g., Bernstein and Trahiotis (1985), Henning (1974, 1980), McFadden and Pasanen (1976), Nuetzel and Hatter (1976, 1981), Stern *et al.* (1988a)]. The general configuration of a bandpass filter followed in turn by a nonlinear rectifier and a low-pass filter is used in several other models of auditory physiology and perception [e.g., Duifhuis (1973), Colburn and Esquissaud (1976), Blauert and Cobben (1978), Lindemann (1986)], which all provide reasonable descriptions of the auditory-nerve data.

## II. MODIFICATIONS TO THE MODEL OF BINAURAL PROCESSING

In this section we discuss modifications to the model for binaural processing that are needed to enable it to describe the lateralization of tones at frequencies other than 500 Hz, without degrading the model's ability to describe relative binaural detection thresholds. The most important modification concerns the distribution of the coincidence-counting units, which we refer to as  $p(\tau|f_c)$ , where  $\tau$  indicates the internal interaural delay and  $f_c$  represents characteristic frequency (CF). We were also concerned with the precise order  $\nu$  of the half-wave  $\nu$ th-law rectifier, and the ranges of CFs that are relevant to a particular lateralization or detection task. Our goals in this effort have been both to gain general insight into the dependence of the predictions on  $p(\tau|f_c)$ ,  $\nu$ , and the ranges of CFs considered, and to specify at least one set of specific values for them that provides good agreement between predictions and data, in order to demonstrate that the position-variable model is capable of describing the lateralization and detection phenomena considered.

In this section we propose a new form of  $p(\tau|f_c)$  which allows the position-variable model to describe both the lateralization of pure tones as a function of their frequency [as described by Schiano *et al.* (1986)] and the difference between  $N_\pi S_0$  and  $N_0 S_\pi$  detection thresholds as a function of target frequency [as summarized by Durlach and Colburn (1978)]. Sections II A and II B describe in a general sense how predictions for the lateralization and detection experiments are affected by the shape of the probability density function for the coincidence-counting units with respect to internal delay. We introduce the new function for  $p(\tau|f_c)$  in Sec. II C and comment on some of its properties. A more detailed discussion of the development of  $p(\tau|f_c)$  is provided in Stern and Shear (1996), a companion publication submitted to the PAPS archive.<sup>1</sup>

### A. General lateralization considerations

Stern and Colburn (1978) defined the position estimate of the position-variable model to be the center of mass (or centroid) along the internal-delay axis of the number of coincidences observed by the fiber pairs, all averaged over fre-

quency. As a result, the mean of the position estimate of the position-variable model can be approximated by the centroid of the product  $L(\tau, f_c)p(\tau|f_c)$  along the  $\tau$  axis:

$$E[\hat{P}] = \frac{\int_{\mathbf{R}_{\text{lat}}} p(f_c) \int_{-\infty}^{\infty} \tau L(\tau, f_c) p(\tau|f_c) d\tau df_c}{\int_{\mathbf{R}_{\text{lat}}} p(f_c) \int_{-\infty}^{\infty} L(\tau, f_c) p(\tau|f_c) d\tau df_c}. \quad (3)$$

In the above expression  $\mathbf{R}_{\text{lat}}$  is defined to be the range of CFs over which lateralization predictions are calculated;  $L(\tau, f_c)$  is equal to the average number of coincidences observed by a fiber pair with internal delay  $\tau$  and CF  $f_c$ , and  $p(f_c)$  is the probability density function that specifies the distribution of CFs.

The actual rate of coincidences for a given fiber pair depends on whether zero, one, or two of the auditory-nerve fibers that are input to the fiber pair are firing synchronously in response to the stimulus. [As discussed in Stern and Shear (1996) the fraction of fibers that are synchronous or "active" in their response to a given stimulus depends on the frequency and intensity of the stimulus, the CF of the fiber pair, and the threshold of the fibers that are input to the pair.] The rate of coincidences recorded by a fiber pair depends on its internal delay only when both fibers are active in their response to the stimulus.

The function  $L(\tau, f_c)$ , which describes the expected rate of coincidences as a joint function of characteristic frequency and internal delay for the  $m$ th fiber, can be expressed as

$$L(\tau, f_c) = \eta_2(f_c)L_2(\tau, f_c) + \eta_1(f_c)L_1 + \eta_0(f_c)L_0, \quad (4)$$

where

$$L_2(\tau, f_c) \triangleq E[L_m(\tau_m, f_m) | \tau_m = \tau, f_m = f_c, \text{ 2 fibers are active}]. \quad (5)$$

The constant  $L_0$  in Eq. (4) above indicates the expected number of coincidences recorded by fiber pairs with both input fibers firing spontaneously. The constant  $L_1$  indicates the expected number of coincidences recorded by fiber pairs with one input fiber firing spontaneously and one input fiber active in its response to the stimulus. Finally, the function  $L_2(\tau, f_c)$  indicates the expected number of coincidences recorded by fiber pairs with internal delay  $\tau$  and CF  $f_c$  for which both input fibers are active in their response to the stimulus. While the contributions of the  $L_0$  and  $L_1$  terms are needed to describe the dependence of subjective lateral position on IID [as in Stern and Colburn (1978)], these terms of Eq. (4) have little effect on the predictions described in the present paper.

As noted above, the original function  $p(\tau|f_c)$  was assumed to be frequency independent (Colburn, 1977). The modifications to the function  $p(\tau|f_c)$  described in this paper were initially motivated by the inability of the original model to describe the data of Schiano *et al.* (1986), who measured the perceived lateralization of tones with small ITDs as a function of frequency. The symbols in Fig. 2 indicate estimates of subjective lateral position for tones presented with a 150- $\mu$ s ITD produced by each of three subjects in Schiano's experiments. The solid curve in Fig. 2 shows the corresponding predictions of the original position-variable model of Stern and Colburn (1978). (The responses of each of the

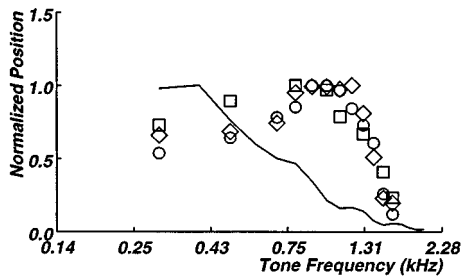


FIG. 2. Comparisons of normalized lateralization predictions of the original position-variable model with data adapted from Schiano *et al.* (1986) describing the lateralization of tones with ITDs of  $150 \mu\text{s}$  as a function of frequency. Data from three subjects are denoted by the square, circular, and triangular symbols, while the predictions are indicated by the smooth curve.

three subjects and the predictions were vertically scaled to produce a maximum value of 1.) The original model predicted that perceived position would shift toward the center of the head for frequencies above about 500 Hz, while the data of Schiano *et al.* (1986) indicate that perceived image position remains approximately constant for tones with frequencies up to about 1200 Hz.<sup>2</sup>

In order to better understand why the original position-variable model does not describe the data at the higher frequencies shown in Fig. 2, consider a binaural tone of frequency  $f_0$  (and no interaural intensity difference [IID]) for which the signal in the left ear is leading the signal in the right by a small ITD  $\tau_s$ . Figure 3 shows, as a function of the fixed internal delay of the fiber pair, the functions  $L_2(\tau, f_c)$  (the relative number of predicted coincidences when a fiber pair has input fibers from both ears that are “active” in their response to the stimulus),  $p(\tau|f_c)$  (the relative number of fiber pairs with internal delay  $\tau$  and CF  $f_c$ ), and the product of  $L_2(\tau, f_c)$  and  $p(\tau|f_c)$  [the total number of coincidences from “active” fiber pairs, which is designated  $L_T(\tau, f_c)$ ]. As

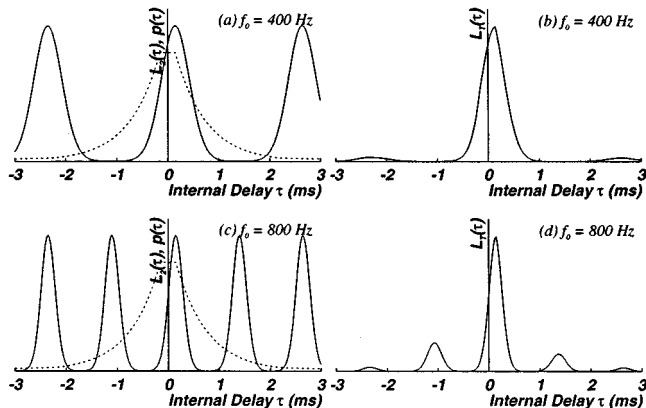


FIG. 3. The functions  $L_2(\tau, f_c)$  (the number of coincidences per fiber pair from binaurally stimulated fiber pairs) and  $L_T(\tau, f_c)$  (the number of coincidences from all binaurally-stimulated fiber pairs) of the original position-variable model plotted as a function of the internal delay parameter  $\tau$  for [(a) and (b)] 400-Hz and [(c) and (d)] 800-Hz tones with ITDs of  $150 \mu\text{s}$ . In each case  $f_c$  is equal to the frequency of the tone.  $L_T(\tau, f_c)$  is the product of  $L_2(\tau, f_c)$  and  $p(\tau|f_c)$ , the relative distribution of internal delays, as originally specified by Colburn (1977). The dotted curve in panels (a) and (c) displays  $p(\tau|f_c)$  (the function specifying the distribution of fiber pairs as a function of internal delay). All functions are plotted with an arbitrary vertical scale factor.

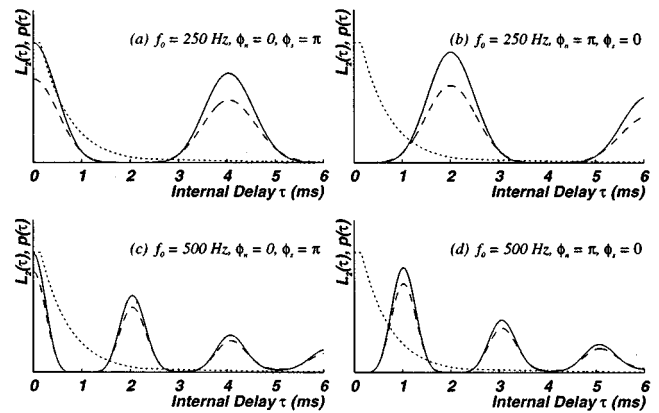


FIG. 4. The function  $L_2(\tau, f_c)$  plotted as a function of  $\tau$  for 250- and 500-Hz tones presented in a broadband noise, with  $f_c$  equal to the target frequency. The solid curve represents the case when the tone is absent and the dashed curve indicates the case when the tone is presented at a target-to-masker ratio near threshold. The dotted curve describes the shape of the original frequency-independent  $p(\tau)$  function specified by Colburn (1977).

noted above, the function  $p(\tau|f_c)$  was independent of CF in Colburn’s original formulation of the model. These functions are plotted using the original position-variable model as a function of the internal delay parameter  $\tau$  for 400- and 800-Hz tones with ITDs of  $150 \mu\text{s}$ . [The functions  $L_2(\tau, f_c)$ ,  $p(\tau|f_c)$ , and  $L_T(\tau, f_c)$  are formally defined in Stern and Shear (1996).] As is illustrated in Fig. 3(a) and (b), the effect of  $p(\tau|f_c)$  in the position calculation is to greatly attenuate all but the most central maxima of  $L_2(\tau, f_c)$  [Fig. 3(b)]. For ITDs less than half the tone period, the primary peak of  $L_2(\tau, f_c)$  occurs when  $\tau = \tau_s$  and the secondary peak occurs when  $\tau = \tau_s - 1/f_0$ . Referring to Fig. 3(c) and (d), we see that as the stimulus frequency increases, the secondary peak of  $L_2(\tau, f_c)$  moves closer to the midline and therefore contributes increasingly more to the centroid computation.

In order to maintain the predicted mean lateral position  $E[\hat{P}]$  more constant for low frequencies, it appears that  $p(\tau|f_c)$  must become narrower as the secondary peak approaches the midline, which occurs as the frequency of the tone increases. This narrowing of  $p(\tau|f_c)$  at higher CFs corresponds to a relative increase in the number of fiber pairs with small internal delays at these frequencies.

## B. General detection considerations

We now review some of the ways in which the shape of  $p(\tau|f_c)$  affects predictions for the difference between  $N_\pi S_0$  and  $N_0 S_\pi$  detection thresholds. We begin by considering the task of detecting the presence of a 250-Hz, out-of-phase tone in a diotic noise masker (i.e., the  $N_0 S_\pi$  configuration). The solid line of Fig. 4(a) describes  $L_2(\tau, f_c)$ , the relative number of predicted coincidences at CFs of 250 Hz, with no target signal present. (The curves in this figure are presented only for positive values of  $\tau$  since these functions are symmetric about  $\tau=0$ .) As the target-to-masker ratio is increased, the peaks of  $L_2(\tau, f_c)$  decrease in amplitude (and some minor changes are noted in the “valleys”). This decrease in peak amplitude is the major useful cue for binaural detection performance. Figure 4(b) describes these same functions for the

$N_\pi S_0$  case. While similar changes occur when the target is present, the locations of the peaks (and valleys) have been shifted by half the period of the tone.

As we have noted, the greatest decrease in the rate of coincidences as the target is added to the masker occurs for coincidence-counting units with internal delays  $\tau$  that are at or near the peaks of  $L_2(\tau, f_c)$ . In general, if a given decrease in coincidences per fiber pair is recorded by a greater number of fiber pairs, detection thresholds will tend to be lower because there is a greater net decrease in putative neural activity, and because a greater amount of statistical averaging is possible. Since the locations of the peaks of the  $L_2(\tau, f_c)$  function are at smaller values of  $|\tau|$  for stimuli in the  $N_0 S_\pi$  configuration than in the  $N_\pi S_0$  configuration, a greater number of coincidence-counting units will record the decrease in the rate of coincidences when targets are added to maskers in the  $N_0 S_\pi$  configuration. As a result, predicted  $N_0 S_\pi$  thresholds are lower than predicted  $N_\pi S_0$  thresholds, as is observed in the experimental data.

As the tone frequency is increased to 500 Hz [see Fig. 4(c) and (d)], the peaks for the  $N_\pi S_0$  case move closer to the midline, and thus the difference between predicted  $N_0 S_\pi$  and  $N_\pi S_0$  detection thresholds decreases. The exact rate with respect to target frequency at which this difference in thresholds decreases is dependent on the shape of  $p(\tau|f_c)$ . If the distribution of internal delays is broad with respect to the period of the tone, there will be only a small difference between the thresholds for the two cases. On the other hand, if the function  $p(\tau|f_c)$  is narrow relative to half of the period of the tone, then the  $N_0 S_\pi$  threshold should be significantly less than the  $N_\pi S_0$  threshold.

At very high frequencies, the loss of synchrony on the auditory nerve fibers causes  $L_2(\tau, f_c)$  to become a constant independent of  $\tau$ . For these stimuli, detection now appears to be related to decreases in the interaural correlation of the stimulus envelope (Bernstein and Trahiotis, 1992), and interaural time disparities do not significantly affect threshold levels [cf. Durlach and Colburn (1978)]. Thus, detection data at frequencies above about 2000 Hz are not particularly useful in constraining  $p(\tau|f_c)$ .

### C. A new distribution of fiber pairs

Shear (1987) developed several constraints on the shape of  $p(\tau|f_c)$  that are implied by the lateralization data of Schiano *et al.* (1986). Although the detailed arguments by which the constraints were derived are quite interesting, they are beyond the scope of this paper. The interested reader is encouraged to refer to Chap. 4 of Shear (1987) for a full discussion of these topics. A summary of the major results of this work is included in Stern and Shear (1996), a companion to this paper in the EPAPS archive.

Application of these constraints led to a form of  $p(\tau|f_c)$  that best describes both the detection data and lateralization data considered. This function, referred to as  $p_{LF}(\tau|f_c)$ , is defined below:

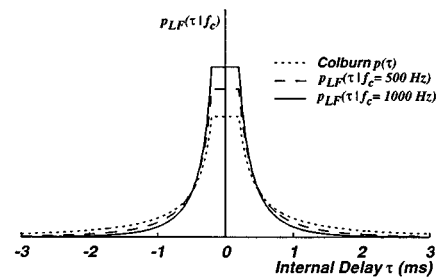


FIG. 5. Comparison of the shapes of  $p_{LF}(\tau|f_c)$  and the original frequency-independent density function  $p(\tau)$  specified by Colburn (1977).

$$p_{LF}(\tau|f_c) = \begin{cases} C_{LF}(f_c), & \text{for } |\tau| \leq 200 \mu\text{s}, \\ C_{LF}(f_c)(e^{-2\pi k_l(f_c)|\tau|} - e^{-2\pi k_h|\tau|})/|\tau|, & \text{otherwise,} \end{cases} \quad (6)$$

where

$$k_l(f_c) = \begin{cases} 0.1f_c^{1.1}, & \text{for } f_c \leq 1200 \text{ Hz} \\ 0.1(1200)^{1.1}, & \text{for } f_c > 1200 \text{ Hz.} \end{cases} \quad (7)$$

The parameter  $k_h$  is set equal to  $3000 \text{ s}^{-1}$ , and  $C_{LF}(f_c)$  is chosen to normalize  $p_{LF}(\tau|f_c)$  as a valid probability density function.

The expression above for  $p_{LF}(\tau|f_c)$  implicitly assumes that the overall number of fiber pairs is the same for all CFs. This assumption was made for simplicity, as no study has addressed this issue explicitly. We remind the reader that these coincidence-counting fiber pairs can be useful for extracting information about ITDs of envelopes of high-frequency binaural stimuli, as well as information about the ITDs of the fine structure of low-frequency stimuli, as discussed by Colburn and Esquissaud (1976) and Stern *et al.* (1988a).

Figure 5 compares  $p_{LF}(\tau|f_c)$ , evaluated at  $f_c$  equal to 500 and 1000 Hz, with the original frequency-independent distribution proposed by Colburn (1977). (The three curves are normalized so that the areas beneath them are equal.) As CF increases,  $p_{LF}(\tau|f_c)$  specifies an increase in the relative number of fiber pairs with internal delays of less than  $200 \mu\text{s}$  in magnitude compared to the number of pairs with delays of larger magnitudes. However, the distribution of delays between  $-200 \mu\text{s}$  and  $+200 \mu\text{s}$  remains constant, independent of CF. Note that  $p_{LF}(\tau|f_c)$  has less area in its “tails” than the original density function  $p(\tau)$  proposed by Colburn (1977) for both frequencies shown.

Predictions obtained using  $p_{LF}(\tau|f_c)$  for both experiments of interest are shown in Fig. 6. These predictions were obtained using a decision statistic that is the optimal weighting (for a particular stimulus configuration and target-to-masker ratio) of the outputs of coincidence-counting units over a range of CFs. This decision statistic is formally defined in Stern and Shear (1996). The data for the detection experiments are a composite of results from nine studies cited in Fig. 54 of Durlach and Colburn (1978). [We also describe in Shear (1987) and Stern and Shear (1996) a second decision statistic that develops detection predictions by summing the outputs of coincidence-counting units over a

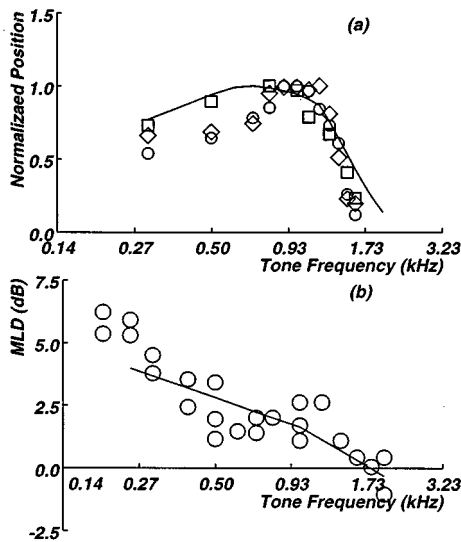


FIG. 6. Comparisons of predictions and data obtained using the function  $p_{LF}(\tau|f_c)$  specified in Eq. (6) for (a) the lateralization of tones with an ITD of  $150 \mu\text{s}$  as a function of frequency, and (b) the ratio of  $N_{\pi}S_{\pi}$  thresholds to  $N_0S_{\pi}$  thresholds as a function of target frequency, using the optimal range of CFs for each stimulus configuration. The data in panel (a), indicated by the square, circular, and triangular symbols, are from Schiano *et al.* (1986). The data in panel (b), indicated by the circular symbols, are from nine studies summarized in Fig. 54 of Durlach and Colburn (1978). In each case, predictions of the extended position-variable model are indicated by the smooth curves.

range of CFs that is centered at the target frequency. There are no major differences between predictions obtained using these two decision variables.] Comparisons of data and predictions in Fig. 6 demonstrate that  $p_{LF}(\tau|f_c)$  enables the revised position-variable model to provide a reasonably accurate description of both the lateralization data of Schiano *et al.* (1986) and the ratios of  $N_{\pi}S_0$  and  $N_0S_{\pi}$  binaural detection thresholds.

### III. PREDICTIONS OF THE MODIFIED MODEL FOR OTHER LOW-FREQUENCY PHENOMENA

We now describe comparisons of predictions of the modified position-variable model to the results of several additional experiments that measure the subjective lateral position of binaural stimuli. These comparisons are developed to provide additional support for the modified model, and to provide further insight into which features of the model determine the nature of a given set of results. Lateralization predictions were obtained using Eqs. (1)–(5) in this paper and Eqs. (B.2), (B.5), (B.7), or (B.13) in Appendix B of Stern and Shear (1996). Lateralization predictions were calculated only over the range of CFs for which auditory-nerve fibers in both ears are actively firing in response to the stimulus. Theoretical predictions are plotted using an arbitrarily normalized vertical scale. Some additional implementation details are summarized in Appendix B of Stern and Shear (1996).

#### A. Pure tones

In the previous section we described modifications of the model that were needed to describe the data of Schiano

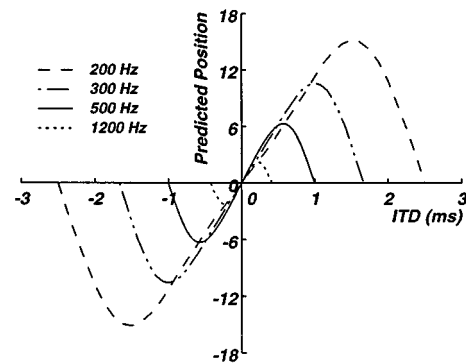


FIG. 7. Predicted lateral position of pure tones of various frequencies as a function of ITD. These and all other lateralization predictions are plotted with an arbitrary vertical scale.

*et al.* (1986). Figure 7 shows predictions of the position-variable model for a wider range of stimulus ITDs than were considered by Schiano *et al.* The data in Fig. 7 are plotted as a function of the ITD of the tone, using the frequencies in the study of Sayers (1964), except that we provide predictions at 500 Hz rather than 600 Hz to facilitate comparison with the predictions of Stern and Colburn (1978) and other papers. Since the lateralization of pure tones is periodic with respect to ITD, we plot only a single period at each frequency. The predictions at 500 Hz had been vertically normalized to describe the data of Schiano *et al.* (1986) during the development of the function  $p(\tau|f_c)$  as described in Sec. II. This vertical scale was then used for all other predictions described in this paper as well.

The most salient feature of the predictions is that all of the curves exhibit approximately the same slope as they approach the origin of the graph, and that the maximum amplitude of the predicted curves decreases with increasing frequency. These attributes are consistent with the findings of Schiano *et al.* that perceived lateral position for a given ITD is approximately constant over a broad range of frequencies, provided that the ITD is less than one-quarter of the stimulus period. [The good correspondence of predictions to these data is to be expected, as the data were used to constrain the shape of  $p_{LF}(\tau|f_c)$ .]

These predictions can be compared to the results of the study of Sayers (1964), who used a mechanical pointer to record the perceived laterality of tones as a joint function of ITD and frequency. Sayers presented only a single frequency during each block of trials in the experiments for which data were plotted in the 1964 paper, and he reported that under these circumstances the subjects tended to renormalize their response strategy so as to make use of the full scale of the mechanical pointer at each frequency. Nevertheless, when tones of two different frequencies are presented simultaneously during the same experiment, Sayers (1964, p. 924) reports that “the apparent maximum shift of image-position judgments decreased with frequency. However, the slope of the judged position against time delay appeared to be independent of frequency; signal-frequency increase caused a decrease in the extent of maximum image displacement with ITD, reached just prior to the transition across to contralateral judgments.” In other words, we find the data of Sayers

(1964) to be consistent with the data of Schiano *et al.* (1986) in that the position of the tones depends on their ITD at least for ITDs of less than one-quarter of a period. This is the form of predictions that are obtained (by design) using the position-variable model. There have also been some measurements of low-frequency pure tones [e.g., Yost (1981) and Shackleton *et al.* (1991)] that imply that their lateral position depends more on interaural phase difference than ITD, but differences in procedures makes comparisons across these experiments difficult.

As described above, the position-variable model assumes that subjective lateral position is determined by the location of the centroid of the function  $L_2(\tau, f_c)$ . As a result, the model predicts that pure tones presented interaurally out of phase (i.e., with an ITD equal to half the stimulus period) will be perceived in the center of the head. Several experimenters [e.g., Sayers (1964), Yost (1981), Bernstein and Trahiotis (1985) and Shackleton *et al.* (1991)] have reported that pure tones presented interaurally out of phase may produce a pair of images, one near each ear, which can be perceived independently in some cases, which may be more consistent with the locations of the peak of  $L_2(\tau, f_c)$  than the centroid. Similarly, lateralization models based on the centroid of  $L_2(\tau, f_c)$  cannot easily describe either the lateral position of perceptual images produced using the multiple phase-shift method for dichotic-pitch stimuli (Raatgever, 1980) or the various binaural stimuli that produce multiple images [e.g., Whitworth and Jeffress (1961) and Hafter and Jeffress (1968)].

There are other ways, of course, to predict lateral position besides the centroid computation. One plausible alternative is to assume that position can be related to the location of the peaks of the function  $L_2(\tau, f_c)$  (as opposed to its centroid along the  $\tau$  axis). The locations of the peaks of  $L_2(\tau, f_c)$  may allow one to account for the multiple images which can occur both for tonal stimuli presented interaurally out of phase, and for the secondary “time image” observed for some stimuli presented with conflicting ITDs and IIDs. Lateralization mechanisms based on the peaks of functions characterizing the response of the coincidence counters have been discussed by Lindemann (1986) in conjunction with some of the predictions of his model. More recently, Shackleton *et al.* (1992) described a model that assumes that the listener computes either the centroid along the  $\tau$  axis or the locations of the peaks of the responses of coincidence-counting units, choosing the statistic that more accurately describes the results for a given experiment. While definitely not parsimonious, this type of approach may be necessary to account for the data in all their complexity.

We chose to base predictions on the centroid rather than the peaks of the function  $L_2(\tau, f_c)$  for two reasons. From the beginning of our work in binaural modeling we have been strongly influenced by the data of Domnitz and Colburn (1977), who instructed subjects to match the center of the target tone to the center of a pointer tone. Using this method, pure tones presented interaurally out of phase were reported to be perceived at or near the midline consistently on a trial-by-trial basis, although these data are labile and quite dependent on experimental paradigm and the specific instructions

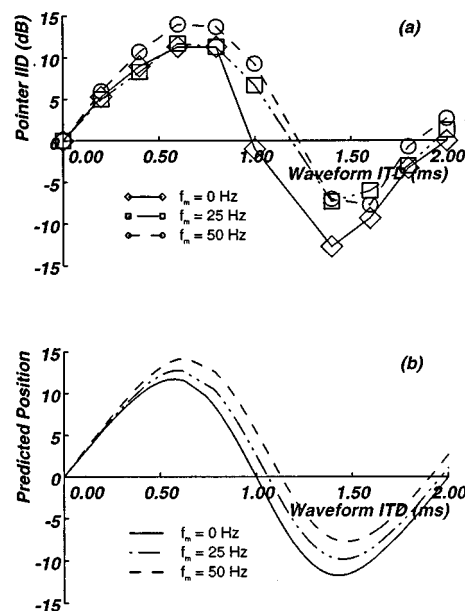


FIG. 8. Comparison of (a) data by Bernstein and Trahiotis (1985) and (b) predictions of the extended position-variable model describing the dependence of the perceived laterality of 500-Hz amplitude-modulated tones on  $f_m$ , the modulation frequency, and waveform ITD.

given to the subjects. In addition, the use of the centroid of  $L_2(\tau, f_c)$  rather than the locations of its peaks renders the model much more analytically tractable. It should be borne in mind that the centroid-based model predicts many experimental results, including the lateralization predictions described in Stern and Colburn (1978) and the interaural discrimination predictions described in Stern and Colburn (1985). The centroid assumption is also needed to predict other binaural phenomena such as the results of Schiano *et al.* (1986), Hafter and Shelton (1991), and the dominant-region data of Raatgever (1980, 1986), as will be discussed below. We continue to use the centroid as the basis of our predictions because of the generality and simplicity that it provides, despite its inability to describe some binaural phenomena including the individual trials of Sayers' data for interaural phase differences that are close to  $180^\circ$ .

## B. Amplitude-modulated low-frequency tones

Several experimenters have measured the perceived lateralization of amplitude-modulated (AM) tones and band-pass noise as a function of the carrier frequency, modulator frequency, interaural carrier frequency difference, different types of ITD of the stimulus, and other stimulus parameters. Lateralization has been measured either directly using subjective methods [e.g., Bernstein and Trahiotis (1985)], by considering the percentage of consistent lateralization estimates in a task [e.g., Henning (1983)], or by inference from measurements of lateralization performance in objective discrimination experiments [e.g., Nuetzel and Hafter (1981)].

Figure 8(a) summarizes data obtained by Bernstein and Trahiotis (1985), showing the joint dependence of lateral position of an AM tone with a carrier frequency of 500 Hz on modulation frequency and waveform ITD. (In this experiment the carrier and modulator components of the waveform

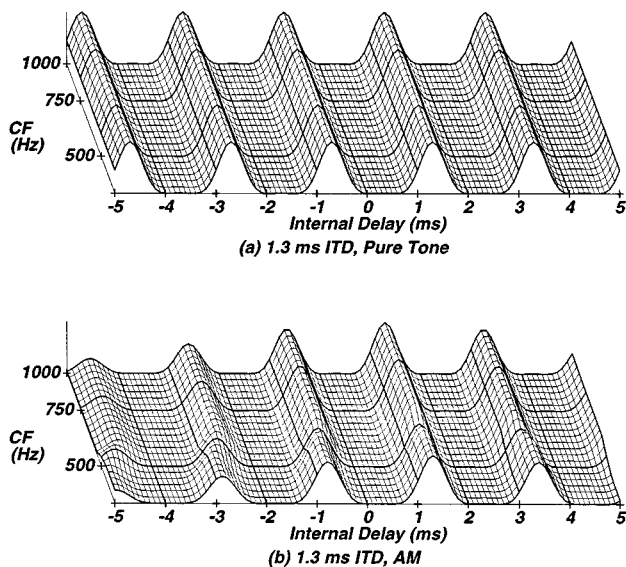


FIG. 9. Cross-correlation patterns showing the expected number of coincidences,  $L_2(\tau, f_c)$ , of an ensemble of doubly active binaural fiber pairs to (a) a pure tone with an ITD of 1.3 ms, and (b) an amplitude-modulated tone with an ITD of 1.3 ms and a modulation frequency of 50 Hz. The horizontal axis indicates the internal interaural delay of the fiber pairs (in ms), and the oblique axis indicates the characteristic frequency of the auditory-nerve fibers (in Hz).

were subjected to the same ITD.) It can be seen that the modulation frequency affects the perceived lateral position for AM tones by increasing amounts as waveform ITDs are increased from 1 to 2 ms. Stimuli with ITDs of approximately 1.3 ms are lateralized increasingly toward the side of the head as the modulation frequency is increased from 0 Hz (producing an unmodulated pure tone) to 50 Hz. The corresponding theoretical predictions are shown in Fig. 8(b).

Bernstein and Trahiotis (1985) suggested that these data indicate that the binaural system can lateralize low-frequency AM tones on the basis of the ITD of their envelopes. Stern *et al.* (1988b) reexamined the same data and suggested that the differences in lateral position that were observed as the modulation frequency increased from 0 to 50 Hz indicate that the binaural system is weighting more heavily those modes of the cross-correlation function  $L_2(\tau, f_c)$  that occur at the same internal delay over a range of frequencies. Although these observations by Bernstein and Trahiotis (1985) and Stern *et al.* (1988b) are not without merit, a simpler explanation based on the form of  $L_2(\tau, f_c)$  can also account for the form of the observed data, as demonstrated by the example in Fig. 9.

Figure 9 shows the response of an ensemble of binaural coincidence-counting units [the function  $L_2(\tau, f_c)$ ] responding to a pure tone with an ITD of 1.3 ms [Fig. 9(a)], and to an amplitude-modulated tone with the same carrier frequency and ITD, and a modulation frequency of 50 Hz [Fig. 9(b)]. The horizontal axis indicates the internal interaural delay of the fiber pairs (in ms), and the oblique axis indicates the characteristic frequency of the auditory-nerve fibers (in Hz). The relative number of coincidences for the pure tone [Fig. 9(a)] is a periodic function of internal delay because the stimulus is periodic with a period of 2 ms. The amplitudes of

the peaks in Fig. 9 are independent of CF because the model of auditory-nerve activity used in these calculations (Colburn, 1973) assumes a constant average rate of firing for “active” auditory-nerve fibers. The stimulus is actually lateralized toward the left side because the function  $p(\tau|f_c)$  gives greatest emphasis to the modes of  $L_2(\tau, f_c)$  that are closest to the midline, for reasons discussed in the previous section and in Stern *et al.* (1988b). Note that as the modulation frequency increases from 0 to 50 Hz, the function  $L_2(\tau, f_c)$  becomes aperiodic, with the mode of  $L_2(\tau, f_c)$  that appears at the true ITD (1.3 ms) exhibiting the greatest amplitude. [This is most easily seen in Fig. 9(b) by examining the relative amplitudes of the peaks of the high-frequency “edge” of  $L_2(\tau, f_c)$  at about 1000 Hz.] This change in shape of  $L_2(\tau, f_c)$  causes the predicted lateralization [the centroid of  $L_2(\tau, f_c)$  multiplied by  $p(\tau|f_c)$ ] to move toward the true ITD as modulation frequency increases because of the increasing disparity in the amplitude of the modes.

These observations can help account for the predictions in Fig. 8(b), by comparing the response of the model to stimuli with ITDs of 0.7 and 1.3 ms. The amount of change in predicted position as the modulation frequency increases from 0 to 50 Hz is greater for ITDs of approximately 1.3 ms than for ITDs of approximately 0.7 ms. This occurs because when the ITD is approximately 0.7 ms, the peak of  $L_2(\tau, f_c)$  at  $\tau = +0.7$  ms is given greatest emphasis by  $p(\tau|f_c)$  for low modulation frequencies, and it is also the peak that takes on the greatest amplitude as modulation frequency increases. When the ITD is 1.3 ms, on the other hand, the peak at  $\tau = -0.7$  ms is the most “central,” but the true peak at  $\tau = +1.3$  ms develops the greatest amplitude as modulation frequency increases. This causes the predicted position to move toward the right side of the head, as is seen in the data.

### C. “Dominant region” experiments

The role that  $p(\tau|f_c)$  plays in the lateralization of broadband low-frequency stimuli is also illustrated elegantly in the experiments that produce the “dominant region” effect (Raatgever, 1980; Raatgever and Bilsen, 1986), using broadband stimuli with different ITDs applied to different spectral regions. Subjects in these experiments were presented a binaural stimulus consisting of three contiguous bands of noise, with either the high- and low-frequency bands or the mid-frequency band presented with an ITD of  $+T$ . The overall amplitude of the mid-frequency band is incremented (relative to that of the flanking bands) by an intensity difference that causes the total stimulus to have the same subjective lateral position when the ITD of  $+T$  is applied to the mid-frequency band as when the ITD of  $+T$  is applied to the two flanking bands. This increment in intensity is referred to as  $\Delta I$ , by Raatgever and Bilsen.

Figure 10(a) shows typical experimental data from three subjects for this experiment from Raatgever (1980) and Fig. 10(b) shows the corresponding predictions of the extended position-variable model. The data and predictions were obtained using a value of 0.3 ms for the ITD  $+T$ , and a bandwidth of one-third octave for the mid-frequency band. The ordinate of Fig. 10 indicates the value of  $\Delta I$  that is needed for the total stimulus to exhibit the same lateral position,



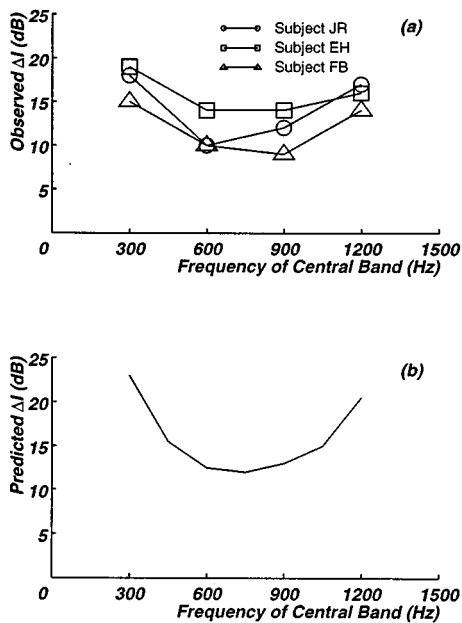


FIG. 10. Comparison of (a) data by Raatgever (1980) and (b) theoretical predictions for experiments whose results implied the existence of a “dominant frequency region” for binaural lateralization. The increment in overall intensity,  $\Delta I$ , is plotted that is needed for a midfrequency region of critical bandwidth to dominate the lateralization mechanism when frequency components in the two flanking bands are presented with a conflicting ITD.

regardless of whether the mid-frequency band or the two flanking bands receives the ITD+T. The abscissa of Fig. 10 indicates the center frequency of the mid-frequency band. As can be seen,  $\Delta I$  achieves a minimum for some value of center frequency that is approximately 600–700 Hz for each subject, which caused Raatgever to suggest that this “dominant” mid-frequency region contributes more heavily to the binaural lateralization process. The theoretical predictions in Fig. 10(b) were generated using the position-variable model in the form specified in this paper and in Stern and Shear (1996). The predictions describe the form of Raatgever’s data quite well, indicating that the binaural system appears to be maximally sensitive to ITDs of stimuli in this mid-frequency region.

We believe that the ability of the model to predict these data without any further assumptions is a consequence of the interaction between the functions  $L_2(\tau, f_c)$  and  $p(\tau|f_c)$ . The relevant aspects of these functions are illustrated by the curves plotted in Fig. 11, using a broadband noise with an ITD of  $3 \mu\text{s}$  as the stimulus. The three panels of Fig. 11 show the relative number of coincidences as a function of internal delay [ $L_2(\tau, f_c)$ , solid curves] and the relative number of fiber pairs [ $p(\tau|f_c)$ , dotted curves] at three CFs: 300, 750, and 1200 Hz. In each case, the location of the centroid of the product of these two functions is indicated by the solid vertical bar. It can be seen that if only one CF at a time were considered, coincidences from fiber pairs at 750 Hz produce an estimate of subjective lateral position that is farther from the midline than coincidences from fiber pairs at either 300 Hz or 1200 Hz. It is easy to understand why this is the case because of the form of the function  $L_2(\tau, f_c)$  at each of the three frequencies. At very low frequencies such as 300 Hz,

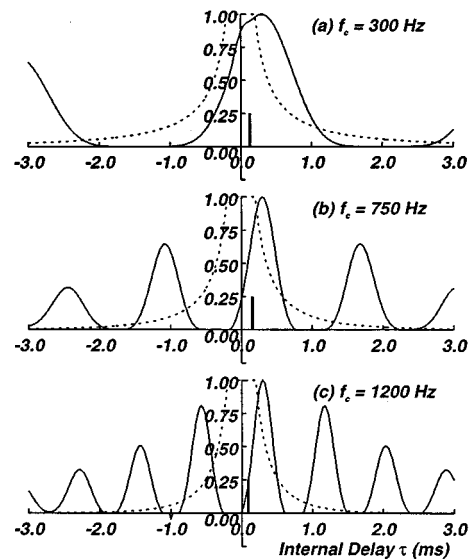


FIG. 11. The functions  $L_2(\tau, f_c)$  (solid curve) and  $p_{LF}(\tau|f_c)$  (broken curve) plotted as a function of internal delay  $\tau$  in response to broadband noise presented with an ITD of  $300 \mu\text{s}$ . Three values of  $f_c$  are shown: (a) 300 Hz, (b) 750 Hz, and (c) 1200 Hz. The vertical bar in each panel indicates the location of the centroid along the  $\tau$  axis of  $L_2(\tau, f_c)$ , which would be the predicted lateral position obtained if only fiber pairs at the frequency plotted were considered in lateralization calculations.

the modes of  $L_2(\tau, f_c)$  for noise stimuli are broad, and there are significant numbers of coincidences at negative values of  $\tau$  that fall within the major mode of the density function  $p(\tau|f_c)$ . These coincidences at negative values of  $\tau$  tend to move the centroid of the product of  $L_2(\tau, f_c)$  and  $p(\tau|f_c)$  toward the midline ( $\tau=0$ ). At higher CFs such as 1200 Hz, secondary modes of the function  $L_2(\tau, f_c)$  will appear within the central region of  $p(\tau|f_c)$  for negative values of  $\tau$ , which also drives the centroid of the product of these two functions back toward the midline. At the intermediate frequency of 750 Hz there are relatively few coincidences at negative values of  $\tau$  within the major mode of  $p(\tau|f_c)$ , so fiber pairs at this CF produce an estimate of lateral position that is farther from the midline. Since position predictions are averaged over frequency, the CF that by itself produces the greatest predicted laterality is also predicted to be “dominant” in an experiment (like that of Raatgever) in which different stimulus components are set up in a rivalry with one another.

In summary, we believe that the interaction between the number of coincidences produced by the fiber pairs with the distribution of fiber pairs with respect to their internal delay and CF provides a natural explanation for the “dominant region” effect observed by Raatgever (Raatgever, 1980; Raatgever and Bilsen, 1986).

#### IV. SUMMARY

In this paper we describe several modifications to the position-variable model (Stern and Colburn, 1978) that enable it to describe binaural lateralization and detection phenomena over a much wider range of frequencies. The most important of these modifications concerns the function  $p(\tau|f_c)$  which describes the assumed distribution of internal delays in the model. We show that the function  $p(\tau|f_c)$  must

be carefully chosen to enable the model to describe both the lateralization data of Schiano *et al.* (1986) and the observed ratio of  $N_0S_\pi$  to  $N_\pi S_0$  binaural detection thresholds. We then describe a form of  $p(\tau|f_c)$  that is able to describe both the detection and lateralization results with reasonable accuracy.

Finally, we present lateralization predictions of the position-variable model for three types of broad frequency binaural phenomena, pure tones of different frequencies and ITDs, AM tones with carrier frequencies of 500 Hz, and the stimuli producing the “dominant region” phenomenon described by Raatgever (Raatgever, 1980; Raatgever and Bilsen, 1986), and we discuss the bases within the modified model that enable it to describe these data without further assumption.

## ACKNOWLEDGMENTS

This work was supported by the National Science Foundation (Grant No. BNS-87099349). We thank L. R. Bernstein, T. N. Buell, H. S. Colburn, and C. Trahiotis for their very helpful comments on earlier drafts of this manuscript. J. Raatgever, T. Shackleton, and a third anonymous reviewer provided many perceptive comments that led to a number of clarifications of the manuscript. Finally, we thank Sammy Tao, Xiaohong Xu, and Torsten Zeppenfeld for their assistance in implementing the predictions of the model.

<sup>1</sup>See AIP Document No. E-PAPS file via: <http://www.aip.org/epaps/epaps.html> E-JASMA-100-2278-0.175MB for a companion publication “Lateralization and detection of low-frequency binaural stimuli: Specification of the extended position-variable model.” There is no charge for retrieval of electronic PAPS document files from the FTP server. For further information: e-mail: [paps@aip.org](mailto:paps@aip.org) or fax: 516-576-2223.

<sup>2</sup>The inability of subjects to lateralize tones presented at frequencies above 1200 Hz purely on the basis of ITD is believed to be at least partly a result of the loss of synchrony between neural firing patterns and the detailed timing structure of the stimulus. This observation is (qualitatively) consistent with predictions of the auditory-nerve model as modified in Sec. II.

Bernstein, L. R., and Trahiotis, C. (1985). “Lateralization of low-frequency complex waveforms: The use of envelope-based temporal disparities,” *J. Acoust. Soc. Am.* **77**, 1868–1880.

Bernstein, L. R., and Trahiotis, C. (1992). “Discrimination of interaural envelope correlation and its relation to binaural unmasking at high frequencies,” *J. Acoust. Soc. Am.* **91**, 306–316.

Bilsen, F. A., and Goldstein, J. L. (1974). “Pitch of dichotically delayed noise and its possible spectral basis,” *J. Acoust. Soc. Am.* **55**, 292.

Blauert, J., and Cobben, W. (1978). “Some consideration of binaural cross-correlation analysis,” *Acustica* **39**, 96–103.

Carney, L. H. (1993). “A model for the responses of low-frequency auditory nerve fibers in cat,” *J. Acoust. Soc. Am.* **93**, 401–417.

Colburn, H. S. (1973). “Theory of binaural interaction based on auditory-nerve data. I. General strategy and preliminary results on interaural discrimination,” *J. Acoust. Soc. Am.* **54**, 1458–1470.

Colburn, H. S. (1977). “Theory of binaural interaction based on auditory-nerve data. II. Detection of tones in noise,” *J. Acoust. Soc. Am.* **61**, 525–533.

Colburn, H. S. and Esquissaud, P. (1976). “An auditory-nerve model for interaural time discrimination of high-frequency complex stimuli,” *J. Acoust. Soc. Am.* **59**, S23(A).

Domnitz, R. H., and Colburn, H. S. (1977). “Lateral position and interaural discrimination,” *J. Acoust. Soc. Am.* **61**, 1586–1598.

Duifhuis, H. (1973). “Consequences of peripheral frequency selectivity for nonsimultaneous masking,” *J. Acoust. Soc. Am.* **54**, 1471–1488.

Durlach, N. I., and Colburn, H. S. (1978). *Binaural Perception*, edited by E. C. Carterette and M. P. Friedman (Academic, New York).

Hafter, E. R., and Jeffress, L. A. (1968). “Two-image lateralization of tones and clicks,” *J. Acoust. Soc. Am.* **44**, 563–569.

Hafter, E. R., and Shelton, B. R. (1991). “Counterintuitive reversals in lateralization using rectangularly modulated noise,” *J. Acoust. Soc. Am.* **90**, 1901–1907.

Henning, G. B. (1974). “Detectability of interaural delay in high-frequency complex waveforms,” *J. Acoust. Soc. Am.* **55**, 84–90.

Henning, G. B. (1980). “Some observations on the lateralization of complex waveforms,” *J. Acoust. Soc. Am.* **68**, 446–453.

Henning, G. B. (1983). “Lateralization of low-frequency transients,” *Hear. Res.* **9**, 153–172.

Jeffress, L. A. (1948). “A place theory of sound localization,” *J. Comp. Physiol. Psychol.* **41**, 35–39.

Johnson, D. H. (1980). “The relationship between spike rate and synchrony in responses of auditory-nerve fibers to single tones,” *J. Acoust. Soc. Am.* **68**, 1115–1122.

Lindemann, W. (1986). “Extension of a binaural cross-correlation model by contralateral inhibition. Simulation of lateralization for stationary signals,” *J. Acoust. Soc. Am.* **80**, 1608–1622.

McFadden, D., and Pasanen, E. G. (1976). “Lateralization at high frequencies based on interaural time differences,” *J. Acoust. Soc. Am.* **59**, 634–639.

Meddis, R., Hewitt, M. J., and Shackleton, T. M. (1990). “Implementation details of a computational model of the inner hair-cell/auditory-nerve synapse,” *J. Acoust. Soc. Am.* **87**, 1813–1816.

Nuetzel, J. M., and Hafter, E. R. (1976). “Lateralization of complex waveforms: Effects of fine structure, amplitude, and duration,” *J. Acoust. Soc. Am.* **60**, 1339–1346.

Nuetzel, J. M., and Hafter, E. R. (1981). “Discrimination of interaural delays in complex waveforms: Spectral effects,” *J. Acoust. Soc. Am.* **69**, 1112–1118.

Payton, K. L. (1988). “Vowel processing by a model of the auditory prephary: A comparison to eighth-nerve responses,” *J. Acoust. Soc. Am.* **83**, 145–162.

Raatgever, J. (1980). “On the binaural processing of stimuli with different interaural phase relations,” Doctoral dissertation, Technische Hogeschool Delft.

Raatgever, J., and Bilsen, F. A. (1986). “A central spectrum theory of binaural processing. Evidence from dichotic pitch,” *J. Acoust. Soc. Am.* **80**, 429–441.

Sayers, B. McA. (1964). “Acoustic-image lateralization judgments with binaural tones,” *J. Acoust. Soc. Am.* **36**, 923–926.

Sayers, B. McA., and Cherry, E. C. (1957). “Mechanism of binaural fusion in the hearing of speech,” *J. Acoust. Soc. Am.* **29**, 973–987.

Schiano, J. L., Trahiotis, C., and Bernstein, L. R. (1986). “Lateralization of low-frequency tones and narrow bands of noise,” *J. Acoust. Soc. Am.* **79**, 1563–1570.

Shackleton, T. M., Bowsher, J. M., and Meddis, R. (1991). “Lateralization of very-short-duration tone pulses of low and high frequencies,” *J. Exp. Psychol. A* **43**, 503–516.

Shackleton, T. M., Meddis, R., and Hewitt, M. J. (1992). “Across frequency integration in a model of lateralization,” *J. Acoust. Soc. Am.* **91**, 2276–2279(L).

Shamma, S. A., Shen, N., and Gopalswamy, P. (1989). “Binaural processing without neural delays,” *J. Acoust. Soc. Am.* **86**, 987–1006.

Shear, G. D. (1987). “Modeling the dependence of auditory lateralization on frequency and bandwidth,” Master’s thesis, Elec. and Comp. Eng. Dept., CMU.

Stern, R. M., Jr., and Colburn, H. S. (1978). “Theory of binaural interaction based on auditory-nerve data. IV. A model for subjective lateral position,” *J. Acoust. Soc. Am.* **64**, 127–140.

Stern, R. M., and Colburn, H. S. (1985). “Lateral-position-based models of interaural discrimination,” *J. Acoust. Soc. Am.* **77**, 753–755.

Stern, R. M., and Shear, G. D. (1996). “Lateralization and detection of low-frequency binaural stimuli: Specification of the extended position-variable model,” AIP Document No. E-PAPS E-JASMA-100-2278 (see footnote 1).

Stern, R. M., Shear, G. D., and Zeppenfeld, T. (1988a). “High-frequency predictions of the position-variable model,” *J. Acoust. Soc. Am.* **84**, S60(A).

- Stern, R. M., Zeiberg, A. S., and Trahiotis, C. (1988b). "Lateralization of complex binaural stimuli: A weighted image model," *J. Acoust. Soc. Am.* **84**, 156–165.
- Teich, M. C., and Lachs, G. (1979). "A neural-counting model incorporating refractoriness and spread of excitation. I. Application to intensity discrimination," *J. Acoust. Soc. Am.* **66**, 1738–1749.
- Yost, W. A. (1981). "Lateral position of sinusoids presented with intensive and temporal differences," *J. Acoust. Soc. Am.* **70**, 397–409.
- Whitworth, R. H., and Jeffress, L. A. (1961). "Time versus intensity in the localization of tones," *J. Acoust. Soc. Am.* **33**, 925–929.